General Physics: Mechanics

PHYS-101(en)

Lecture 5b: Reference frames, constraints and continuous systems

Dr. Marcelo Baquero
marcelo.baquero@epfl.ch
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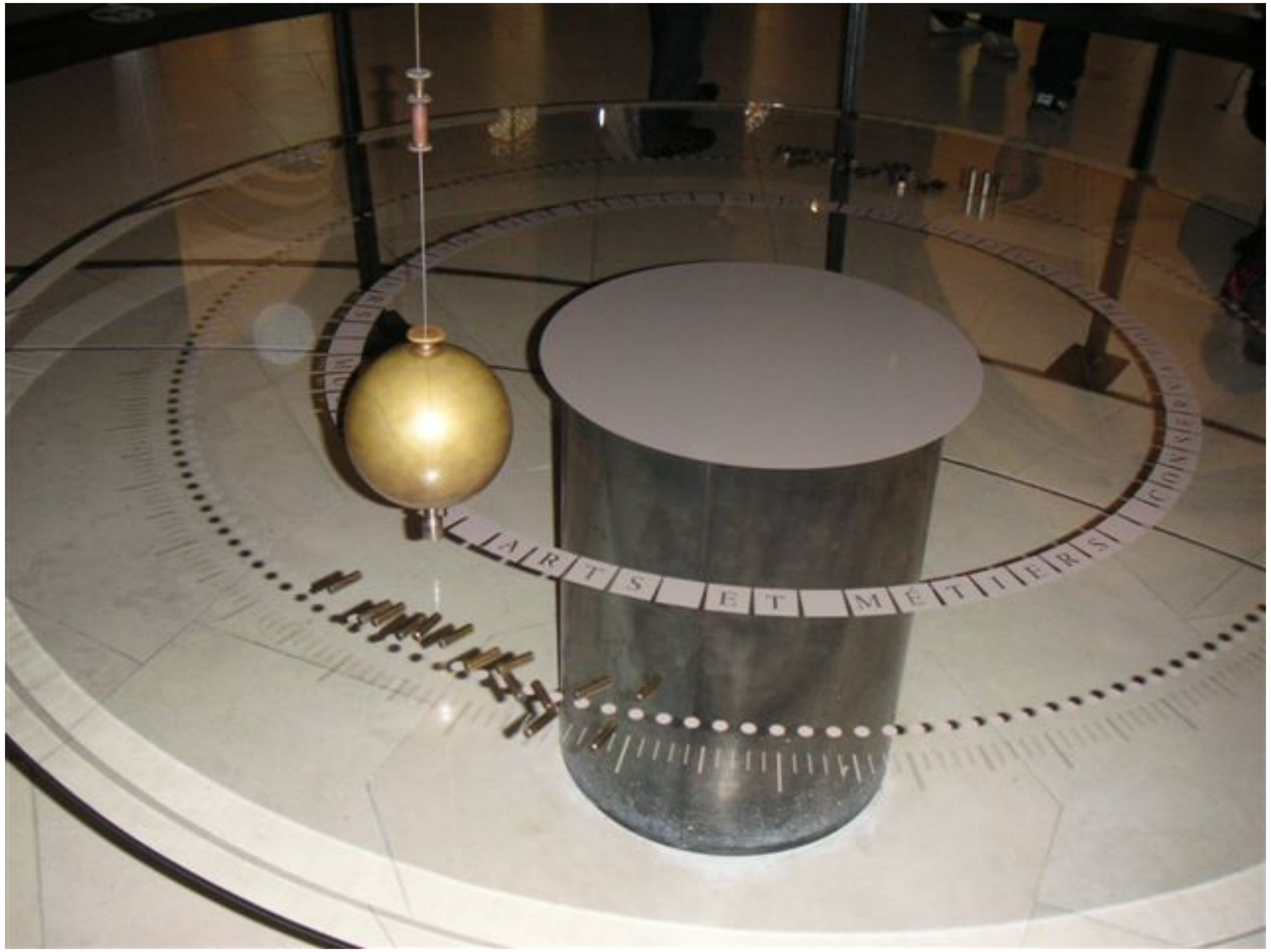

Reminder: Indicative survey

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- Open until Sunday
- Through **IS-Academia**
  - Go to "my courses"
- Opportunity for quick feedback on the course
  - Surveys are anonymous

Conceptual question

Consider two people on opposite sides of a rotating merry-go-round. One of them throws a ball directly towards the other. Consider two reference frames, (i) the frame of a person riding the merry-go-round or (ii) a person standing beside it. In which frame of reference is the horizontal path of the ball straight when viewed from above?

A. (i) only
B. (i) and (ii)
C. (ii) only
D. Neither; because it's thrown while in circular motion, the ball travels along a curved path

