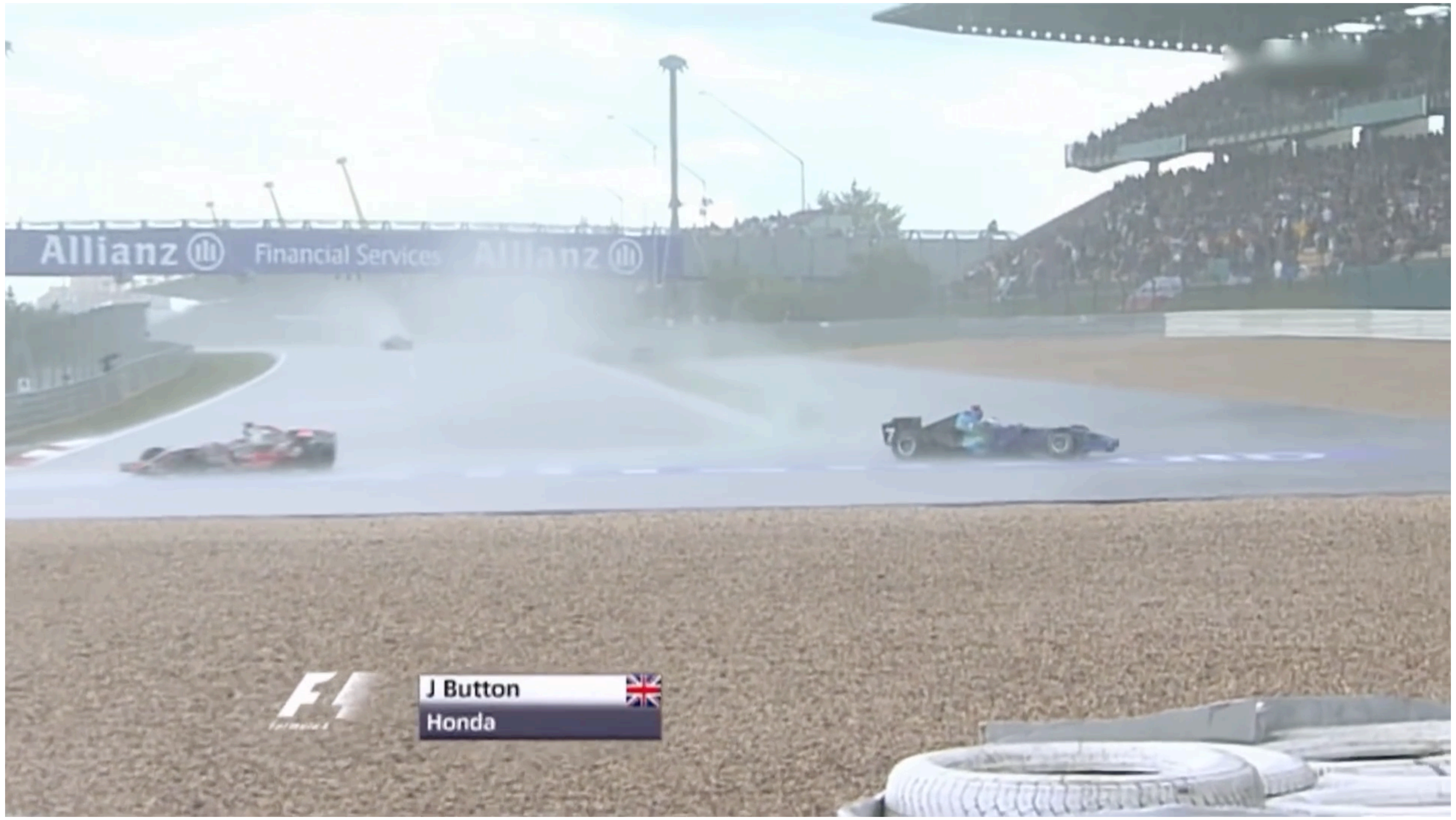
General Physics: Mechanics



PHYS-101(en)

