EPFL

Swiss
Plasma
Center

General Physics: Mechanics

PHYS-101(en)

Lecture 6a:
Drag, momentum, impulse, and center of mass

Dr. Marcelo Baquero
marcelo.baquero@epfl.ch

October 13th, 2025

Announcement

- Next week is "Fall break"
  - There is **no** class Monday nor Tuesday.
  - There are **no** Exercise sessions on Wednesday.
  - There are **no** Office hours.
  - Classes resume Monday, Oct. 27th.

Announcement

- We’ll hold a ***mini* mock exam** on Tuesday October 28th
    - In-class (SG1) during normal lecture hours (10:15-11:00)
    - Does **not** matter at all for your final grade
    - You can bring a “cheat” sheet containing formulas or all of your notes, as you wish
    - Turn in at the end if you want exam to be graded (optional). Graded exams will be returned on Monday November 10th.
    - Exam solutions will be published on the Moodle

Today’s agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. Momentum
    - Conservation of momentum
    - Impulse
3. Center of mass

DEMO (738)

Air drag

Resistive forces, approximately

- Resistive forces oppose the motion of an object

Resistive forces, approximately

- Resistive forces oppose the motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

$$\overrightarrow{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

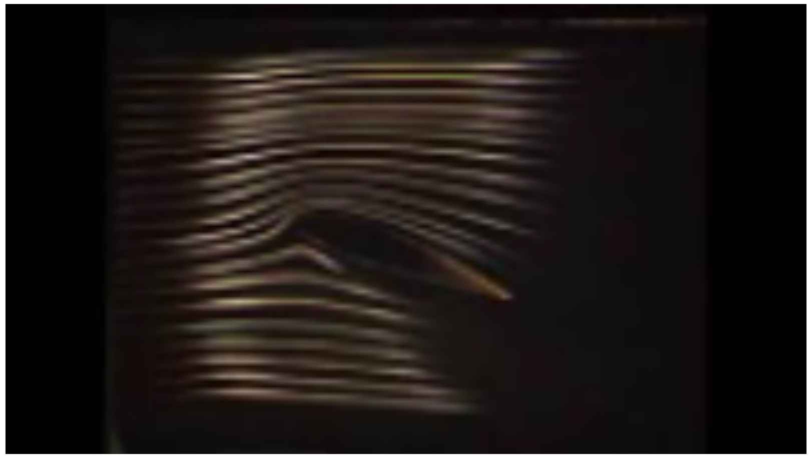
Resistive forces, approximately

- Resistive forces oppose the motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

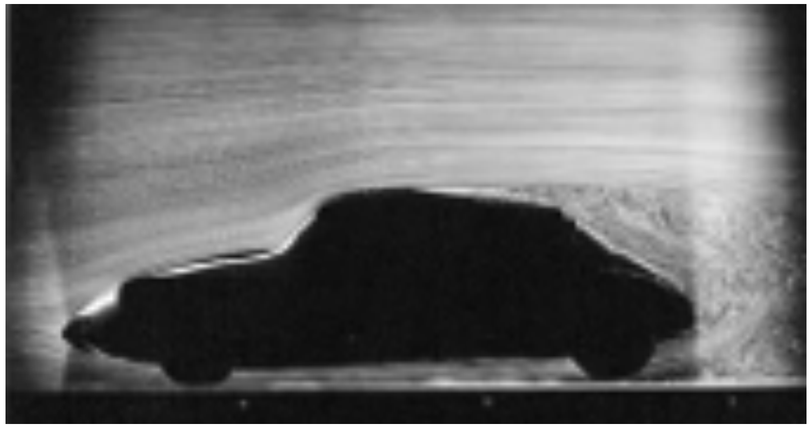
$$\overrightarrow{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

- *Laminar*: for smooth objects at low speeds the flow is steady and $n = 1$



Airplane wing



A car driving

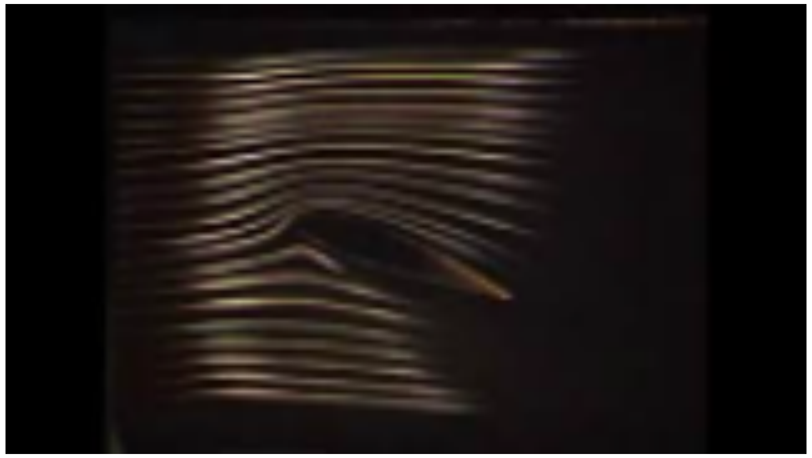
Resistive forces, approximately

- Resistive forces oppose the motion of an object
- A prime example is *viscous drag* with a fluid (e.g. air resistance)

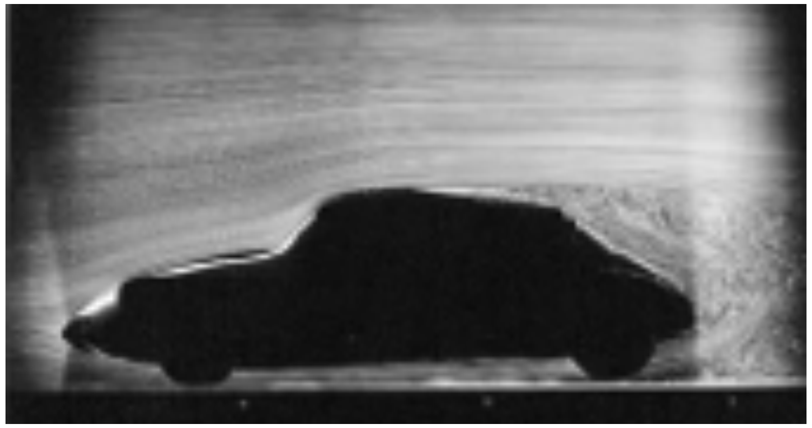
$$\overrightarrow{F}_{drag} = -\beta v^n \hat{v}$$

where β is a constant that depends on the fluid and shape of the object and n characterizes the flow *regime*

- *Laminar*: for smooth objects at low speeds the flow is steady and $n = 1$
- *Turbulent*: for rough objects at high speeds the flow is chaotic and $n = 2$



Airplane wing

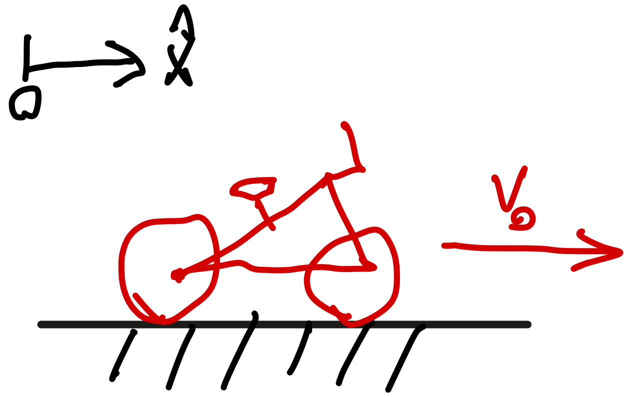


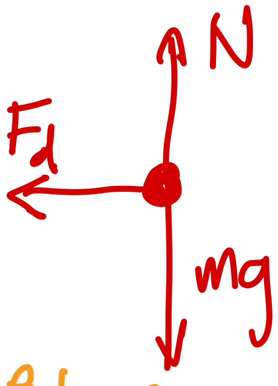
A car driving

Boat wake

Example: Laminar viscous drag

A cyclist is moving at a slow speed v_0 under laminar conditions (i.e. $n = 1$) and has a drag coefficient β. Calculate the bike's speed as a function of time, assuming drag is the only force acting on it.





$$\sum F_x : \overbrace{-F_d}^{-\beta v_x} = m a_x = m\frac{dv_x}{dt}$$

$$\Rightarrow -\beta v_x = m\frac{dv_x}{dt} \Rightarrow \frac{dv_x}{dt} = -\frac{\beta}{m} v_x$$

$$\frac{dv_x}{dt} = -\frac{\beta}{m} v_x \Rightarrow \overbrace{\int \frac{dv_x}{v_x}}^{\ln(v_x)+C_2} = \overbrace{\int -\frac{\beta}{m}\, dt}^{-\frac{\beta}{m}t + C_1} \Rightarrow \ln(v_x) = -\frac{\beta}{m}t + \overbrace{C_1 - C_2}^{=C_3}$$

$$v_x = e^{\ln(v_x)} = e^{-\frac{\beta}{m}t + C_3} = \overbrace{e^{C_3}}^{C_4} e^{-\frac{\beta}{m}t}$$

