

# General Physics: Mechanics

**PHYS-101(en)**

Lecture 7a:

Conservation of momentum,  
center of mass and  
continuous mass transfer

Dr. Marcelo Baquero  
[marcelo.baquero@epfl.ch](mailto:marcelo.baquero@epfl.ch)  
October 27<sup>th</sup>, 2025



# Reminder

---

- Mini mock exam tomorrow
  - In-class (SG1) during normal lecture hours (10:15-11:00)
  - The format is very similar to the exam. You will be given a “booklet” with the questions and space to write your solutions
  - “Cheat” sheet, notes, etc., are all allowed *this time*, just **no talking with neighbors**
  - Turn in at the end if you want exam to be graded (optional). Graded exams will be returned on Monday November 10th

# Midterm course feedback

---

- In general positive.

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes
  - problem sets are difficult

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes
  - problem sets are difficult
  - give final answers of problem sets

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes
  - problem sets are difficult
  - give final answers of problem sets
  - too many details on explanation of some basic concepts vs. going too fast

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes
  - problem sets are difficult
  - give final answers of problem sets
  - too many details on explanation of some basic concepts vs. going too fast
  - long lecture on Mondays. Possibility of doing more examples

# Midterm course feedback

---

- In general positive.
- Some of your comments:
  - my handwriting is too small
  - availability of blank lecture notes
  - problem sets are difficult
  - give final answers of problem sets
  - too many details on explanation of some basic concepts vs. going too fast
  - long lecture on Mondays. Possibility of doing more examples
  - live-streaming of lectures

# Today's agenda (Serway 6, 9 and MIT 8 -12)

---

## 1. Brief review of last week

## 2. Some examples

- Conservation of momentum
- Motion of Center of mass

## 3. Systems with variable mass

- Rocket science and interstellar space travel!

# DEMO (113)

---

Recoil of a cart

# Review of last week — Momentum

---

- **Momentum:** A vector quantity defined as  $\vec{p} = m\vec{v}$

# Review of last week — Momentum

---

- **Momentum:** A vector quantity defined as  $\vec{p} = m\vec{v}$
- **Conservation of momentum:** Total momentum of a system stays constant, if the net external force on it is zero and matter is not exchanged

# Review of last week — Momentum

---

- **Momentum:** A vector quantity defined as  $\vec{p} = m\vec{v}$
- **Conservation of momentum:** Total momentum of a system stays constant, if the net external force on it is zero and matter is not exchanged
- **Impulse:** Integral of net force over a time interval, or change in momentum over a time interval

# Review of last week — Momentum

- **Momentum:** A vector quantity defined as  $\vec{p} = m\vec{v}$
- **Conservation of momentum:** Total momentum of a system stays constant, if the net external force on it is zero and matter is not exchanged
- **Impulse:** Integral of net force over a time interval, or change in momentum over a time interval
- **Center of mass:** For systems and extended bodies, the one point that would move in the same path as a point mass subjected to the same net force

