General Physics: Mechanics

PHYS-101(en)

Lecture 2a: Kinematics, Newton's laws of motion

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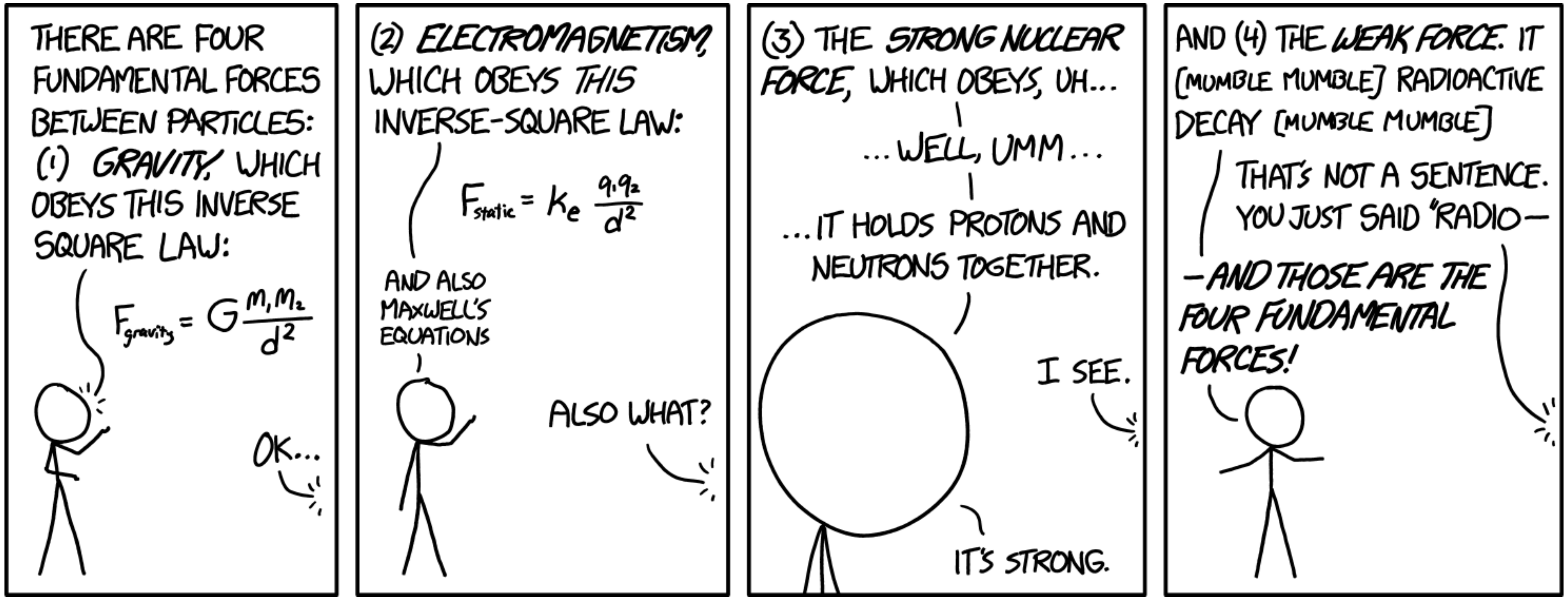
September 15th, 2025



xkcd.com/

Today’s agenda (Serway 3-4, MIT 5)

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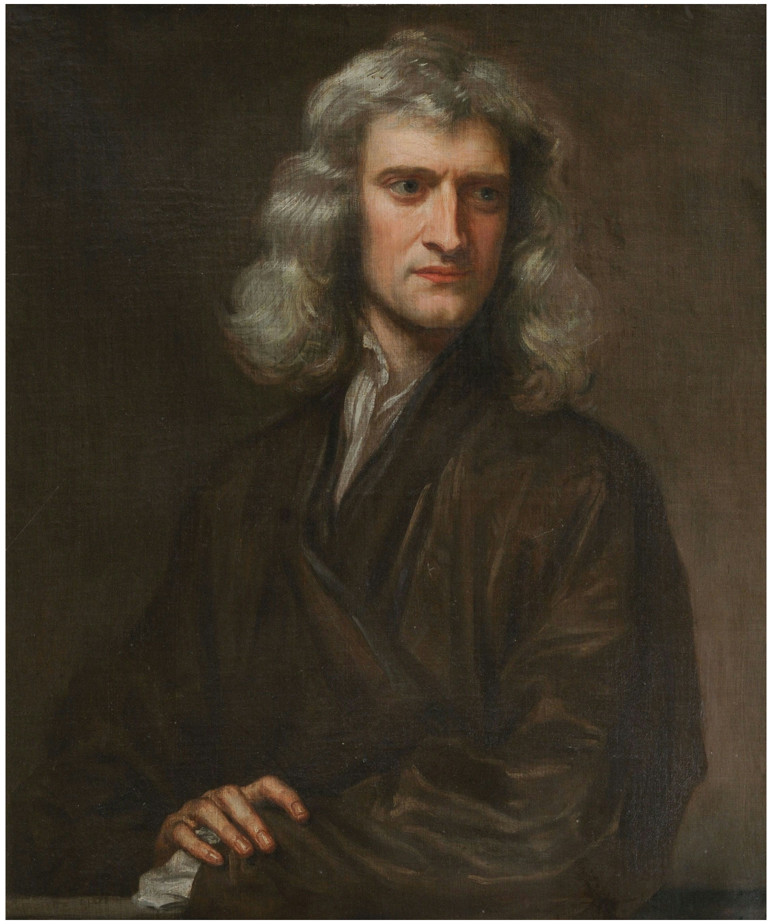
1. Kinematics
   - Position, velocity and acceleration in 3D
   - Projectile motion

Today's agenda (Serway 5, MIT 7-8)

2. Newton's laws of motion:
    1. Newton's 1st law of motion
    2. Newton's 2nd law of motion
    3. Newton's 3rd law of motion



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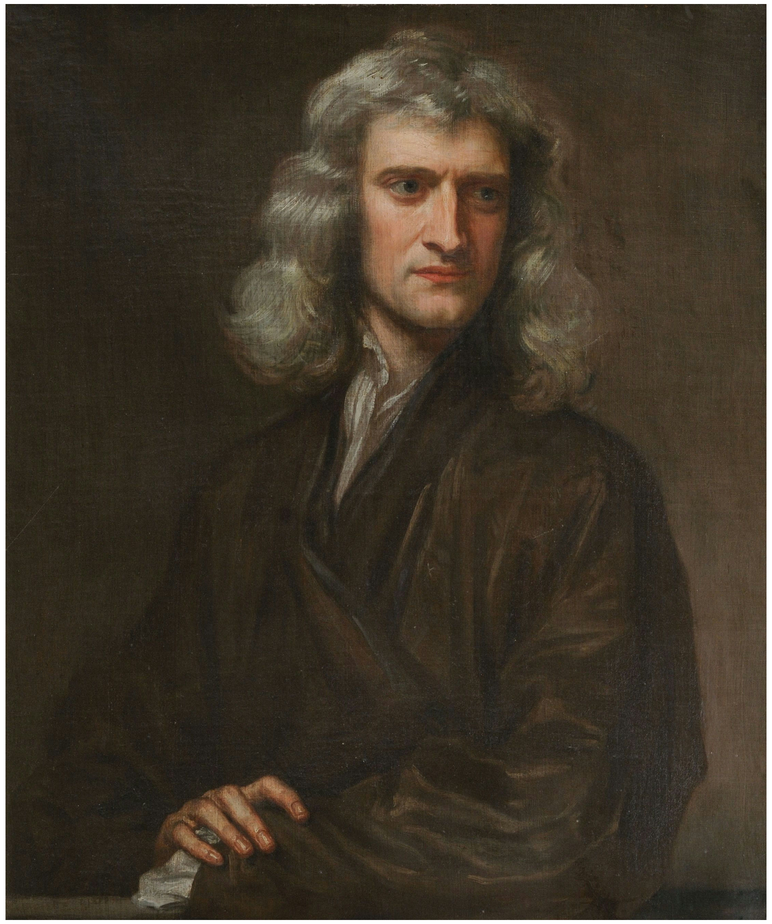

- And along the way we’ll conceptualize
    - Force
    - Mass
    - Frames of reference
    - Free body diagrams

Today's agenda (Serway 5, MIT 7-8)

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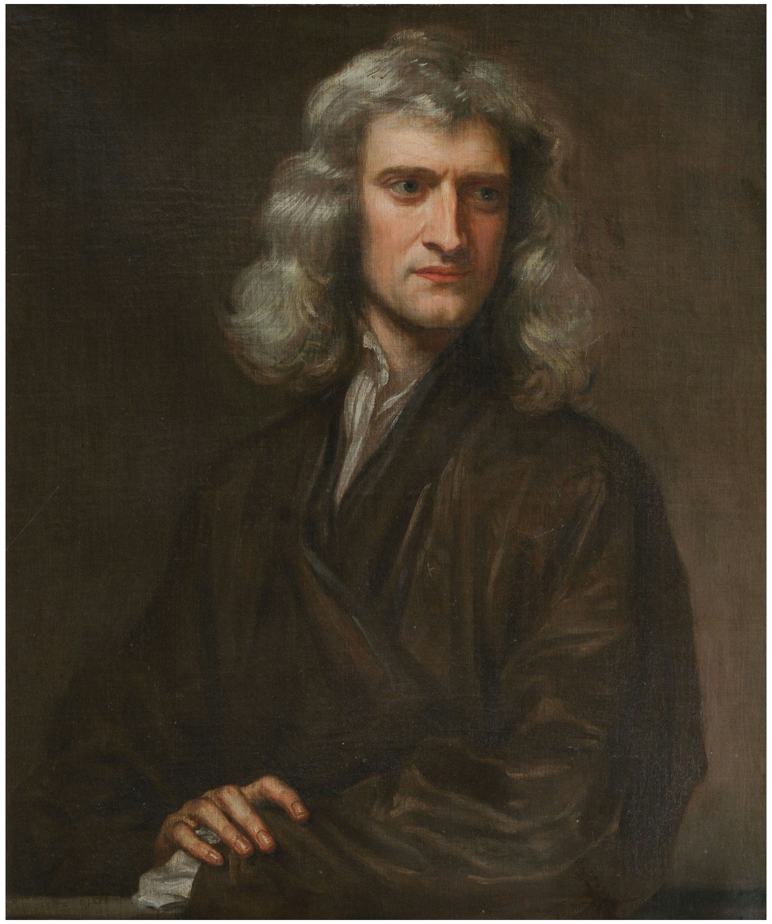

- And along the way we'll conceptualize
    - Force
    - Mass
    - Frames of reference
    - Free body diagrams
    - Friction
    - Springs

Reminder

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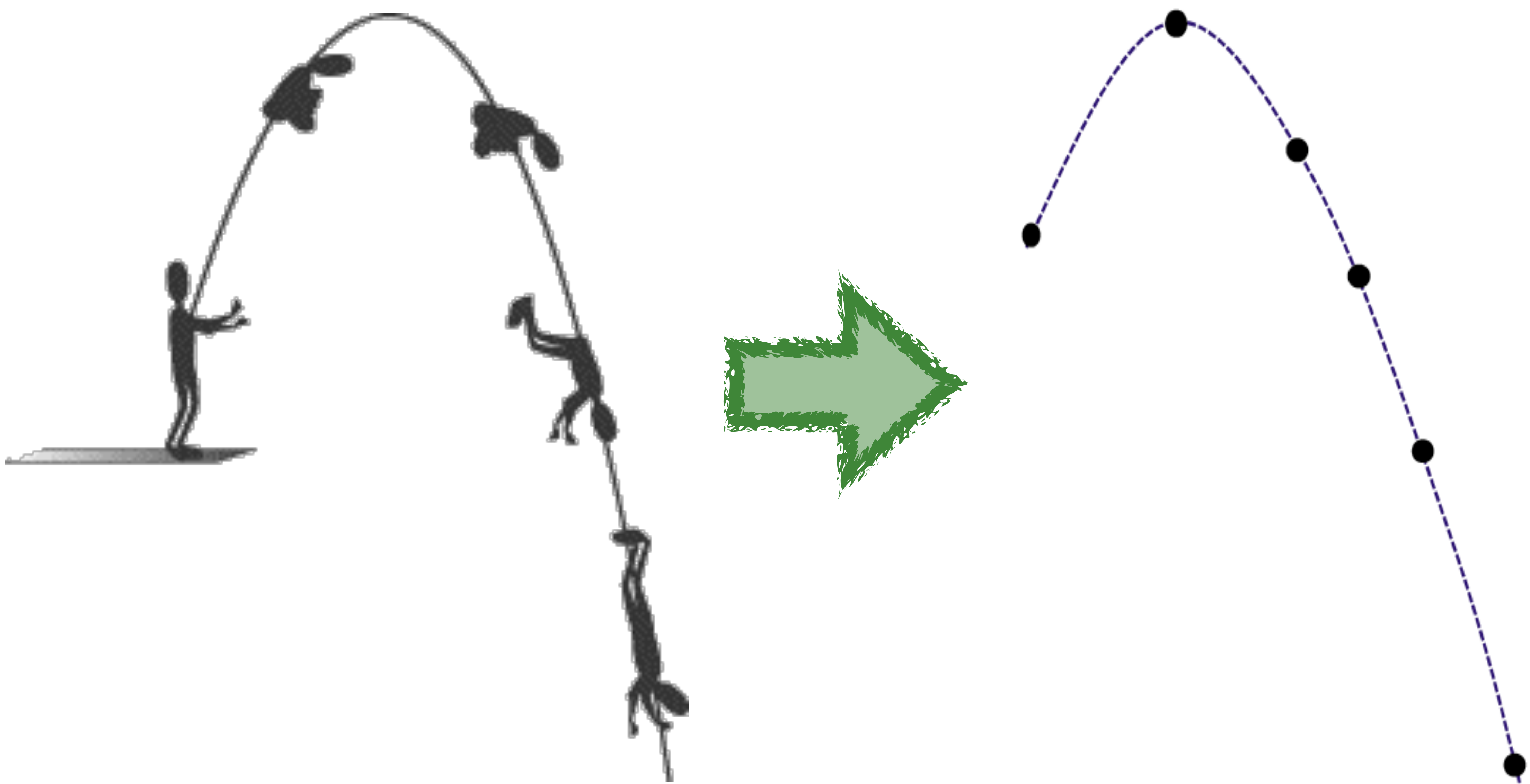
- Wednesday exercise sessions [DO]
  - Wednesdays from 17h15 to 19h
  - One teaching assistant per ~10 students
  - **Please sign up for a tutoring group on Moodle**
  - Several spots still available in rooms BS 160 and BS 170.

