EPFL

Swiss Plasma Center

General Physics: Mechanics

PHYS-101(en)

Lecture 1b: Motion in two and three dimensions

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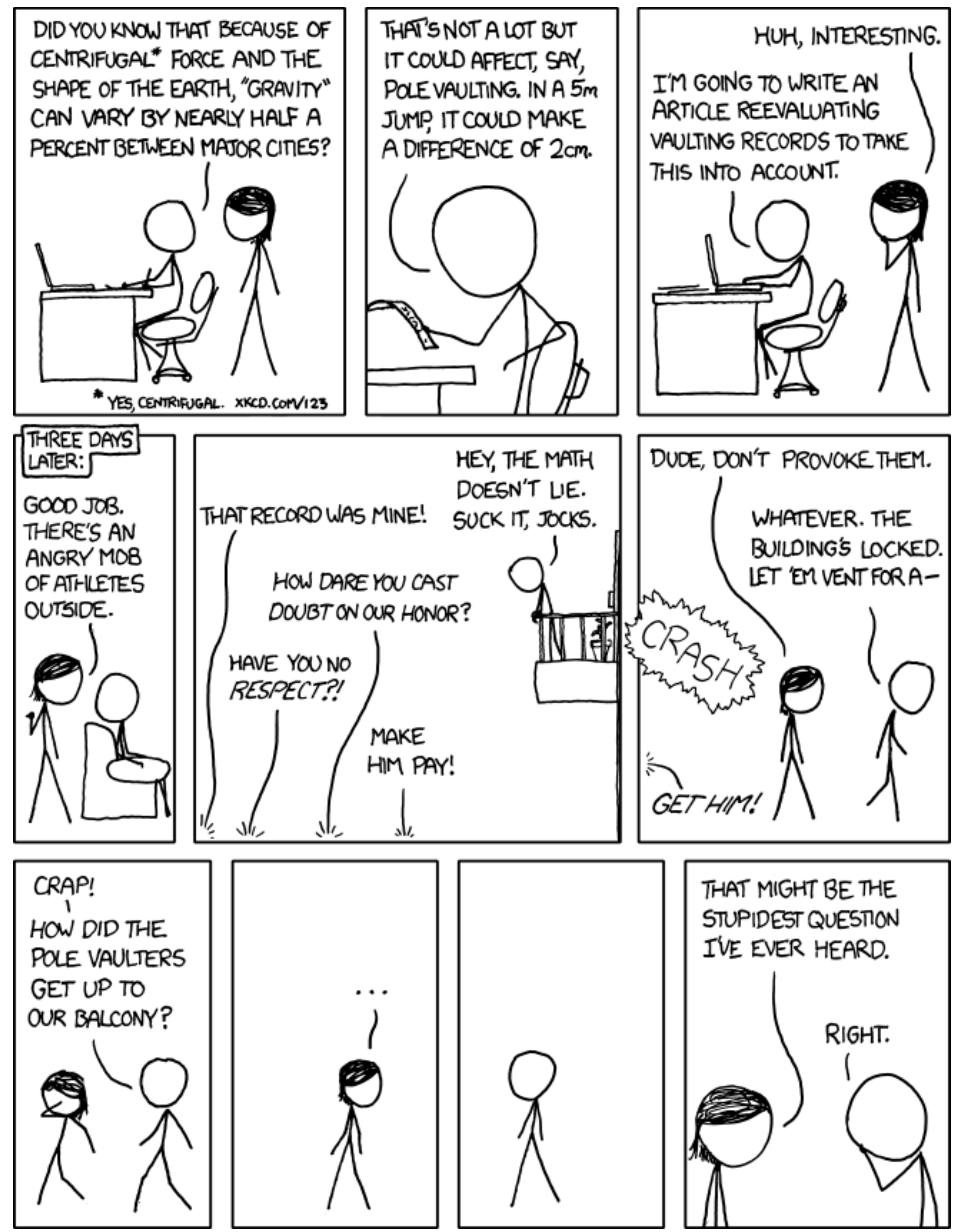

xkcd.com/852

Today's agenda (MIT 3 and 4)

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1. Motion in two and three dimensions in Cartesian coordinates

    - Acceleration due to gravity
    - Introduction to projectile motion
    - Using vectors