$$\vec{F}_{net}^{ext} = \frac{d\vec{p}_{sys}}{dt} = M\vec{A}_{CM}$$

# Conceptual question

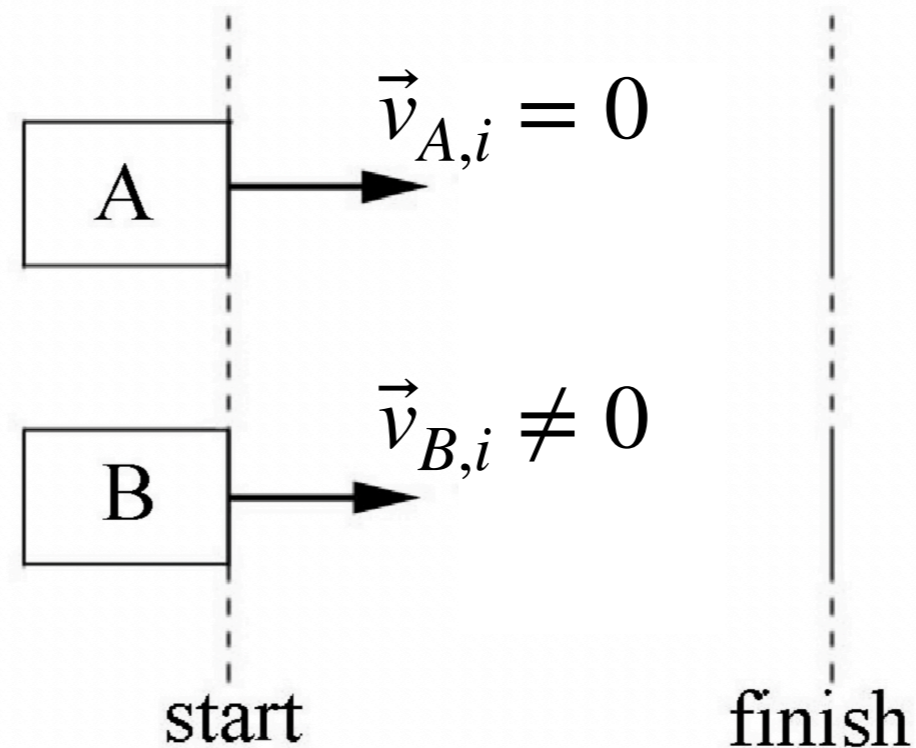
---

You drop a stone from the top of a high cliff. Consider the earth and stone as a system (neglecting the effects of the sun, moon, and other astronomical objects). As the stone falls, the momentum of this system...

- A. increases in the downwards direction.
- B. decreases in the downwards direction.
- C. stays the same.

# Conceptual question

Identical constant forces push two identical objects A and B continuously from a start line to a finish line. If A is initially at rest and B is initially moving to the right...

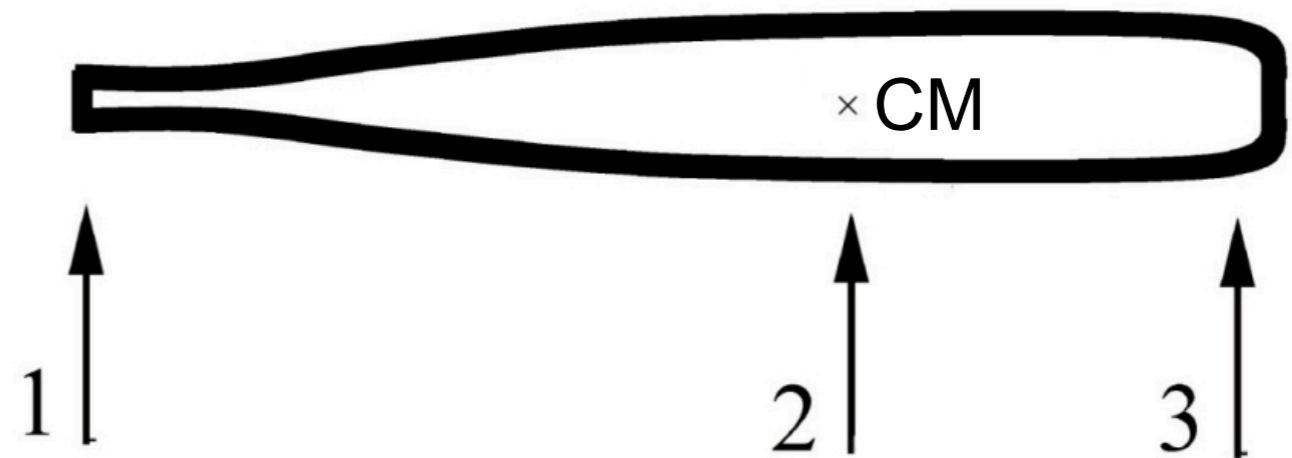


- A. object A has the larger change in momentum.
- B. object B has the larger change in momentum.
- C. Both objects have the same change in momentum.

# Conceptual question

The greatest acceleration of the center of mass (CM) of the baseball bat will be produced by pushing with a force  $F$  at

- A. Position 1
- B. Position 2
- C. Position 3
- D. All are the same.



# Today's agenda (Serway 6, 9 and MIT 8 -12)

---

1. Brief review of last week
  
- 2. Some examples**
  - **Conservation of momentum**
  - Motion of Center of mass
  
3. Systems with variable mass
  - Rocket science and interstellar space travel!

# Example: Conservation of momentum

---

A girl of mass  $m_g$  sees that her dog (mass  $m_d$ ) is playing on a flat icy surface. The dog suddenly falls and starts sliding with velocity  $\vec{v}_d$ . The girl runs after the dog, but upon touching the icy surface she also falls and slides with velocity  $\vec{v}_g$ . Fortunately, she has aimed right and reaches the dog.