Kinematics

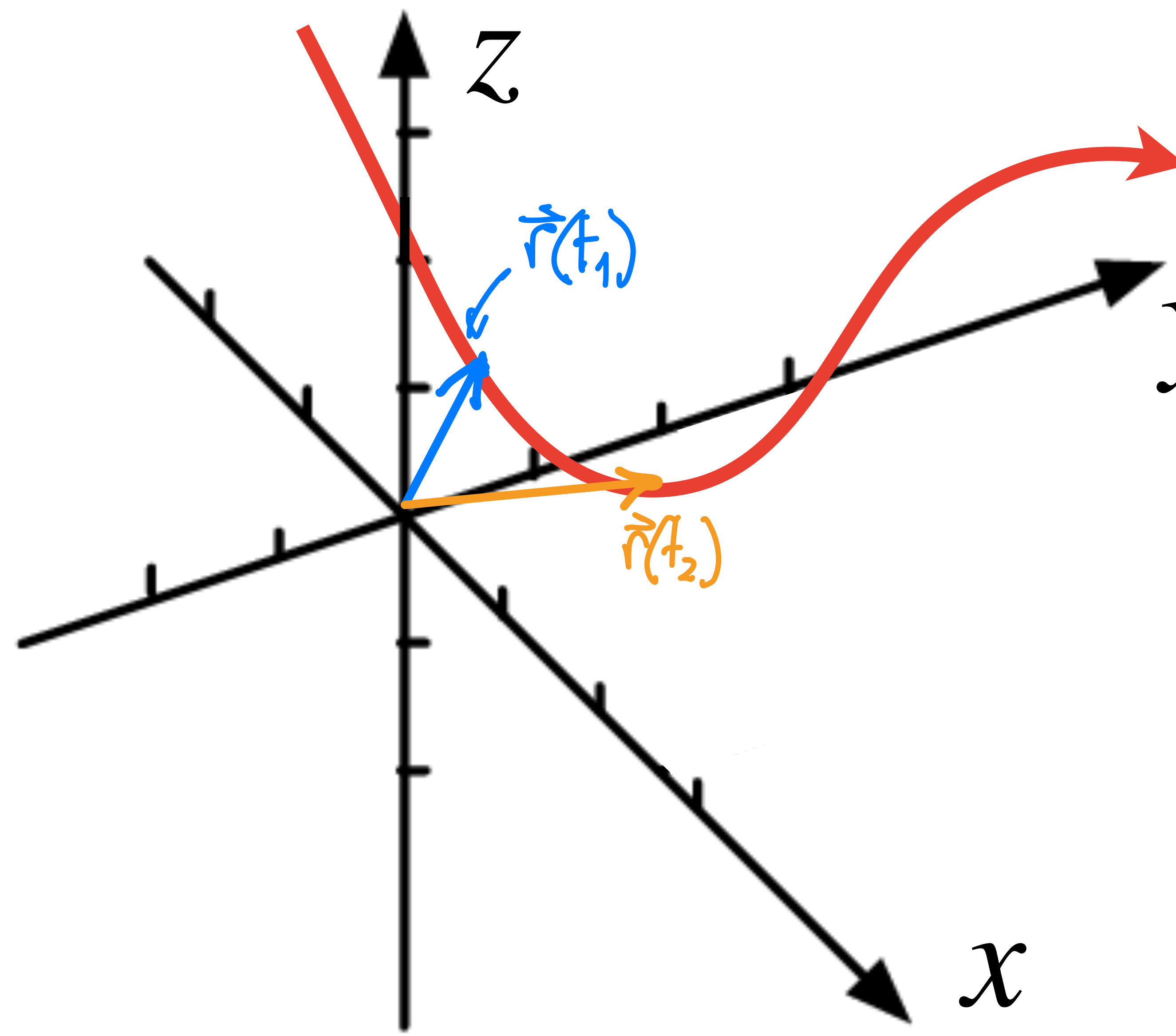
- A description of motion without considering forces
- We will approximate objects as point masses
- Need to go beyond one dimension



Vector position in Cartesian coordinates

- Position in 1D: $x(t)$

$\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z}$



Vector displacement (Cartesian)

- Displacement in 1D: $\Delta x = x(t_2) - x(t_1)$

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

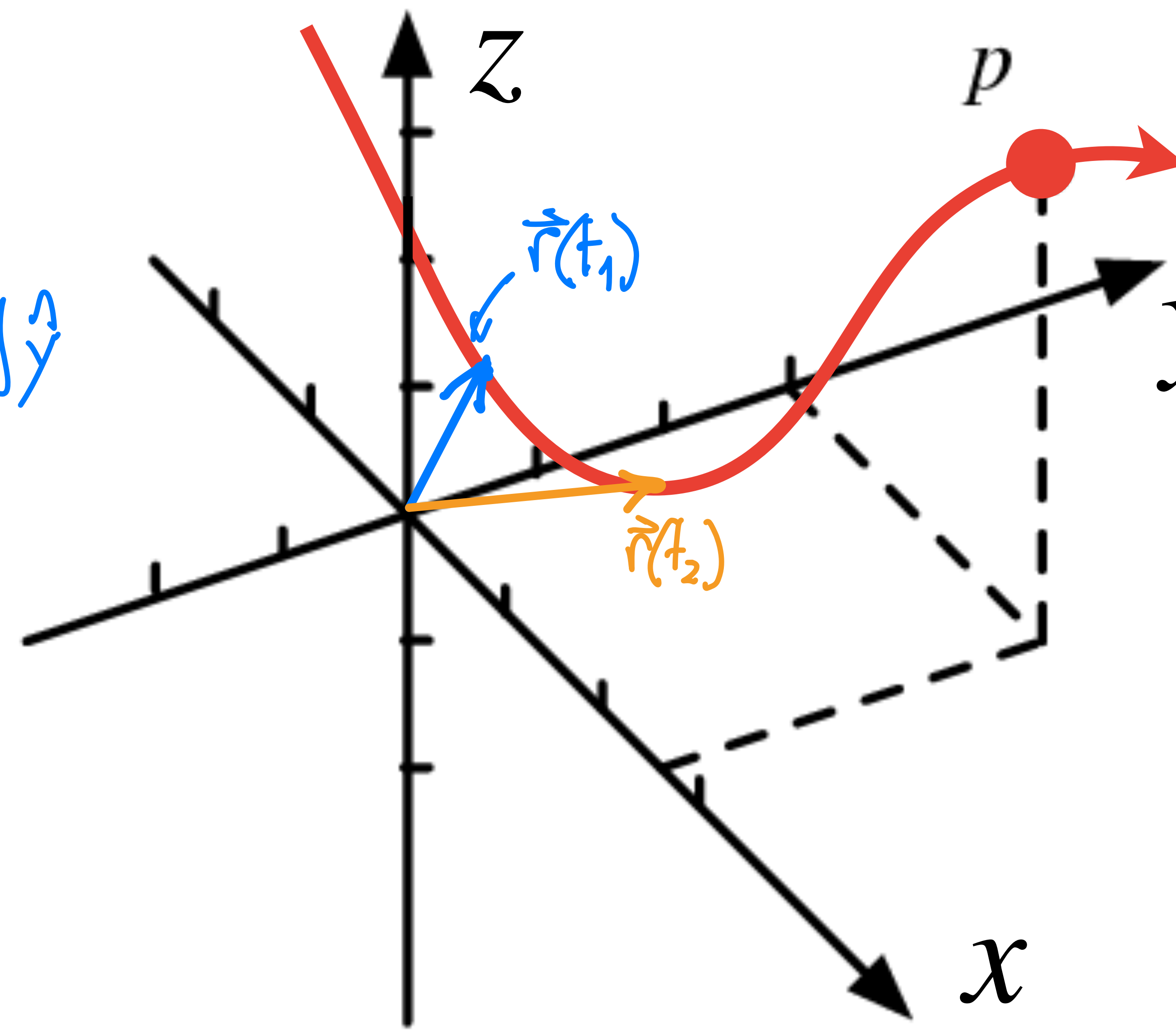
$$= [x(t_2)\hat{x} + y(t_2)\hat{y} + z(t_2)\hat{z}]$$

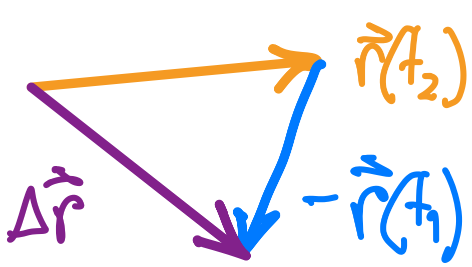
$$- [x(t_1)\hat{x} + y(t_1)\hat{y} + z(t_1)\hat{z}]$$

$$= [x(t_2) - x(t_1)]\hat{x} + [y(t_2) - y(t_1)]\hat{y}$$

$$+ [z(t_2) - z(t_1)]\hat{z}$$

$$= \Delta x \cdot \hat{x} + \Delta y \cdot \hat{y} + \Delta z \cdot \hat{z}$$





Vector velocity (Cartesian)

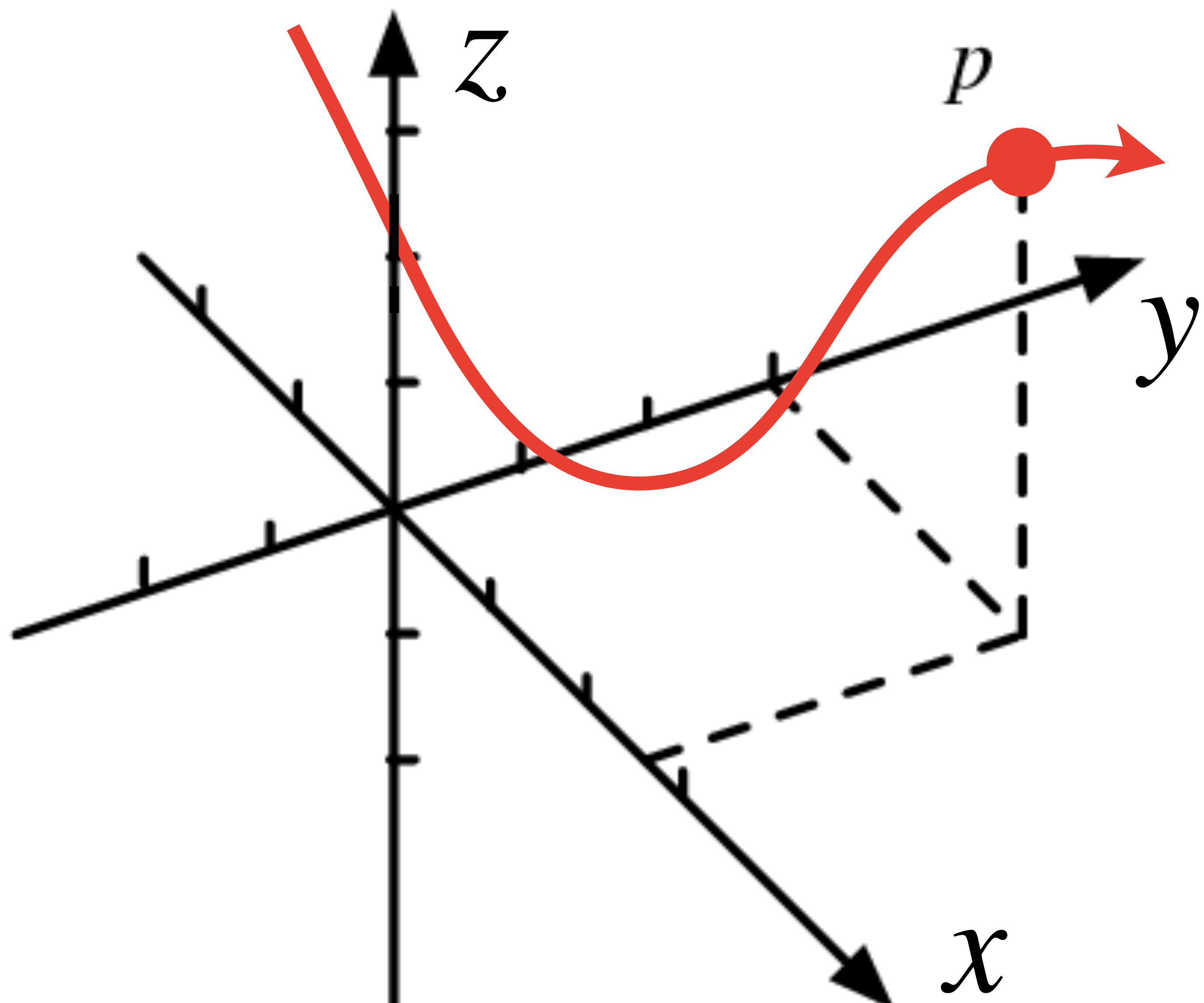
- Average velocity in 1D: $\overline{v} = \dfrac{\text{displacement}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$

$$\overline{\vec{v}} = \frac{1}{\Delta t}\Delta\vec{r}$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\Delta\vec{r} = \frac{d}{dt}\vec{r} \equiv \vec{v}$$

$$\lim_{\Delta t \to 0}\left[\frac{1}{\Delta t}\Delta x\,\hat{x} + \frac{\Delta y}{\Delta t}\hat{y} + \frac{\Delta z}{\Delta t}\hat{z}\right]$$

$$= \underbrace{\frac{dx}{dt}}_{v_x}\hat{x} + \underbrace{\frac{dy}{dt}}_{v_y}\hat{y} + \underbrace{\frac{dz}{dt}}_{v_z}\hat{z}$$



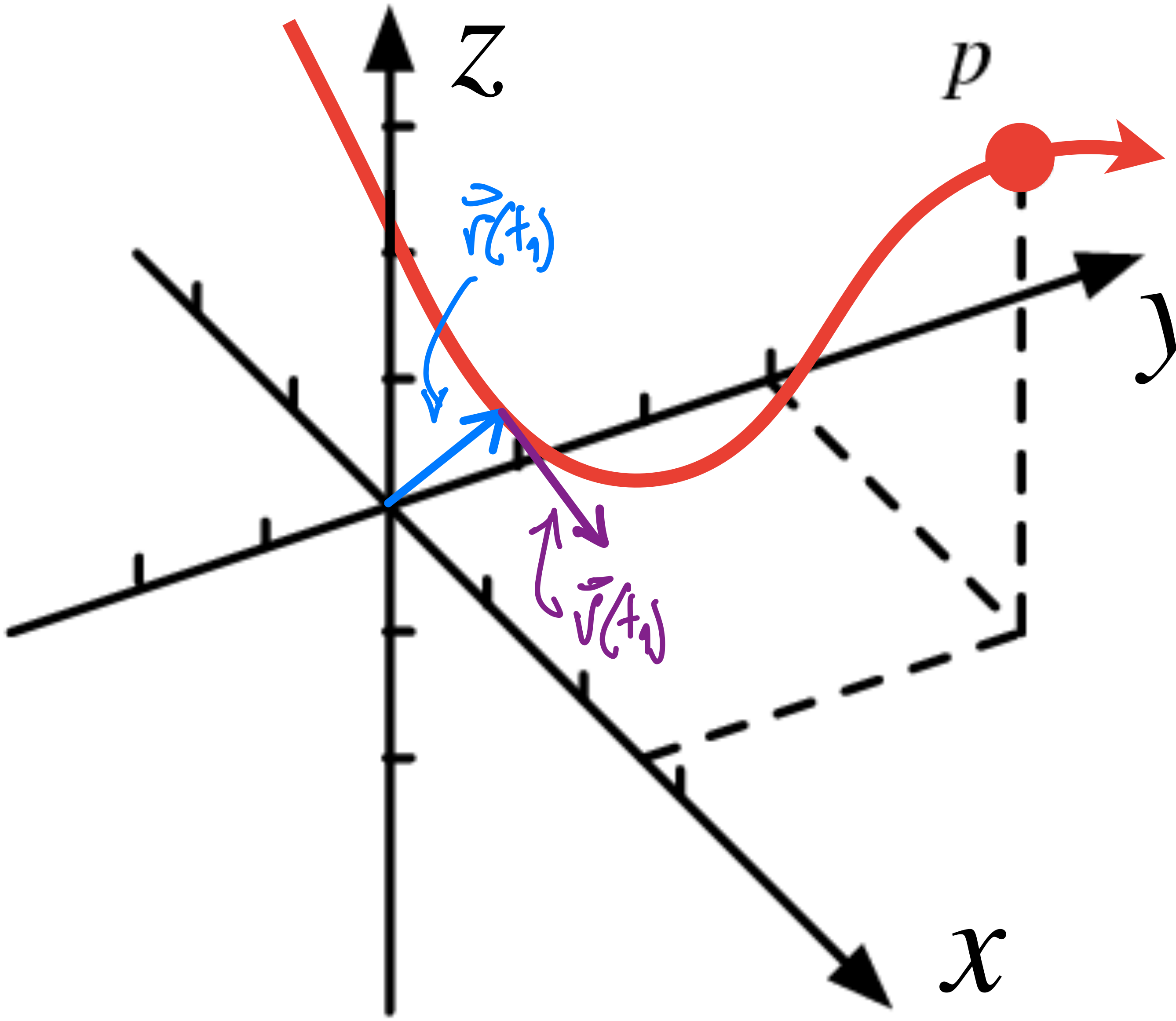
Vector velocity (Cartesian)