Reference frames: Bicycle wheel

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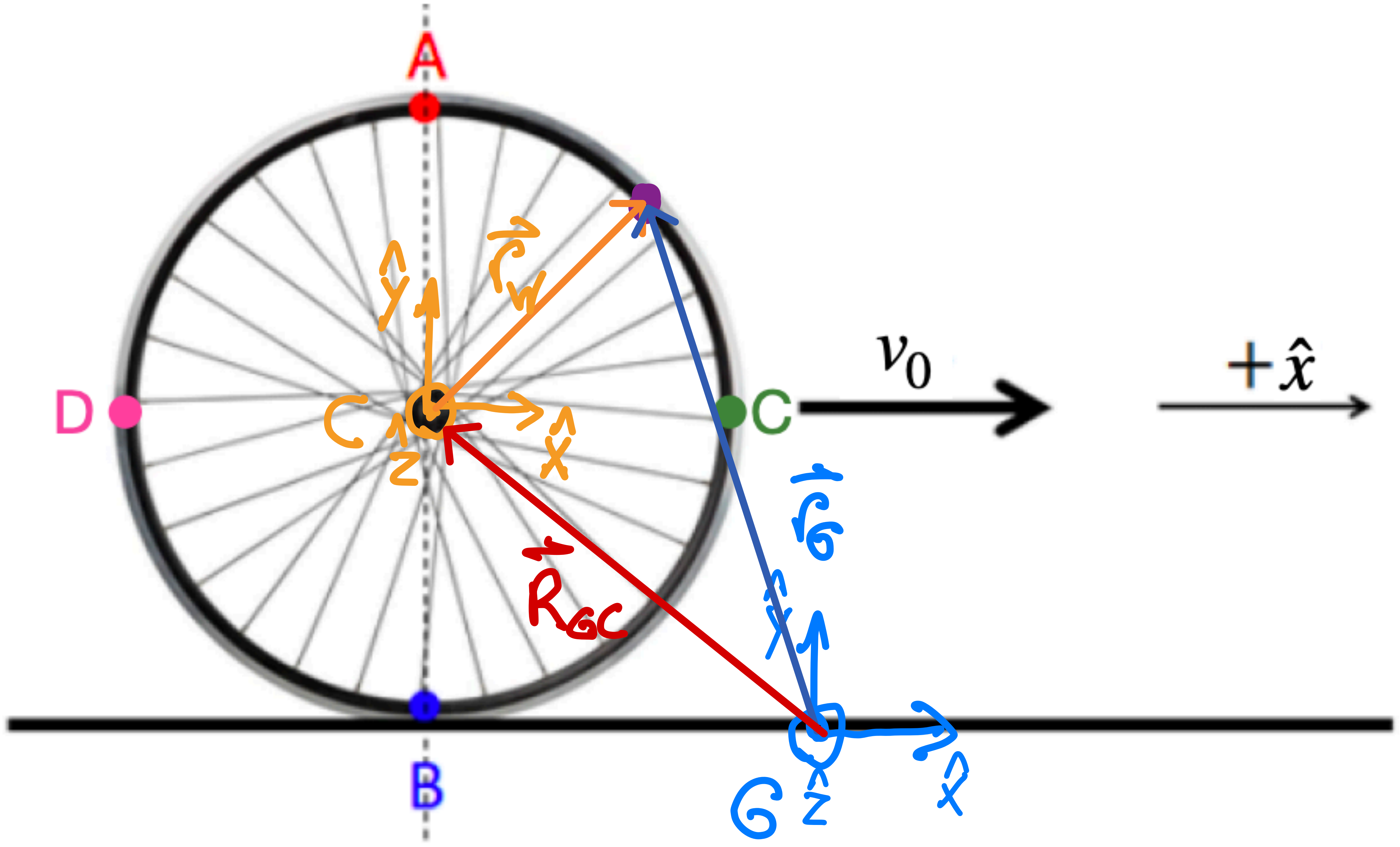
A bicycle wheel of radius R rolls without slipping along the ground. Its center moves with speed v_0. Neglect any resistance. What is the velocity of each of the four marked points on the tire shown in the figure below with respect to the center point? With respect to the ground?



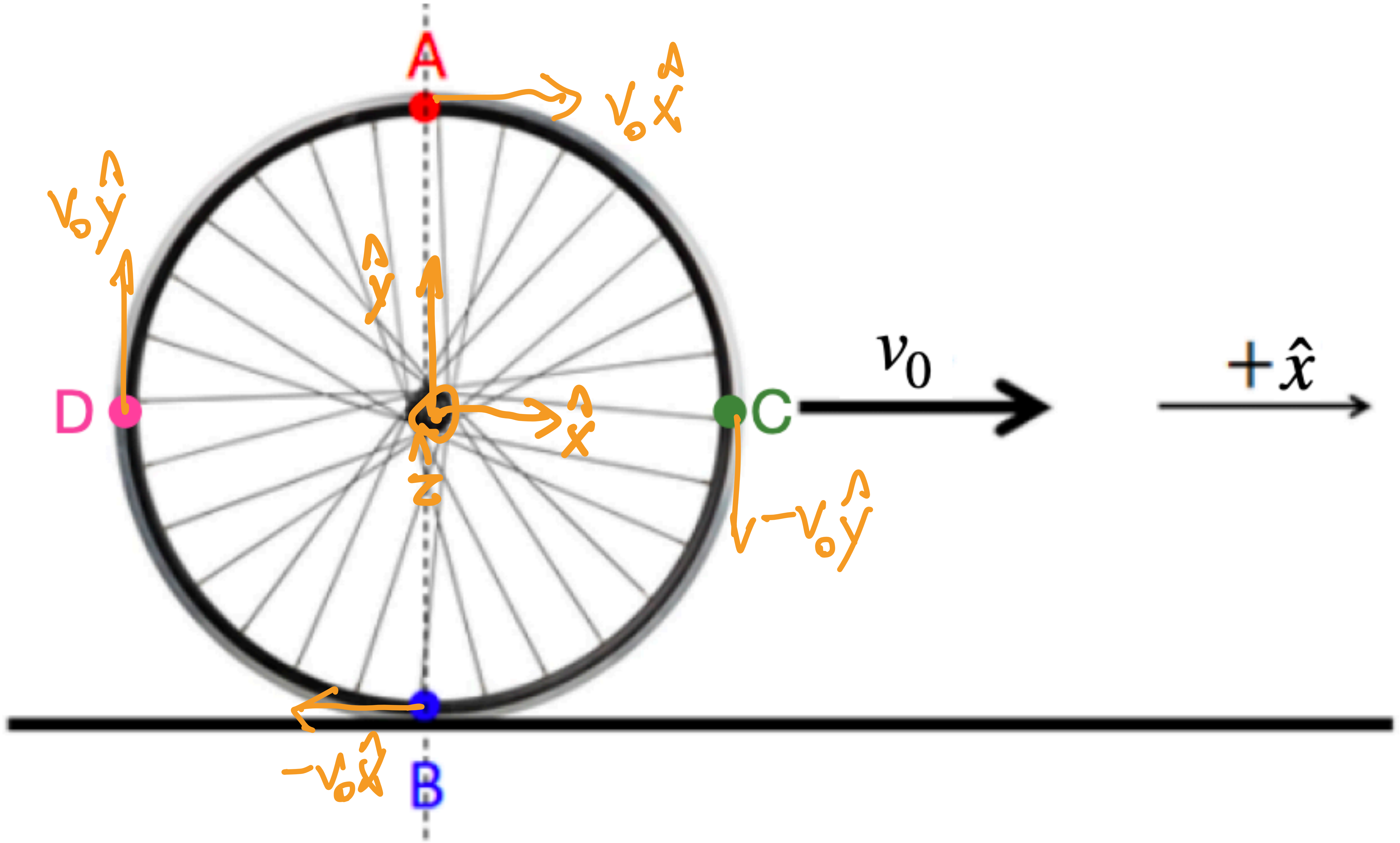
$$\vec{R}_{GC} = (x_0 + v_0 t)\hat{x} + R\hat{y}$$

