General Physics: Mechanics

PHYS-101(en)

Lecture 14a: Damped oscillations, rolling and review

Dr. Marcelo Baquero
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December 15th, 2025

EPFL Swiss Plasma Center

Today's agenda (Serway 10,15; MIT 20)

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1. **Course feedback**
2. Damped oscillations and resonance
3. Review: Rolling, circular motion
4. More review

Mock exam 2: takeaways

Swiss Plasma Center

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- Pick up your exam later today during one of the breaks

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  - **No smelly or noisy food/snacks please!**

Announcement: Review sessions

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- There will be two Review sessions in January
  - First one on Thursday, Jan. 8th, from 10h to 12h
  - Second one on Tuesday, Jan. 13th, 10h to 12h
  - Room numbers to be announced on Moodle
  - You can come and ask any questions you might have (from lectures, exercises, mock exams, etc.)

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Conceptual question

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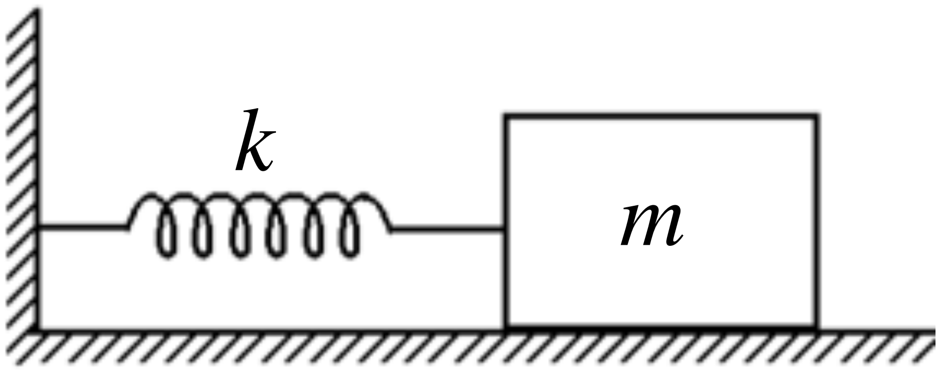
Session ID: epflphys101en

A block of mass m is attached to a spring with spring constant k. It is free to slide along a horizontal frictionless surface. At $t = 0$ the block-spring system is released from rest at a displacement $x_0 > 0$ from the equilibrium position. What is the velocity of the block when it first passes through the equilibrium position?



A. $v = -2x_0\sqrt{k/m}$

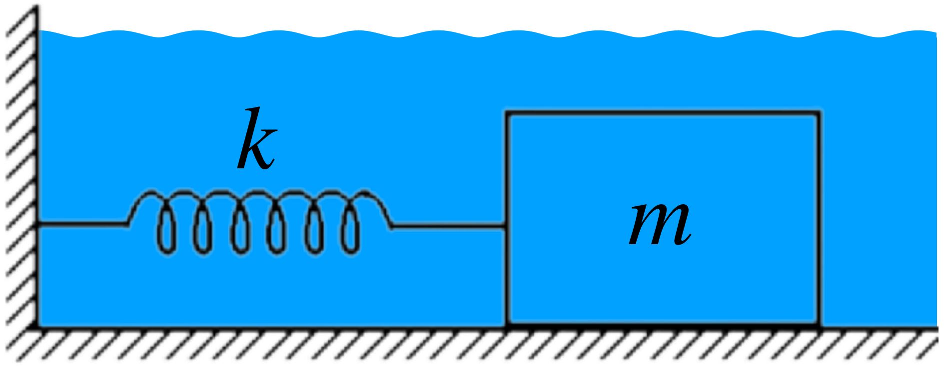
B. $v = -x_0\sqrt{k/m}$

C. $v = x_0\sqrt{k/m}$

D. $v = 2x_0\sqrt{k/m}$

Damped oscillation

- Remember viscous drag (lecture 6)?



$$\vec{F}_{drag} = -\beta v \hat{v}$$

Damped oscillation

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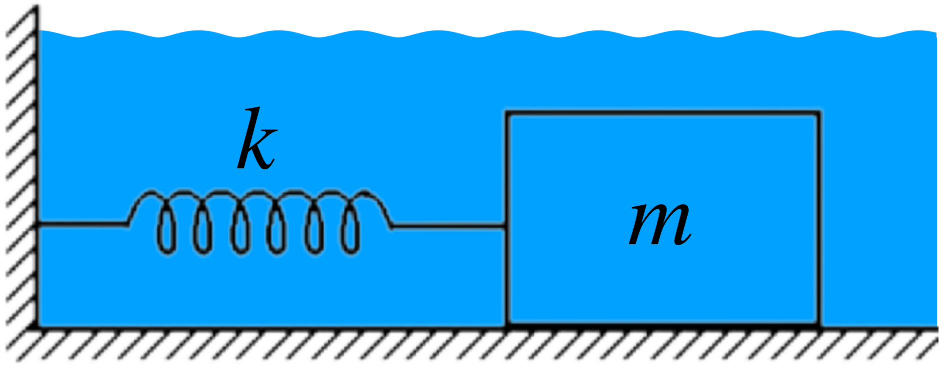