$$\Rightarrow v_x = C_4 e^{-\frac{\beta}{m}t}$$

$$v_x(t=0) = v_0 = C_4 e^0 = C_4$$

$$\Rightarrow \boxed{v_x(t) = v_0 e^{-\frac{\beta}{m}t}}$$

Example: Turbulent viscous drag

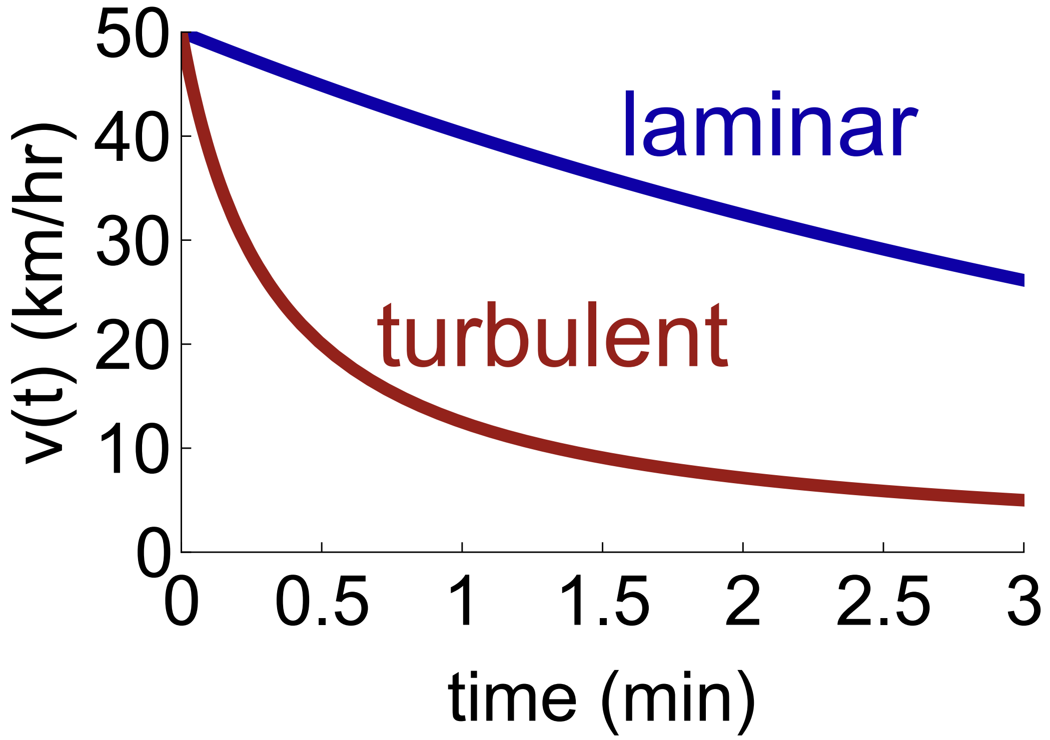
When the cyclist moves at a fast speed v_0 such that they reach the turbulent regime (i.e. $n = 2$), a similar procedure yields the following speed as a function of time:

$$v(t) = \frac{v_0}{1 + \left(\frac{\beta\, v_0}{m}\right) t}$$

Example: Turbulent viscous drag

When the cyclist moves at a fast speed v_0 such that they reach the turbulent regime (i.e. $n = 2$), a similar procedure yields the following speed as a function of time:

$$v(t) = \frac{v_0}{1 + \left(\frac{\beta\, v_0}{m}\right) t}$$



Today’s agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. **Momentum**
   - **Conservation of momentum**
   - **Impulse**
3. Center of mass

DEMO (86 and 113)

Collisions between two spheres

and

the recoil of a cart

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

- It has units of [kg·m/s] (or equivalently [N·s])

Definition of momentum

- Historically called the “quantity of motion”
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

- It has units of [kg·m/s] (or equivalently [N·s])
- Newton’s 2nd law:

$$m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\Sigma\overrightarrow{F} = m\vec{a} \quad \Rightarrow \quad \Sigma\overrightarrow{F} = \frac{d\vec{p}}{dt}$$

- In English, the time rate of change of an object’s momentum is equal to the net force acting on it

Definition of momentum

- Historically called the "quantity of motion"
- Momentum is a vector quantity:

$$\vec{p} = m\vec{v}$$

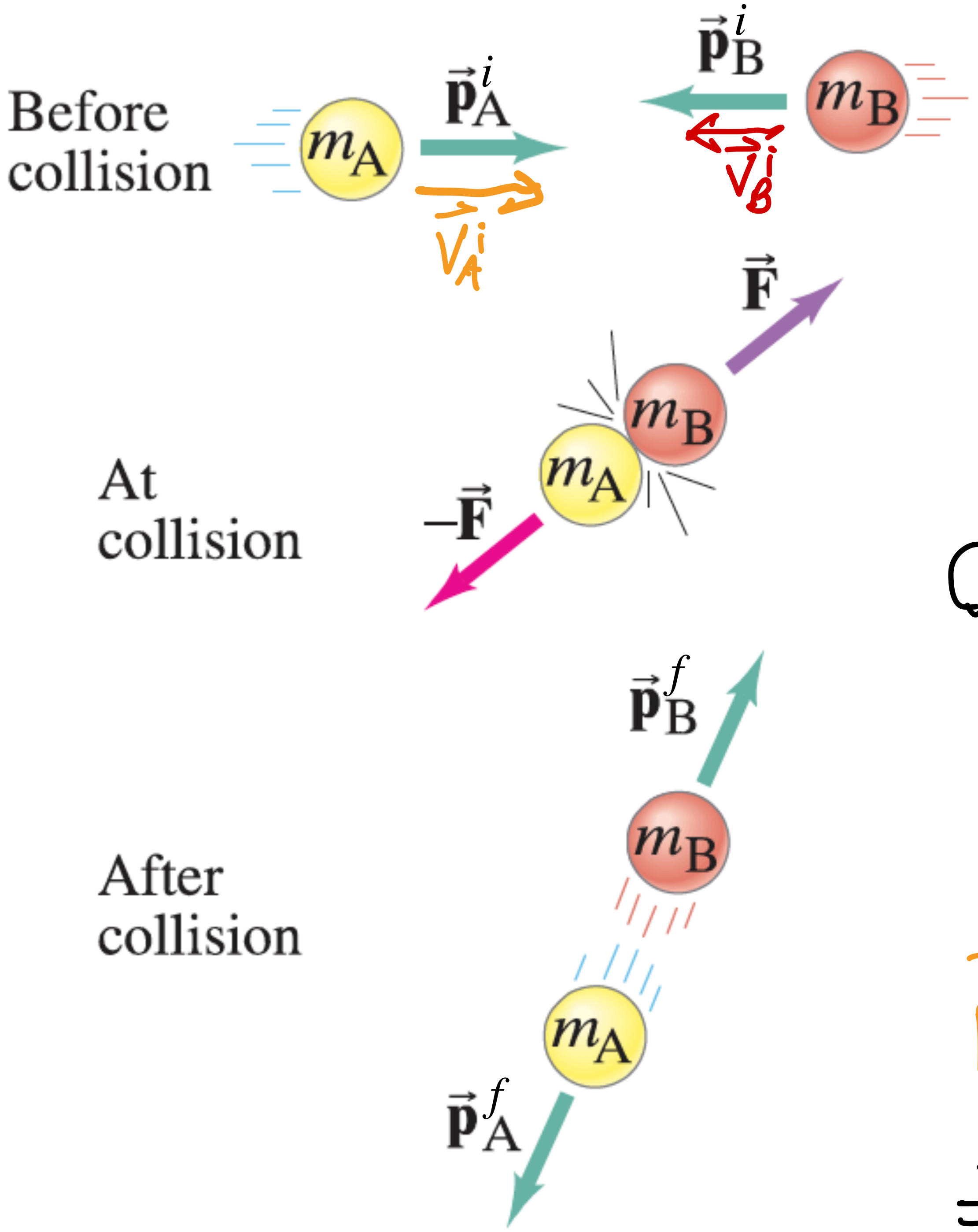
- It has units of [kg·m/s] (or equivalently [N·s])
- Newton's 2nd law:

Most general form

$$\Sigma \overrightarrow{F} = m\vec{a} \quad \Rightarrow \quad \boxed{\Sigma \overrightarrow{F} = \frac{d\vec{p}}{dt}}$$

- In English, the time rate of change of an object's momentum is equal to the net force acting on it

Momentum conservation from Newton's laws



$\vec{p}_A^{\,i} = m_A \vec{v}_A^{\,i}$ $\quad$ $\vec{p}_B^{\,i} = m_B \vec{v}_B^{\,i}$

$\vec{p}^{\,i} = \vec{p}_A^{\,i} + \vec{p}_B^{\,i}$ = const before interaction (Newton's 1st law)

$\vec{F}_{AB} = \frac{d\vec{p}_A^{\,c}}{dt}$ $\quad$ $\vec{F}_{BA} = \frac{d\vec{p}_B^{\,c}}{dt}$

$$0 = \underbrace{\frac{d\vec{p}_A^{\,c}}{dt}}_{\vec{F}_{AB}} + \underbrace{\frac{d\vec{p}_B^{\,c}}{dt}}_{\vec{F}_{BA} = -\vec{F}_{AB}} = \frac{d}{dt}\left(\vec{p}_A^{\,c} + \vec{p}_B^{\,c}\right) = \frac{d}{dt}\vec{p}^{\,c}$$

(Newton's 3rd law)