Upon touching the dog the girl holds on to it and both keep sliding together. What is their velocity  $\vec{v}_f$ ?

# Example: Conservation of momentum

---

# DEMO (766)

---

Inelastic collision

# Today's agenda (Serway 6, 9 and MIT 8 -12)

---

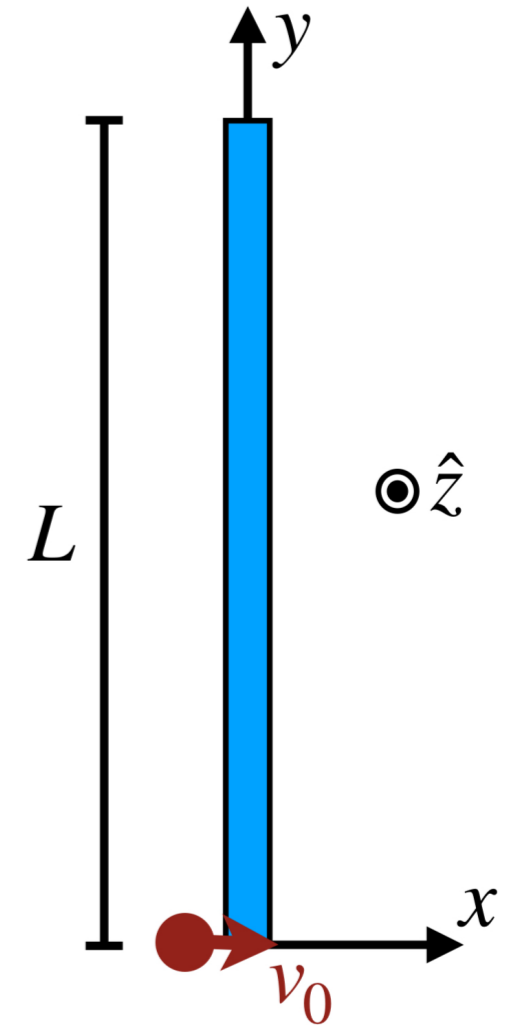
1. Brief review of last week
  
- 2. Some examples**
  - Conservation of momentum
  - **Motion of Center of mass**
  
3. Systems with variable mass
  - Rocket science and interstellar space travel!

# Example: Motion of center of mass (CM)

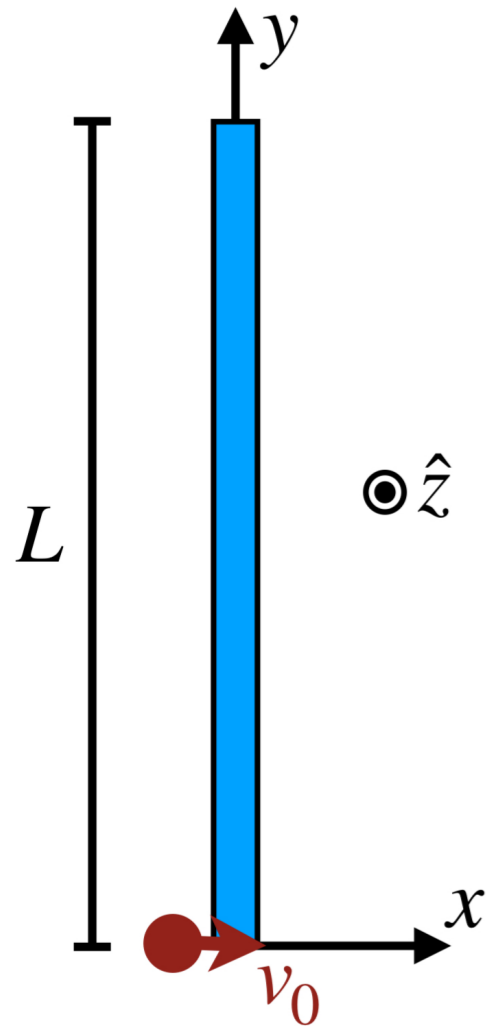
A slender rod of length  $L$  and mass  $M$  rests along the  $y$ -axis on a flat icy surface. The linear mass density of the rod  $\lambda(y)$  varies quadratically with the distance from the origin.

A particle of mass  $2M$  that moves along the  $x$ -axis with speed  $v_0$  strikes the rod at the instant of time  $t = t_0$ .