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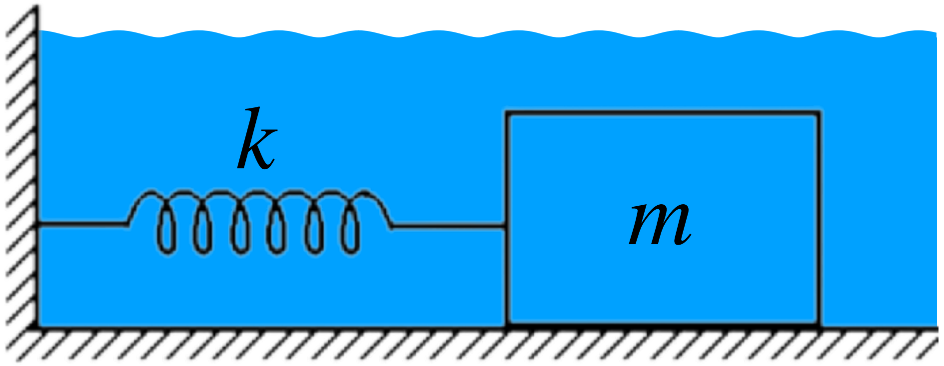

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- Equation of motion becomes

$$-\beta\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

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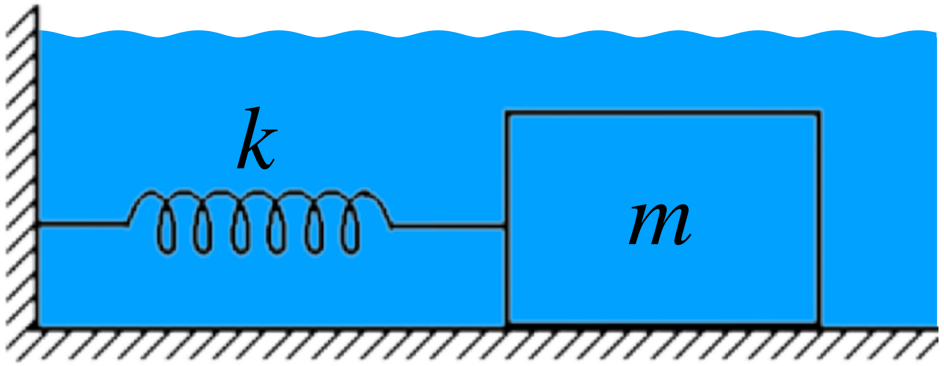

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$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = 0$$

Damped oscillation

Solutions of

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Damped oscillation

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- If $\lambda^2 < \omega_0^2$ I define $\omega_f^2 = \omega_0^2 - \lambda^2$. Then

$$x(t) = A\,e^{-\lambda t}\cos(\omega_f\,t + \varphi)$$

Damped oscillation

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- If $\lambda^2 = \omega_0^2 \Rightarrow$ $$x(t) = (A_1 + A_2 t)\, e^{-\lambda t}$$

Damped oscillation

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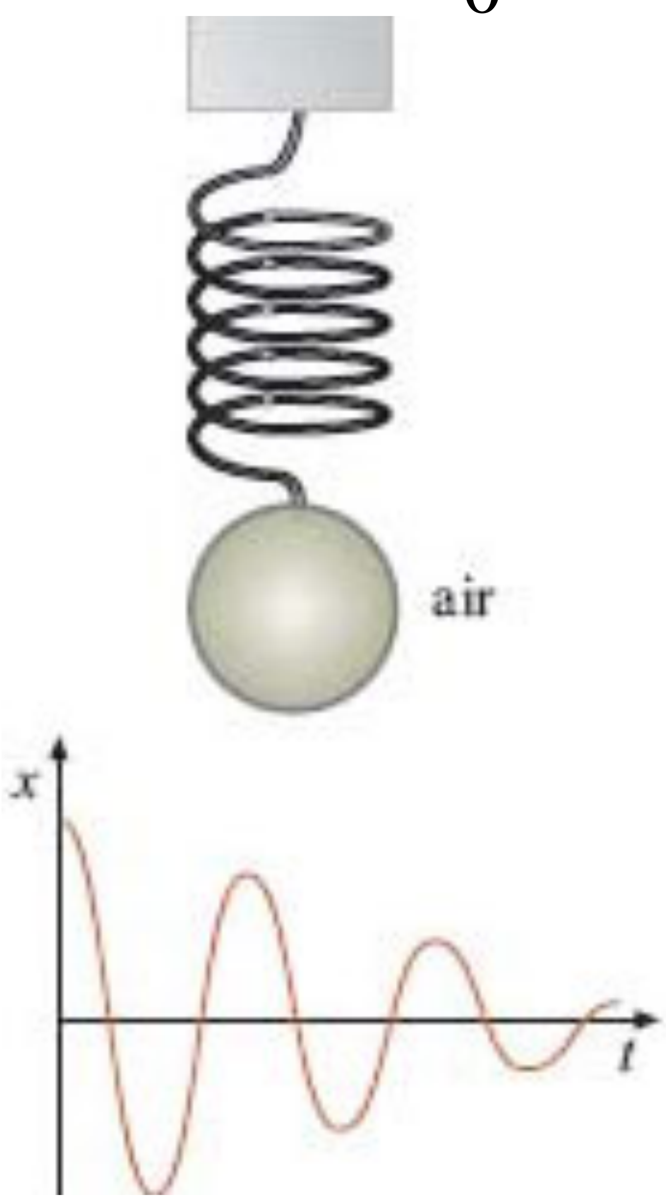
- If $\lambda^2 > \omega_0^2$ I define $-\lambda_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \omega_0^2}$. Then