$$\vec{r}_G = \vec{R}_{GC} + \vec{r}_W$$

$$\Rightarrow \vec{v}_G = \frac{d}{dt}\vec{R}_{GC} + \frac{d}{dt}\vec{r}_W = v_0\hat{x} + \vec{v}_W$$

$$\Rightarrow \vec{v}_W = \vec{v}_G - v_0\hat{x}$$

Bicycle wheel



$$\vec{V}_G = V_0\hat{x} + \vec{V}_W \quad (1)$$

$$\vec{V}_W = \vec{V}_G - V_0\hat{x} \quad (2)$$

$$\vec{V}_G^B = 0 \Rightarrow \vec{V}_W^B = 0 - V_0\hat{x} = -V_0\hat{x}$$

$$\vec{V}_G^D = V_0\hat{x} + V_0\hat{y} \Leftarrow \vec{V}_W^D = V_0\hat{y}$$

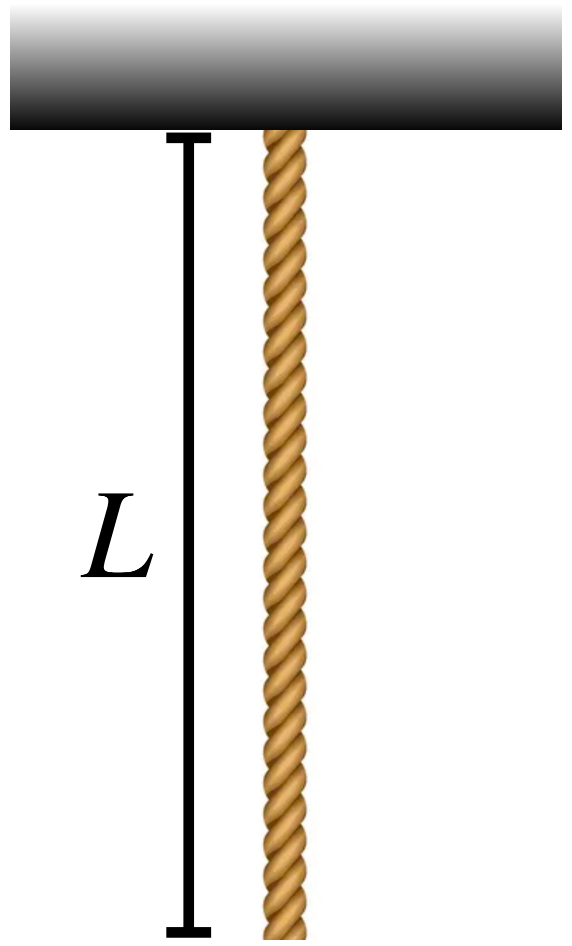
$$\vec{V}_G^C = V_0\hat{x} - V_0\hat{y} \qquad \vec{V}_W^C = -V_0\hat{y}$$

$$\vec{V}_G^A = 2V_0\hat{x} \qquad \vec{V}_W^A = V_0\hat{x}$$

Continuous systems

Example: Massive hanging rope

A uniform rope of mass M and length L is hanging from the ceiling. What is its tension (as a function of position)?



Differential elements

- Let’s go beyond the point mass and consider an object that is extended in space

Differential elements

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- Decompose the object in an enormous number of tiny bits, called *differential elements*