Summary of motion in one dimension

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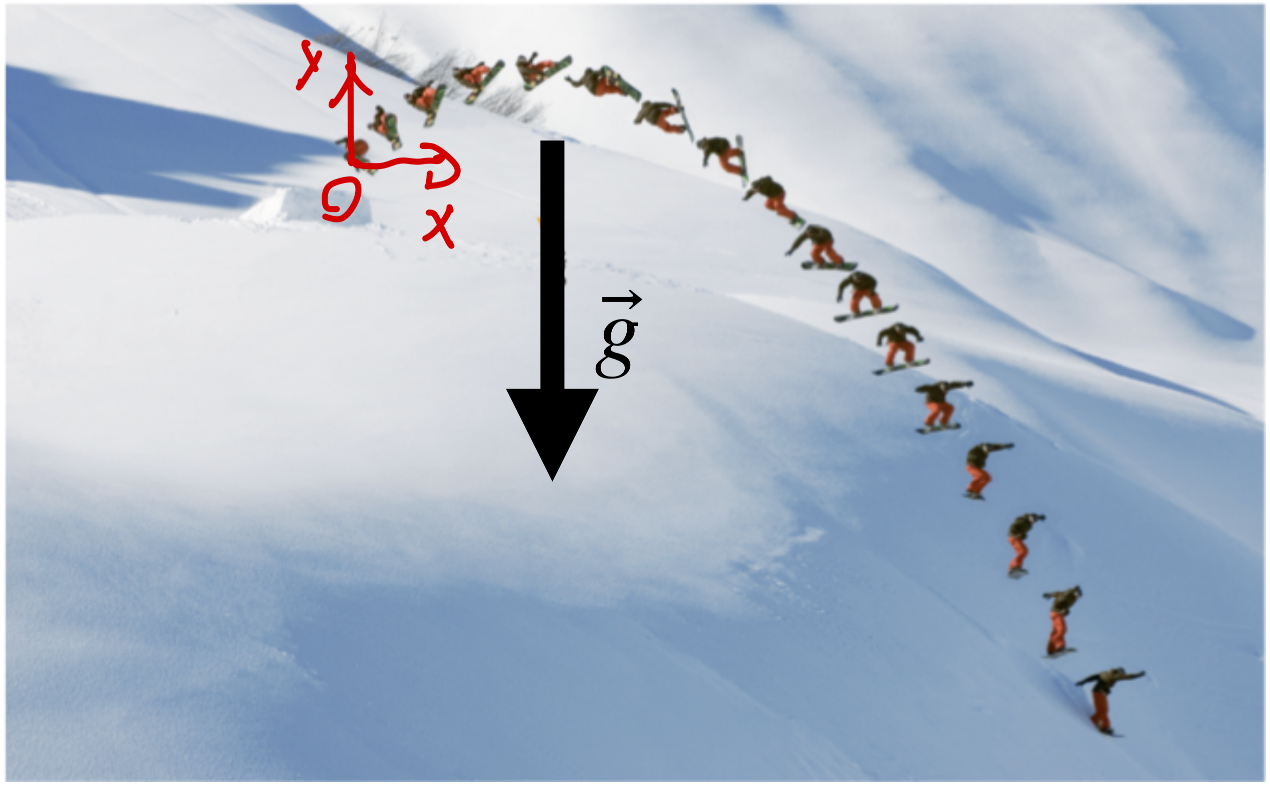
- Position of an object as a function of time denoted by $x(t)$
- Average velocity: $\bar{v} = \dfrac{\text{change of position}}{\text{time elapsed}} = \dfrac{\Delta x}{\Delta t}$
- Instantaneous velocity: $v(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
- Average acceleration: $\bar{a} = \dfrac{\text{change of velocity}}{\text{time elapsed}} = \dfrac{\Delta v}{\Delta t}$
- Instantaneous acceleration:

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Projectile motion

1D → **2D**



A projectile is an object moving in 2D under the sole influence of the Earth's gravity