$$x(t) = A_1\, e^{-\lambda_1 t} + A_2\, e^{-\lambda_2 t}$$

Three cases of damped oscillation

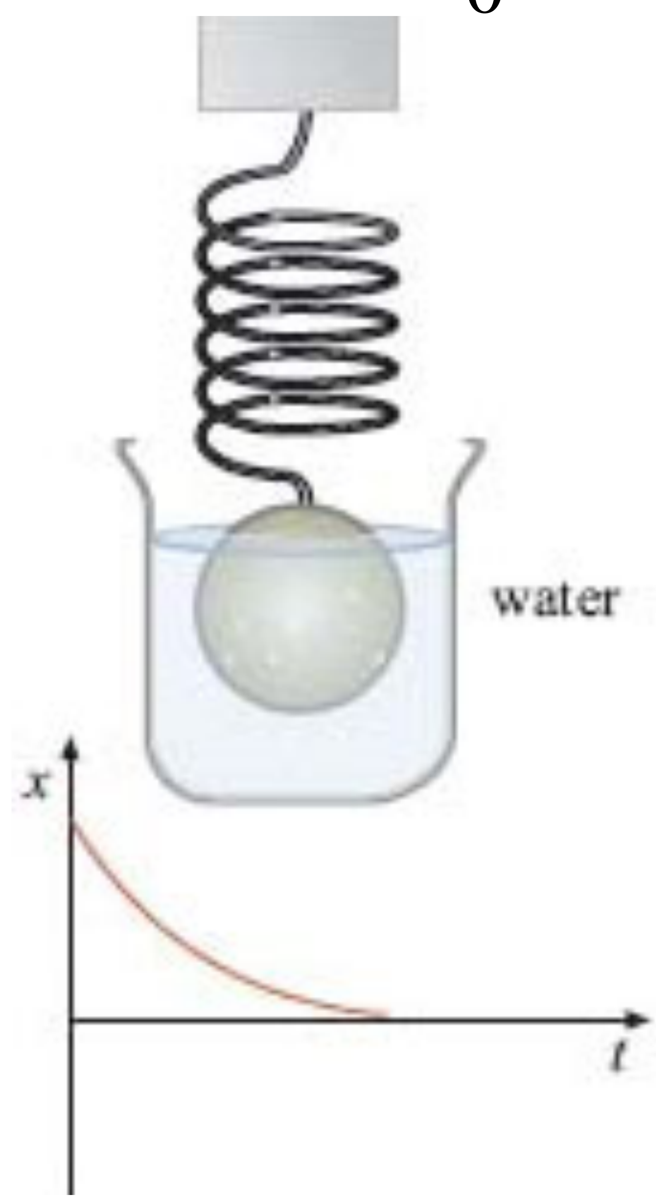
Weak damping

$$\lambda^2 < \omega_0^2$$



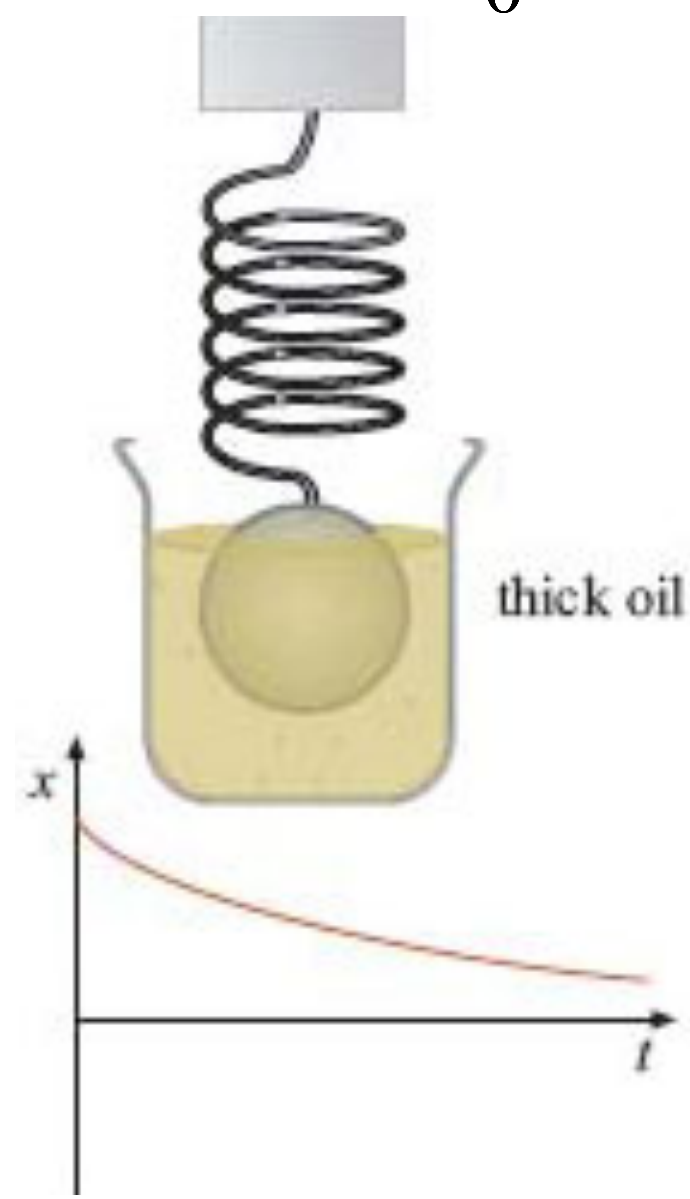
Critical damping

$$\lambda^2 = \omega_0^2$$



Strong damping

$$\lambda^2 > \omega_0^2$$



Forced oscillation

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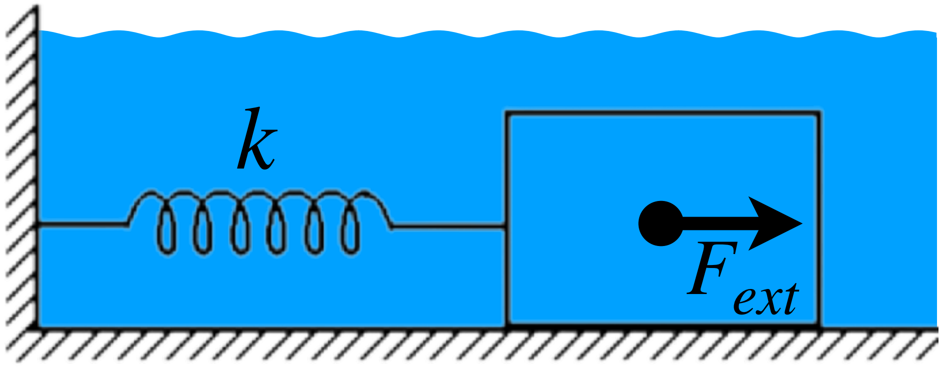

- Now let's add yet another term, an external driving force

$$F_{ext}(t) = F_d \cos(\omega_d t)$$

- This represents the influence of an externally applied force with amplitude F_d and angular frequency ω_d

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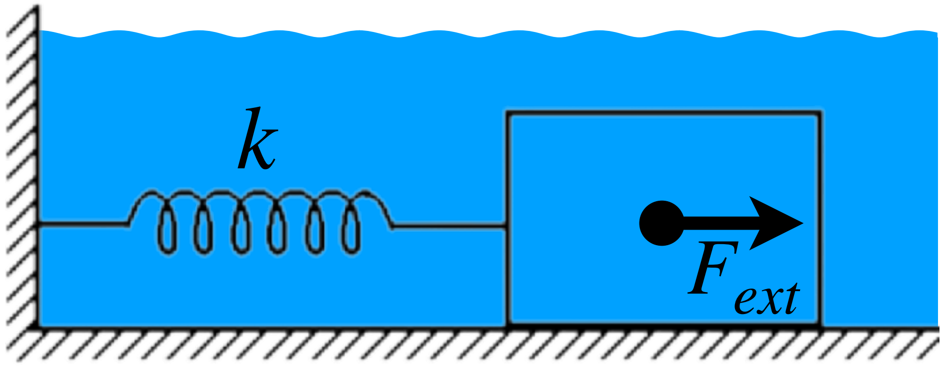

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- Equation of motion becomes

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = \frac{F_d}{m}\cos(\omega_d t)$$