$\vec{p}_A^{\,f} + \vec{p}_B^{\,f}$ = const $= \vec{p}^{\,f} = \vec{p}^{\,c} = \vec{p}^{\,i}$

$$\Rightarrow \boxed{\vec{p}^{\,f} = \vec{p}^{\,i}}$$

Conservation of momentum

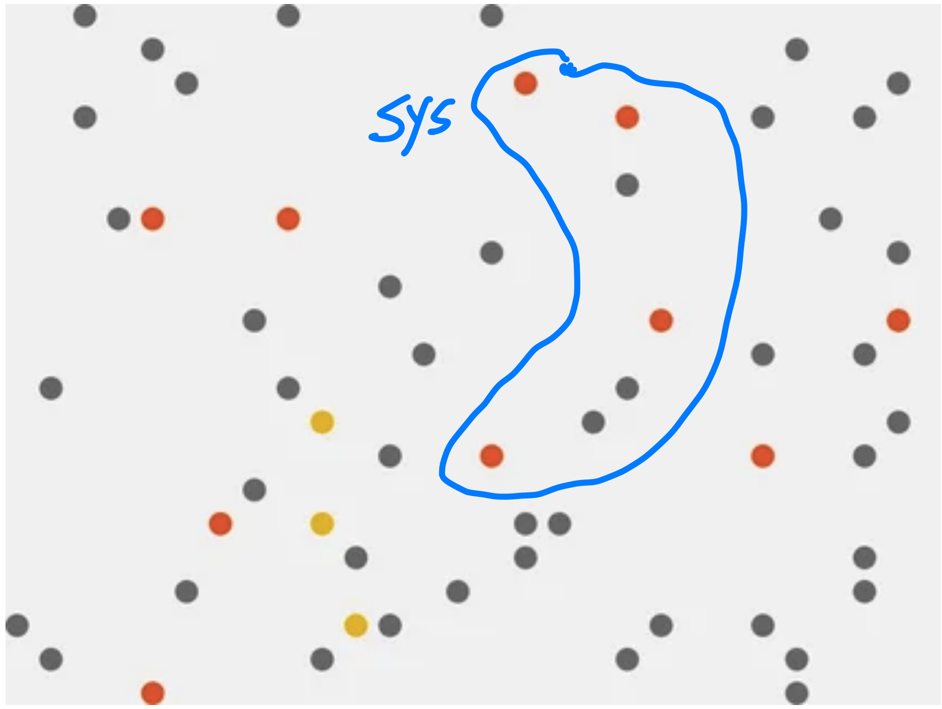
In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

Conservation of momentum

In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

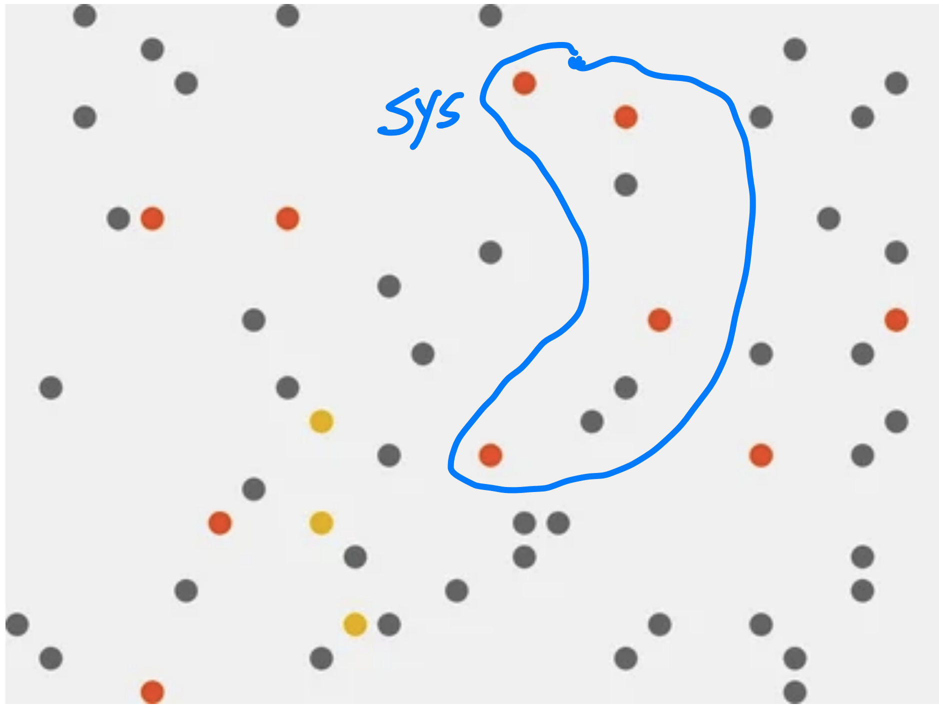
What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want



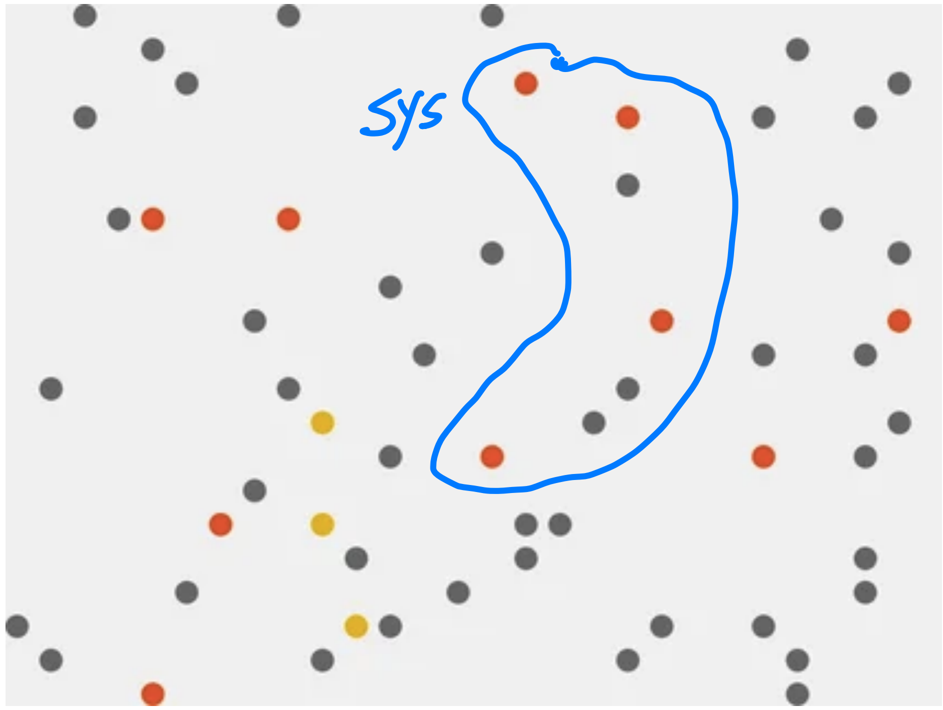
What is a system?

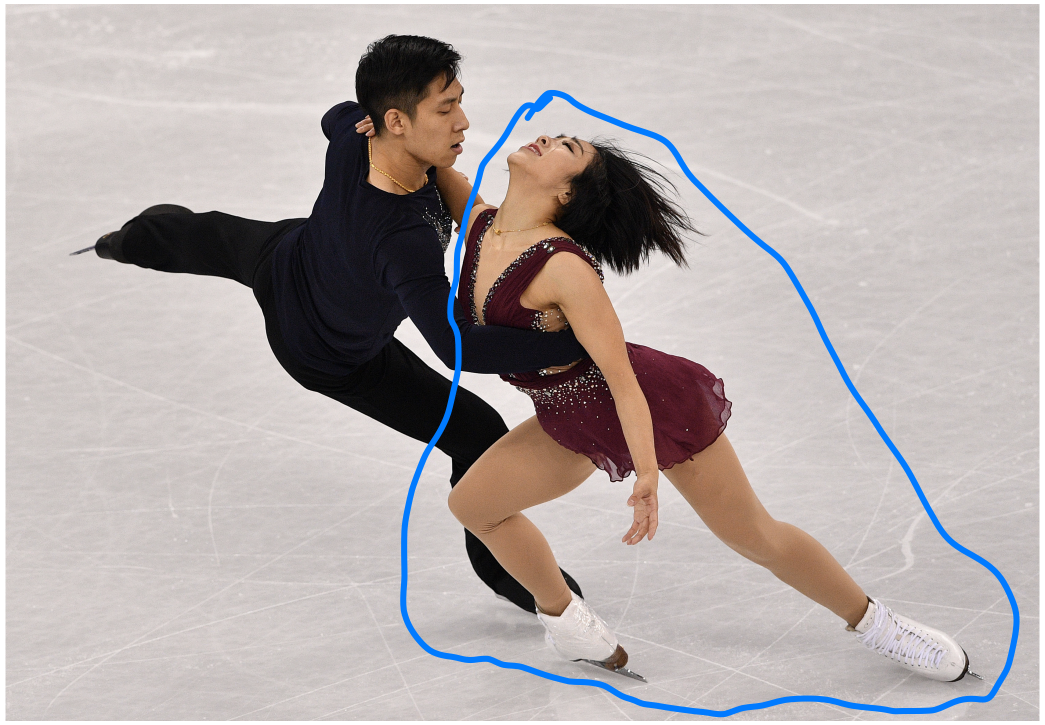
- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want
- If there is no net force from the outside and all the particles remain in the system, then the system is said to be *isolated*



What is a system?

- For a collection of particles, it is any subset that you wish to consider
- You can think about it as the particles within the region delimited by a border drawn anyway you want
- If there is no net force from the outside and all the particles remain in the system, then the system is said to be *isolated*





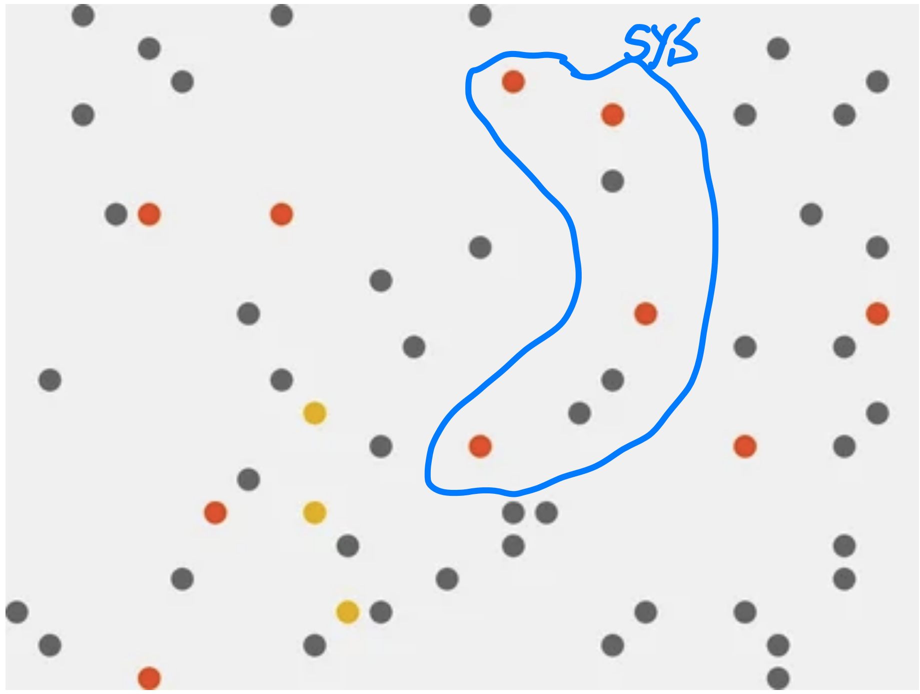
Conservation of momentum

In a given inertial reference frame, the total momentum of an isolated system stays constant.