- Speed (i.e. magnitude of velocity):

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

- Direction:

Tangent to trajectory



Vector acceleration (Cartesian)

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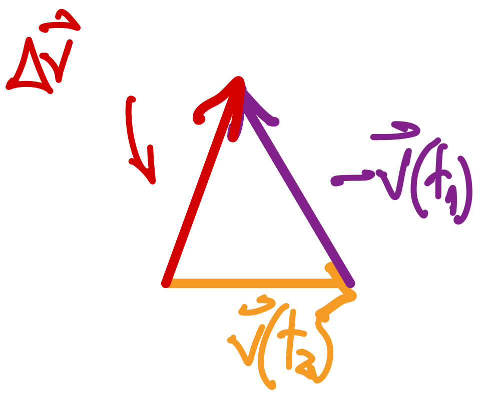
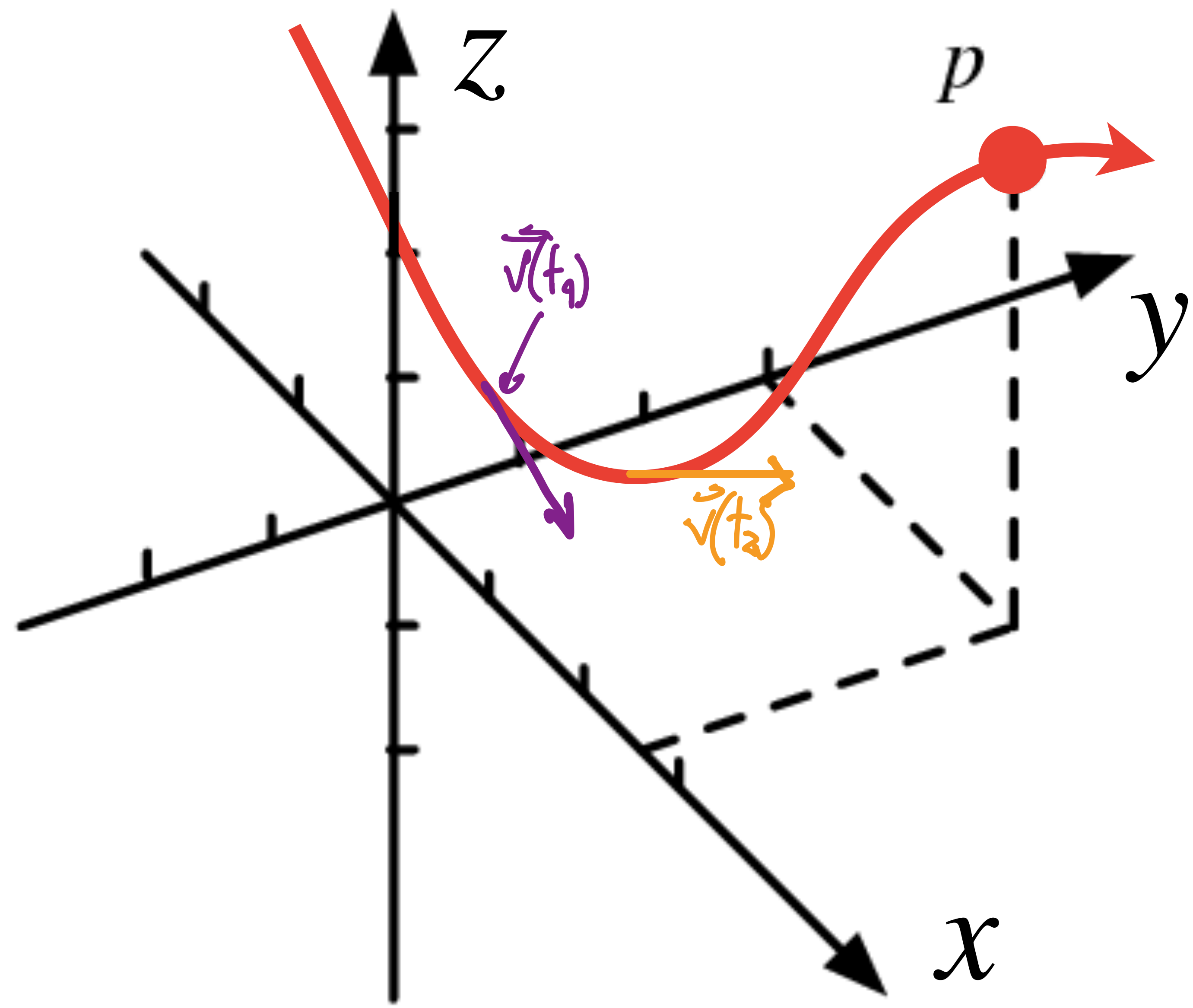
- Average acceleration in 1D: $\bar{a} = \dfrac{\text{change in velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$

$\Delta\vec{v} = \vec{v}(t_2) - \vec{v}(t_1)$



$$\vec{a} = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\Delta\vec{v} = \frac{d\vec{v}}{dt}$$

$$= \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} + \frac{dv_z}{dt}\hat{z}$$



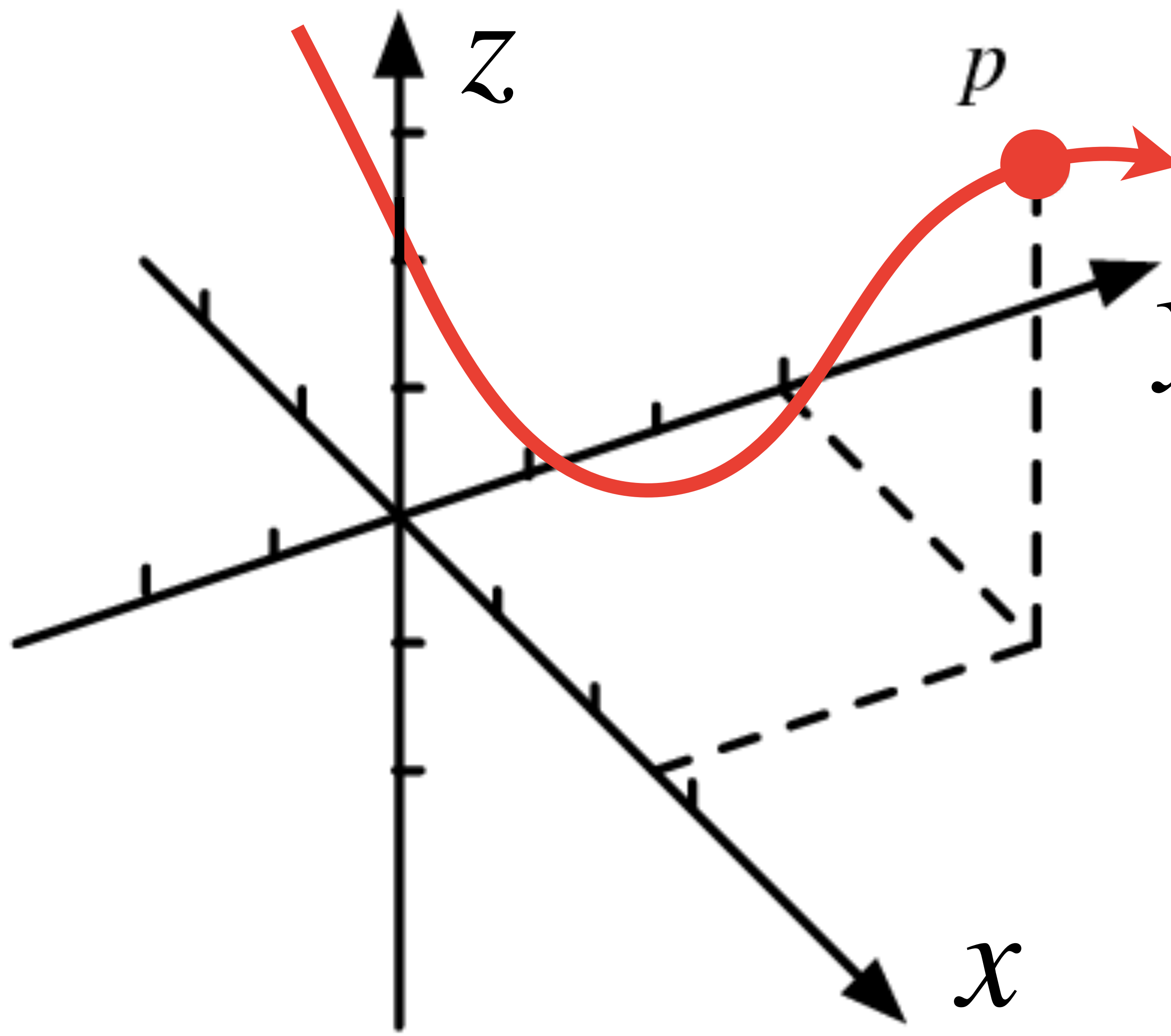
Vector acceleration (Cartesian)

- Magnitude of the acceleration:

$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

- Direction:

change in velocity



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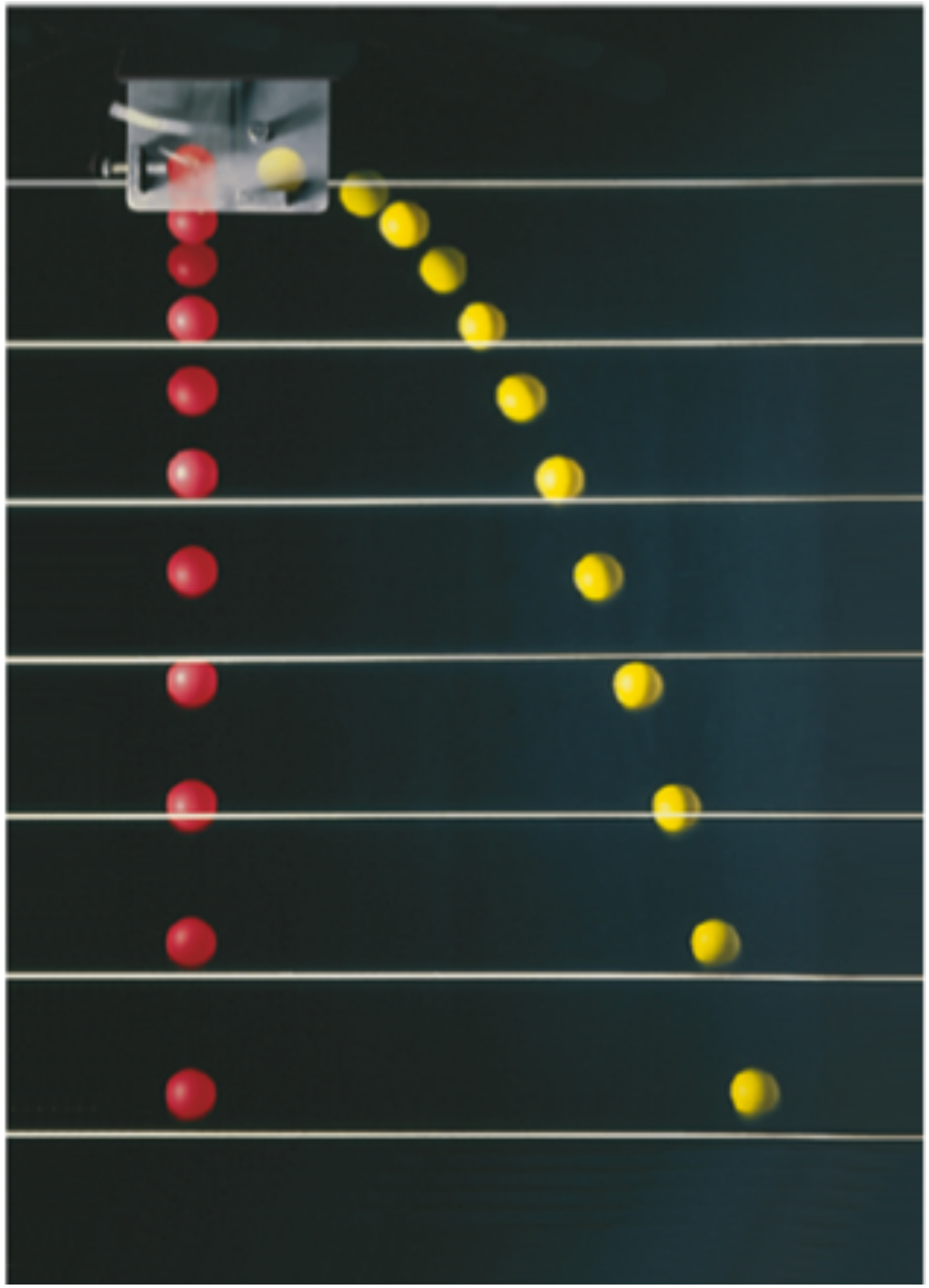
Projectile motion

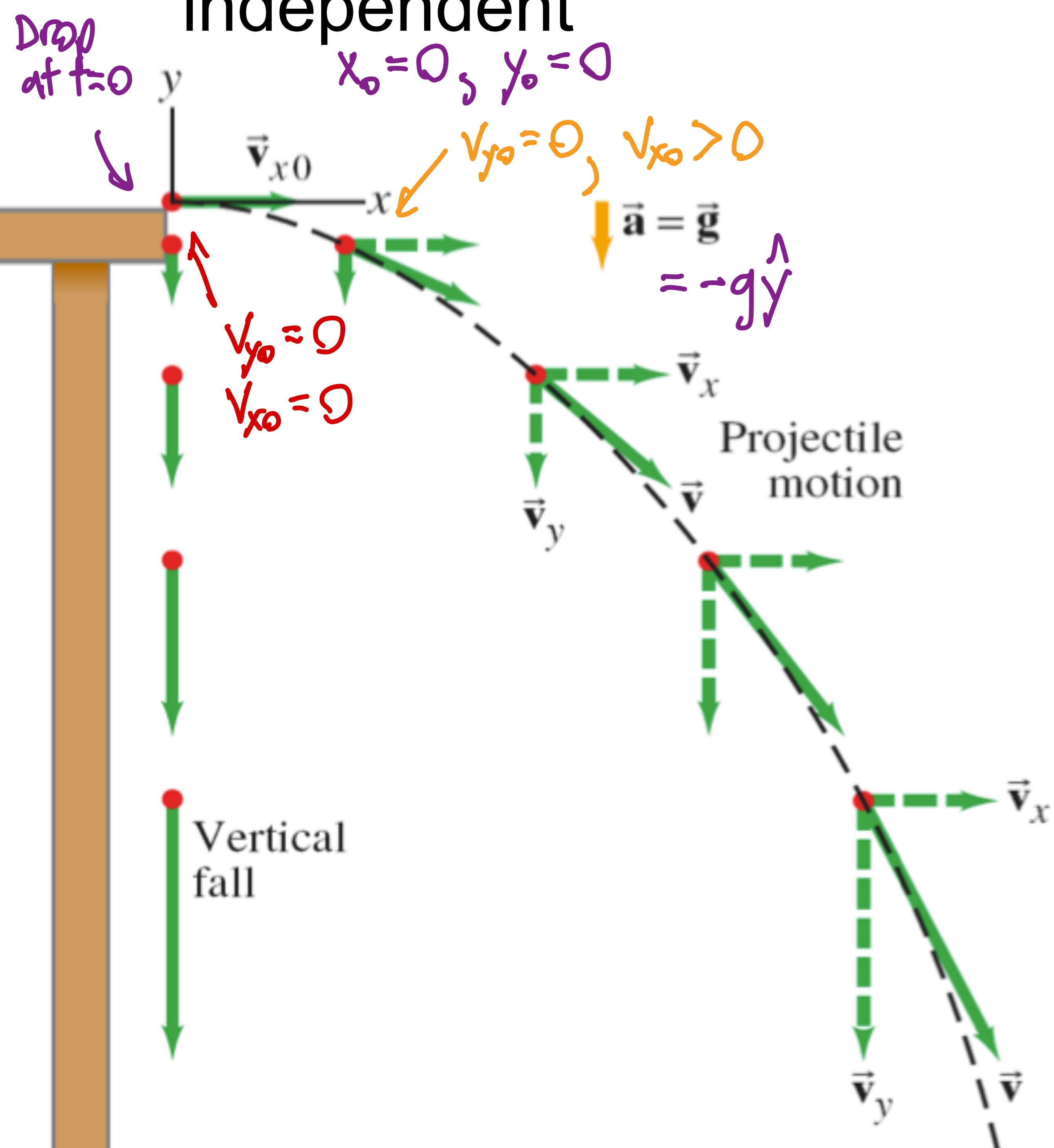
Projectile motion

- Two balls are released simultaneously under gravity
- What causes the difference in their motions?
- What equations of motion need modified?