Forced oscillation

- The solution is

$$x(t) = \left(\begin{matrix}\text{homogeneous} \\ \text{solution}\end{matrix}\right) + A_d \cos(\omega_d t + \varphi_d)$$

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$$x(t) = \begin{pmatrix}\text{homogeneous}\\ \text{solution}\end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

$$A_d = A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

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and the *forced* phase is

$$\varphi_d = \varphi_d(\omega_d) = \tan^{-1}\left(\frac{2\lambda\omega_d}{\omega_d^2 - \omega_0^2}\right)$$

Resonance

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- When the driving frequency and natural frequency are very close, $\omega_d \approx \omega_0$, weird stuff can happen

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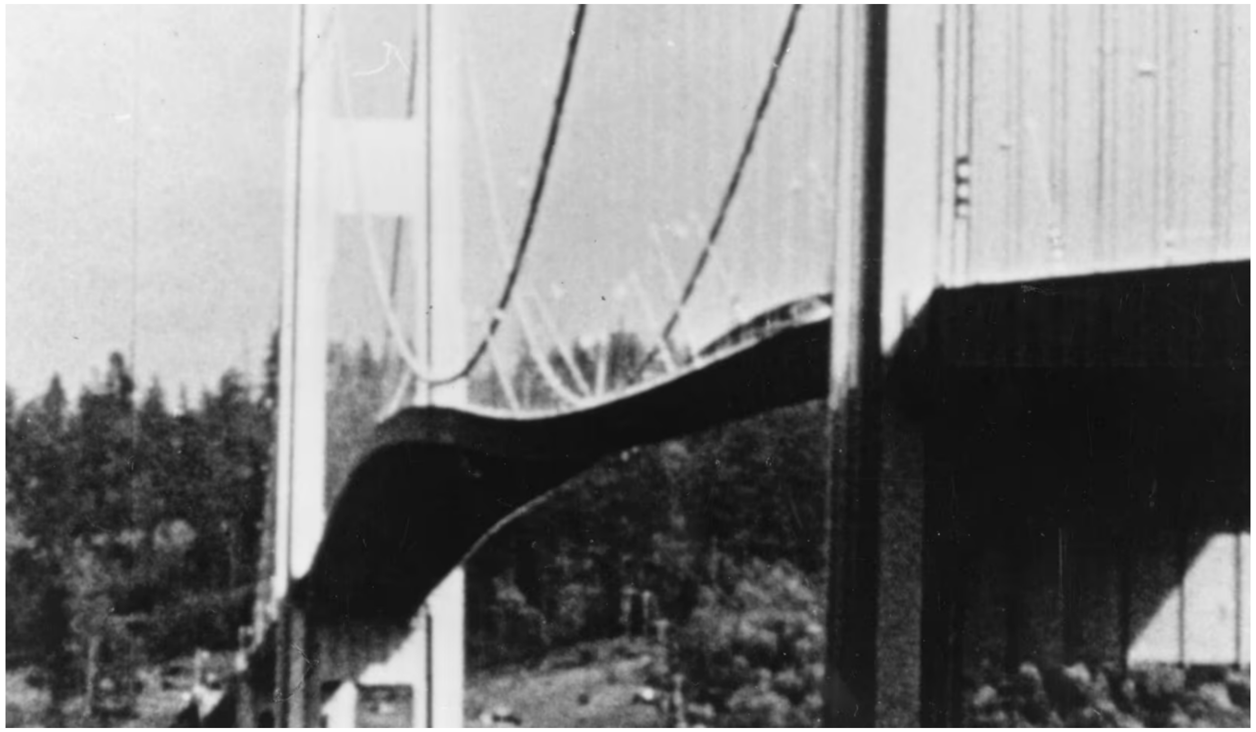