What is the total momentum?

- Momentum of a point mass i is $\vec{p}_i = m_i \vec{v}_i$
- Total momentum of a system of N point masses is simply

$$\vec{p}_{sys} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$



Conservation of momentum

In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

The net force on the system is zero and matter is not exchanged

Non-inertial reference frames create fictitious forces that can change the momentum

Conservation of momentum

In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

The net force on the system is zero and matter is not exchanged

Non-inertial reference frames create fictitious forces that can change the momentum

Discussed next lecture

Conservation of momentum

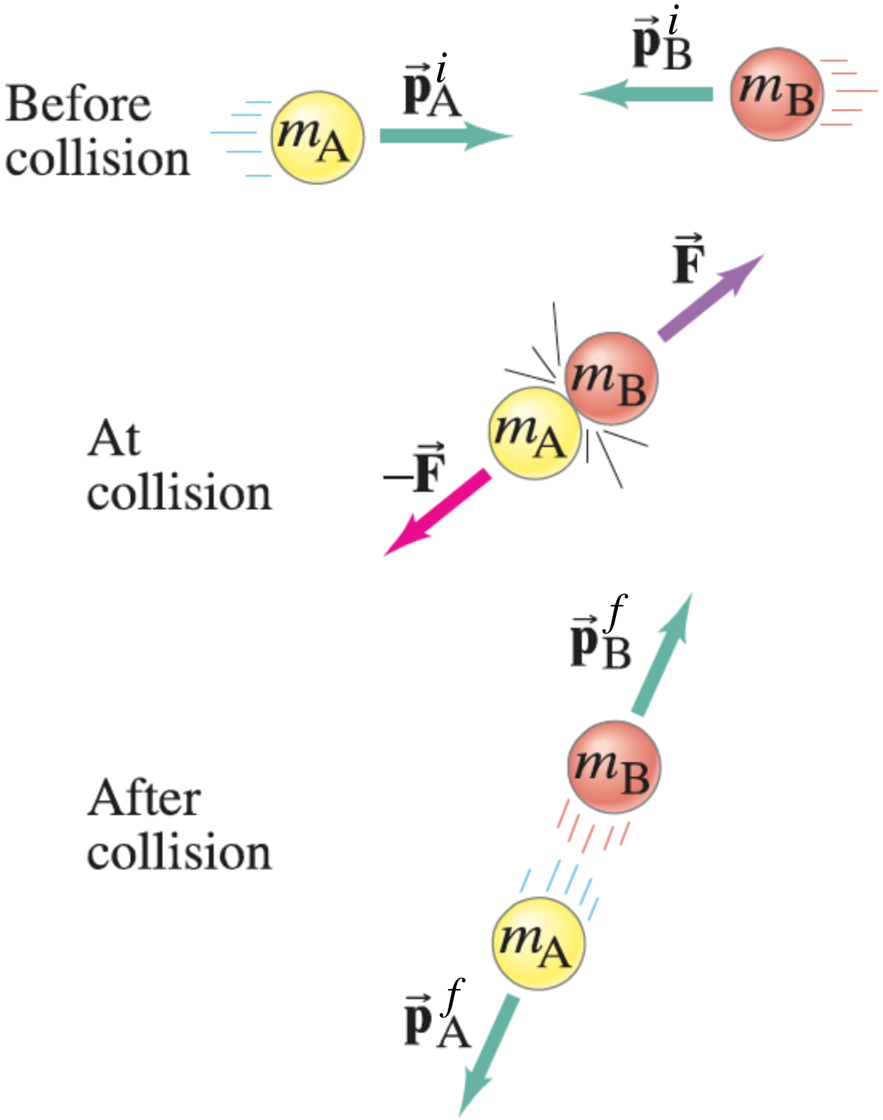
In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

- Veeeeeeeeeeeeerrrrrrrrrrrrrrrrrrrry profound

Conservation of momentum

In a given inertial reference frame,
the total momentum of
an isolated system stays constant.

- Veeeeeeeeeeeeeerrrrrrrrrrrrrrrrry profound



$$\vec{p}^{\,i} = \vec{p}^{\,f}$$

$$\Rightarrow \boxed{\vec{p}_A^{\,i} + \vec{p}_B^{\,i} = \vec{p}_A^{\,f} + \vec{p}_B^{\,f}}$$

Conceptual question

responseware.eu
Session ID: epflphys101en

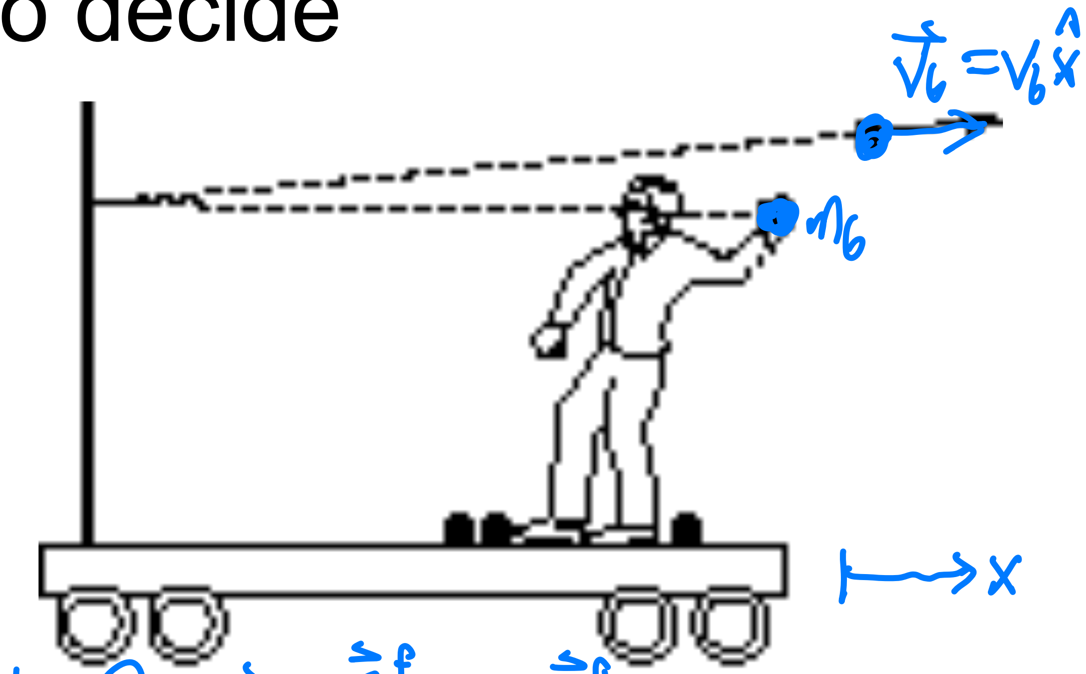
Suppose you are on a cart, initially at rest. Neglect friction. You throw a ball at a partition that is rigidly mounted on the cart and the ball bounces straight back as shown in the figure. After the ball bounces, is the cart moving?

A. Yes, it moves to the right.
[✓ B.] Yes, it moves to the left.
C. No, it remains in place.
D. Not enough information is given to decide

$\vec{p}_M^f = M\vec{v}_M^f$

$\vec{v}_M^f = -\frac{m_b}{M} v_b \hat{x}$

System has two objects:
1) Ball of mass m_B
2) Everything else (cart, guy, etc.): mass M



$\vec{p}_b^i = 0$ $\vec{p}_M^i = 0$ ⇒ $\vec{p}^i = \vec{p}_b^i + \vec{p}_M^i = 0$

$\vec{p}_b^f = m_b\vec{v}_b = m_b v_b \hat{x}$ $\vec{p}_M^f = M\vec{v}_M^f$ ⇒ $\vec{p}^f = \underline{\vec{p}_M^f + \vec{p}_b^f} = \vec{p}^i = 0$ ⇒ $\vec{p}_M^f = -\vec{p}_b^f$

How do we move (i.e. change momentum)?