Lecture 4b: Circular motion

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Reminder: Supplementary Q&A

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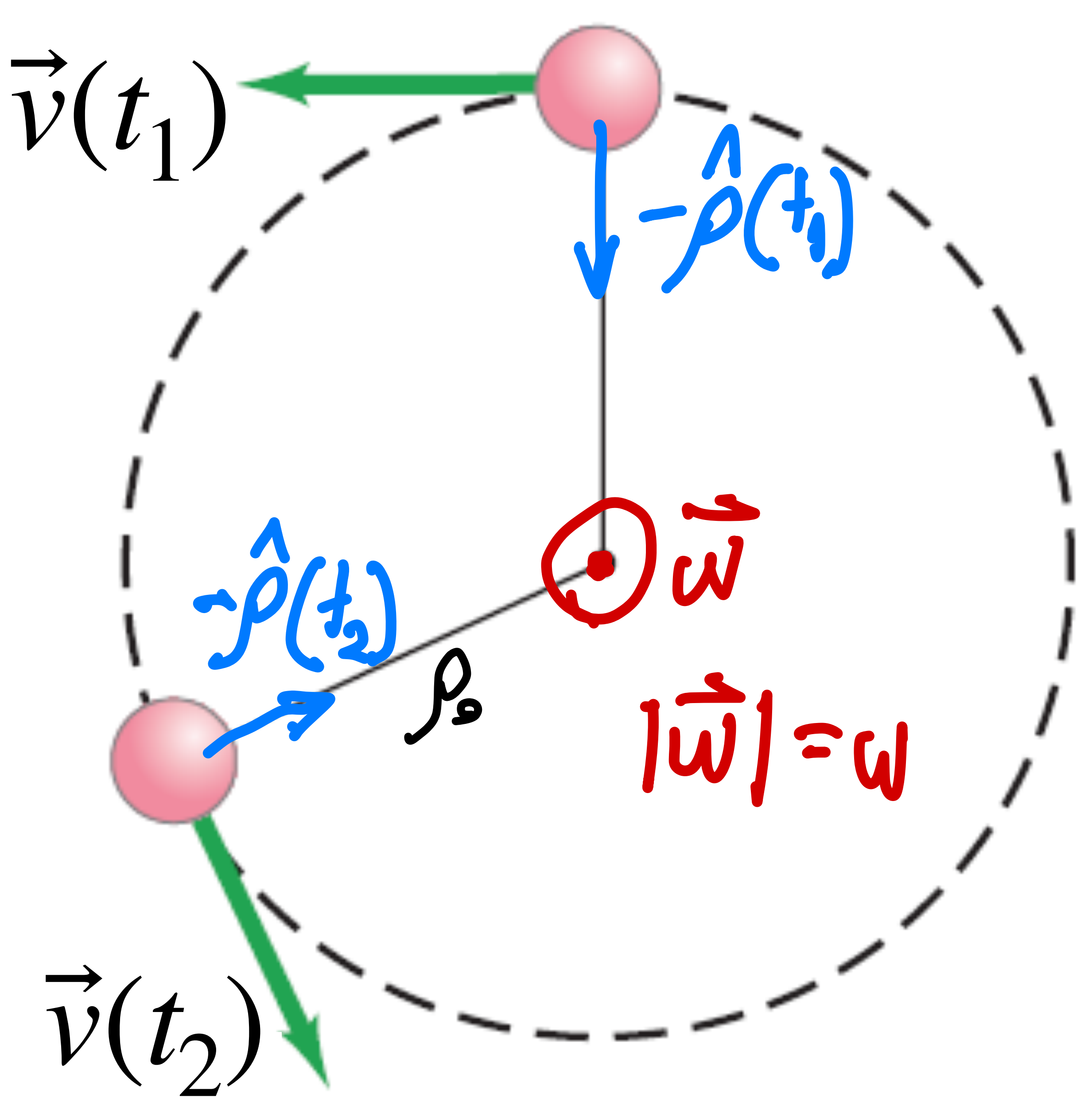
- Sessions start today and will take place
  - Tuesdays 17h30-19h in room **CE 1 101**
  - Thursdays 18h15-19h45 in room **MA A1 10**
  - Additional resource for those of you who want to further discuss current and past problem sets.

Reminder: Exercise sessions

- Classrooms in CE are crowded while the ones in BS have places available!
  - Consider moving to rooms **BS 160** or **BS 170**