Review: scalars and vectors

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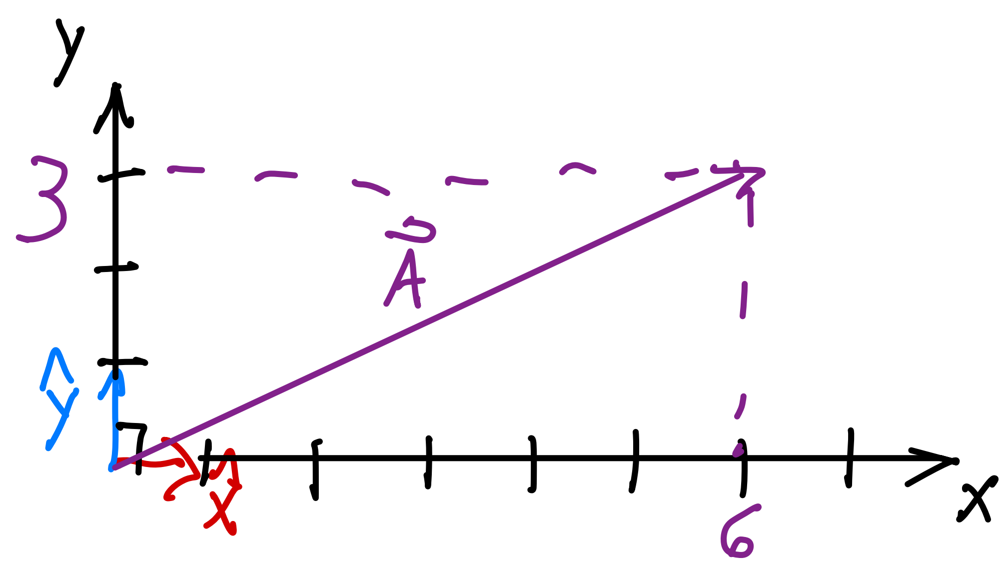
- A **scalar** quantity consists of a single number
  - Examples: distance traveled, speed, mass, time
- A **vector** quantity is a set of numbers, which we will use to give direction
  - A vector quantity is often indicated by putting an arrow over the top (e.g. $\vec{v}$)
  - You can visualize a vector as an arrow, which has a length (i.e. $|\vec{v}| = v$) together with an direction (e.g. $\hat{x}$)
  - Becomes very important for 2D or 3D motion
  - Examples: displacement, velocity, acceleration, force, momentum

Review: define coordinate system by vectors

- A Cartesian coordinate system can be defined using a set of orthogonal *unit vectors* (i.e. vectors of length 1):



$\vec{A} = 6\hat{x} + 3\hat{y}$

- Any vector can be expressed as a sum of its components parallel to the unit vectors

$$\vec{R} = \underline{R_x}\,\hat{x} + \underline{R_y}\,\hat{y} = \begin{bmatrix} R_x \\ R_y \end{bmatrix}$$

Review: getting components of vectors

- Here $A = |\vec{A}|$ is the "norm" (i.e. length or magnitude) of a vector:

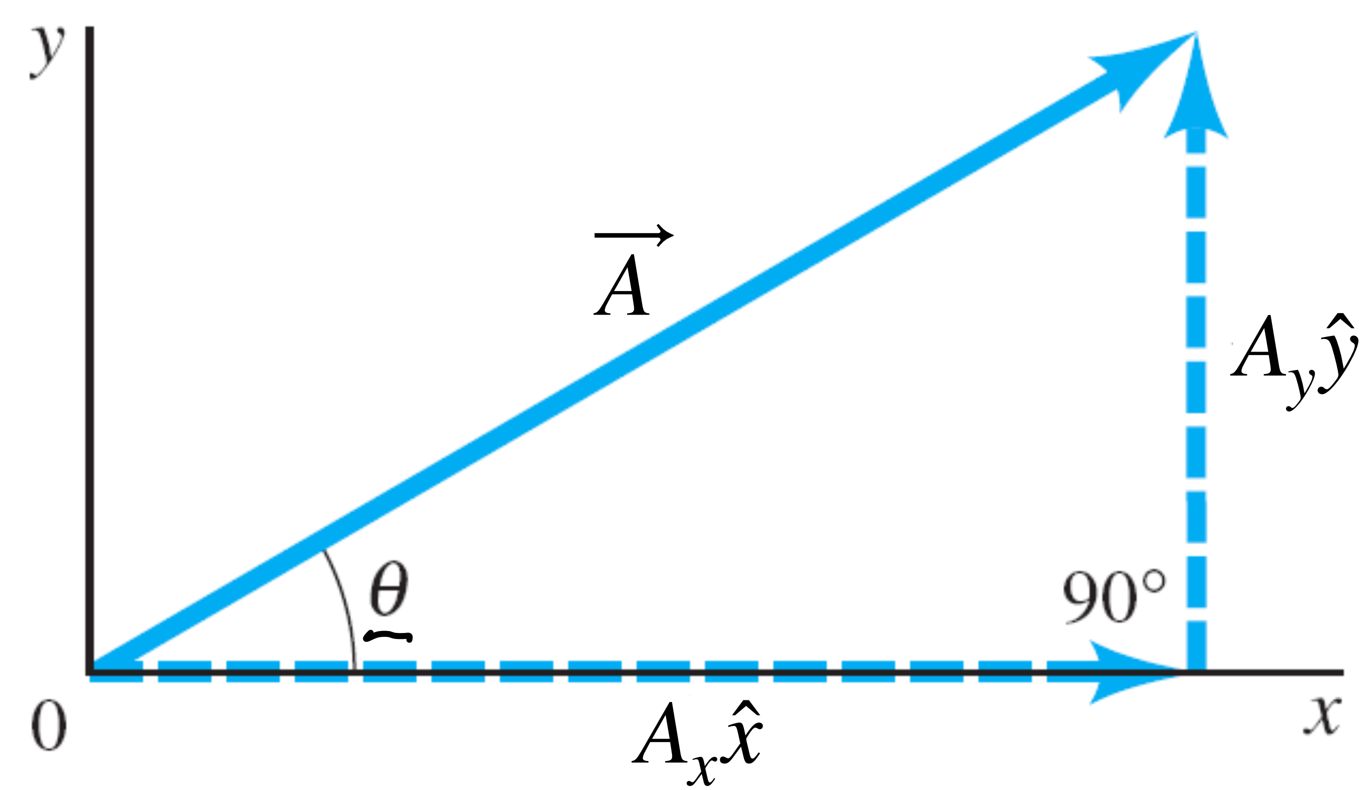
$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

- Since the components are orthogonal, they are related by simple trigonometric functions:

$$\cos(\theta) = \frac{A_x}{A}$$

$$\sin(\theta) = \frac{A_y}{A}$$

$$\tan(\theta) = \frac{A_y}{A_x}$$



Review: math with vectors

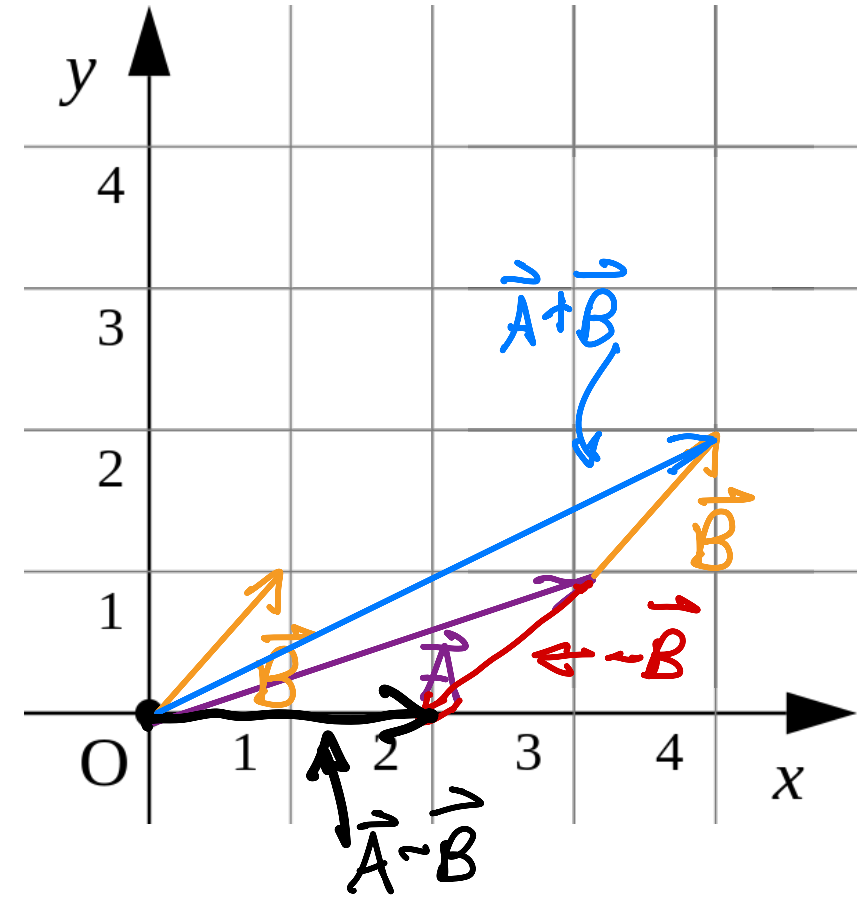
- Vector addition and subtraction are accomplished by adding or subtracting component-by-component