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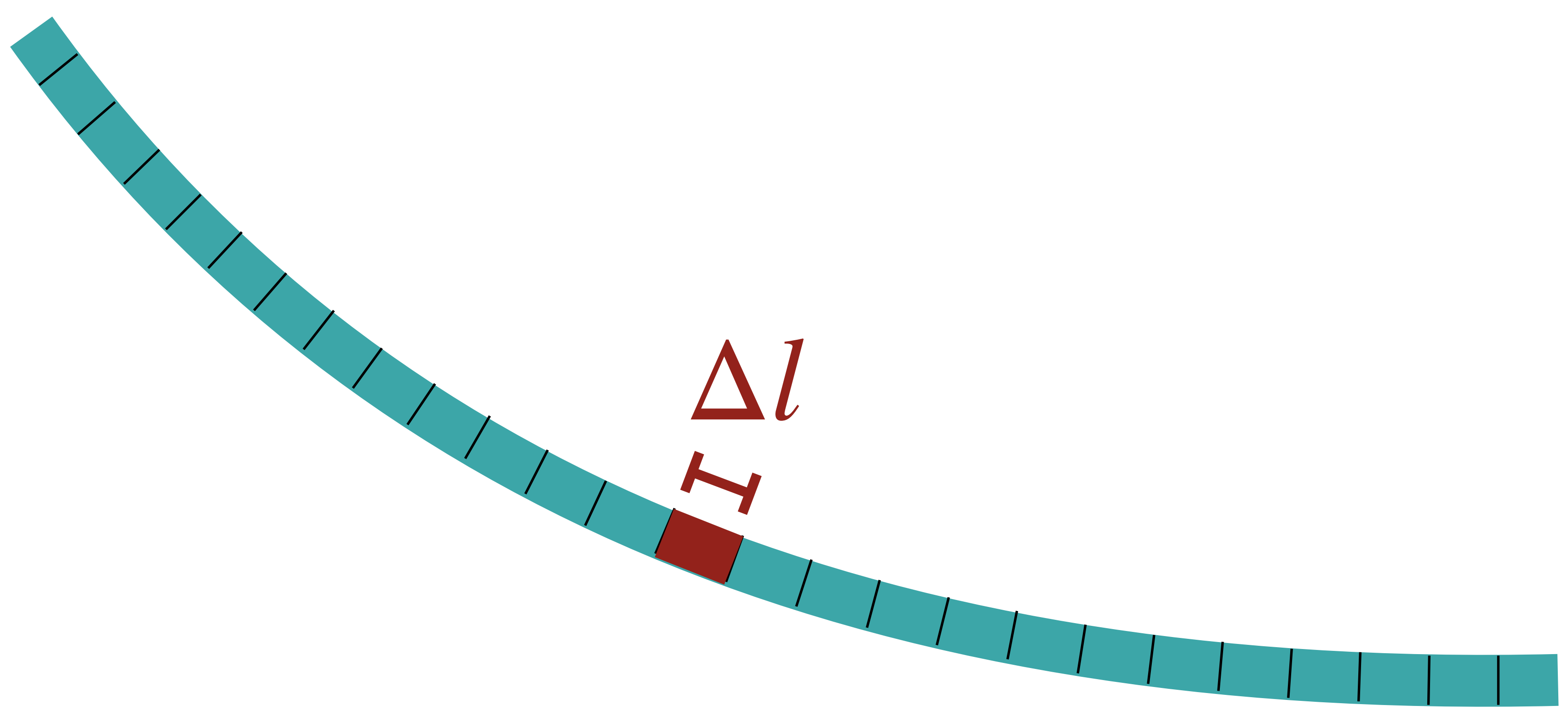

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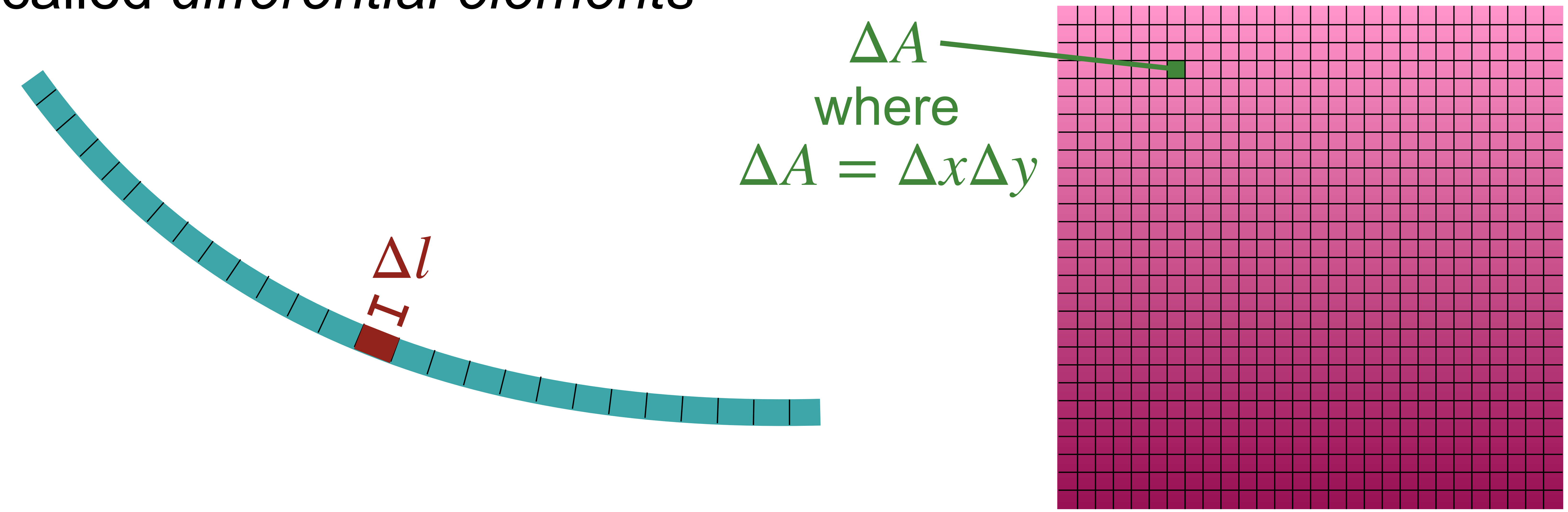

