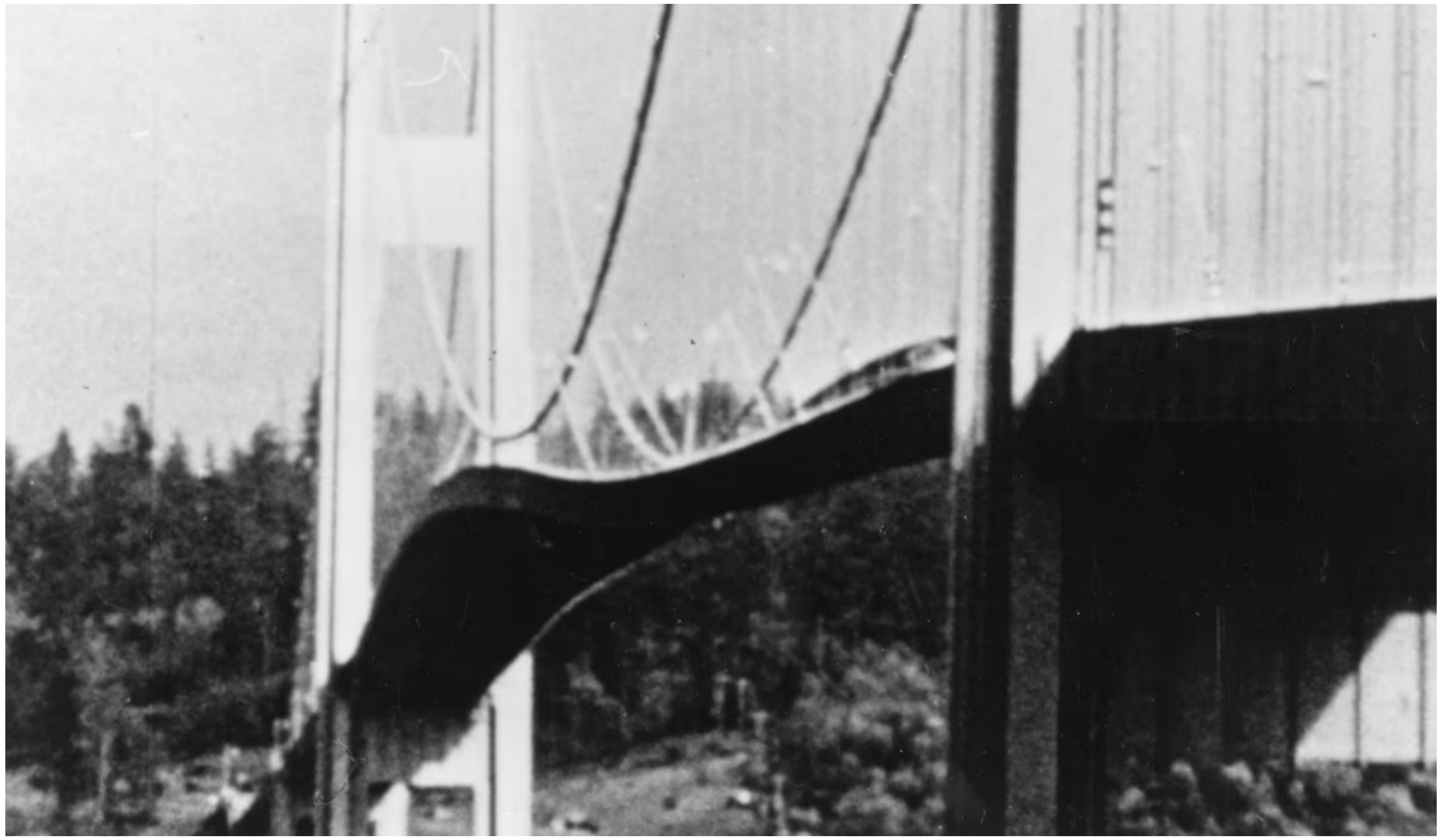
General Physics: Mechanics

PHYS-101(en)

Lecture 14a: Damped oscillations, rolling and review



Dr. Marcelo Baquero
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December 15th, 2025

EPFL
Swiss Plasma Center

Today's agenda (Serway 10,15; MIT 20)

Swiss Plasma Center

1. **Course feedback**
2. Damped oscillations and resonance
3. Review: Rolling, circular motion
4. More review

Mock exam 2: takeaways

Swiss Plasma Center

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  - More of you turned in exam for grading!
  - Check solutions on Moodle and comments written by the TAs on your graded exams to find weaknesses that you can work on
- Pick up your exam later today during one of the breaks

Final exam: guidelines

Swiss Plasma Center

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  - **No smelly or noisy food/snacks please!**

Announcement: Review sessions

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- There will be two Review sessions in January
  - First one on Thursday, Jan. 8th, from 10h to 12h
  - Second one on Tuesday, Jan. 13th, 10h to 12h
  - Room numbers to be announced on Moodle
  - You can come and ask any questions you might have (from lectures, exercises, mock exams, etc.)