$$\vec{A} = 3\hat{x} + 1\hat{y} = 3\hat{x} + \hat{y}$$

$$\vec{B} = \hat{x} + \hat{y}$$

$$\vec{A} + \vec{B} = (3\hat{x} + \hat{y}) + (\hat{x} + \hat{y})$$
$$= 3\hat{x} + \hat{x} + \hat{y} + \hat{y} = 4\hat{x} + 2\hat{y}$$

$$\vec{A} - \vec{B} = (3\hat{x} + \hat{y}) - (\hat{x} + \hat{y})$$
$$= 3\hat{x} - \hat{x} + \hat{y} - \hat{y} = 2\hat{x}$$



- Multiplying (or dividing) by a scalar

$$c\vec{A} = c(A_x\hat{x} + A_y\hat{y}) = (cA_x)\hat{x} + (cA_y)\hat{y}$$

Review: dot product between two vectors

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- Mathematical definition of a dot (scalar) product

$$\hat{x}\cdot\hat{x} = 1 = \hat{y}\cdot\hat{y}$$
$$\hat{x}\cdot\hat{y} = 0 = \hat{y}\cdot\hat{x}$$

$$\vec{A}\cdot\vec{B} = (A_x\hat{x} + A_y\hat{y})\cdot(B_x\hat{x} + B_y\hat{y})$$
$$= \underbrace{(A_x\hat{x})\cdot(B_x\hat{x})}_{A_xB_x(\underbrace{\hat{x}\cdot\hat{x}}_{=1})} + \underbrace{(A_x\hat{x})\cdot(B_y\hat{y})}_{A_xB_y\cancel{(\hat{x}\cdot\hat{y})}} + \underbrace{(A_y\hat{y})\cdot(B_x\hat{x})}_{A_yB_x\cancel{(\hat{y}\cdot\hat{x})}} + \underbrace{(A_y\hat{y})\cdot(B_y\hat{y})}_{A_yB_y(\underbrace{\hat{y}\cdot\hat{y}}_{=1})}$$
$$= A_xB_x + A_yB_y$$

- Dot product with a unit vector gives the component in that direction

$$\vec{A}\cdot\hat{x} = (A_x\hat{x} + A_y\hat{y})\cdot x = A_x\underbrace{\hat{x}\cdot\hat{x}}_{=1} + A_y\cancel{\hat{y}\cdot\hat{x}} = A_x$$