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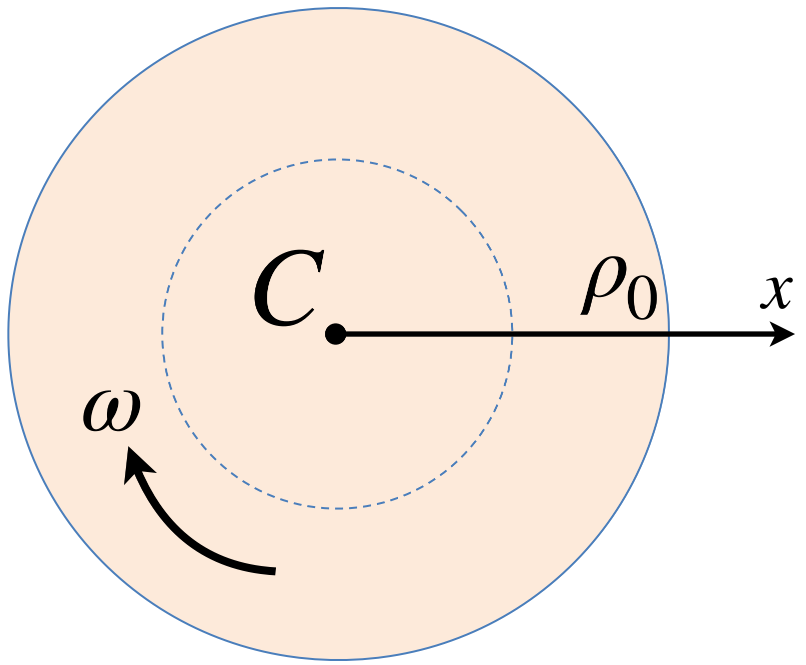
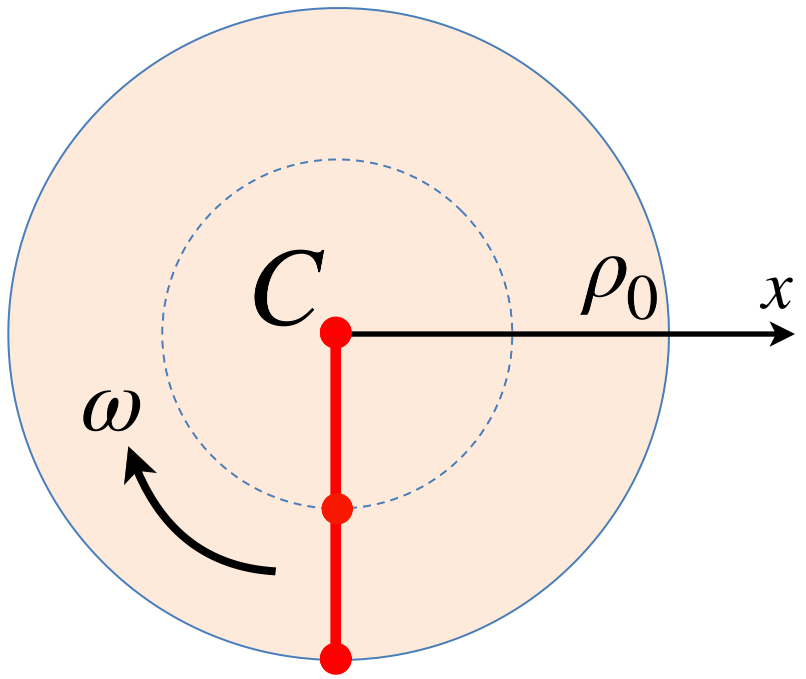
Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C



Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt



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Pure rotation of a rigid body

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- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α
- The distance they move is the arc length $\Delta\ell(\rho) = \rho\Delta\phi$, so any point p has $\vec{v}_{Cp} = \dfrac{\Delta\ell}{\Delta t}\hat{\phi} = \dfrac{\rho\Delta\phi}{\Delta t}\hat{\phi} = \rho\omega\,\hat{\phi}$ <u>in the CM frame</u>