Today's agenda (Serway 10,15; MIT 20)

Swiss Plasma Center

1. Course feedback
2. **Damped oscillations and resonance**
3. Review: Rolling, circular motion
4. More review

Conceptual question

responseware.eu
Session ID: epflphys101en

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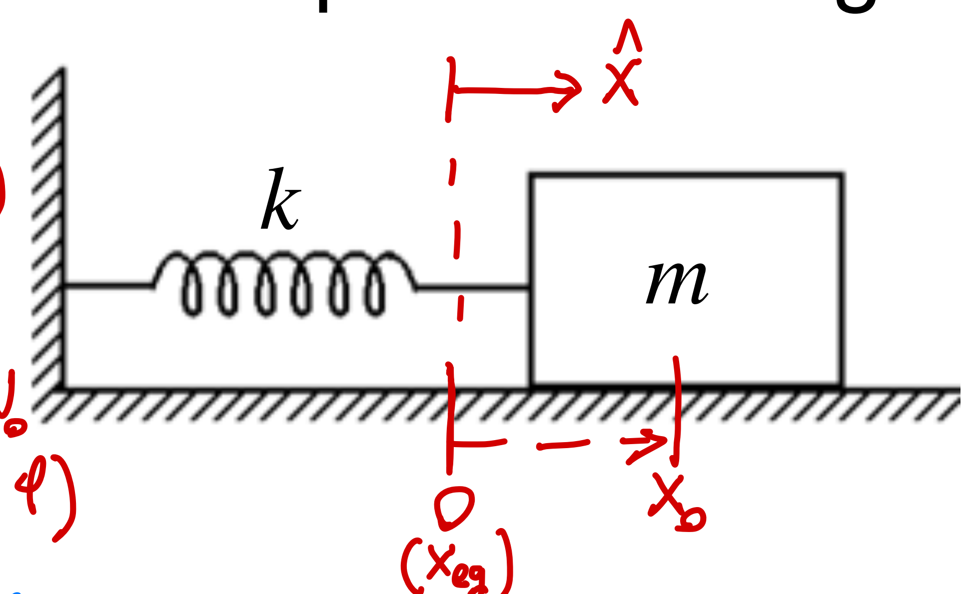
A block of mass m is attached to a spring with spring constant k. It is free to slide along a horizontal frictionless surface. At $t = 0$ the block-spring system is released from rest at a displacement $x_0 > 0$ from the equilibrium position. What is the velocity of the block when it first passes through the equilibrium position?



A. $v = -2x_0\sqrt{k/m}$

B. $v = -x_0\sqrt{k/m}$

C. $v = x_0\sqrt{k/m}$

D. $v = 2x_0\sqrt{k/m}$

$\omega_0 = \sqrt{\frac{k}{m}}$

$x(t) = A\cos(\omega_0 t + \varphi)$

$v(t) = \dot{x}(t)$

$= -A\sin(\omega_0 t + \varphi)\,\omega_0$

$= -A\omega_0 \sin(\omega_0 t + \varphi)$

$v(t=0) = 0 = -A\omega_0 \sin(\varphi) \Rightarrow \varphi = 0$ or $\varphi = \pi$

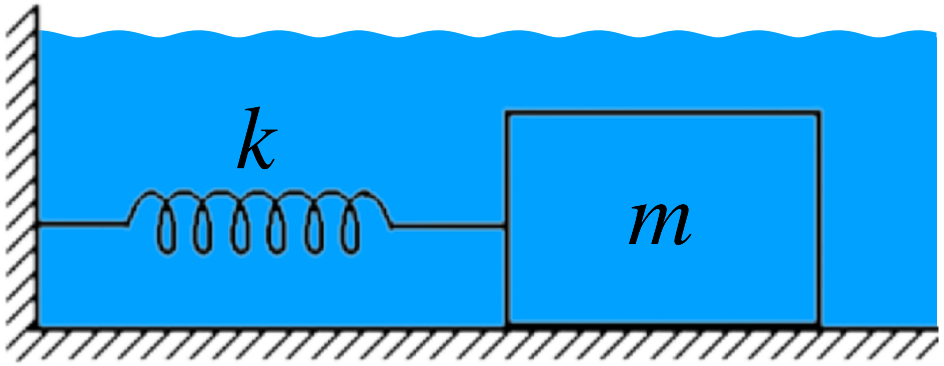
$x(t=0) = x_0 = A\cos(\varphi) \Rightarrow \varphi$ has to be 0 and then $A = x_0$