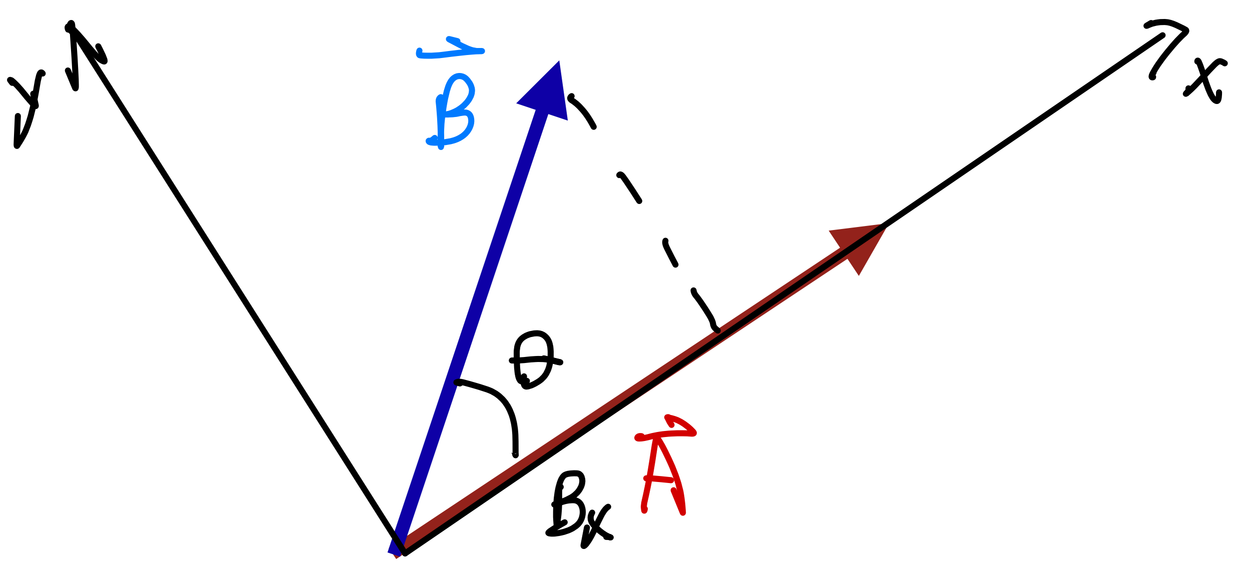
Review: dot product between two vectors

- Geometric interpretation of dot product

In this case $\vec{A} = A\hat{x} \Rightarrow \vec{A}\cdot\vec{B} = (A\hat{x})\cdot(B_x\hat{x} + B_y\hat{y})$

$= AB_x$ but $\cos(\theta) = \frac{B_x}{B}$

$= \boxed{AB\cos(\theta)}$



Conceptual question

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Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are shown below. The vector sum of these three vectors is $\vec{S} = \vec{A} + \vec{B} + \vec{C}$.

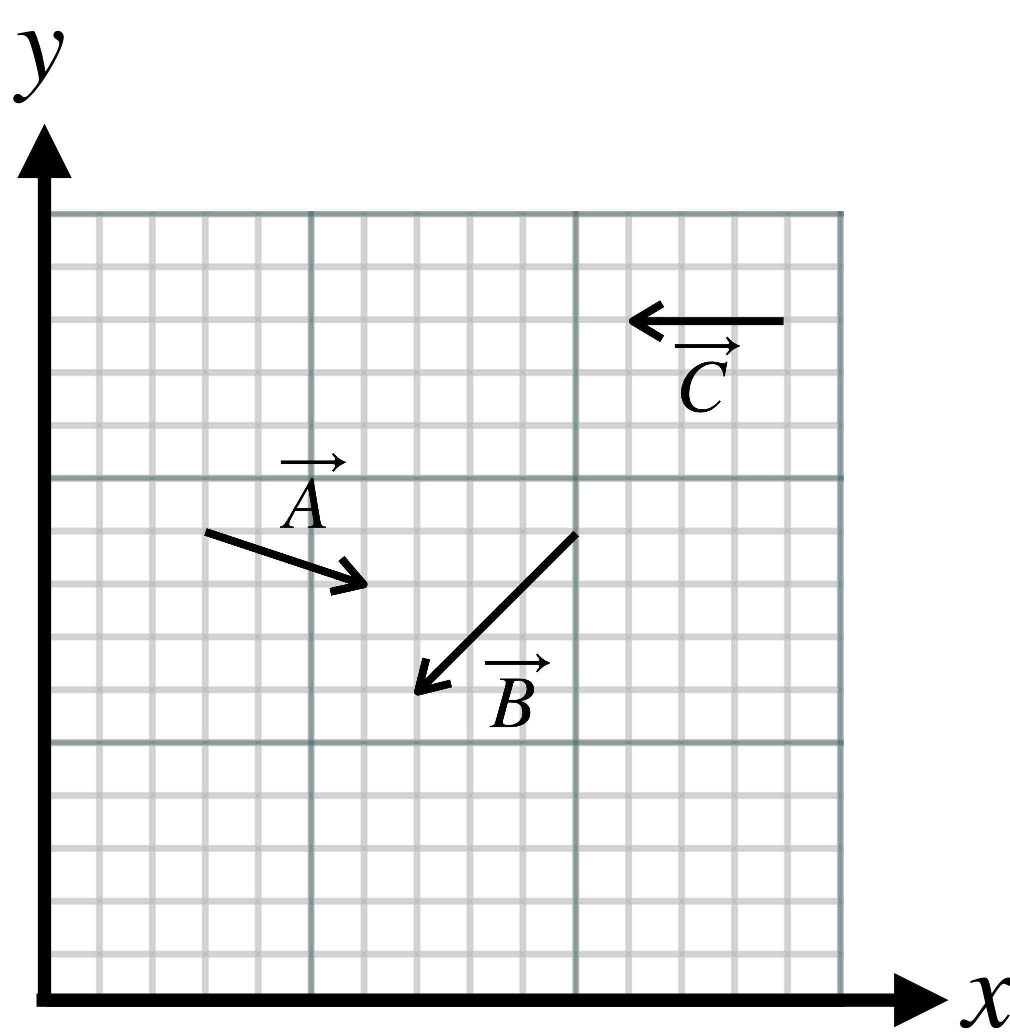
What is the value of S_x, the x component of $\vec{S}$?