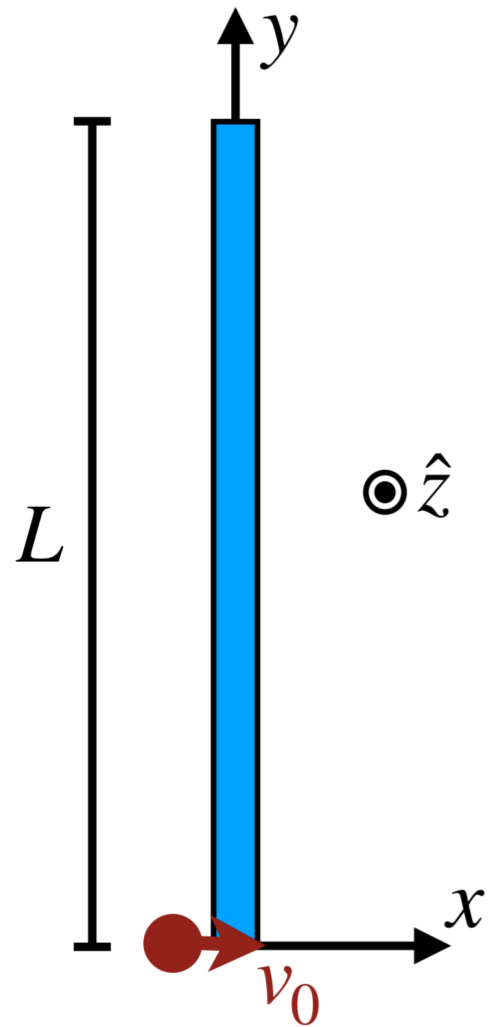
Find the position  $\vec{R}_{CM}$  of the center of mass as a function of  $t$ .



# Example: Motion of center of mass (CM)



# Example: Motion of center of mass (CM)



# Today's agenda (Serway 6, 9 and MIT 8 -12)

---

1. Brief review of last week
  
2. Some examples
  - Conservation of momentum
  - Motion of Center of mass
  
3. **Systems with variable mass**
  - Rocket science and interstellar space travel!