Velocity throughout projectile motion

- Motion in horizontal and vertical components are decoupled and independent



Case $v_{x0} = 0$

- $v_x(t) = a_x t + v_{x0} = 0$
- $v_y(t) = a_y t + v_{y0} = -gt$

$\Rightarrow \vec{v} = v_x \hat{x} + v_y \hat{y} = -gt\hat{y}$

Case $v_{x0} > 0$

- $v_x(t) = a_x t + v_{x0} = v_{x0}$
- $v_y(t) = a_y t + v_{y0} = -gt$

$\Rightarrow \vec{v} = v_{x0}\hat{x} - gt\hat{y}$

Position throughout projectile motion

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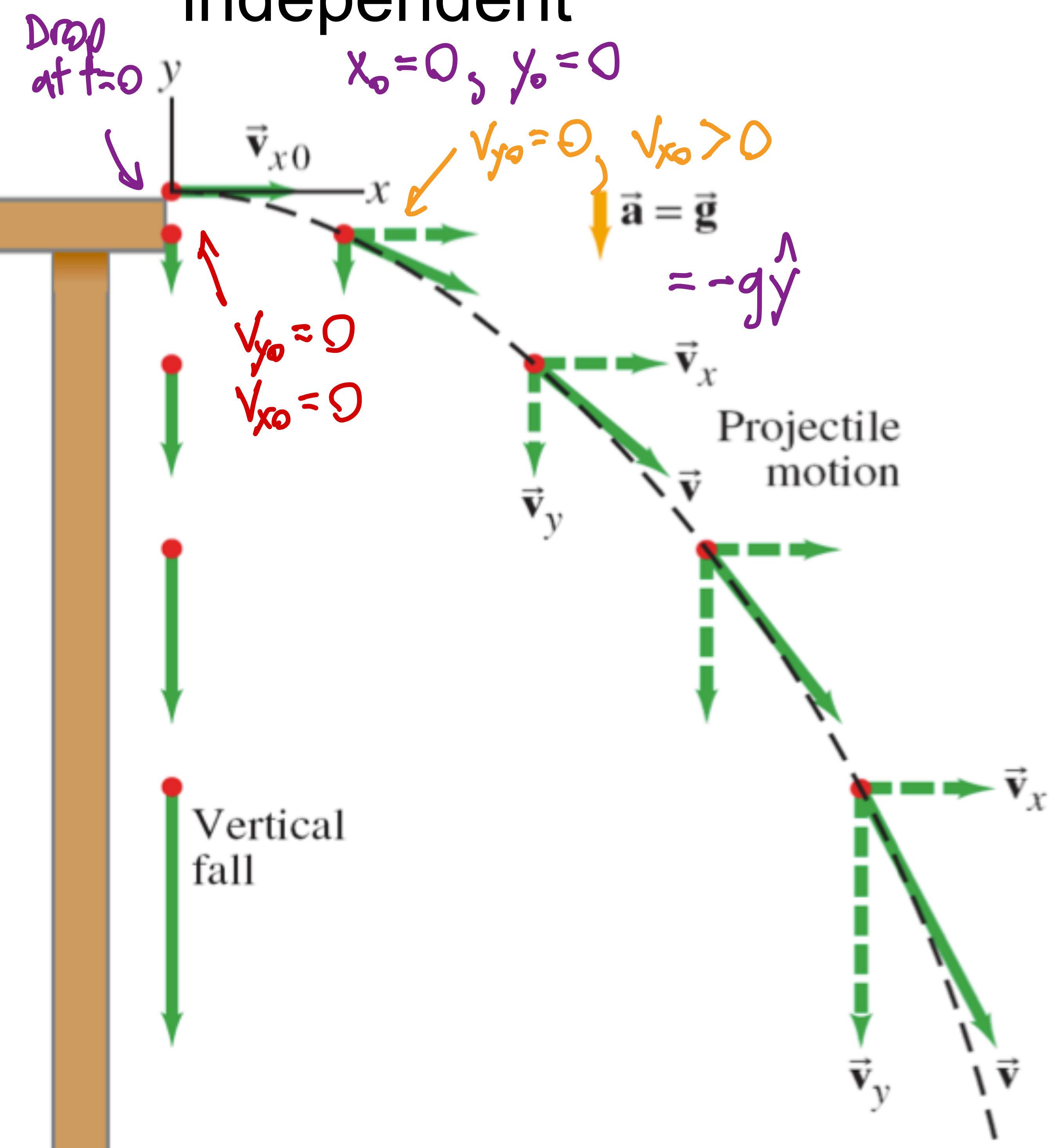
- Motion in horizontal and vertical components are decoupled and independent



Case $v_{x0}=0$

$$x(t) = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 = 0$$

$$y(t) = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 = -\frac{1}{2}gt^2 \quad (a_y=-g)$$

$$\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} = -\frac{1}{2}gt^2\hat{y}$$

Case $v_{x0}\neq 0$ ($v_{x0}>0$)

$$x(t) = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$$

$$y(t) = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 = -\frac{1}{2}gt^2 \quad (a_y=-g)$$

$$\vec{r}(t) = v_{x0}t\hat{x} - \frac{1}{2}gt^2\hat{y}$$

Trajectory of projectile motion

- The path of a projectile is always a parabola

$$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 = v_{x0} t \Rightarrow t = \frac{x}{v_{x0}}$$

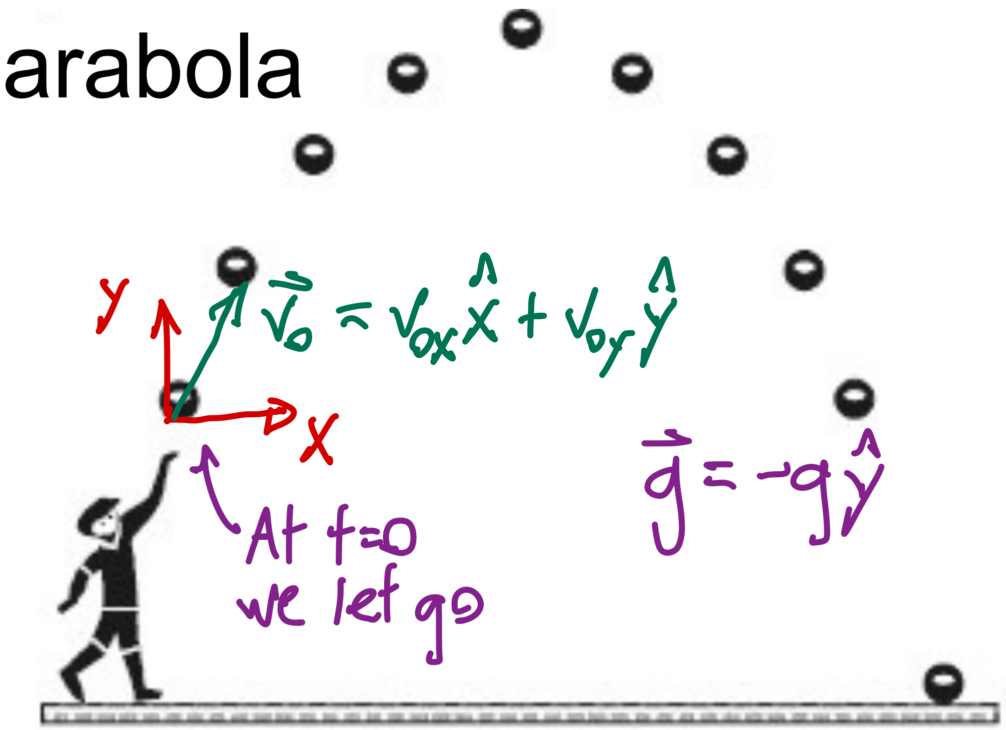
($x_0 = 0$, $a_x = 0$)

$$y(t) = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 = v_{y0} t - \frac{1}{2} g t^2$$

($y_0 = 0$, $a_y = -g$)

$$y = v_{y0}\left(\frac{x}{v_{x0}}\right) - \frac{1}{2} g \left(\frac{x}{v_{x0}}\right)^2$$

$$= \boxed{\frac{v_{y0}}{v_{x0}} x - \frac{g}{2 v_{x0}^2} x^2}$$



Summary of projectile motion

- The acceleration is

$$\vec{a}(t) = -g\hat{y}$$

- By integrating, we find the velocity is

$$\vec{v}(t) = v_{x0}\hat{x} + (-gt + v_{y0})\hat{y}$$

where v_{x0} and v_{y0} are the initial velocities in each direction

- Integrating again, the position is

$$\vec{r}(t) = (v_{x0}t + x_0)\hat{x} + (-\frac{g}{2}t^2 + v_{y0}t + y_0)\hat{y}$$

where x_0 and y_0 are the initial positions in each dimension

DEMO (∞)

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DEMO (∞)

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An object at rest, stays at rest…

DEMO (766)

An object in motion at a constant velocity stays in motion at a constant velocity…

Newton’s 1st law of motion

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In an inertial reference frame, an object will remain at rest, or in motion at a constant speed in a straight line, unless acted upon by a net force.

Newton's 1st law of motion

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In an inertial reference frame, an object will remain at rest, or in motion at a constant speed in a straight line, unless acted upon by a net force.

- Expresses the idea of "inertia"

The concept of force

- A force is an influence that can change the motion of an object
- Whenever an object is accelerating in an inertial reference frame, there must be a force behind it
- It is a vector quantity, often denoted by $\overrightarrow{F}$
- Measured in units of *Newtons* ($[\mathrm{N}] = \left[\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right]$)

Newton's 1st law of motion

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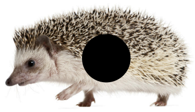
In an inertial reference frame, an object will remain at rest, or in motion at a constant speed in a straight line, unless acted upon by a net force.

- Expresses the idea of "inertia"
- In mathematics:

$$\Sigma \overrightarrow{F} = 0 \quad \Longleftrightarrow \quad \vec{v} = \text{constant}$$

Inertial frames of reference

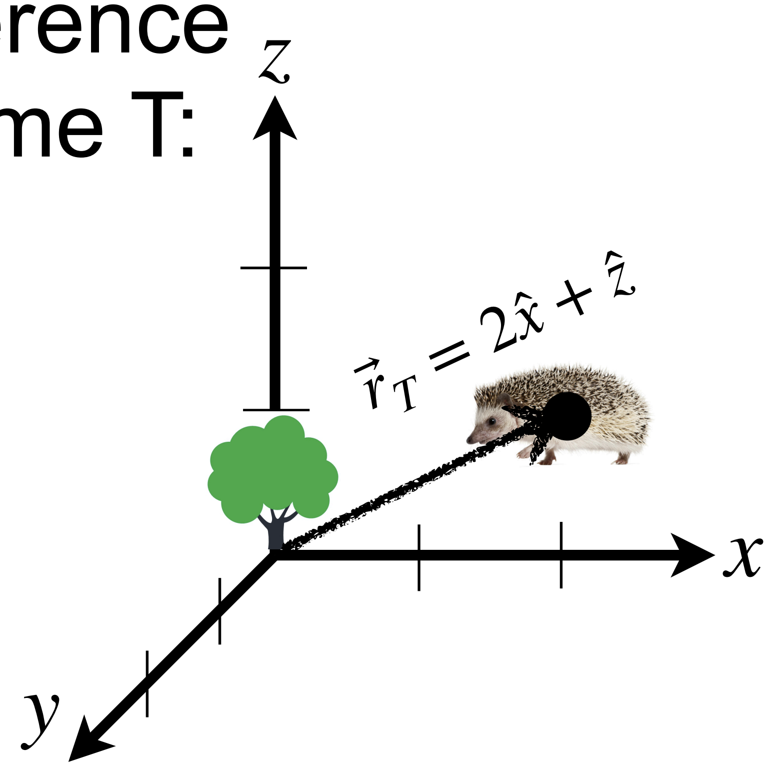
- A frame of reference is a *coordinate system*
- Consider a hedgehog, he's still a *point mass*



Inertial frames of reference

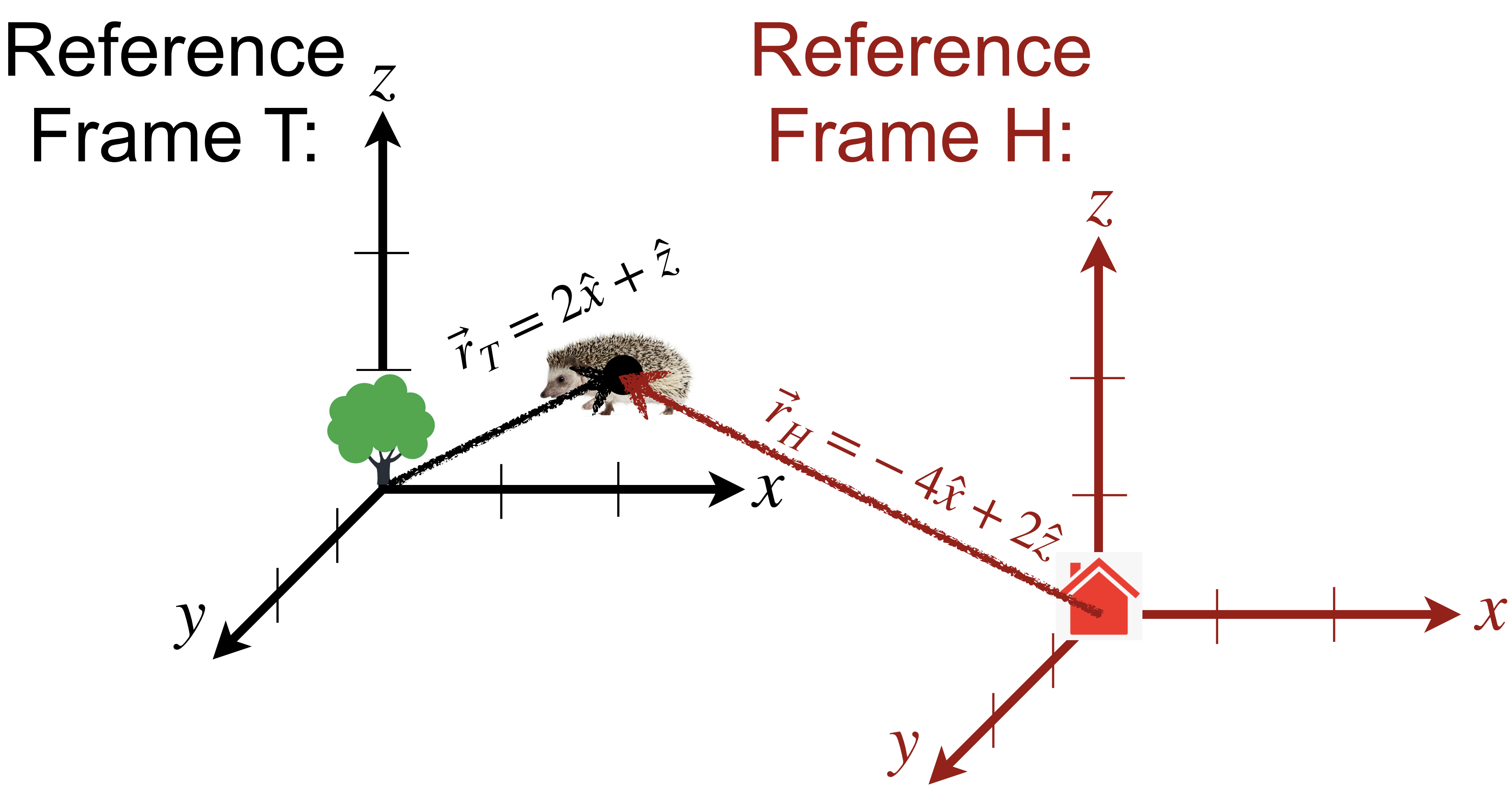
- A frame of reference is a *coordinate system*
- Consider a hedgehog, he's still a *point mass*
- You can quantify his position relative to another object, like a tree