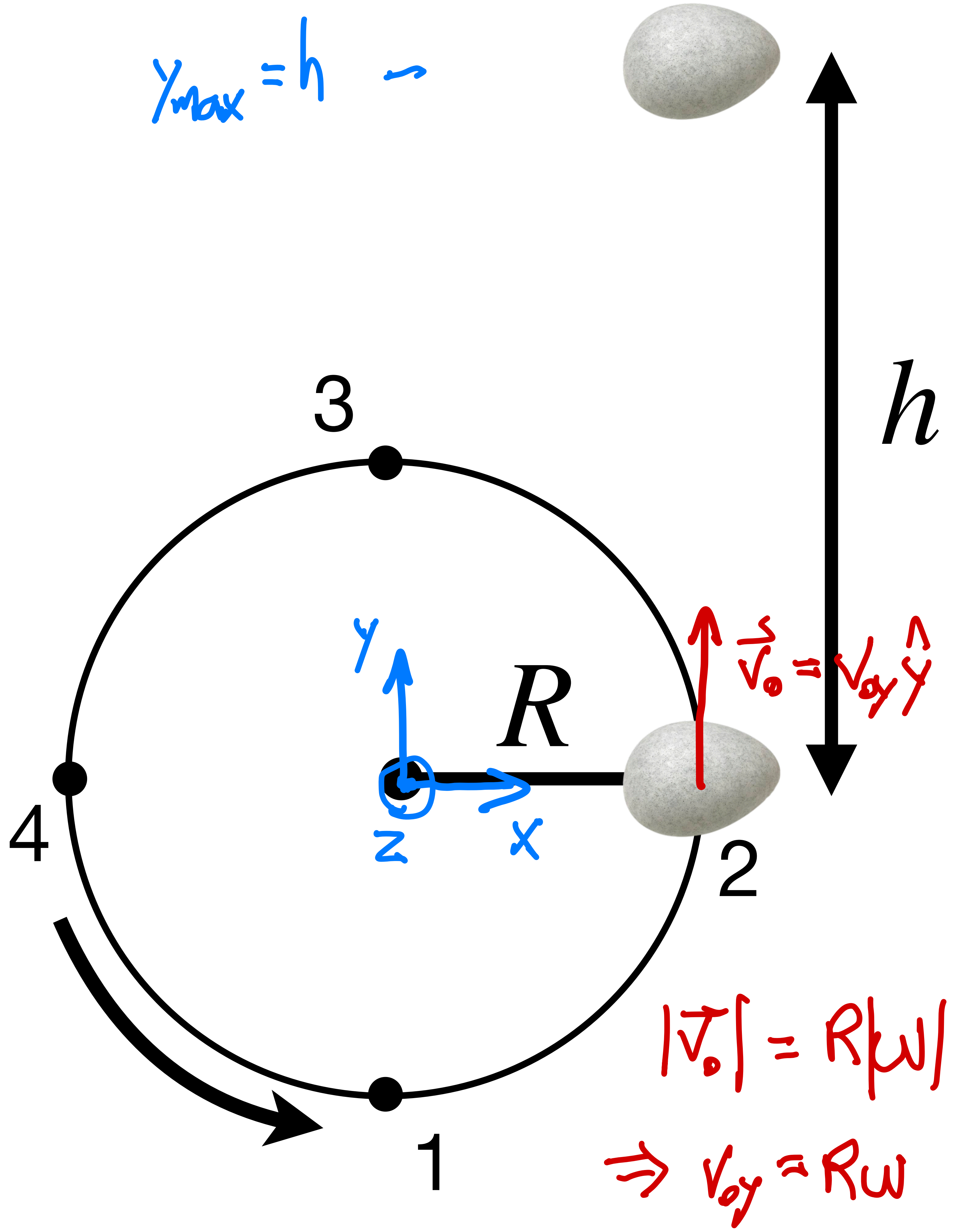
Summary: **Uniform** circular motion

- Motion in a circle of constant radius ρ_0 at constant angular velocity $\vec{\omega}$ (in rad/s)
- Instantaneous velocity is always tangent to circle and has magnitude $v = \rho_0 |\omega|$
- Acceleration points towards axis of rotation. It is called *centripetal acceleration:* $\vec{a}_{cent} = \rho_0 \omega^2 (-\hat{\rho})$
- Cylindrical coordinates are extremely convenient to describe this motion!



Example: Whirling stone

A stone is attached to a wheel and held in place by a string. It is whirled in circular trajectory of radius R in a vertical plane. Suppose the string is cut when the stone is at position 2, and the stone then rises to a maximum height h above the point at position 2. What was the angular velocity of the stone when the string was cut? Give your answer in terms of R, h, and g.