# DEMO (178)

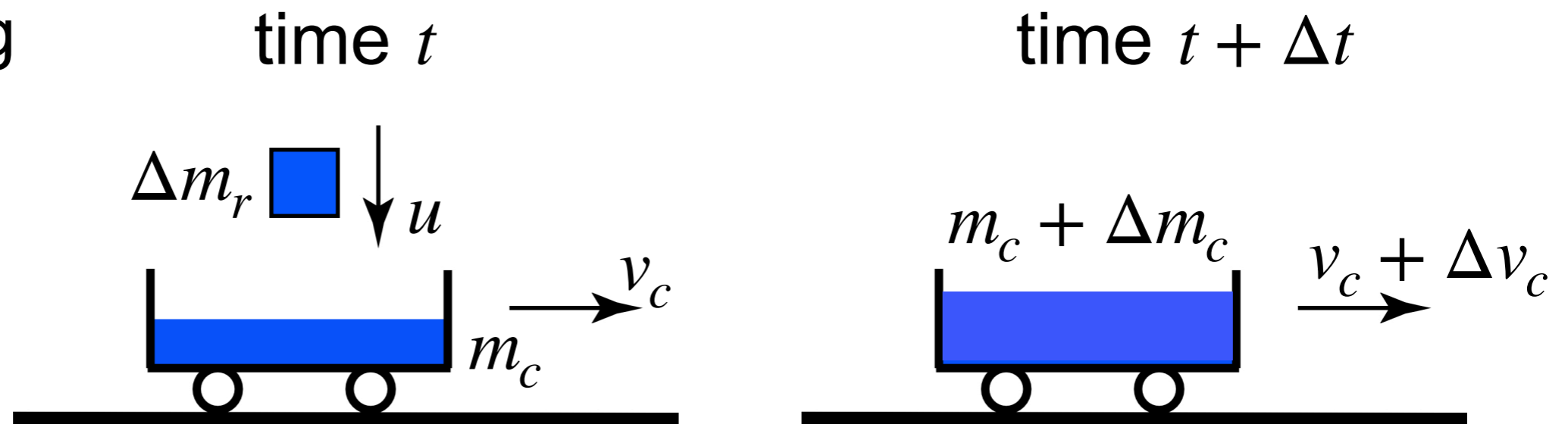
---

Falling chain

# Types of systems with changing mass

1. Transfer of mass to an object, but no transfer of momentum along the axis of motion

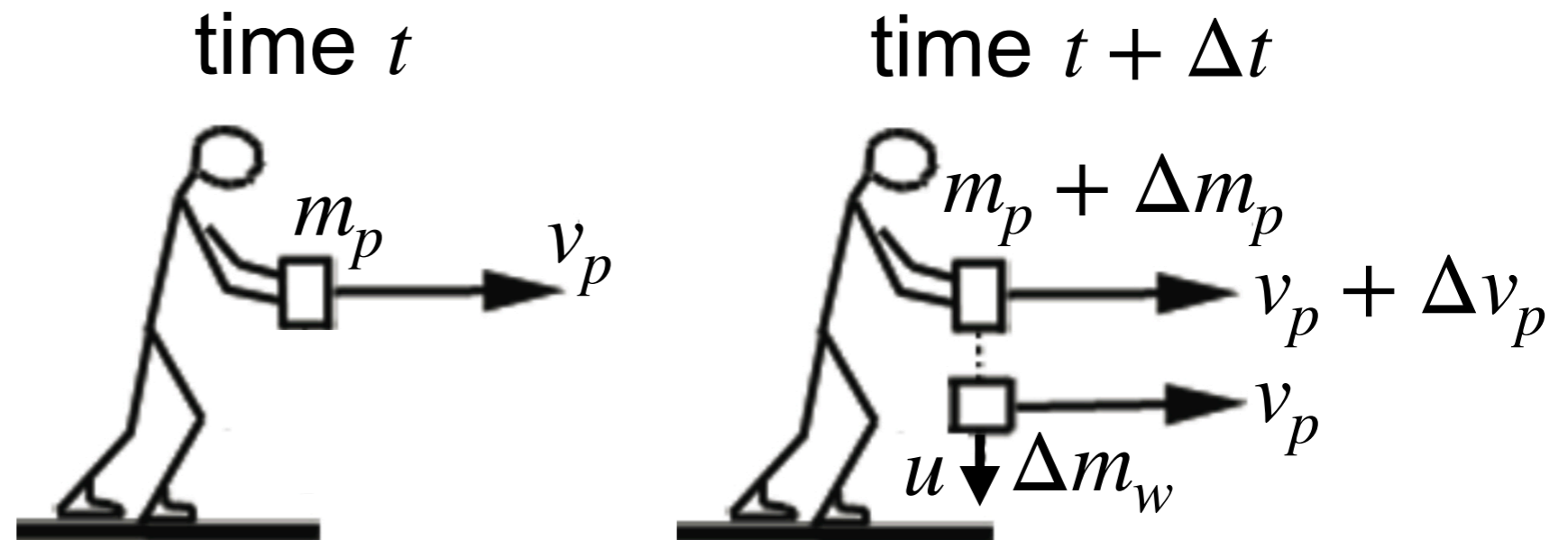
Rain falling  
into a cart:



# Types of systems with changing mass

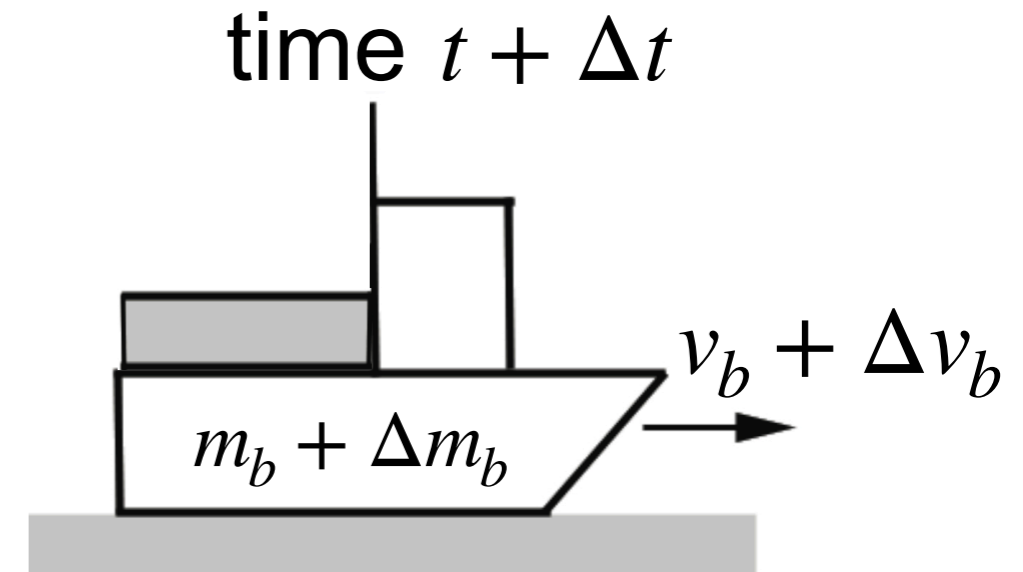
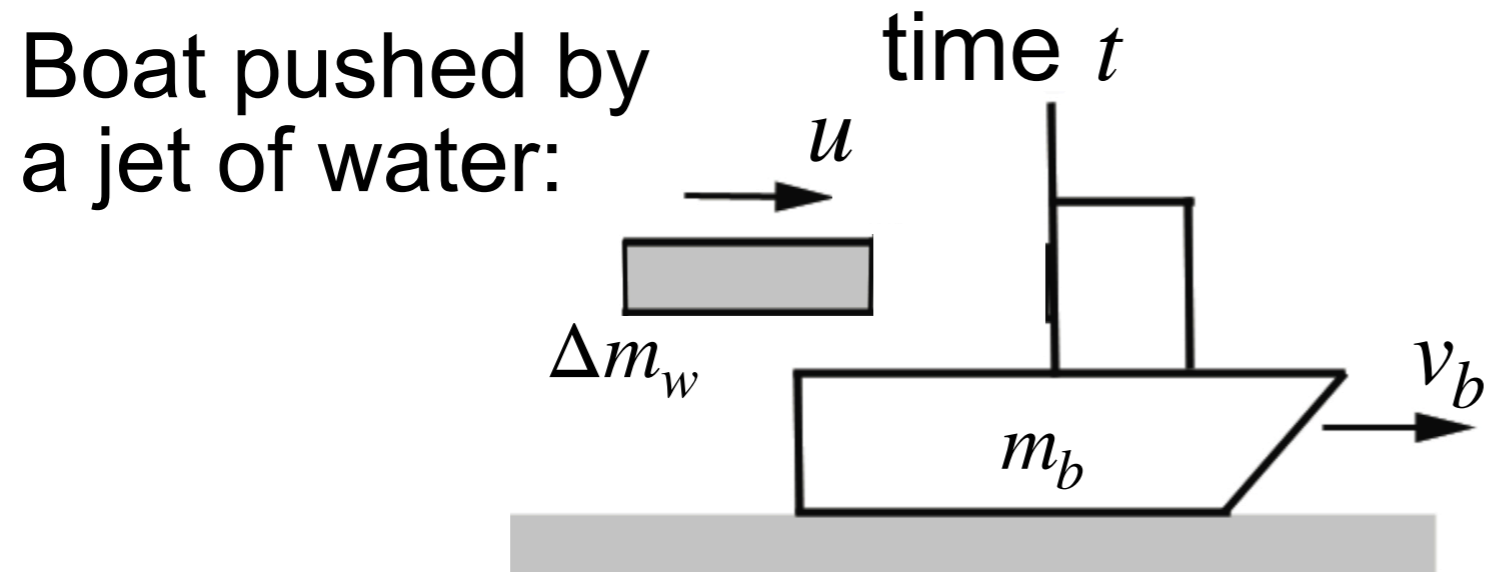
2. Transfer of mass from an object, but no transfer of momentum along the axis of motion

Ice skater with a leaky water bottle:



# Types of systems with changing mass

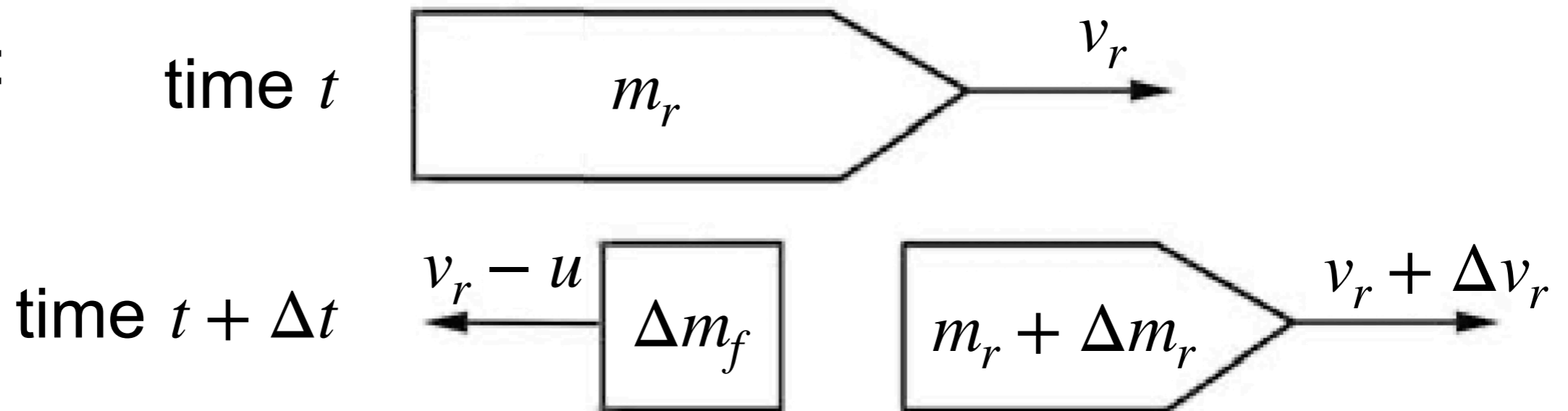
3. Mass hits an object, providing an impulse that transfers momentum along the direction of motion



# Types of systems with changing mass

4. Mass is ejected from an object, resulting in a recoil along the direction of motion

Rocket:



# DEMO (113)

---

Recoil of a cart  
(ejection of gas)

# Tackling systems with changing mass

- Determine the velocity of an object with changing mass



# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
1. Choose reference frame and define system



# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
  1. Choose reference frame and define system
  2. Draw **momentum diagrams** at different times



# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
  1. Choose reference frame and define system
  2. Draw **momentum diagrams** at different times
  3. Apply conservation of mass



# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
  1. Choose reference frame and define system
  2. Draw **momentum diagrams** at different times
  3. Apply conservation of mass
  4. Use generalized version of Newton's 2nd law



$$\vec{F}_{net}^{ext} = \frac{d\vec{p}_{sys}}{dt}$$

# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
  1. Choose reference frame and define system
  2. Draw **momentum diagrams** at different times
  3. Apply conservation of mass
  4. Use generalized version of Newton's 2nd law



$$\vec{F}_{net}^{ext} = \frac{d\vec{p}_{sys}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{p}_{sys}(t + \Delta t) - \vec{p}_{sys}(t)}{\Delta t}$$

# Tackling systems with changing mass

- Determine the velocity of an object with changing mass
1. Choose reference frame and define system
  2. Draw **momentum diagrams** at different times
  3. Apply conservation of mass
  4. Use generalized version of Newton's 2nd law



$$\vec{F}_{net}^{ext} = \frac{d\vec{p}_{sys}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{p}_{sys}(t + \Delta t) - \vec{p}_{sys}(t)}{\Delta t}$$

5. Find and solve the resulting differential equation

# Momentum diagrams

- A simple graphical summary of ALL the masses and velocities in the system
- Similar to free body diagrams
- Draw them at important moments in time or at arbitrary moments



# Today's agenda (Serway 6, 9 and MIT 8 -12)

---

1. Brief review of last week
  
2. Some examples
  - Conservation of momentum
  - Motion of Center of mass
  
3. **Systems with variable mass**
  - **Rocket science and interstellar space travel!**

# Derivation of rocket equation

---

A rocket of dry mass  $M_0$  starts from rest in our Solar System with a fuel mass  $M_f$ . It burns all the fuel, ejecting it backwards with velocity  $u$  relative to the rocket. This exhaust velocity  $u$  is independent of the velocity of the rocket. What is the final speed of the rocket? How long will it take to get to Proxima Centauri?