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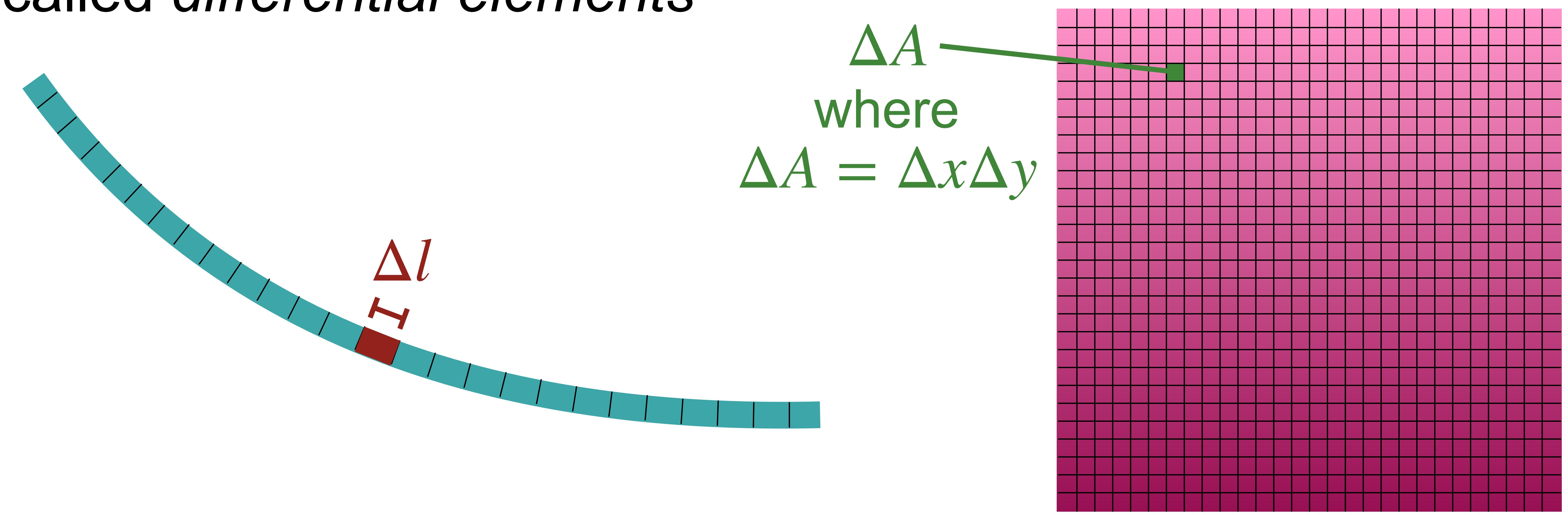


- And ΔV for the volume element of a three-dimensional object

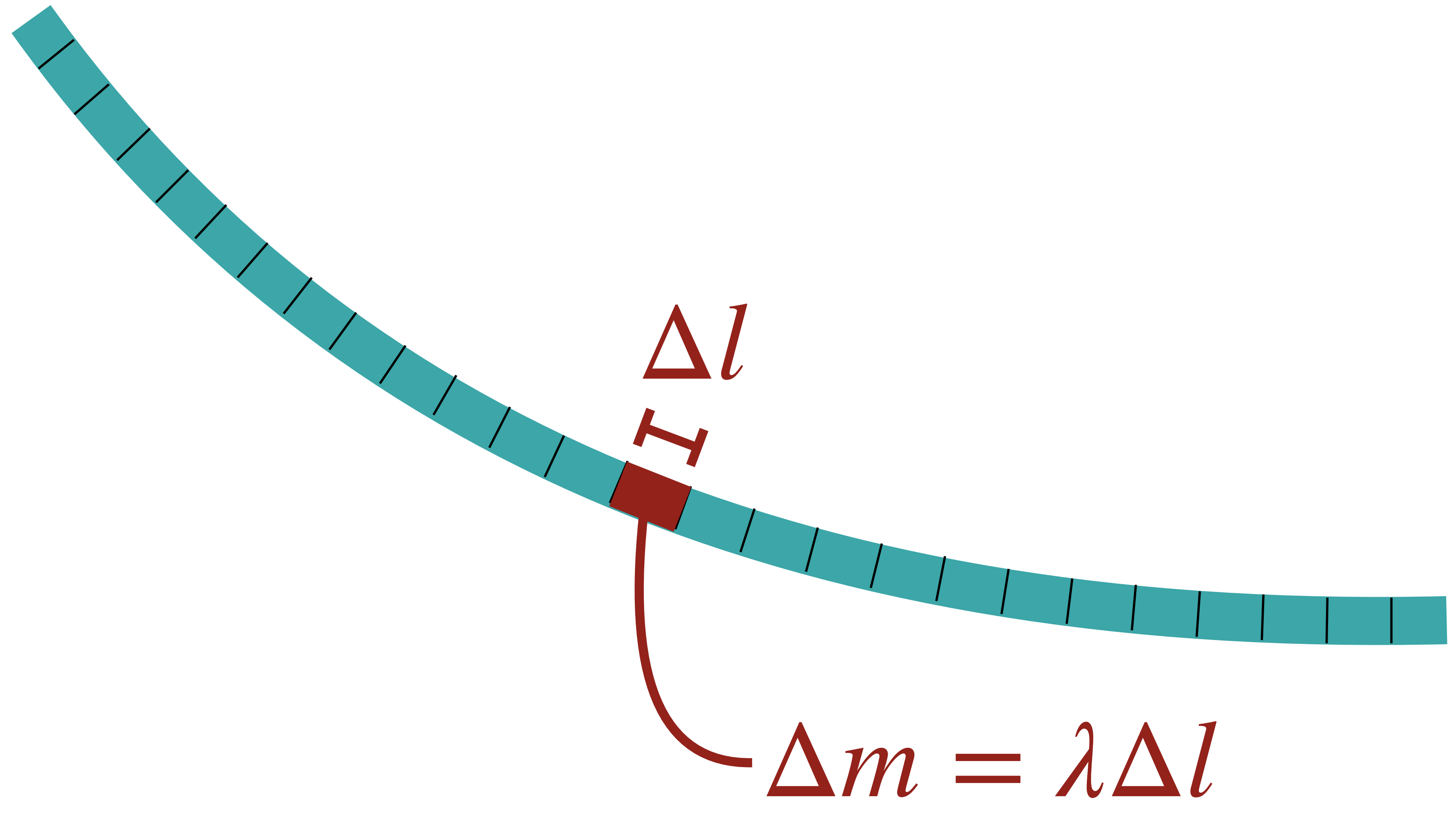
Mass density

- Amount of mass in a given space

- Allows us to calculate the amount of mass in our various differential elements

- A rope has some amount of mass per unit length
  $\lambda = \Delta m / \Delta l$ [kg/m]