Rolling without slipping

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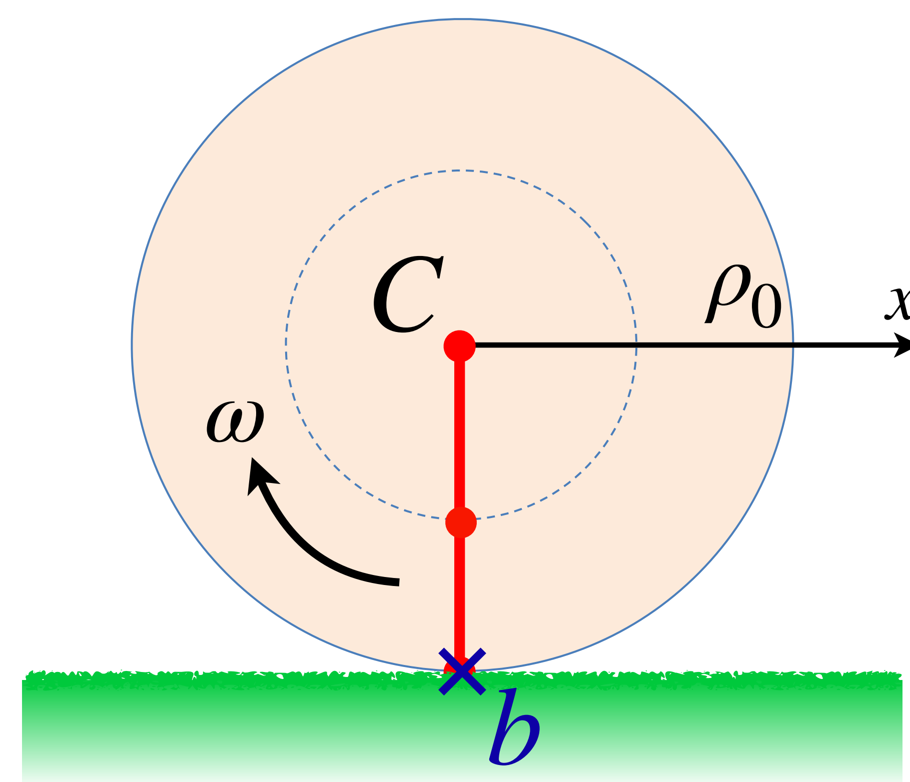
- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

$$\vec{v}_{gb} = 0$$

<u>in the ground frame of reference</u>



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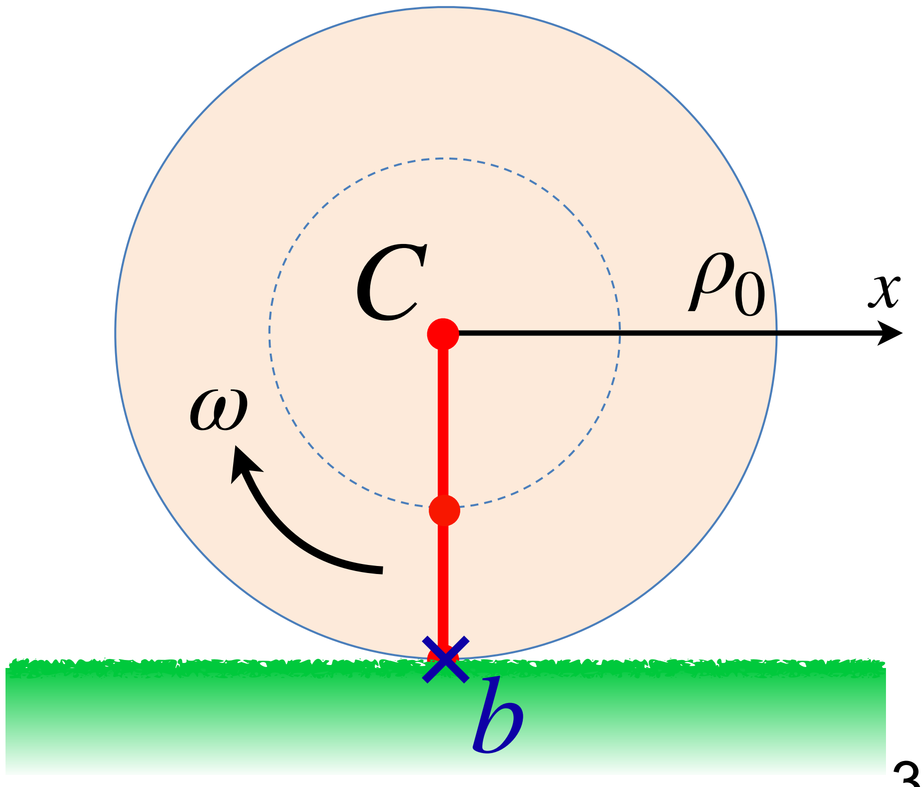
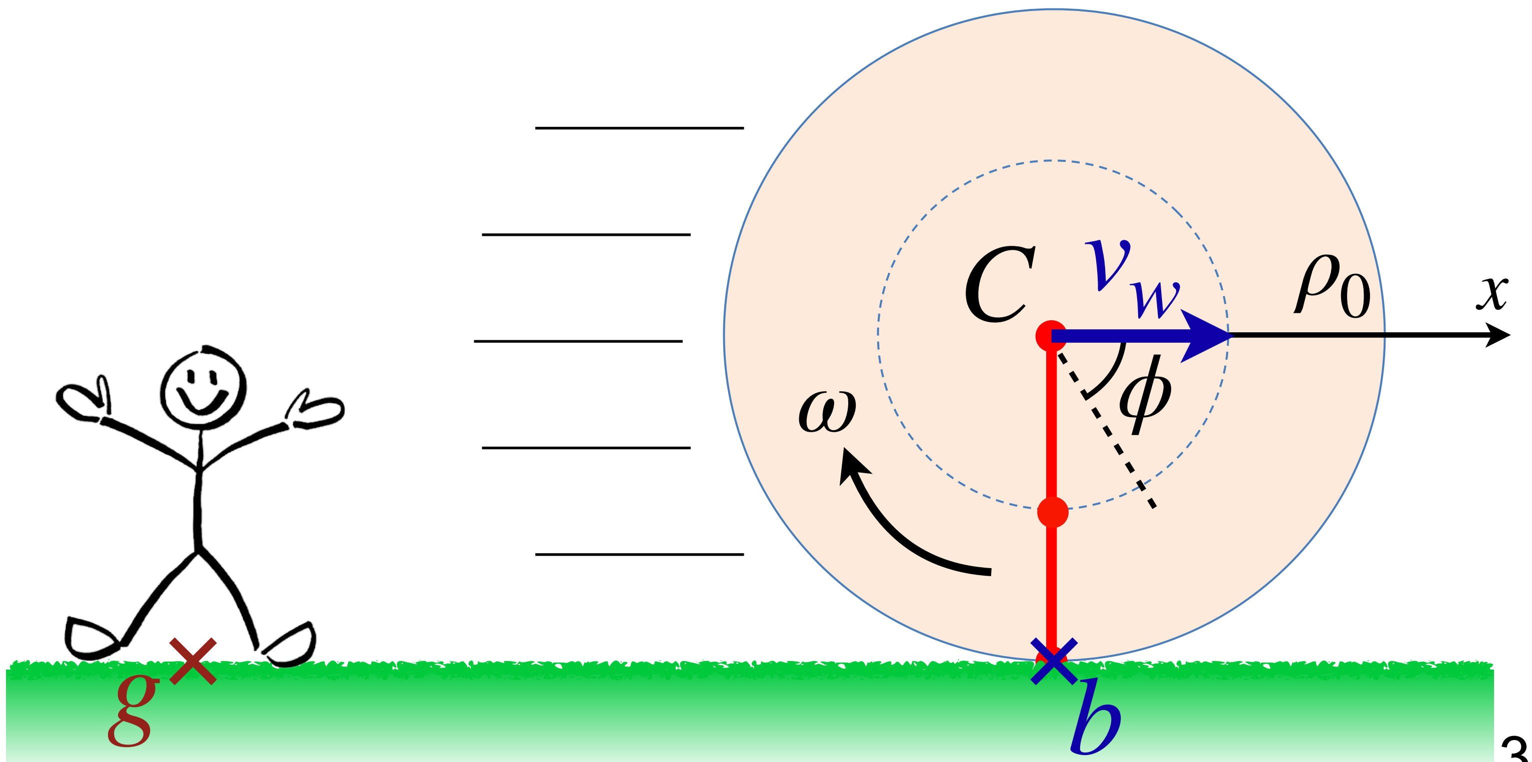
- This means the friction is static, points in the opposite direction to translational acceleration, and does no work