Reference Frame T:



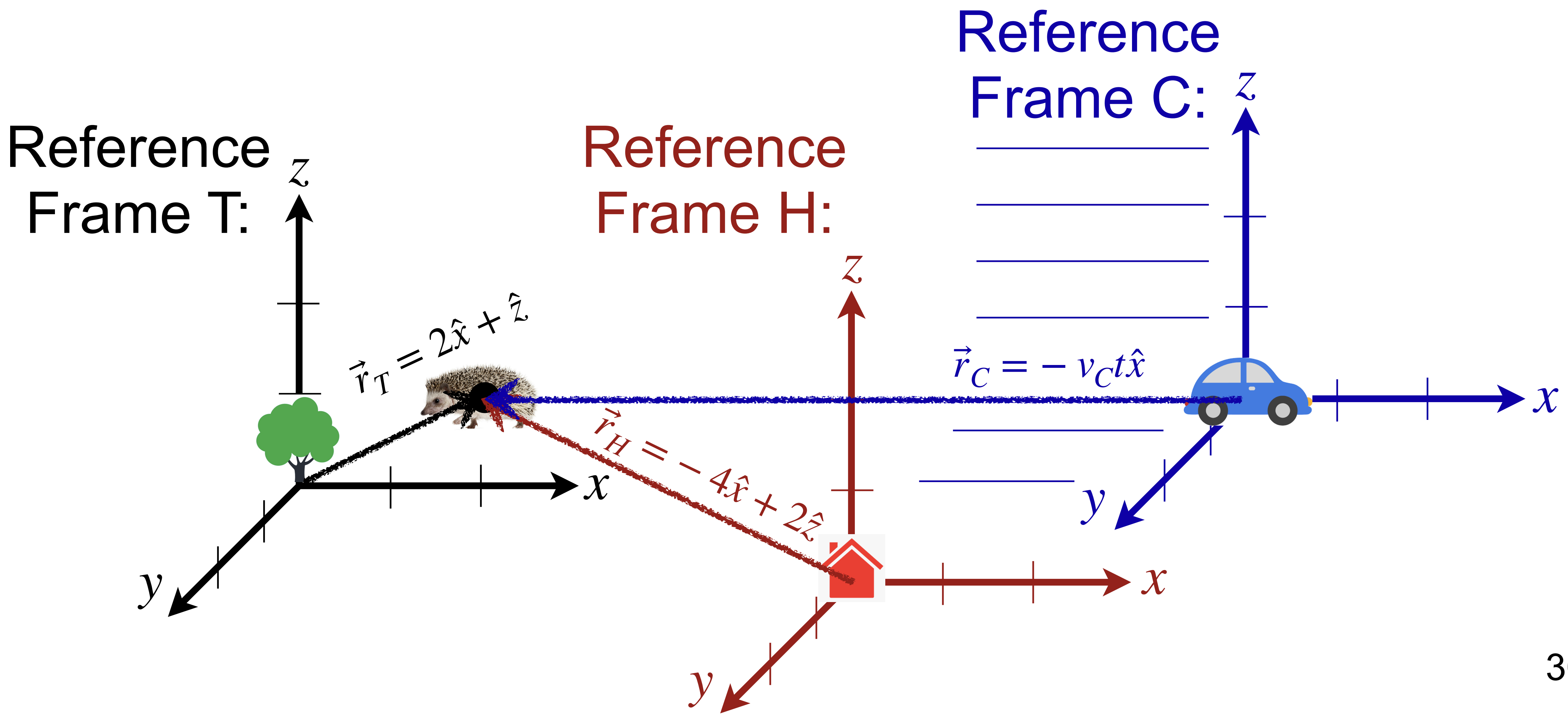
Inertial frames of reference

- A frame of reference is a *coordinate system*
- Consider a hedgehog, he's still a *point mass*
- You can quantify his position relative to another object, like a tree or a house



Inertial frames of reference

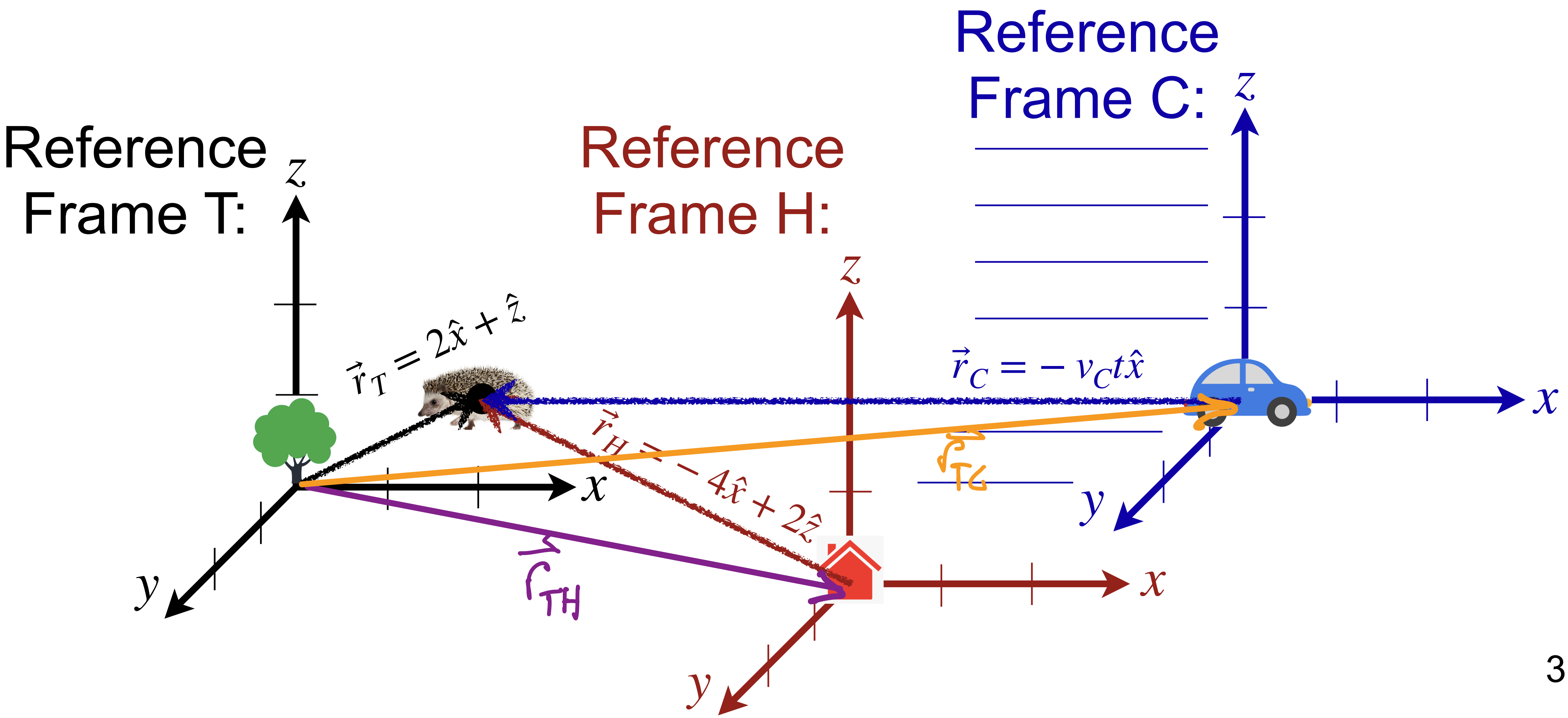
- A frame of reference is a *coordinate system*
- Consider a hedgehog, he's still a *point mass*
- You can quantify his position relative to another object, like a tree or a house or even a moving car



Inertial frames of reference

- You can convert between frames of reference by comparing origins

$\vec{r}_T = \vec{r}_{TH} + \vec{r}_H$ $\qquad$ $\vec{r}_T = \vec{r}_{TC} + \vec{r}_C$

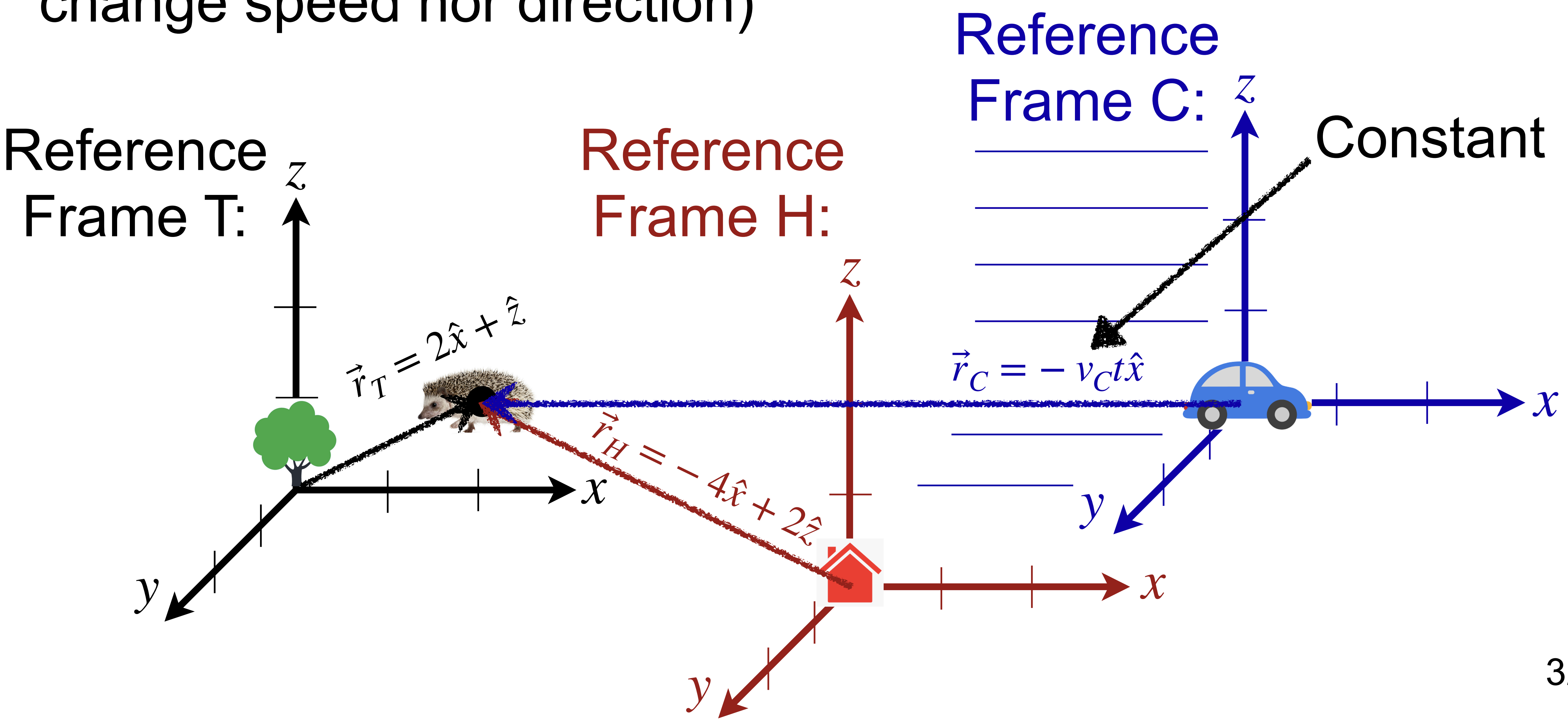


Inertial frames of reference

- You can convert between frames of reference by comparing origins

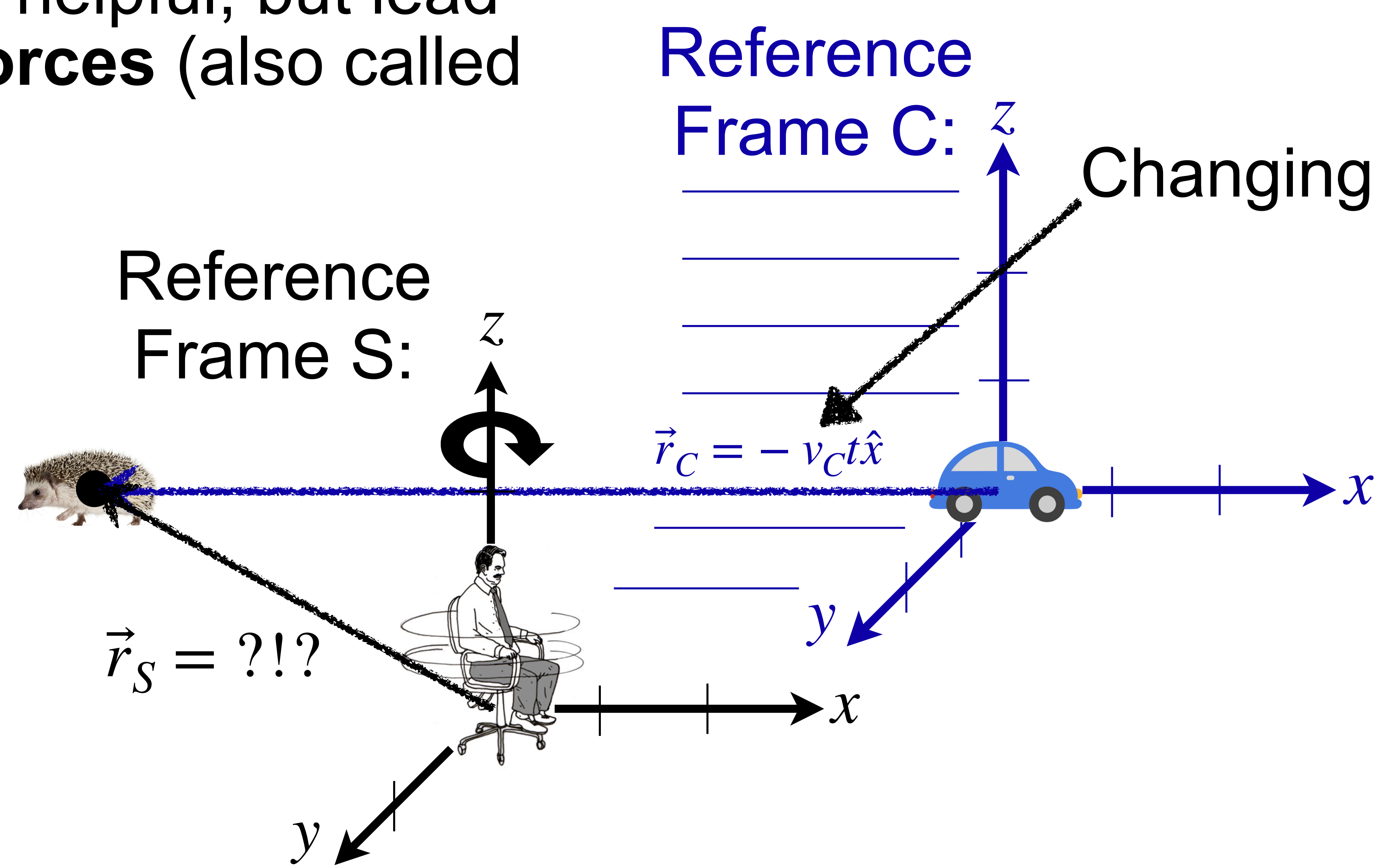
$$\vec{r}_T = \vec{r}_{TH} + \vec{r}_H \quad \text{or} \quad \vec{r}_T = \vec{r}_{TC}(t) + \vec{r}_C$$