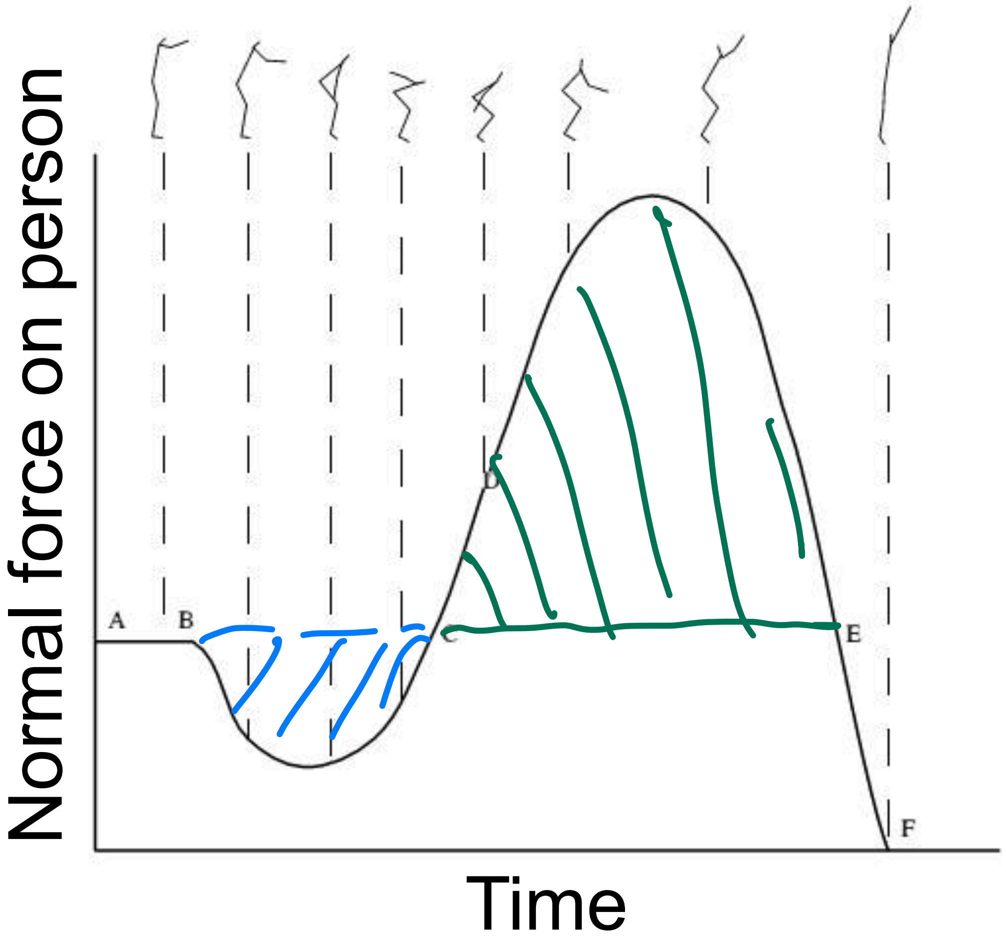
- If momentum is conserved, how do we move? We push off of things, usually the ground

How do we move (i.e. change momentum)?

- If momentum is conserved, how do we move? We push off of things, usually the ground
- Consider a “system” to include only your body, then exert a net force on an external object
- We change momentum by applying force for a time interval

How do we move (i.e. change momentum)?

- If momentum is conserved, how do we move? We push off of things, usually the ground
- Consider a “system” to include only your body, then exert a net force on an external object
- We change momentum by applying force for a time interval



Impulse and momentum

- Defined as the integral of a net force over a time period:

$$\vec{I} = \int_{t_i}^{t_f} \overrightarrow{F}_{net}\, dt$$

Impulse and momentum

- Defined as the integral of a net force over a time period:

$$\vec{I} = \int_{t_i}^{t_f} \overrightarrow{F}_{net}\, dt$$

- Using Newton's 2nd law $\overrightarrow{F}_{net} = \frac{d\vec{p}}{dt}$

$$\vec{I} = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt \quad \Rightarrow \quad \vec{I} = \vec{p}(t_f) - \vec{p}(t_i) = \Delta\vec{p}$$

- It is simply a change of momentum in time

- It has the same units as momentum of [kg·m/s] (or equivalently [N·s])

Neglecting the details

- Often we don’t care about the details of the force (e.g. when it’s very short)

Neglecting the details

- Often we don’t care about the details of the force (e.g. when it’s very short)
- Model the impulse as an average force applied over the same time interval

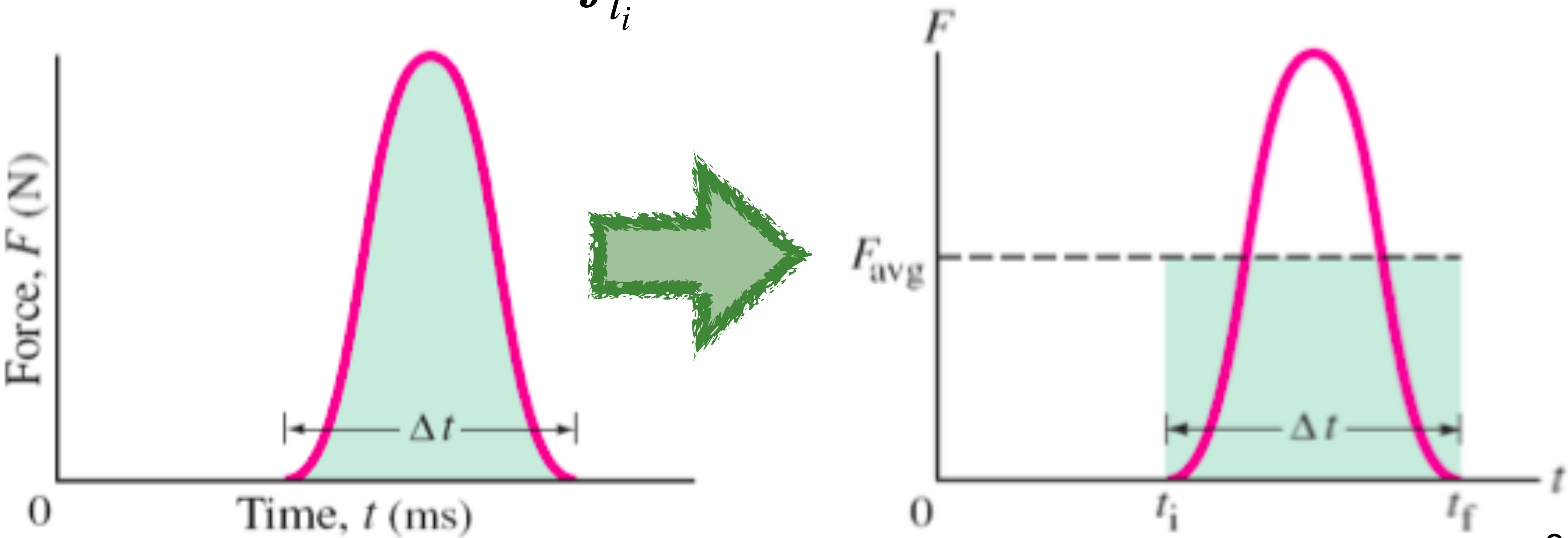
$$\vec{I} = \int_{t_i}^{t_f} \overrightarrow{F}\, dt = \overrightarrow{F}_{avg} \overbrace{\Delta t}^{\Delta t = t_f - t_i}$$

Neglecting the details

- Often we don't care about the details of the force (e.g. when it's very short)

- Model the impulse as an average force applied over the same time interval

$$\vec{I} = \int_{t_i}^{t_f} \overrightarrow{F} dt = \overrightarrow{F}_{avg} \Delta t$$



DEMO (84)

$$\vec{I}_H = \Delta\vec{p}_H = m_H\overbrace{\vec{v}_H^f}^{\approx -\vec{v}_H^i} - m_H\overbrace{\vec{v}_H^i}^{-v_H^i\hat{y}} = 2m_H v_H^i\hat{y}$$

$\hat{y}\uparrow$

$$\vec{F}_{HS} = \frac{1}{\Delta t}\vec{I}_H = \hat{y}\,\frac{2m_H v_H^i}{\Delta t} = -\vec{F}_{SH}$$

Duration of a collision

Impulse approximation

- During a very short collision, the force of the collision is usually much larger than all other forces

- Thus, all other forces can be ignored during the collision

Impulse approximation

- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision
- Example: A tennis ball is coming at you at $\vec{v}_1$. You hit it with a tennis racket and it departs leaving at $\vec{v}_2$.
  - What is the change in momentum? $\Delta\vec{p} = m_b\vec{v}_2 - m_b\vec{v}_1$
  - What is the impulse? $\vec{I} = \Delta\vec{p}$
  - What is the force and how long was it applied?

Impulse approximation

- During a very short collision, the force of the collision is usually much larger than all other forces
- Thus, all other forces can be ignored during the collision
- Example: A tennis ball is coming at you at $\vec{v}_1$. You hit it with a tennis racket and it departs leaving at $\vec{v}_2$.
    - What is the change in momentum?
    - What is the impulse?
    - What is the force and how long was it applied? 🤷‍♂️
    - Impulse approximation: don’t consider the effect of gravity *<u>during</u>* the collision with the tennis racket

Conceptual question

The figure below depicts the paths of two colliding blue circles, 1 and 2. Which of the following arrows best represents the direction of the impulse applied to circle 2 by circle 1 during the collision?

A. 