Example: Whirling stone

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The string is cut at $t=0$ $\Rightarrow$ $y(t) = \cancel{y_0}^{\,0} + v_{0y} t - \frac{1}{2} g t^2$ $\quad v_y(t) = v_{0y} - gt$

At max height: $y(t_h) = h$ $\quad v(t_h) = 0 = v_{0y} - g t_h \Rightarrow t_h = \frac{v_{0y}}{g}$

$$h = v_{0y} t_h - \frac{1}{2} g t_h^2 = v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2 = \frac{v_{0y}^2}{g} - \frac{1}{2}\frac{v_{0y}^2}{g} = \frac{1}{2}\frac{v_{0y}^2}{g}$$

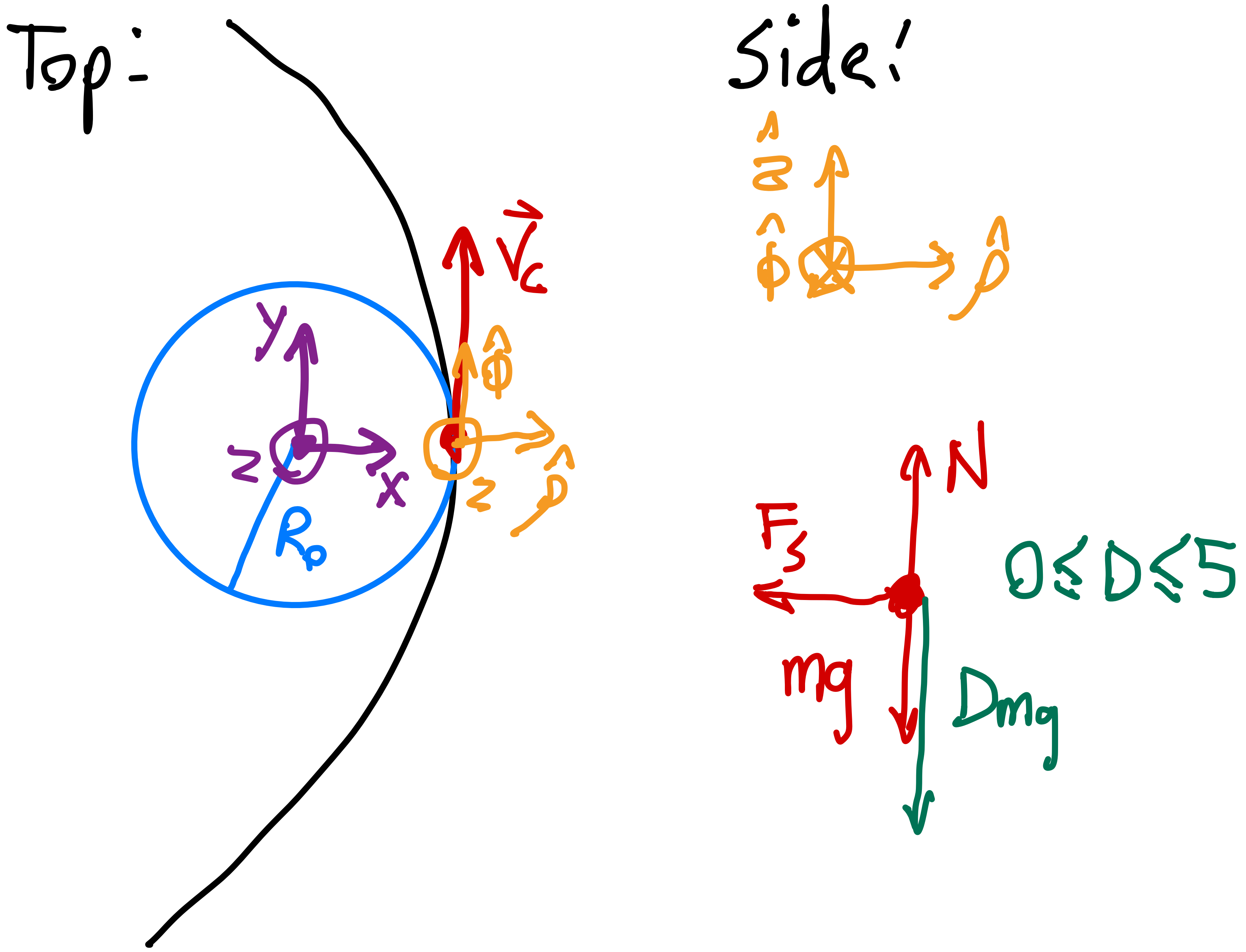
$$\Rightarrow h = \frac{1}{2}\frac{v_{0y}^2}{g}$$

From circ motion: $v_{0y} = R\omega \Rightarrow 2gh = v_{0y}^2 = R^2\omega^2 \Rightarrow \omega = \frac{\sqrt{2gh}}{R}$

$$\boxed{\vec{\omega} = \omega \hat{z} = \frac{\sqrt{2gh}}{R}\hat{z}}$$

Example: Formula 1 downforce

At top speed, an F1 car can generate as much as 5 g's of *downforce* (i.e. a downwards force 5 times its own weight). This comes from *airfoils* designed into the car, which are effectively upside down airplane wings. Imagine navigating a flat turn of radius R_0. How much faster can an F1 car *with* airfoils drive, compared to one *without*?