- This could vary along the rope or, for a uniform rope, be
  $\lambda = M/L$ everywhere



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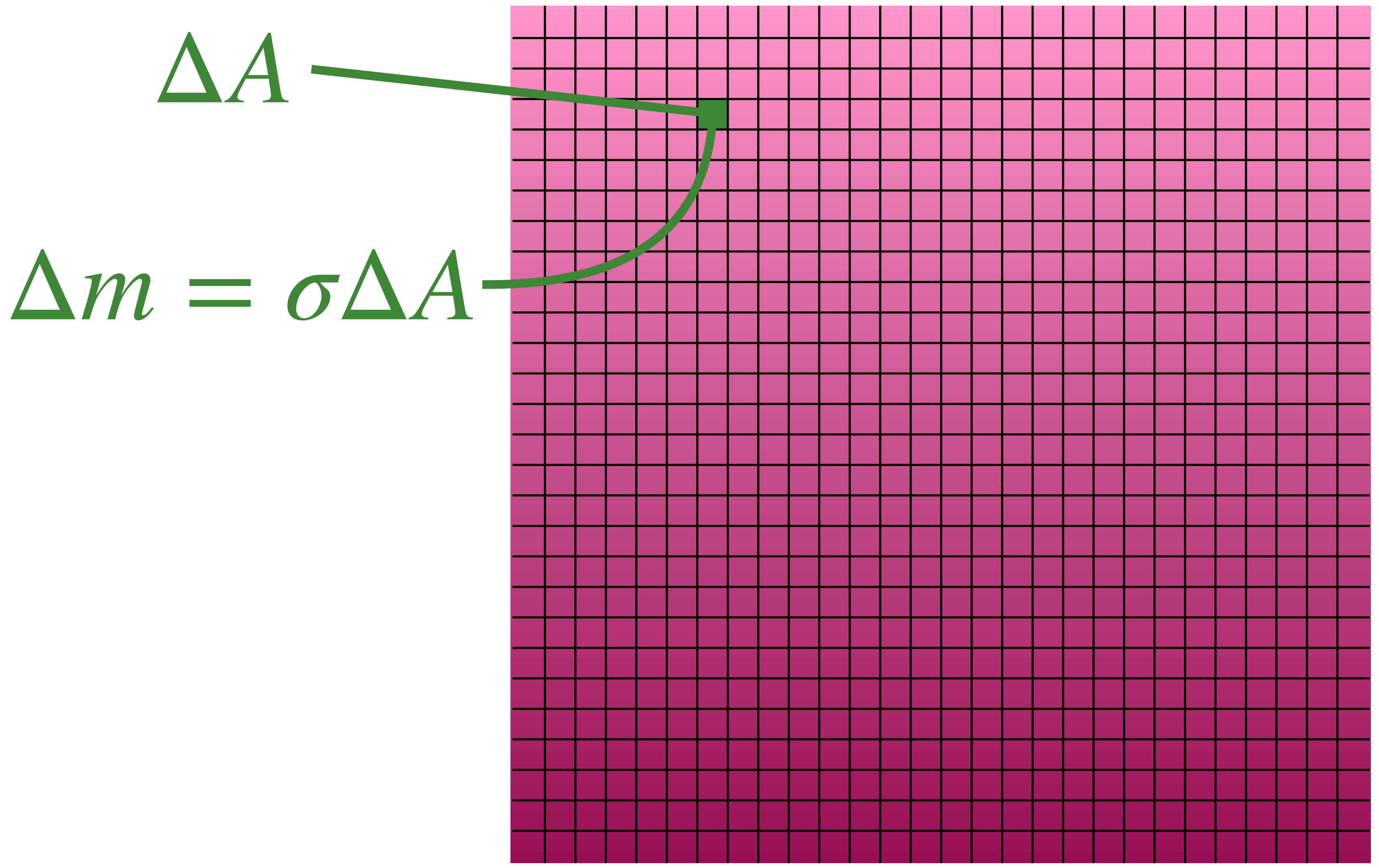
- In 2D:

$$\sigma = \Delta m / \Delta A \text{ [kg/m}^2\text{]}$$



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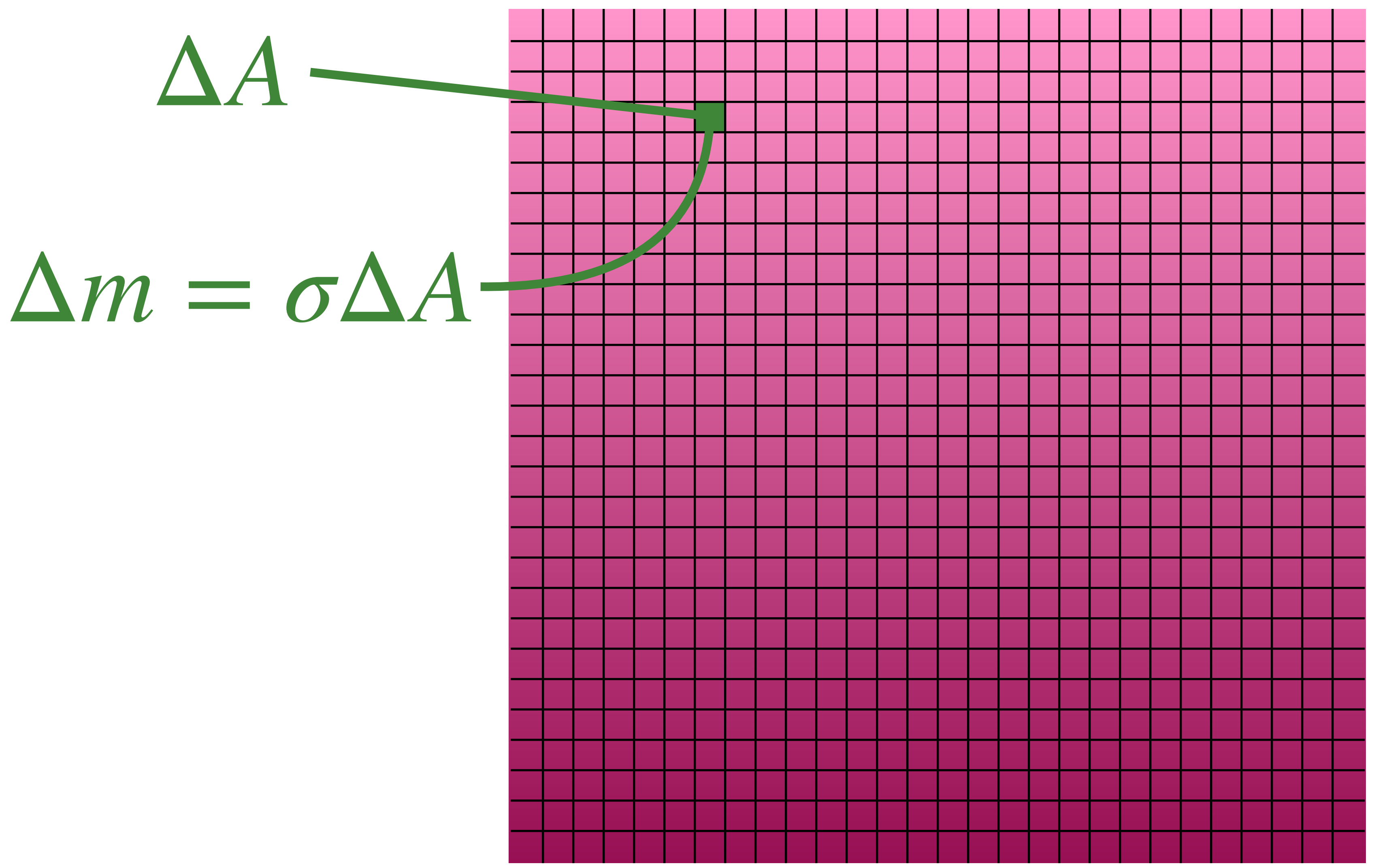
- In 3D:
  $$\rho_V = \Delta m / \Delta V \text{ [kg/m}^3\text{]}$$



Example: Massive hanging rope

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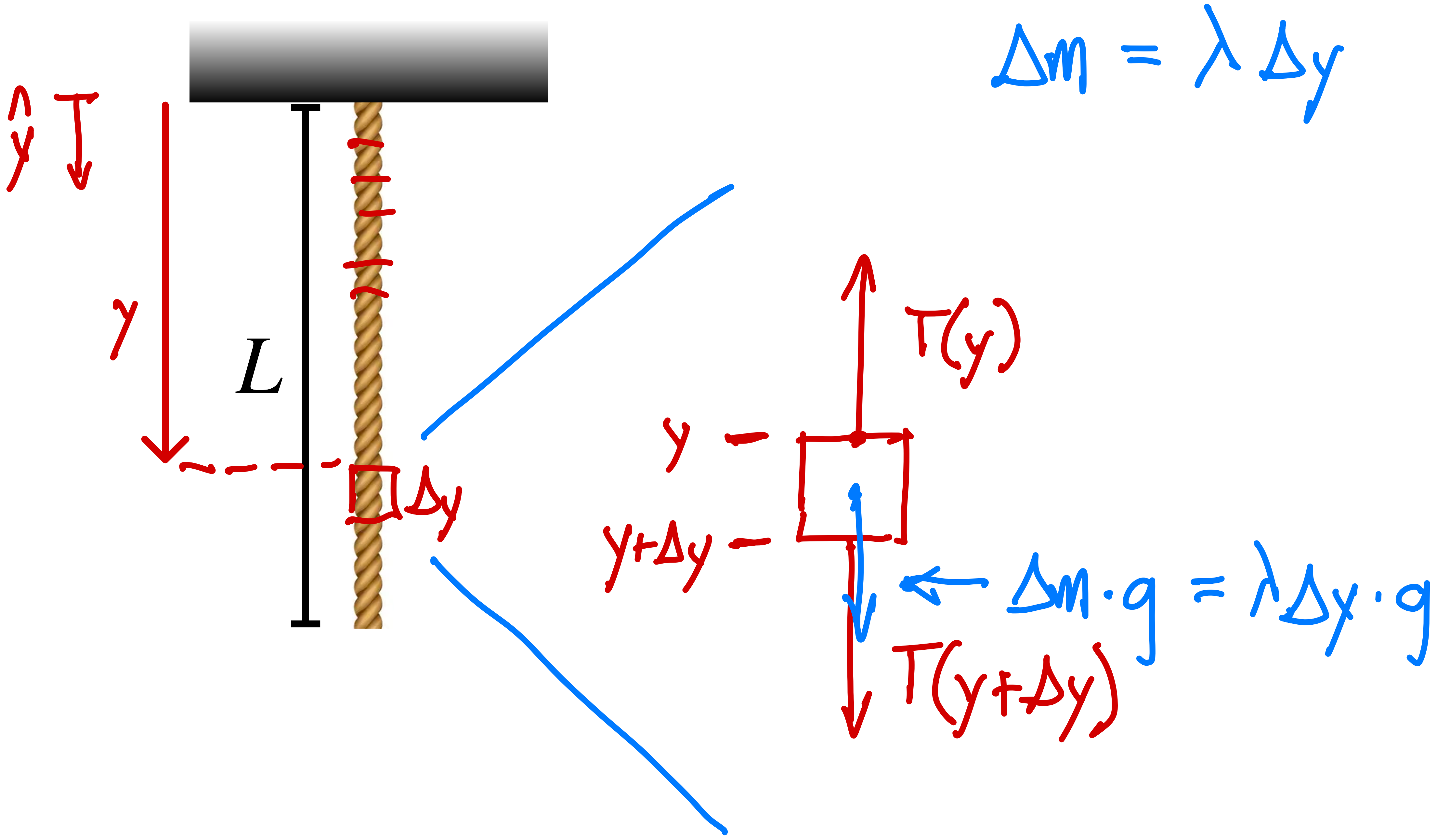
A uniform rope of mass M and length L is hanging from the ceiling. What is its tension (as a function of position)?