- Inertial reference frames do NOT accelerate (i.e. don't change speed nor direction)



Non-inertial frames of reference

- Non-inertial reference frames can accelerate (i.e. change speed or direction)
- Such coordinate systems can sometimes be helpful, but lead to **fictitious forces** (also called inertial forces)



Newton's 1st law of motion

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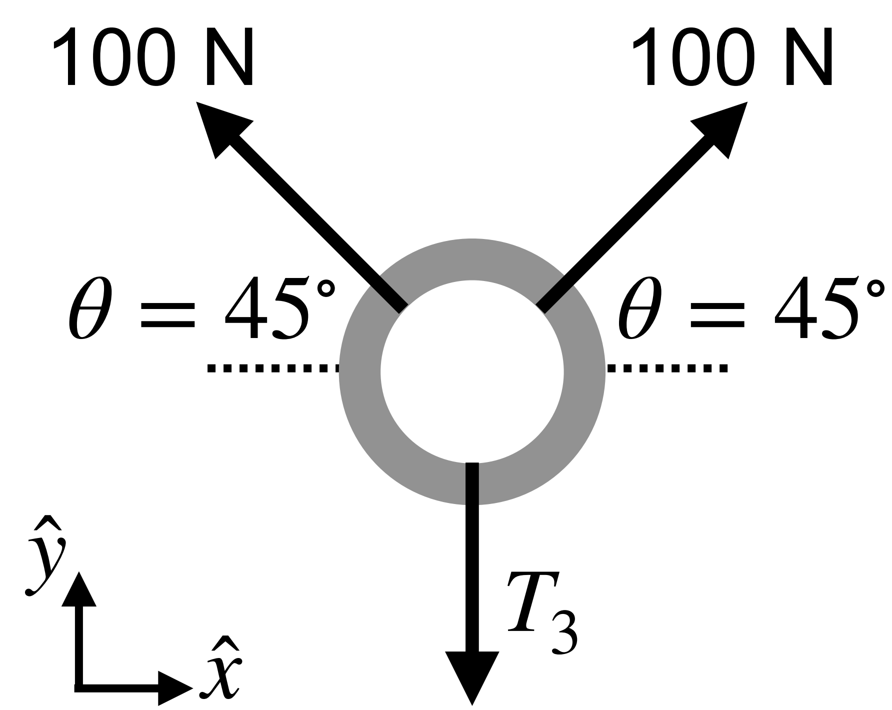
In an inertial reference frame, an object will remain at rest, or in motion at a constant speed in a straight line, unless acted upon by a net force.

- Expresses the idea of "inertia"
- In mathematics:

$$\Sigma \overrightarrow{F} = 0 \quad \Longleftrightarrow \quad \vec{v} = \text{constant}$$

Conceptual question

Three people are pulling on a ring in a two-dimensional tug of war. Shown is a "top view”. No one is winning, so the ring is sitting still. The pulls are configured as shown. Teams 1 and 2 are each pulling with a force of 100 N, while team 3 pulls with unknown force T_3.



Which expression gives the sum of the forces in the $\hat{y}$ direction?

A. 100 N
B. 200 N
C. 200 N * cos(45) (=141 N)
D. 0 N
E. None of these

$\vec{v} = 0 \Rightarrow \vec{v} = \text{constant}$
$\Rightarrow \Sigma \vec{F} = 0$

Newton’s 2nd law of motion

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In an inertial reference frame,
an object accelerates when it is acted upon by a net force.

The acceleration is directly proportional to the net force,
and inversely proportional to the mass.

Newton's 2nd law of motion

$$\Sigma \overrightarrow{F} = m\vec{a}$$

Newton's 2nd law of motion

$$\Sigma \overrightarrow{F} = m\underbrace{\vec{a}}$$

acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

Newton's 2nd law of motion

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$$\Sigma \overrightarrow{F} = \underbrace{m}_{\text{mass}} \vec{a}$$

The concept of mass

- Mass is an intrinsic property of an object
- It quantifies the inertia of an object, i.e. its resistance to changing its motion
- It is sometimes understood as the “amount” of matter in an object
- Mass is measured in *kilograms* [kg]
- It is not weight — mass is a property of an object, while weight is the force exerted on an object by gravity
  - If you go to the moon (which has 6 times weaker gravity than Earth), your weight will decrease by a factor of 6 while your mass will stay the same

Newton's 2nd law of motion

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$$\Sigma \underbrace{\overrightarrow{F}}_{\text{force}} = m\vec{a}$$

Some forces

Force	Magnitude	Direction
Gravitational (in general)	$\lvert\vec{F}\rvert = G\dfrac{m_1 m_2}{r^2}$	In a straight line between the centers of the two masses, pulling them together
Gravitational (at Earth's surface)	$\lvert\vec{F}\rvert = mg$ where $g = \dfrac{Gm_E}{r_E^2}$	Downwards
Electrostatic (Coulomb's Law)	$\lvert\vec{F}\rvert = k_e\dfrac{\lvert q_1\rvert\lvert q_2\rvert}{r^2}$	In a straight line between the centers of the two charges
Elastic (Hooke's Law)	$\lvert\vec{F}\rvert = k\Delta x$	Restores to equilibrium position
Normal	$\lvert\vec{F}\rvert = N$	Perpendicular to the surface
Static Friction	$\lvert\vec{F}\rvert < \mu_s N$	Opposing the direction of <u>impending</u> motion
Kinetic Friction	$\lvert\vec{F}\rvert = \mu_k N$	Opposing the direction of motion
Viscous Drag	$\lvert\vec{F}\rvert = bv^n$	Opposing the direction of motion
Tension	$\lvert\vec{F}\rvert = T$	Against the direction of opposing forces

The four fundamental forces

- Modern physics now recognizes four fundamental forces

    1. Gravity
    2. Electromagnetism
    3. Strong nuclear (confines quarks in protons, neutrons and other subatomic particles)
    4. Weak nuclear ("responsible for some forms of nuclear decay")

The four fundamental forces

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- Modern physics now recognizes four fundamental forces
    1. Gravity
    2. Electromagnetism
    3. Strong nuclear (confines quarks in protons, neutrons and other subatomic particles)
    4. Weak nuclear ("responsible for some forms of nuclear decay")
- What about friction, the normal force, tension, etc?
    - Except for gravity, all other "everyday" forces are due to electromagnetism acting at the atomic scale

Newton's 2nd law of motion

$$\underbrace{\sum}_{\textbf{\underline{NET} force}} \overrightarrow{F} = m\vec{a}$$

Free body diagrams

- Graphically represent ALL of the external forces acting on an object
- Remember that force is a vector
- Label each force and make the magnitudes and directions reasonably accurate
- Draw a separate diagram for each object

Free body diagrams

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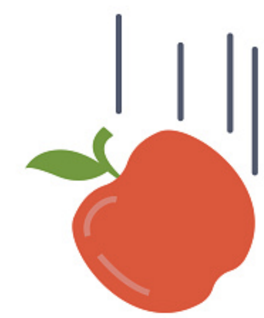
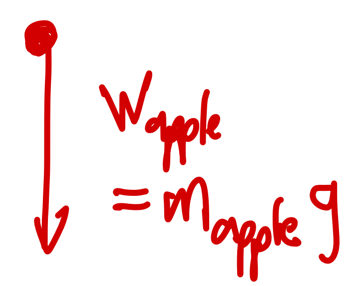
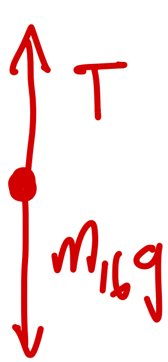
An object in free fall (without drag)



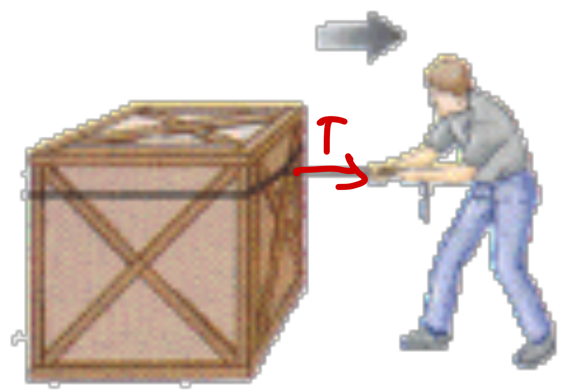


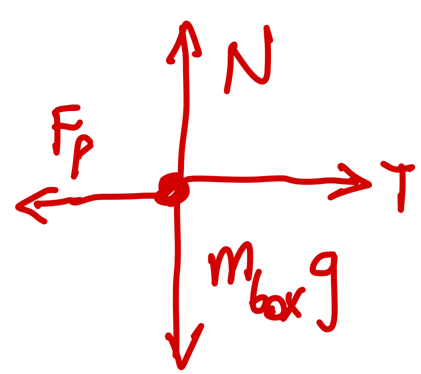
More free body diagrams

A lamp suspended from the ceiling by a chain

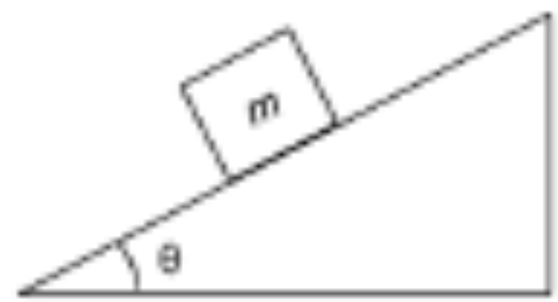


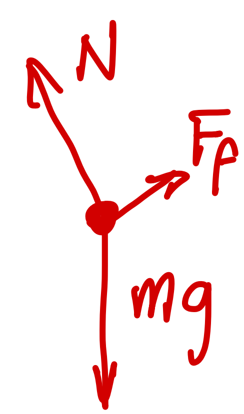
A box pulled along a rough surface



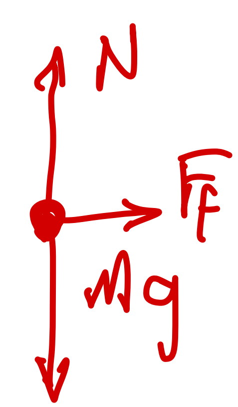


A block on an inclined plane





A foot in contact with the ground when walking



Conceptual question

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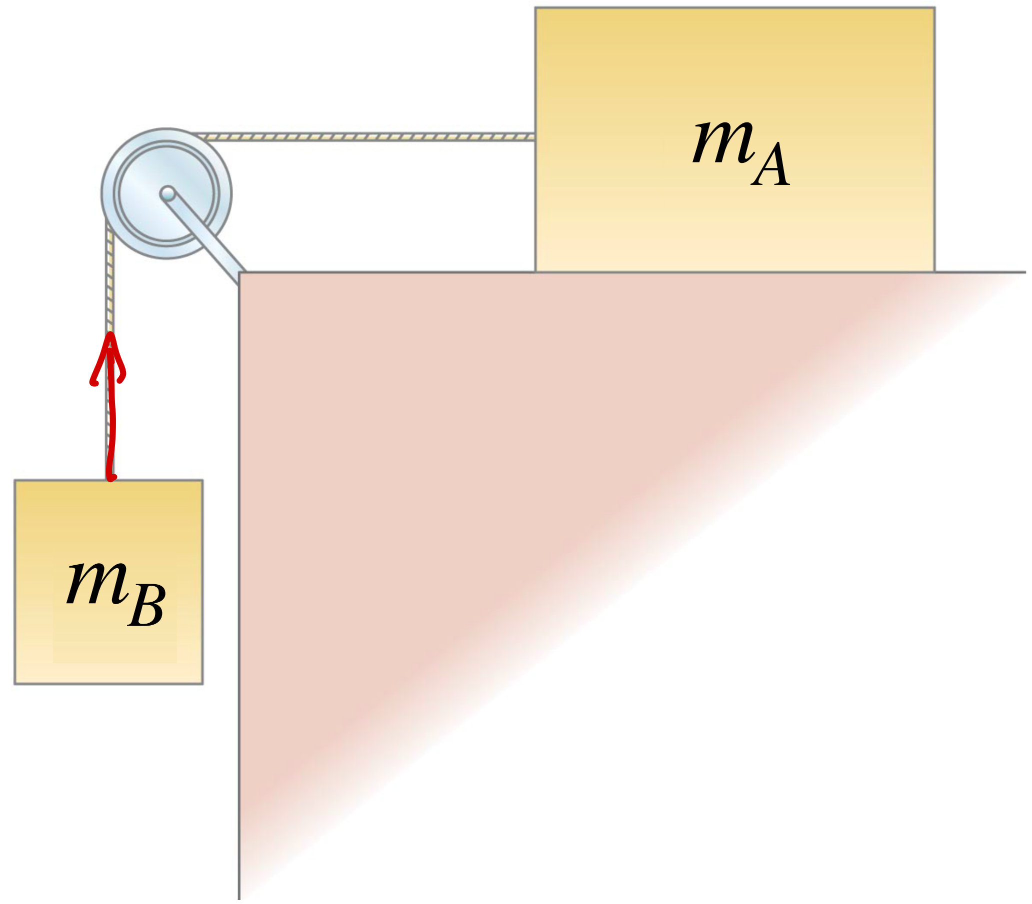
How does the force exerted on block A by the string (T) compare with the weight of block B?

Assume all surfaces are *frictionless*, the pulley is massless, and the rope is *massless* and *inextensible*.