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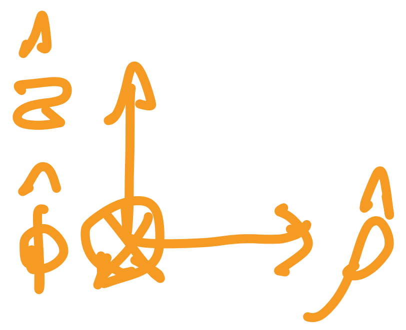
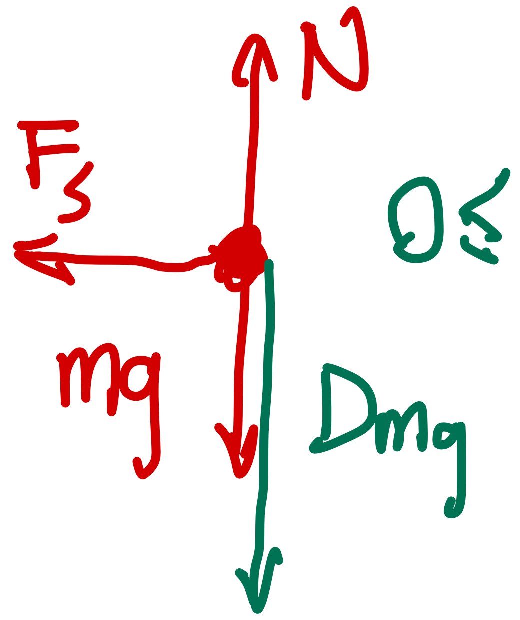
Example: Formula 1 downforce





$0 \leq D \leq 5$

$$\sum F_z: \quad N - mg - Dmg = 0 \quad \Rightarrow \quad N = mg + Dmg = (D+1)mg$$

$$\sum F_\rho: \quad -F_s = m a_{cent} = -m R_0 \omega_c^2 \quad \Rightarrow \quad F_s = m R_0 \omega_c^2$$

Now $F_s \leq \mu_s N \Rightarrow m R_0 \omega_c^2 \leq \mu_s N = \mu_s (D+1) mg$

$$\Rightarrow \omega_c^2 \leq \frac{1}{R_0} \mu_s g (D+1)$$

But $v_c = R_0 \omega_c \Rightarrow v_c^2 = R_0^2 \omega_c^2 \leq R_0^2 \cdot \frac{1}{R_0} \mu_s g (D+1) = R_0 \mu_s g (D+1)$

$$\Rightarrow \boxed{v_c \leq \sqrt{R_0 \mu_s g (D+1)}}$$

Example: Formula 1 downforce

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At top speed, an F1 car can generate as much as 5 g's of *downforce* (i.e. a downwards force 5 times its own weight). This comes from *airfoils* designed into the car, which are effectively upside down airplane wings. Imagine negotiating a flat turn of radius R_0. How much faster can an F1 car *with* airfoils drive, compared to one *without*?

$$v_c \leq \sqrt{R_0 \mu_s g (D+1)}$$

$$\left.\begin{aligned} v_D^{max} &= \sqrt{\mu_s g R_0 (D+1)} \\ v_N^{max} &= \sqrt{\mu_s g R_0} \end{aligned}\right\} \quad \frac{v_D^{max}}{v_N^{max}} = \frac{\sqrt{\mu_s g R_0 (D+1)}}{\sqrt{\mu_s g R_0}} = \sqrt{D+1}$$

If $D=5$ then $\frac{v_D^{max}}{v_N^{max}} = \sqrt{6} \approx 2.4$

If $\mu_s = 0.9$, $g = 10\ \frac{m}{s^2}$, $R_0 = 100\,m$ $\Rightarrow$ $v_N^{max} \approx 110\ \frac{km}{h}$ $\quad v_D^{max} \approx 260\ \frac{km}{h}$