$x_{eq} = 0 = x_0\cos(\omega_0 t_{eq}) \Rightarrow \omega_0 t_{eq} = \frac{\pi}{2} \Rightarrow v_{eq} = v(t=t_{eq}) = -x_0\omega_0 \underbrace{\sin(\omega_0 t_{eq})}_{=1} = -x_0\omega_0$

Damped oscillation

- Remember viscous drag (lecture 6)?

$$\overrightarrow{F}_{drag} = -\beta v \hat{v}$$



Damped oscillation

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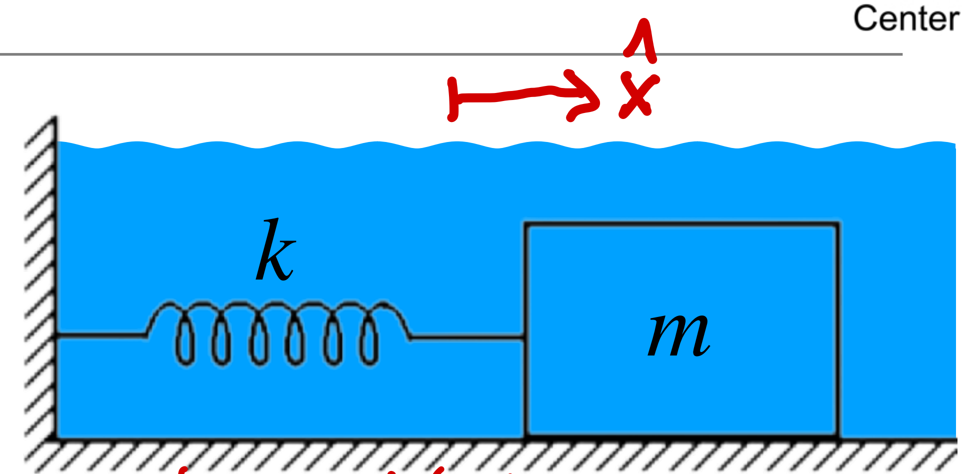

$$\vec{F}_{drag} = -\beta v \hat{v}$$

set equilibrium at $x=0$

- This represents damping, which is realistic for many physical systems

$v = \frac{dx}{dt}$ $\quad a = \frac{d^2x}{dt^2}$

$$\sum F_x: \quad F_s + F_d = -kx - \beta v = ma$$

$$\Rightarrow \quad -kx - \beta \frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Damped oscillation

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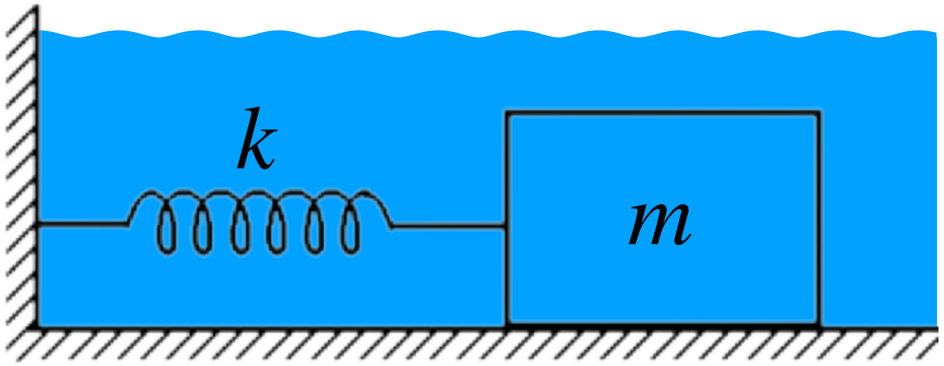

- This represents damping, which is realistic for many physical systems

- Equation of motion becomes

$$-\beta \frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$\Rightarrow m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0 \quad \Rightarrow \frac{d^2x}{dt^2} + \underbrace{\frac{\beta}{m}}_{\equiv 2\lambda}\frac{dx}{dt} + \overbrace{\frac{k}{m}}^{\omega_0^2} x = 0$$

$\lambda = \frac{1}{2}\frac{\beta}{m}$ (def. of λ)