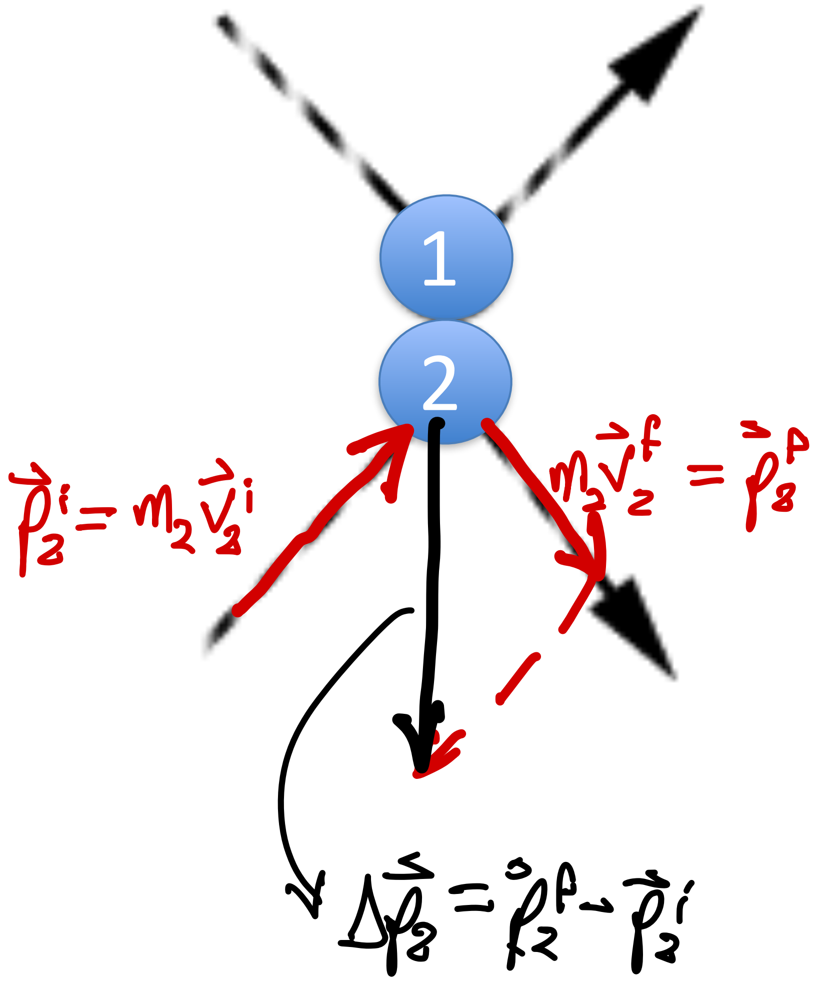
B. 

C. 

D. 

[✓ E.] 

$$\vec{I}_2 = \Delta\vec{p}_2 = \vec{p}_2^{\,f} - \vec{p}_2^{\,i} = m_2\vec{v}_2^{\,f} - m_2\vec{v}_2^{\,i}$$



$\vec{p}_2^{\,i} = m_2\vec{v}_2^{\,i}$

$m_2\vec{v}_2^{\,f} = \vec{p}_2^{\,f}$

$\Delta\vec{p}_2 = \vec{p}_2^{\,f} - \vec{p}_2^{\,i}$

Today’s agenda (Serway 6, 9 and MIT 8, 10)

1. Drag
2. Momentum
   - Conservation of momentum
   - Impulse
3. **Center of mass**

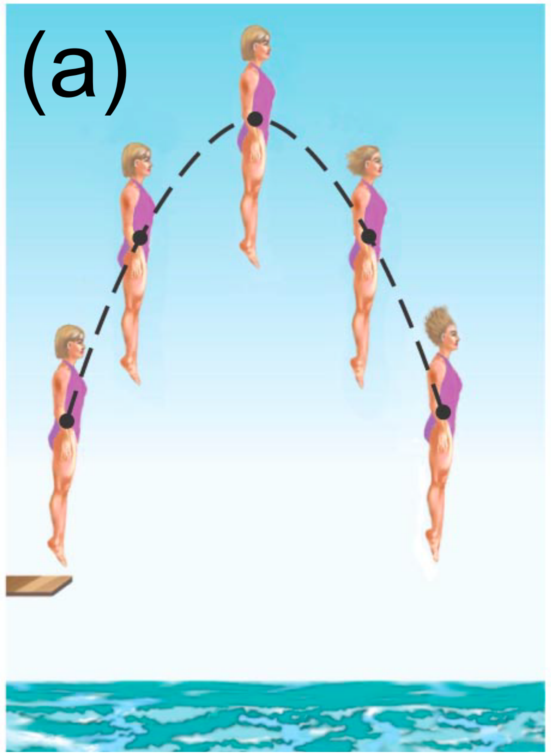
Center of mass

- We’ve used the point mass approximation a lot, but the appropriate point isn’t always obvious

- If we want to summarize this guy's trajectory, which point should we take?

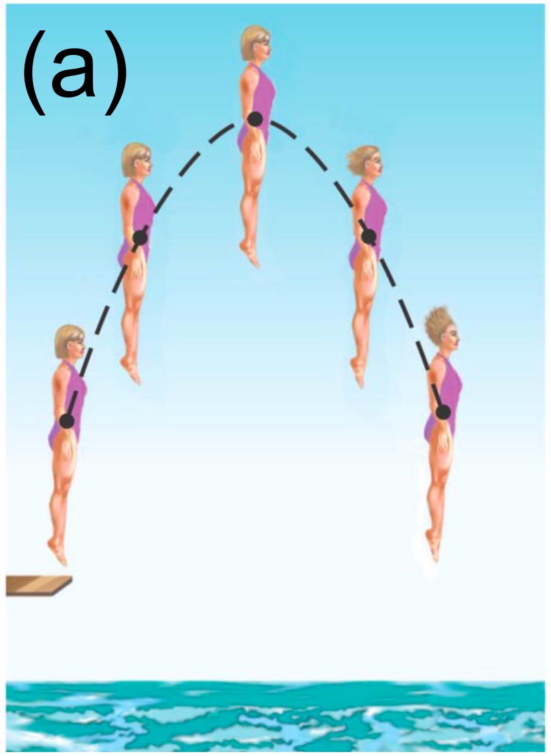
Center of mass

- In (a), the diver’s motion is pure translation
- In (b), the motion is translation plus rotation



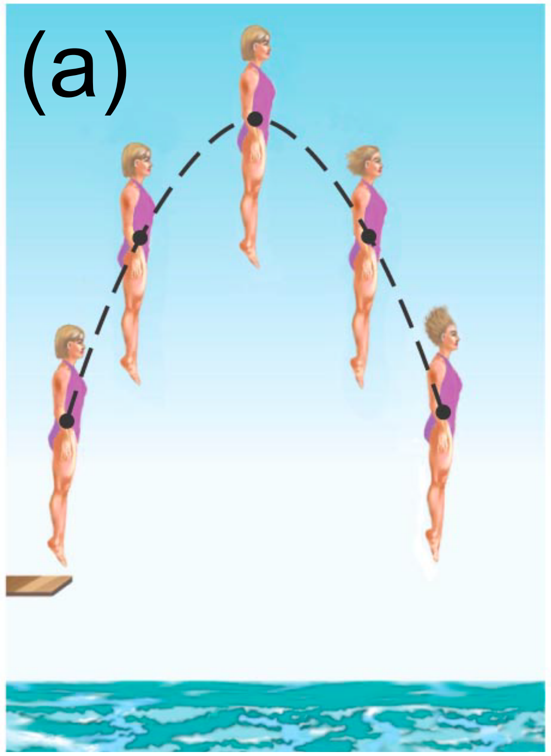
Center of mass

- In (a), the diver’s motion is pure translation
- In (b), the motion is translation plus rotation
- One point, called the *Center of Mass* (CM), that moves the same in both (i.e. it’s the point the diver rotates around)
- It’s the one point that would move in the same path as a point mass subjected to the same net force



Center of mass

- In (a), the diver’s motion is pure translation
- In (b), the motion is translation plus rotation
- One point, called the *Center of Mass* (CM), that moves the same in both (i.e. it’s the point the diver rotates around)
- It’s the one point that would move in the same path as a point mass subjected to the same net force
- The motion of an object can always be decomposed into translational motion of the center of mass, plus rotation, deformation, …
- How do we find it, you ask…



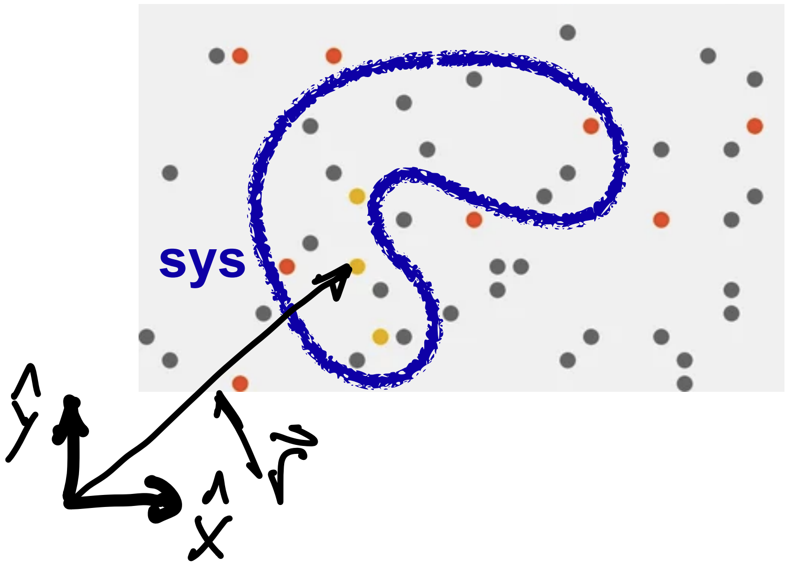
Calculating the center of mass

- It is the “average” position of the system, weighted by mass

$$\overrightarrow{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$

N = num. of particles in systems

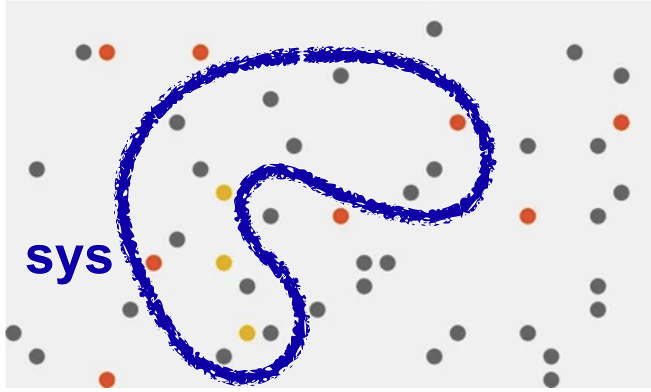
where $M = \Sigma_{i=1}^{N} m_i$ is the total mass