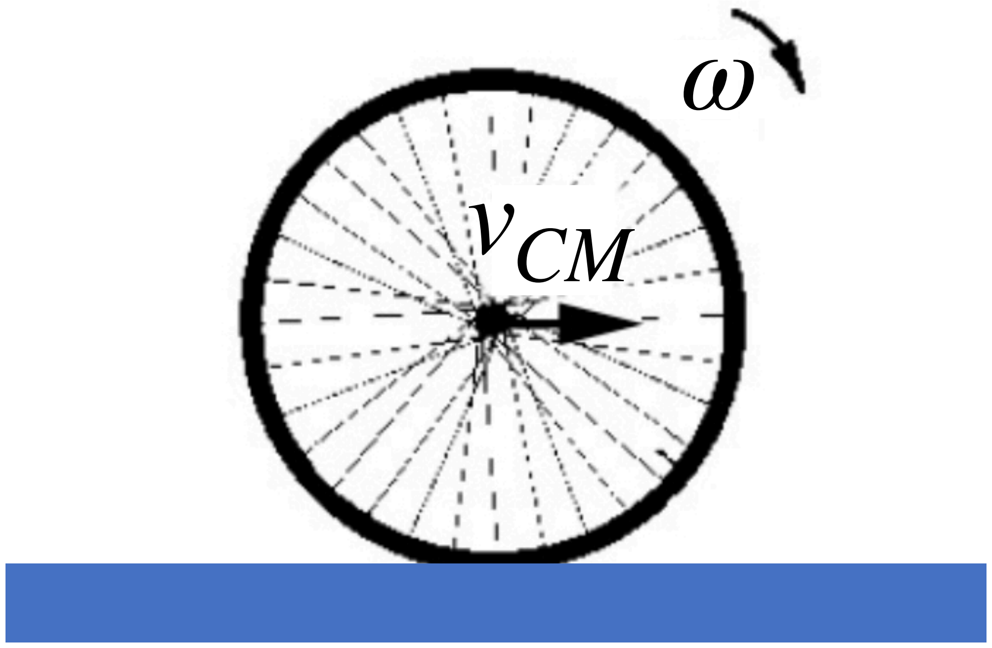
Rolling velocity and switching reference frames

- In the CM frame, all points on the rim have $\vec{v}_{Cp} = \rho_0 \omega\, \hat{\phi}$
- In the frame of reference of the ground, the point touching the ground b has $\vec{v}_{gb} = 0$



Conceptual question

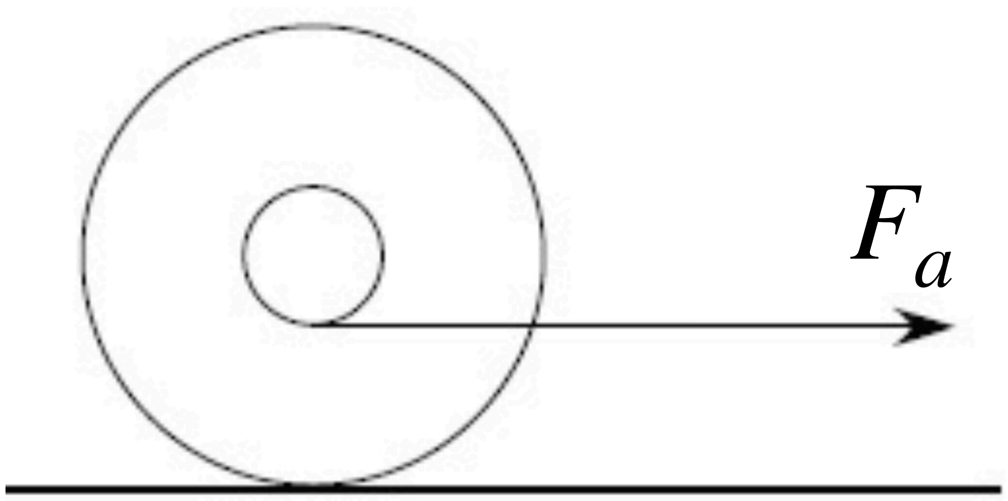
A bicycle wheel is initially spinning in the air and then is put into contact with a rough surface (see figure). It slips against the surface. What is the direction of the kinetic friction force acting on the wheel?