# Derivation of rocket equation

---

# Derivation of rocket equation

---

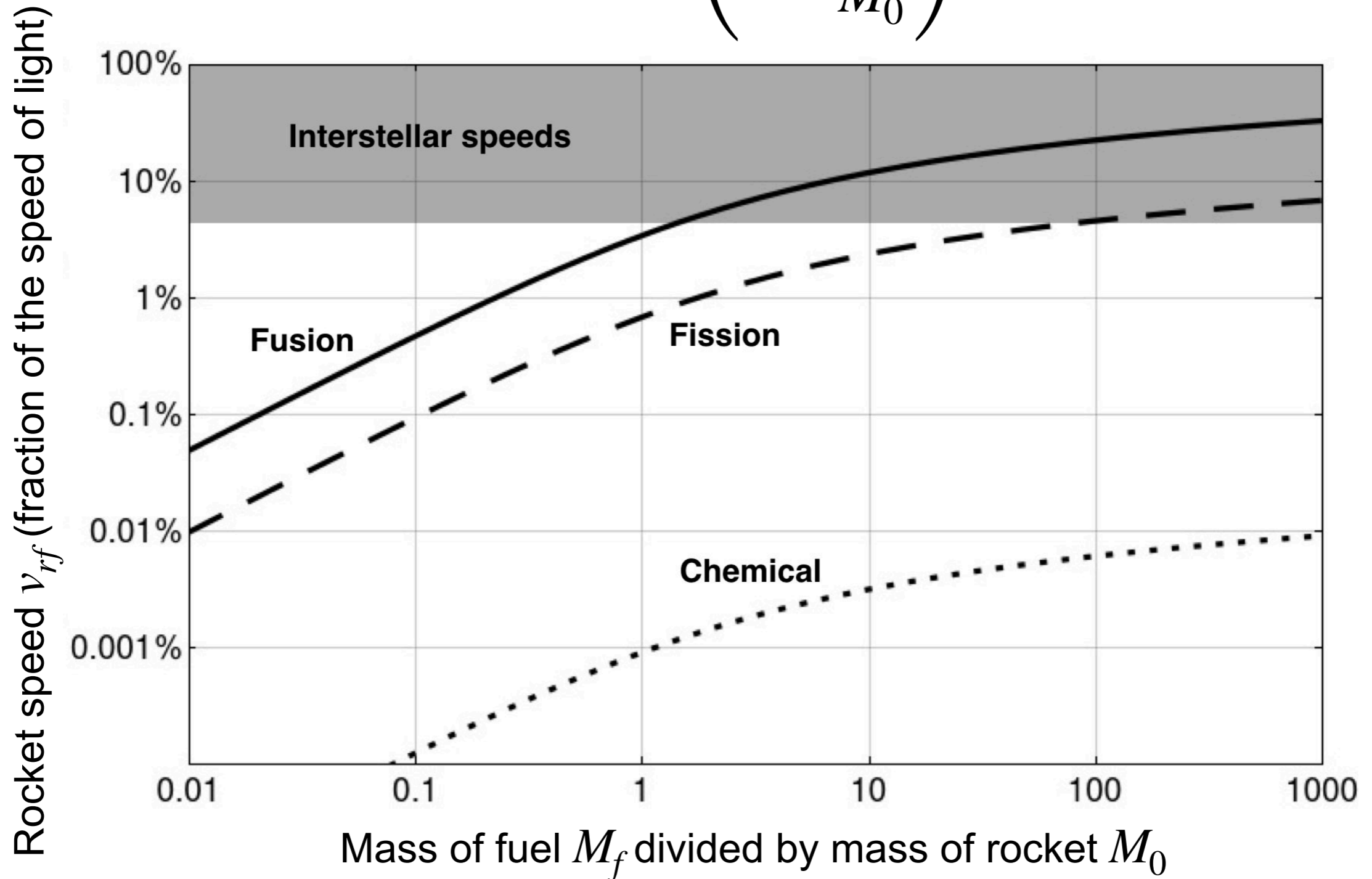
# DEMO (172)

---

Rockets!

# Significance of the rocket equation

$$v_{rf} = u \ln \left( 1 + \frac{M_f}{M_0} \right)$$



# Conceptual question

A rocket is moving in outer space without gravity. It is burning fuel so that its “thrust” is constant (i.e.  $u \frac{dm}{dt} = \text{const}$ ) at all times. Its acceleration is...

- A. constant.
- B. decreasing.
- C. increasing.



# Summary

- Tackling systems with changing mass
  1. Choose reference frame and define system
  2. Apply conservation of mass
  3. Use generalized Newton's 2nd law:  $\vec{F}_{net}^{ext} = \lim_{\Delta t \rightarrow 0} \frac{\vec{p}_{sys}(t + \Delta t) - \vec{p}_{sys}(t)}{\Delta t}$
  4. Find and solve the resulting differential equation

- Rocket equation:  $v_{rf} = u \ln \left( 1 + \frac{M_f}{M_0} \right)$

# Mock exam tomorrow 😲