A. -6

B. -4

C. 3

D. -3



$\vec{A} = 3\hat{x} - \hat{y}$
$\vec{B} = -3\hat{x} - 3\hat{y}$
$\vec{C} = -3\hat{x}$

Conceptual question

responseware.eu
Session ID: epflphys101en

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A particle is moving while experiencing a constant acceleration. The velocity vector is shown below at two different times, an earlier time t_1 and a later time is t_2.

What is the direction of the average acceleration vector?

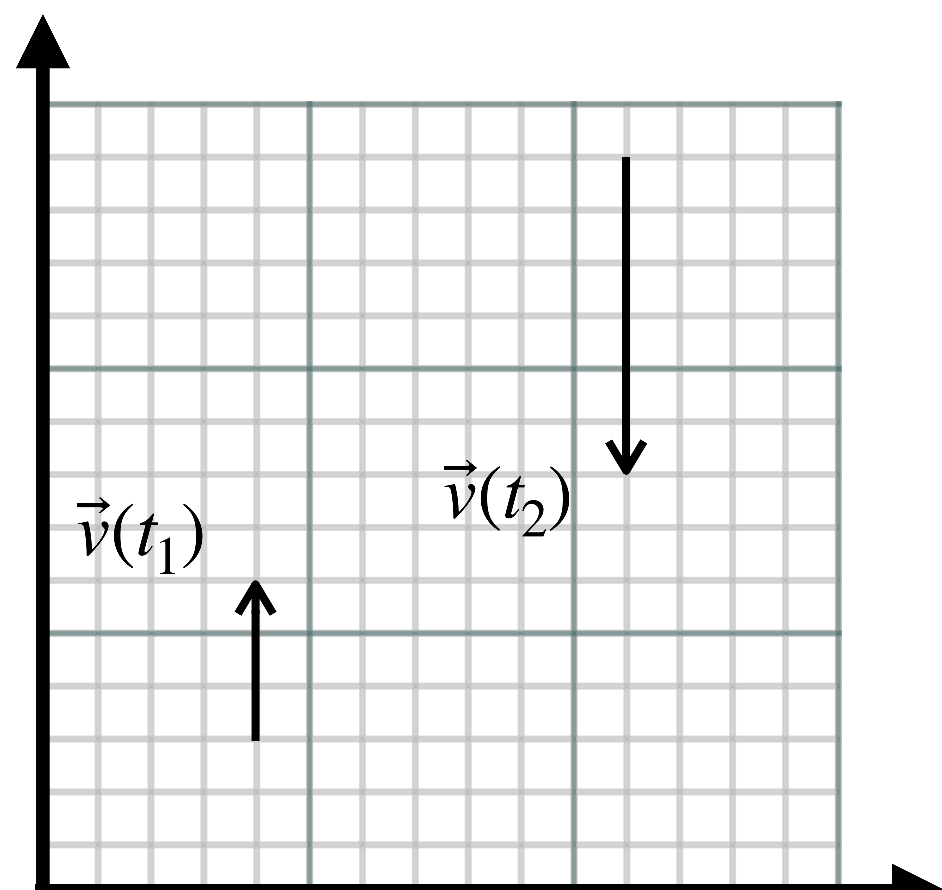
A. ↑

B. →

C. ↓

D. ←

E. None of these.



$\vec{v}(t_2) - \vec{v}(t_1) = \Delta\vec{v}$

See you at the exercises tomorrow!

- Wednesdays from 17:15 to 19:00
  - Don’t forget to sign up for a tutoring group on Moodle
  - Exercises will be found on the Moodle (bring your own paper copy or way to access them digitally)