A. $T = m_B g$
B. $T < m_B g$
C. $T > m_B g$



Conceptual solution

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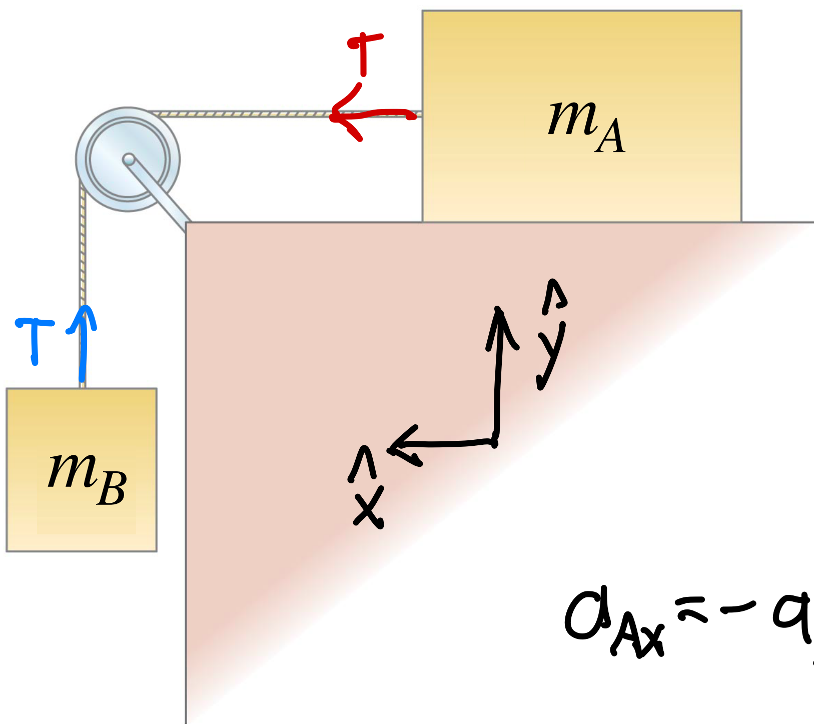
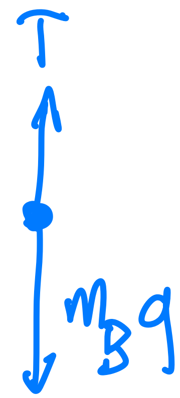


$$a_{Ax} = -a_{By}$$

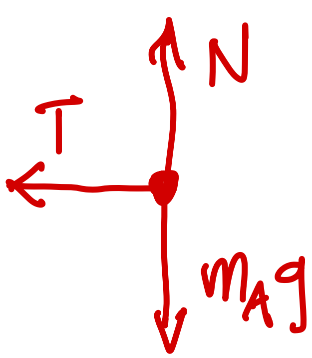


Block B:

In $\hat{y}$: $T - m_B g = m_B a_{By}$

$$\Rightarrow T = m_B g + m_B a_{By}$$
$$= m_B g - m_B a_{Ax}$$

Block A:



$$\sum \vec{F} = m\vec{a}$$

In $\hat{y}$: $N - m_A g = m_A a_{Ay}$ ($a_{Ay} = 0$)

$$\Rightarrow N = m_A g$$

In $\hat{x}$: $T = m_A a_{Ax}$

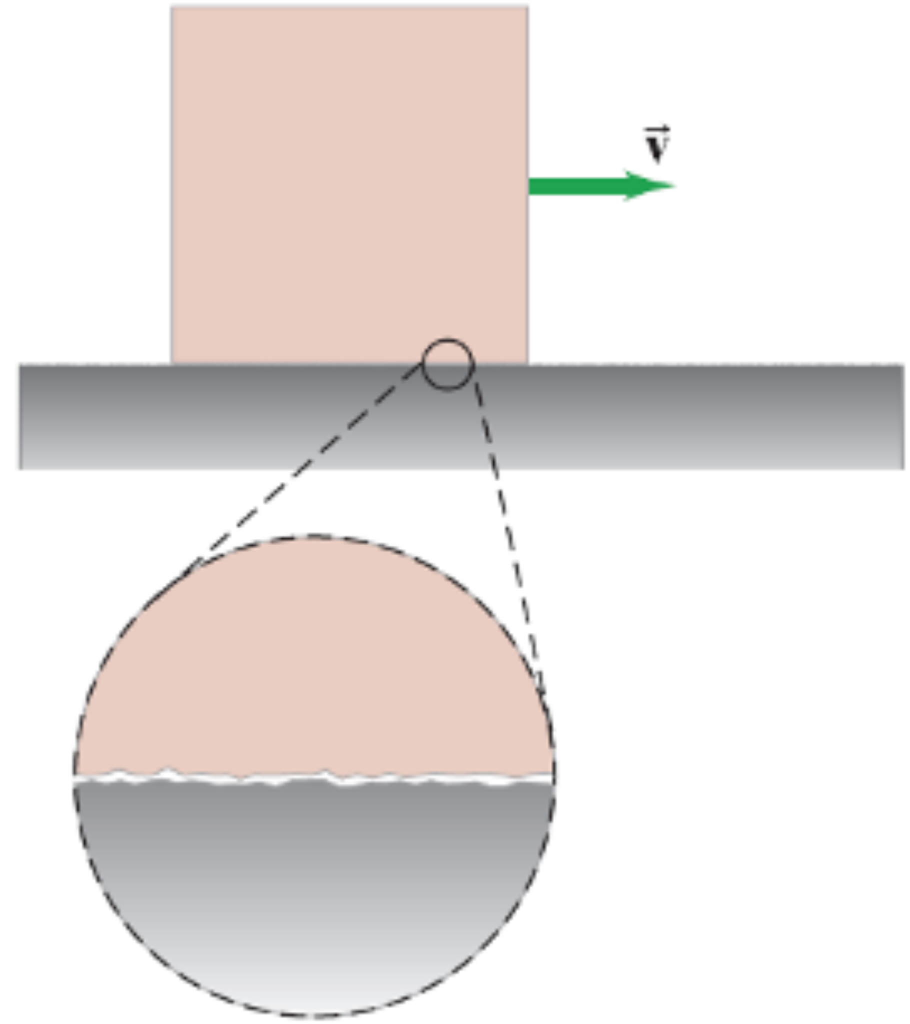
$$m_A a_{Ax} = m_B g - m_B a_{Ax}$$
$$(m_A + m_B)\, a_{Ax} = m_B g$$
$$\Rightarrow a_{Ax} = \frac{m_B}{m_A + m_B} g$$

$$T = m_A a_{Ax} = \frac{m_A m_B}{m_A + m_B} g$$
$$= \boxed{\frac{m_A}{m_A + m_B} m_B g}$$

$\frac{m_A}{m_A+m_B} < 1 \Rightarrow T < m_B g$

Friction

- Friction is always present when two solid surfaces are in contact



Friction

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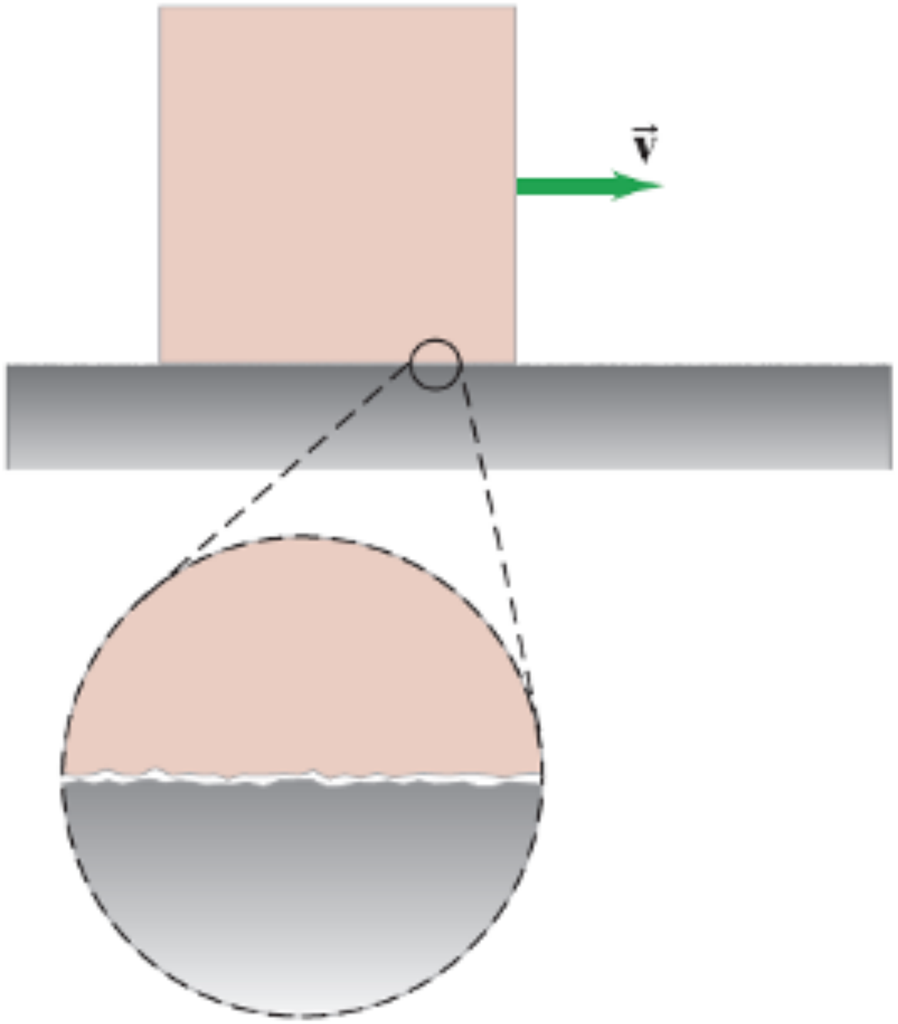
- Friction is always present when two solid surfaces are in contact

- Friction while sliding is called kinetic friction and approximately follows

$$|F| = \mu_k N$$



Friction

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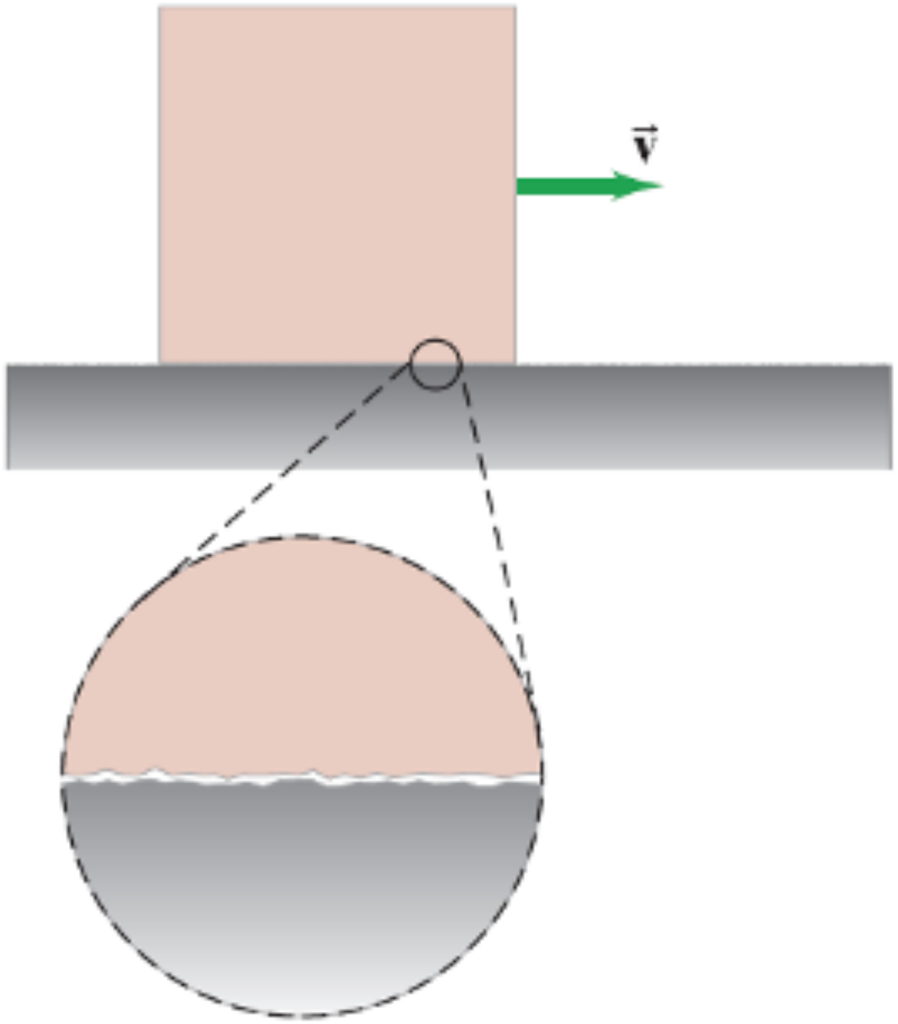
- Static friction is when the two surfaces are at rest relative to each other, which balances an applied force up to the maximum value of

$$|F| \leq \mu_s N$$



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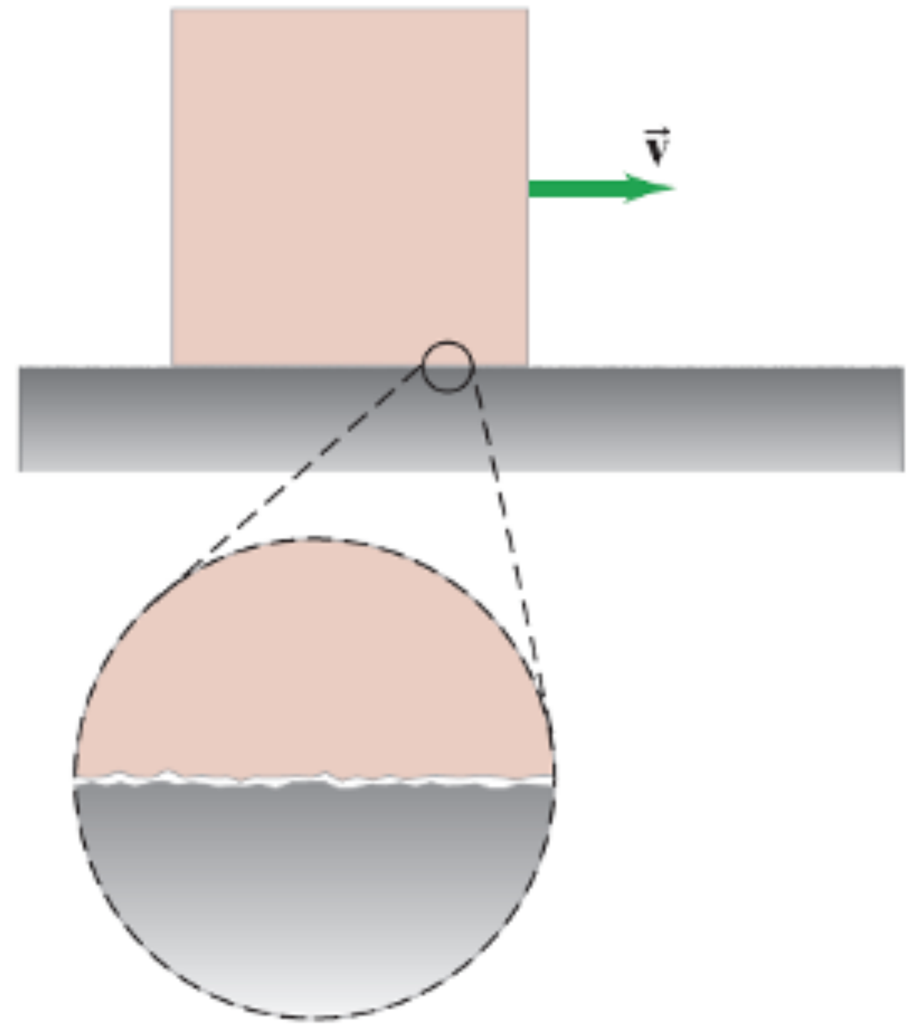
- It is easier to keep an object sliding than it is to get it started
- Surface area doesn't enter



Surfaces	μ_s	μ_k
Rubber tires on pavement	0.9	0.8
Metal on ice	0.022	0.02
Steel on steel	0.6	0.4
Steel on steel (with grease)	0.1	0.05

Direction of frictional forces

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✦ If an object is sliding on a surface, kinetic friction applies:

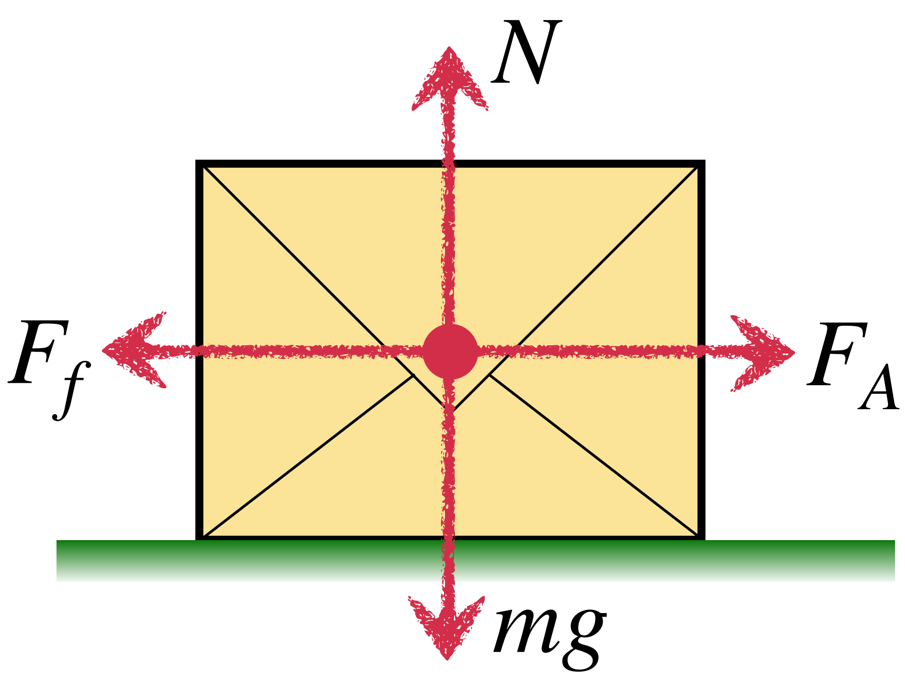
- The friction force *experienced by the object* is in the direction opposite to its velocity (relative to the surface)
- The friction force *experienced by the surface* is equal and opposite