Calculating the center of mass

- It is the “average” position of the system, weighted by mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$

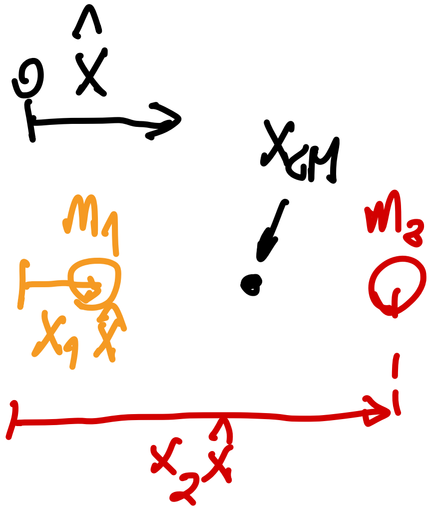


where $M = \sum_{i=1}^{N} m_i$ is the total mass

- For just two particles in 1D, the center of mass lies closer to the one with more mass

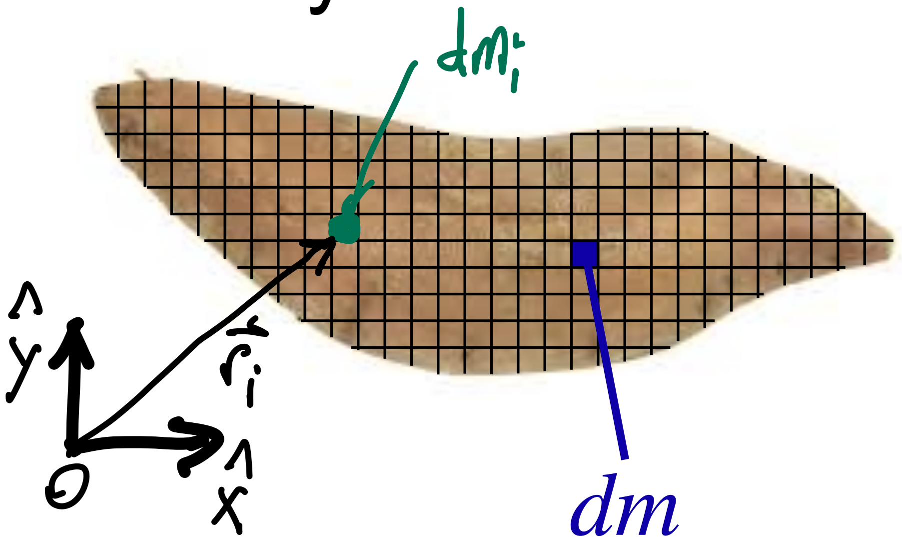
$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \hat{x}$$

$$= \left[\frac{m_1}{M} x_1 + \frac{m_2}{M} x_2 \right] \hat{x}$$



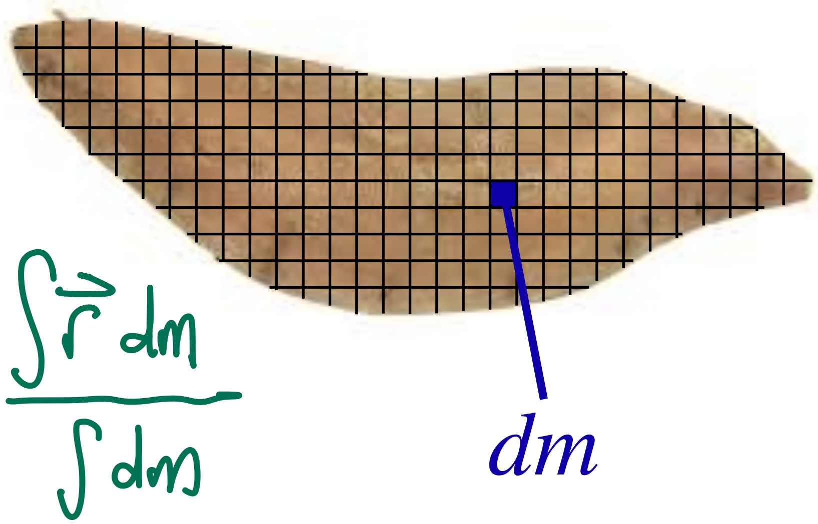
Center of mass for continuous systems

- What about for an extended object? Like this yam



Center of mass for continuous systems

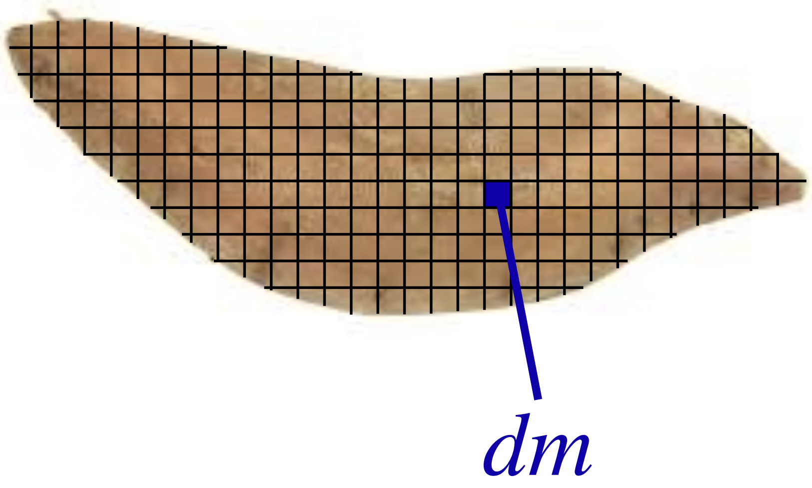
- What about for an extended object? Like this yam
- The previous formula actually still holds



$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_i \vec{r}_i \, dm_i}{\sum_i dm_i} \xrightarrow{dm_i \to 0} \frac{\int \vec{r} \, dm}{\int dm}$$

Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds



$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i}$$

- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\vec{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r}\, \underline{dm}}{M}$$

1D: $\frac{dm}{dx} = \lambda$

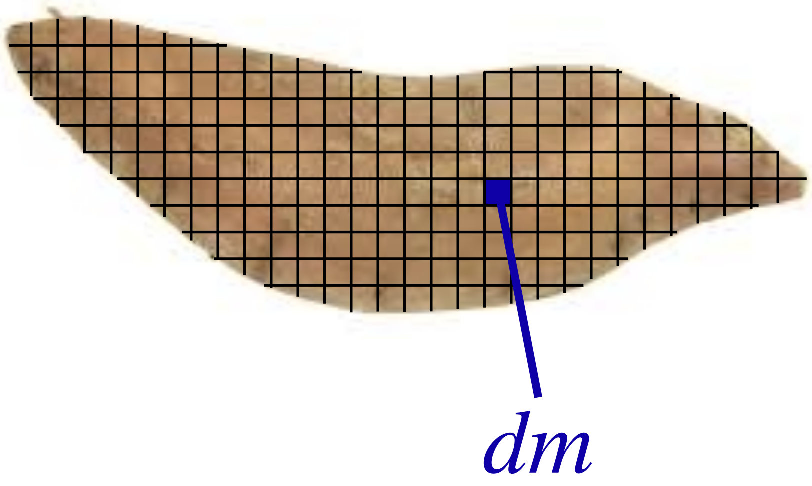
$\Rightarrow dm = \lambda dx$

2D: $dm = \sigma dA$

3D: $dm = \rho_V dV$

Center of mass for continuous systems

- What about for an extended object? Like this yam
- The previous formula actually still holds



$$\overrightarrow{R}_{CM} = \frac{\sum_{i=1}^{N} \vec{r}_i m_i}{\sum_{i=1}^{N} m_i}$$

- Imagine it is made up of differential elements, which each have a tiny mass dm

$$\overrightarrow{R}_{CM} = \frac{\int_M \vec{r} dm}{\int_M dm} = \frac{\int_M \vec{r} dm}{M}$$

- Use density to convert mass to a spatial integral

DEMO (148)

Finding the center of mass:

Hanging

Kinematics of the center of mass

- Given a system with constant masses, take the derivative of

$$\vec{V}_{CM} = \frac{d}{dt}\left[\vec{R}_{CM}\right] = \frac{d}{dt}\left[\frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}\right] \quad \text{or} \quad \vec{R}_{CM} = \frac{\int_M \vec{r}\,dm}{M}$$

$$= \frac{1}{M}\frac{d}{dt}\left[\sum_i m_i\vec{r}_i\right] = \frac{1}{M}\sum_i m_i\frac{d\vec{r}_i}{dt} = \frac{1}{M}\sum_i m_i\vec{v}_i$$

Kinematics of the center of mass

- Given a system with constant masses, take the derivative of

$$\overrightarrow{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} \quad \text{or} \quad \overrightarrow{R}_{CM} = \frac{\int_M \vec{r} dm}{M}$$

to find

$$\overrightarrow{V}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M} \quad \text{or} \quad \overrightarrow{V}_{CM} = \frac{\int_M \vec{v} dm}{M}$$

Kinematics of the center of mass

- Given a system with constant masses, take the derivative of

$$\overrightarrow{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} \quad \text{or} \quad \overrightarrow{R}_{CM} = \frac{\int_M \vec{r} dm}{M}$$

to find

$$\overrightarrow{V}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M} \quad \text{or} \quad \overrightarrow{V}_{CM} = \frac{\int_M \vec{v} dm}{M}$$