$$\Delta m = \lambda \Delta y \qquad \lambda = \frac{\Delta m}{\Delta y} = \frac{M}{L}$$



$$\Sigma F_y: \; T(y+\Delta y) - T(y) + \lambda \Delta y g = 0$$

$$\Rightarrow \; T(y+\Delta y) - T(y) = -\lambda g \Delta y$$

$$\Rightarrow \; \frac{T(y+\Delta y) - T(y)}{\Delta y} = -\lambda g$$

$$\lim_{\Delta y \to 0} \Rightarrow \; \frac{dT}{dy} = -\lambda g$$

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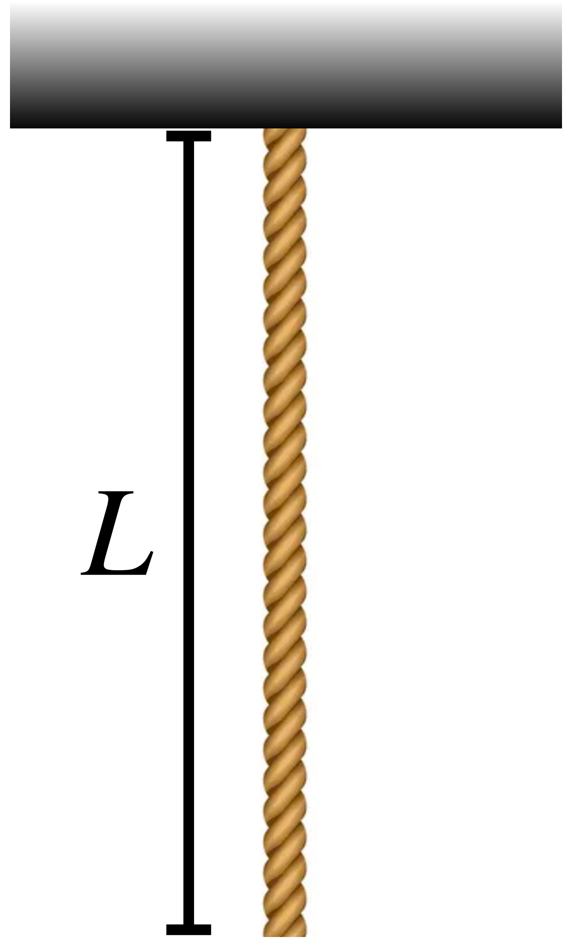


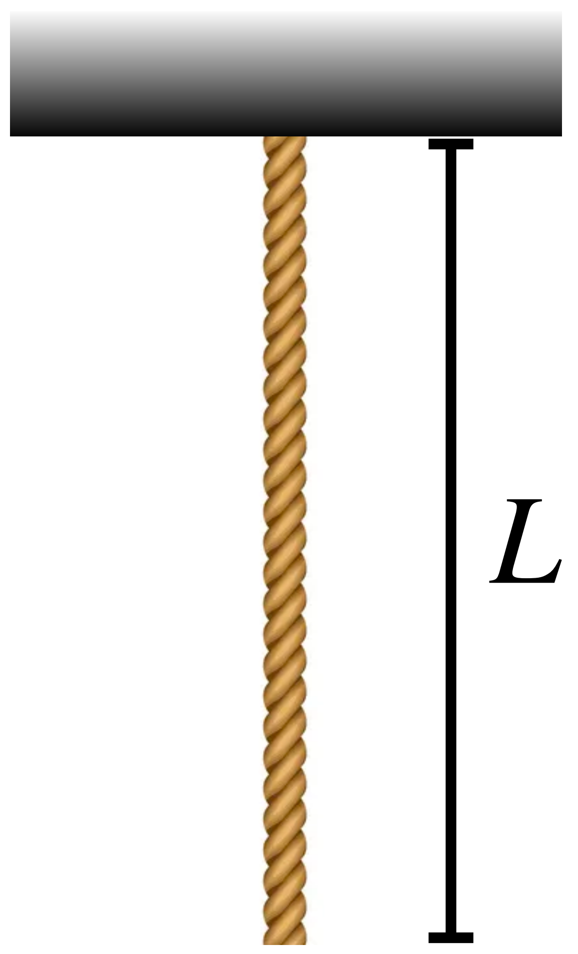
$$\frac{dT}{dy} = -\lambda g \Rightarrow \int dT = \int -\lambda g \, dy$$

$$T = -\lambda g \int dy = -\lambda g y + C$$

$$T(L) = 0 \Rightarrow 0 = -\lambda g L + C \Rightarrow C = \lambda g L$$

$$T(y) = -\lambda g y + \lambda g L = \lambda g (L - y) = Mg\left(1 - \frac{y}{L}\right), \quad \lambda = \frac{M}{L}$$

Summary



1. Chop up the system into differential elements (e.g. of size Δy in 1D), each containing a mass Δm that can be calculated through the density
2. Analyze the forces acting on the elements (probably with a free body diagram)
3. Apply Newton's 2nd law and take the limit as the differential element shrinks in size (e.g. $\Delta y \to 0$ in 1D)
4. Separate variables and integrate the differential equation
5. Apply the appropriate boundary conditions