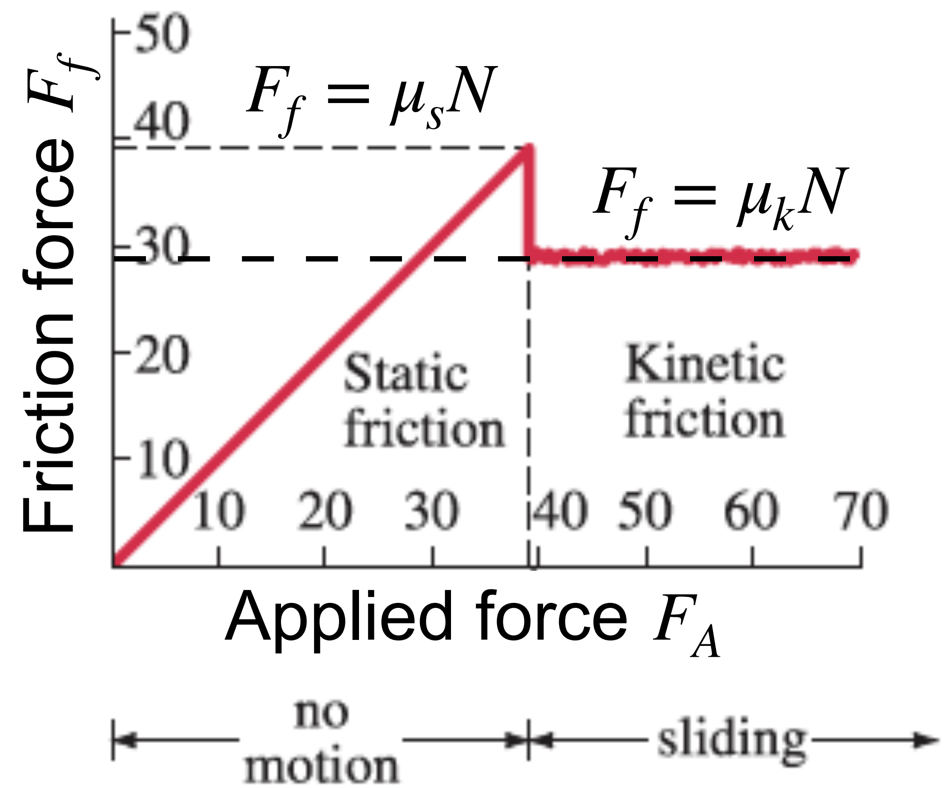
Direction of frictional forces

- If an object is sliding on a surface, kinetic friction applies:
  - The friction force *experienced by the object* is in the direction opposite to its velocity (relative to the surface)
  - The friction force *experienced by the surface* is equal and opposite
- If an object is at rest on a surface, static friction applies:
  - The static friction force *experienced by the object* is in the direction opposite to what its velocity (relative to the surface) would be if there was no friction
  - The friction force *experienced by the surface* is equal and opposite

Example of friction

- The dependence of the friction force on the applied force for a 10 kg box on a horizontal floor with the coefficients of static friction $\mu_s = 0.4$ and kinetic friction $\mu_k = 0.3$



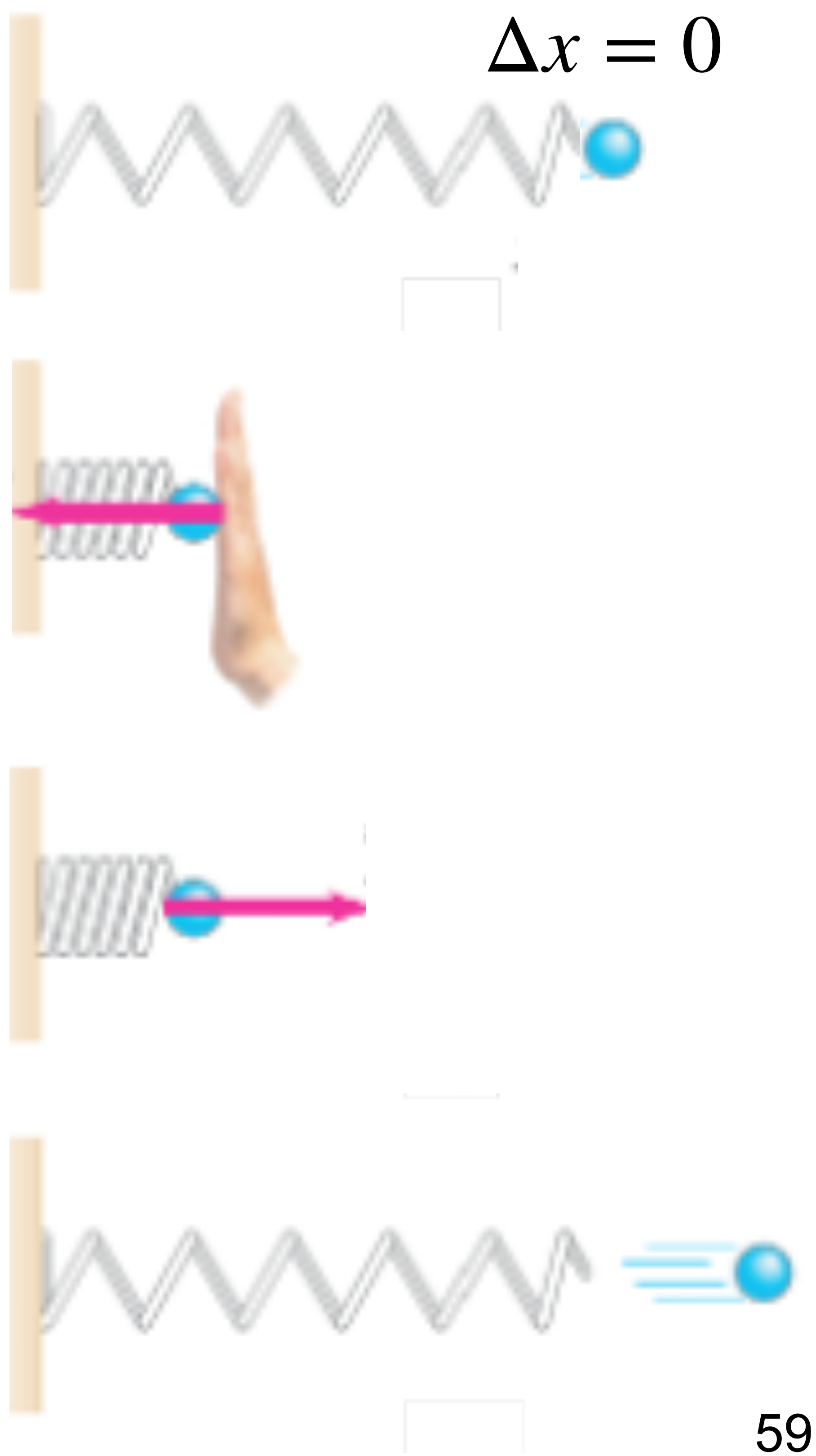


DEMO (69)

Friction

Springs

- The force required to compress or stretch a spring is
$$\vec{F} = -k\Delta\vec{x},$$
where k is the spring constant and $\Delta\vec{x}$ is the displacement from the equilibrium position



Springs

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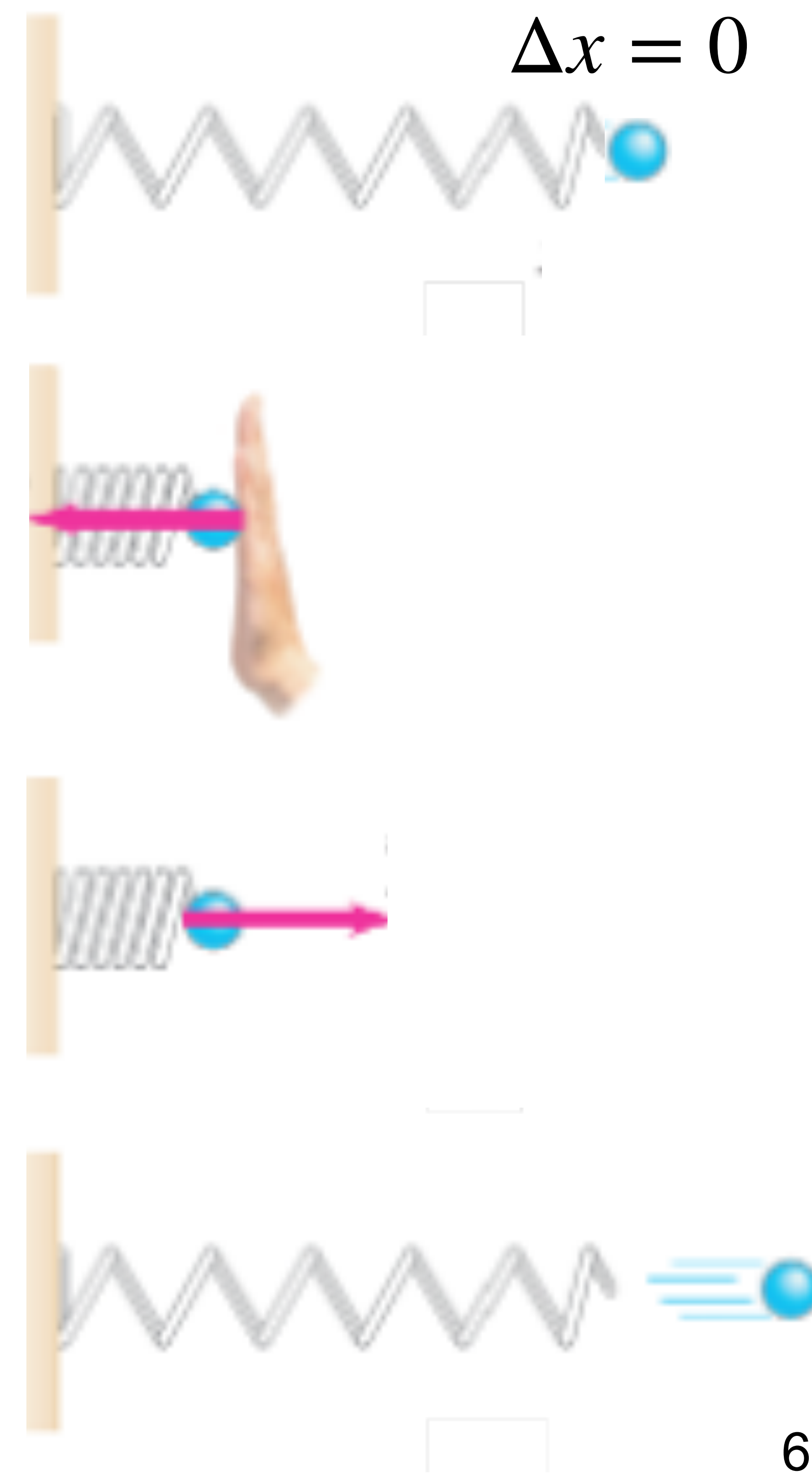
- "Restoring" force is proportional to the displacement



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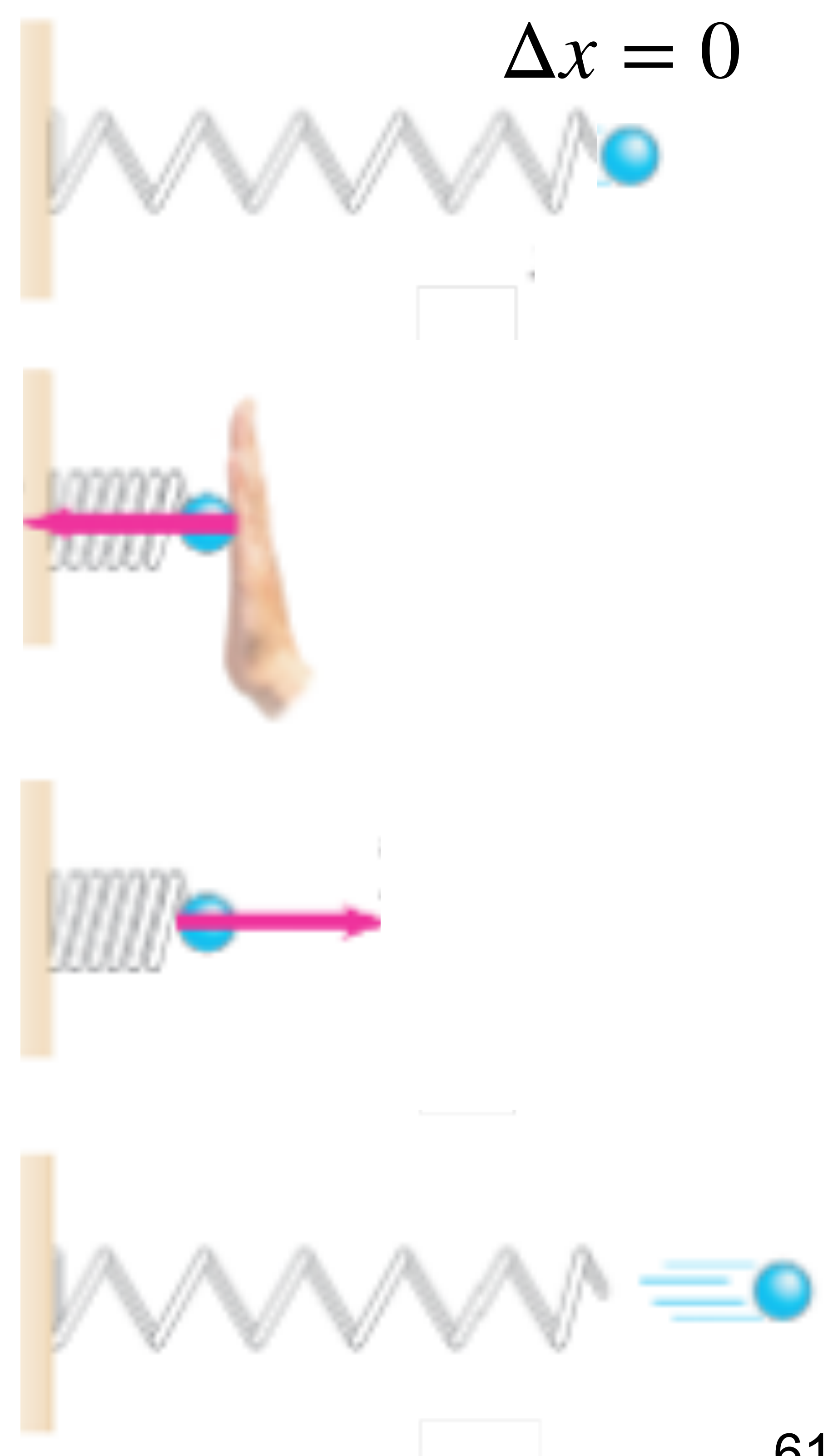
- Springs might seem weird to focus on, but they are a good model for many applications

  - E.g. rubber bands, pendulums, electrical circuits, nuclear physics!



DEMO (20)

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Stretching a spring

See you tomorrow at 10h15 in room SG1!