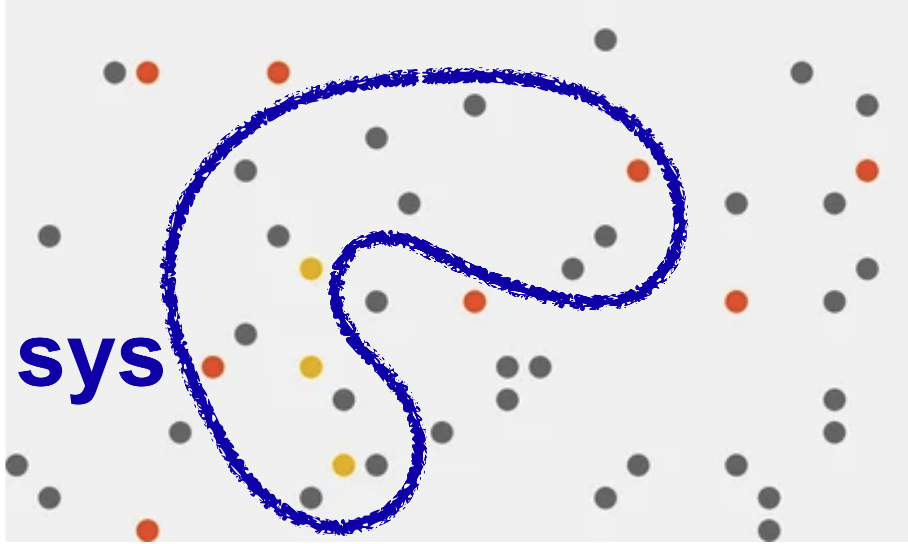
and again to find

$$\overrightarrow{A}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M} \quad \text{or} \quad \overrightarrow{A}_{CM} = \frac{\int_M \vec{a} dm}{M}$$

Forces are applied at the center of mass

- The center of mass can prove it's own usefulness

$$\vec{A}_{CM} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{M} = \frac{\sum_{i\in sys} m_i\vec{a}_i}{M}$$



$M\vec{A}_{CM} = \sum_{i\in sys} m_i\vec{a}_i$ By Newton's 2nd law

$$m_i\vec{a}_i = \sum_{j} \vec{F}_{ij} = \sum_{j\in sys} \vec{F}_{ij} + \sum_{j\notin sys} \vec{F}_{ij}$$

$$M\vec{A}_{CM} = \sum_{i\in sys}\left[\sum_{j\in sys}\vec{F}_{ij} + \sum_{j\notin sys}\vec{F}_{ij}\right] = \underbrace{\sum_{i\in sys}\sum_{\in sys}\vec{F}_{ij}}_{=0} + \sum_{i\in sys}\sum_{j\notin sys}\vec{F}_{ij}$$

(by Newton's 3rd law and $\vec{F}_{ii} = 0$ for all i)

$$\Rightarrow \boxed{M\vec{A}_{CM} = \sum_{i\in sys}\sum_{j\notin sys}\vec{F}_{ij} = \vec{F}^{ext}_{net}}$$

Forces are applied at the center of mass

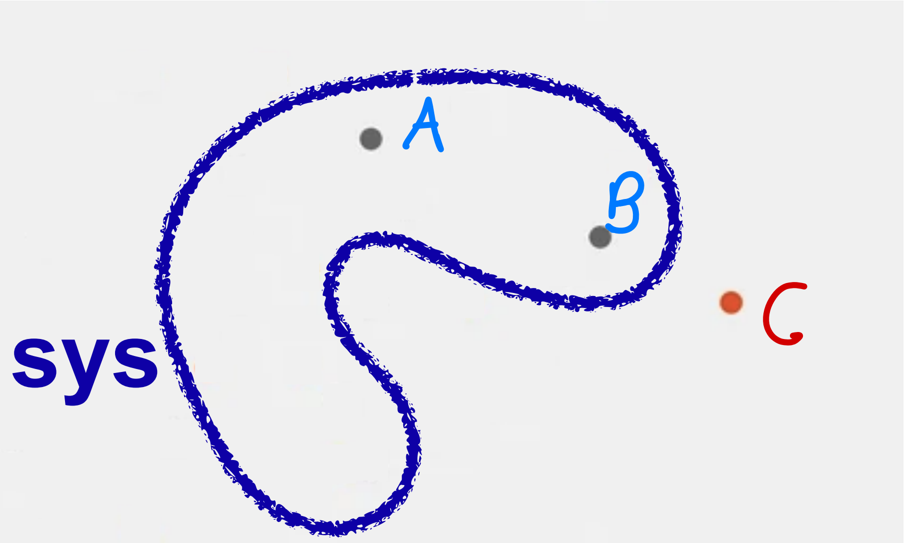
$$M\overrightarrow{A}_{CM} = \overrightarrow{F}^{ext}_{net}$$

- Thus, we can pretend the entire system is located at the center of mass

Forces are applied at the center of mass

- Separate **internal** and **external** forces

$$M\vec{A}_{CM} = \sum_{i\in sys}\left(\sum_{j\in sys}\vec{F}_{ij} + \sum_{j\notin sys}\vec{F}_{ij}\right)$$



$$= \cancelto{0}{\vec{F}_{AA}} + \underline{\vec{F}_{AB}} + \cancelto{0}{\vec{F}_{BB}} + \underbrace{\vec{F}_{BA}}_{=-\vec{F}_{AB}} + \vec{F}_{AC} + \vec{F}_{BC}$$

$$= \boxed{\vec{F}_{AC} + \vec{F}_{BC} = \vec{F}^{ext}_{net}}$$

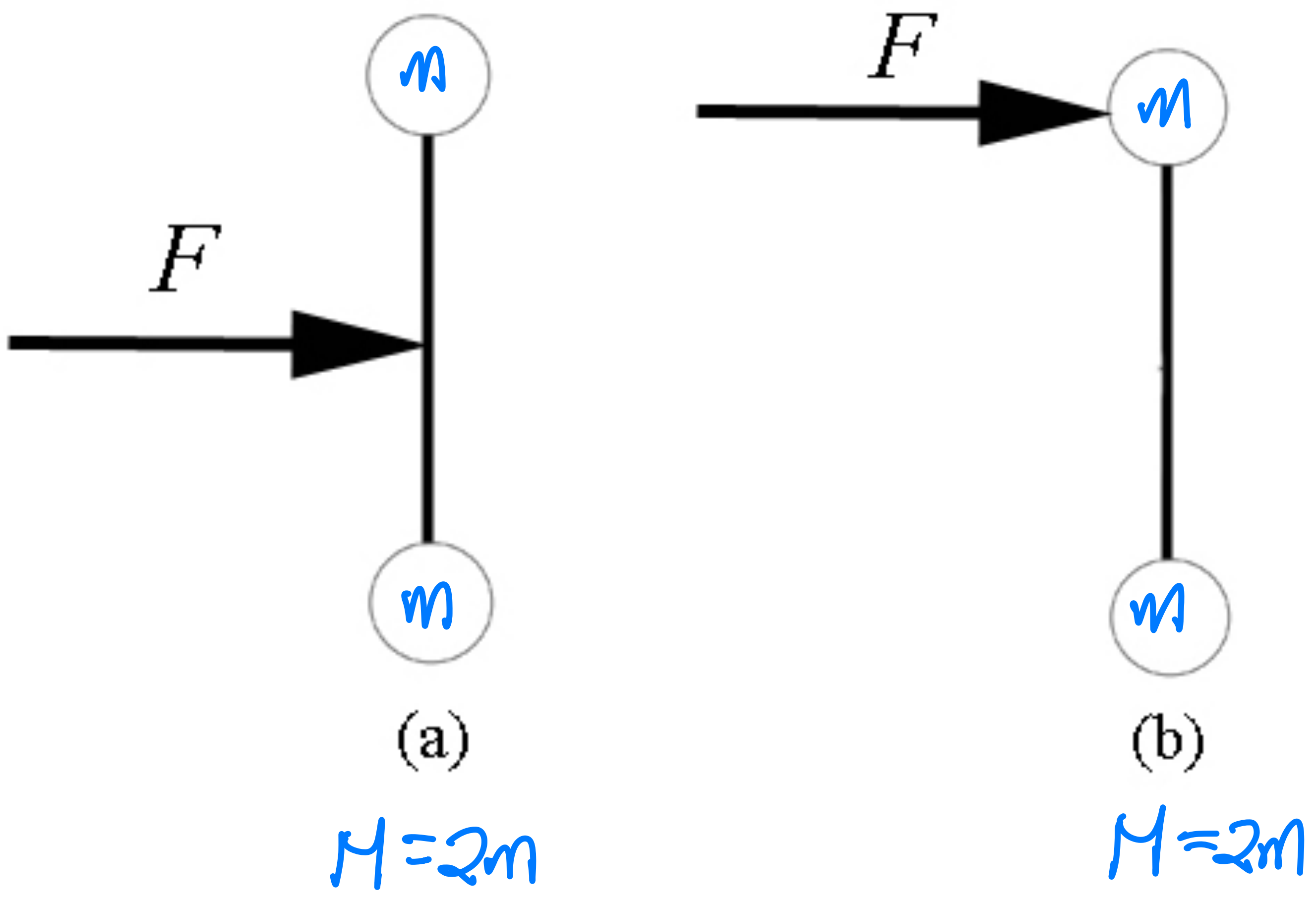
Conceptual question

A force of magnitude F is applied for a short time to a dumbbell, either as in (a) or as in (b). In which case does the dumbbell gain more **momentum**?

A. (a)
B. (b)
[✓ C. They are the same]

$$M\vec{A}_{CM} = \vec{F}^{ext}_{net} = \vec{F}$$

$$\Rightarrow \vec{A}_{CM} = \frac{1}{M}\vec{F} = \frac{1}{2m}\vec{F}$$



See you tomorrow!