A. Points to the right
B. Points to the left
C. Points up
D. Points down

Example: Yo-yo

A yo-yo of mass m is placed on a rough surface and rolls without slipping. It is composed of two disks separated by a spindle with a smaller diameter. A string is wound around the spindle and pulled with a force F_a. In which direction does it move? To the right, winding up the string, or to the left, unwinding the string?



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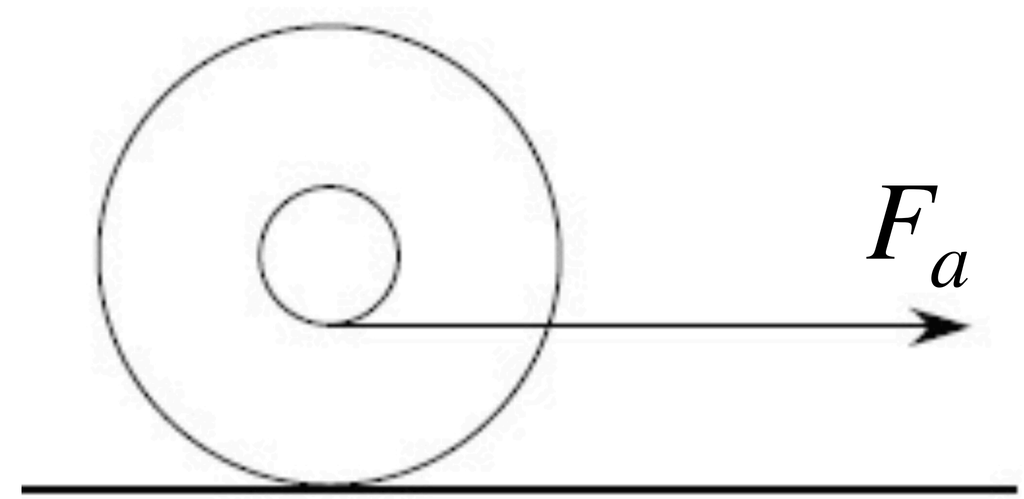

DEMO (45)

Rolling with slipping

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Conceptual question

Which one of the following physical quantities is **not** a *vector*?

A. Position
B. Impulse
C. Torque
D. Work
E. Displacement

Conceptual question

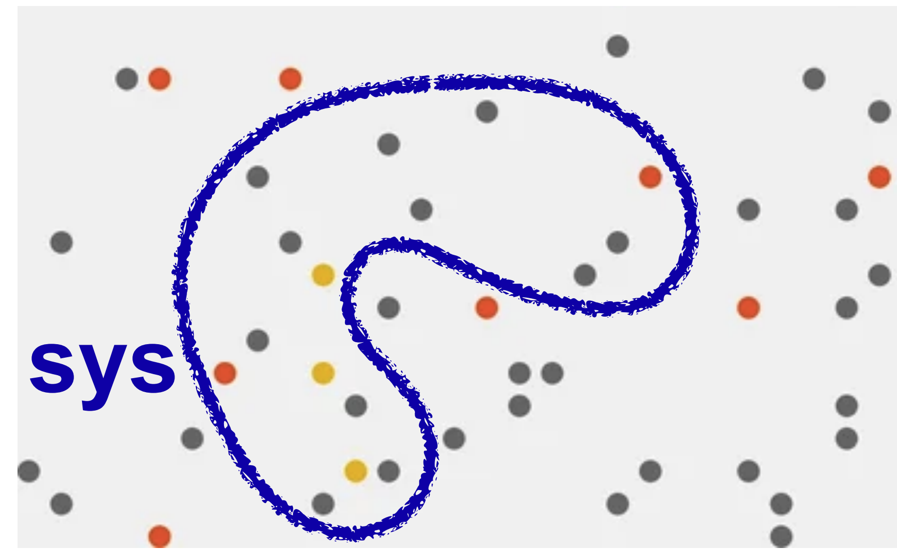
Which one of the following *scalar* quantities **can** be negative?

A. Mass
B. Moment of inertia
C. Work
D. Kinetic energy
E. Spring constant

Conceptual question

The net *force* acting on sys (the physical system on the right) is

A. The largest force exerted on any point anywhere
B. The largest force exerted on any point in sys
C. Zero
D. The sum of all forces internal to sys
E. The sum of all forces exerted by points **not** in sys on points in sys



Conceptual question

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We look at sys from an inertial reference frame and see that $\overrightarrow{F}^{ext}_{net} = 0$. There is no matter exchange with the outside. Which one of the following statements is **not** true?

A. The internal forces add up to zero
B. The total momentum of sys is conserved
C. The momentum of each individual particle is guaranteed to stay constant
D. The position of the center-of-mass (CM) is well defined
E. The velocity of the CM is constant