Damped oscillation

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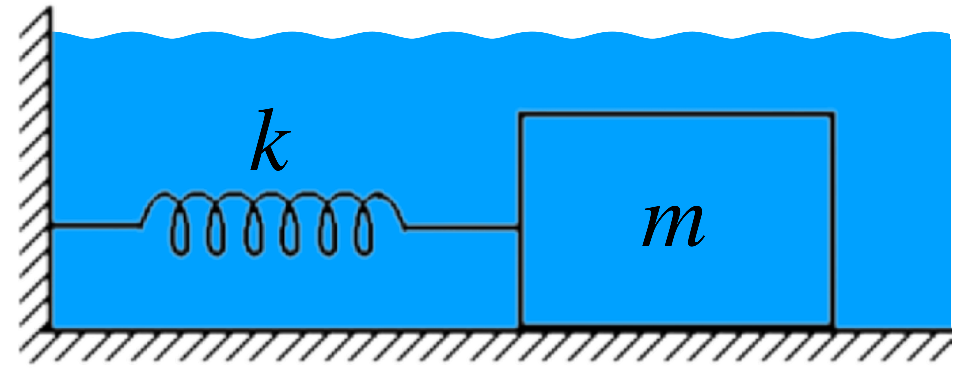

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- This represents damping, which is realistic for many physical systems

- Equation of motion becomes

$$-\beta\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = 0$$

$\omega_0 = \sqrt{\frac{k}{m}}$

$\lambda = \frac{1}{2}\frac{\beta}{m}$

Damped oscillation

Solutions of

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = 0$$

Damped oscillation

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Solutions of

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = 0$$

- If $\lambda^2 < \omega_0^2$ I define $\omega_f^2 = \omega_0^2 - \lambda^2$. Then

$$x(t) = A\,e^{-\lambda t}\cos(\omega_f\,t + \varphi)$$

Damped oscillation

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- If $\lambda^2 = \omega_0^2 \Rightarrow$ $x(t) = (A_1 + A_2 t)\, e^{-\lambda t}$

Damped oscillation

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- If $\lambda^2 > \omega_0^2$ I define $-\lambda_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \omega_0^2}$. Then

$$x(t) = A_1\, e^{-\lambda_1 t} + A_2\, e^{-\lambda_2 t}$$

Three cases of damped oscillation

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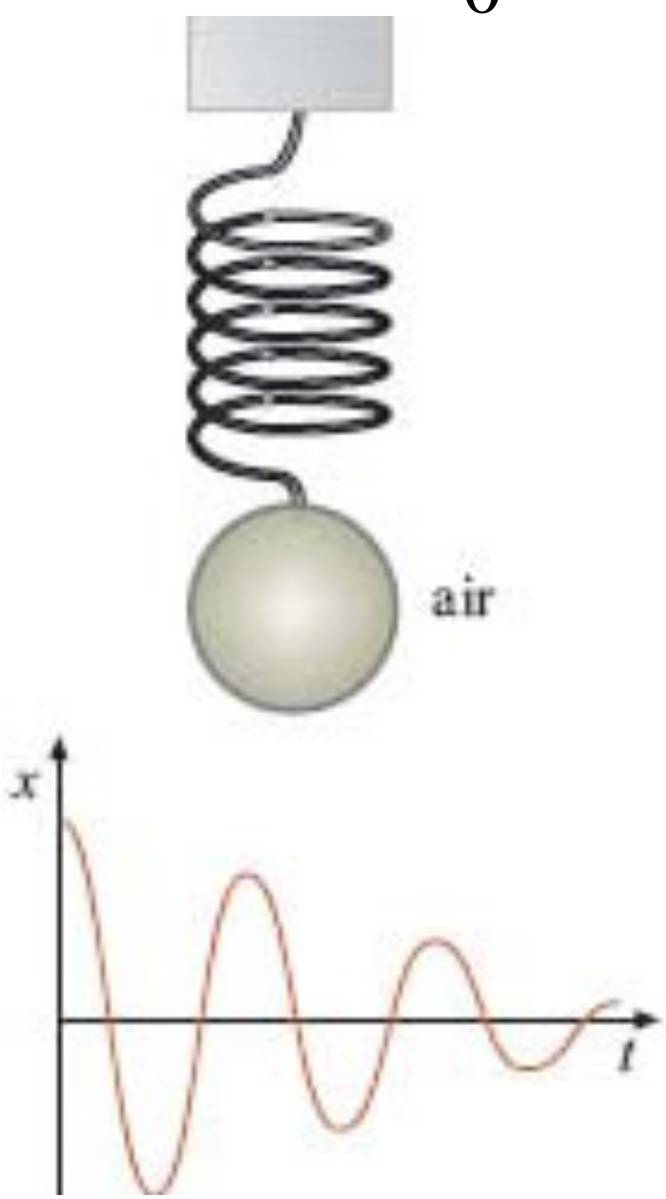
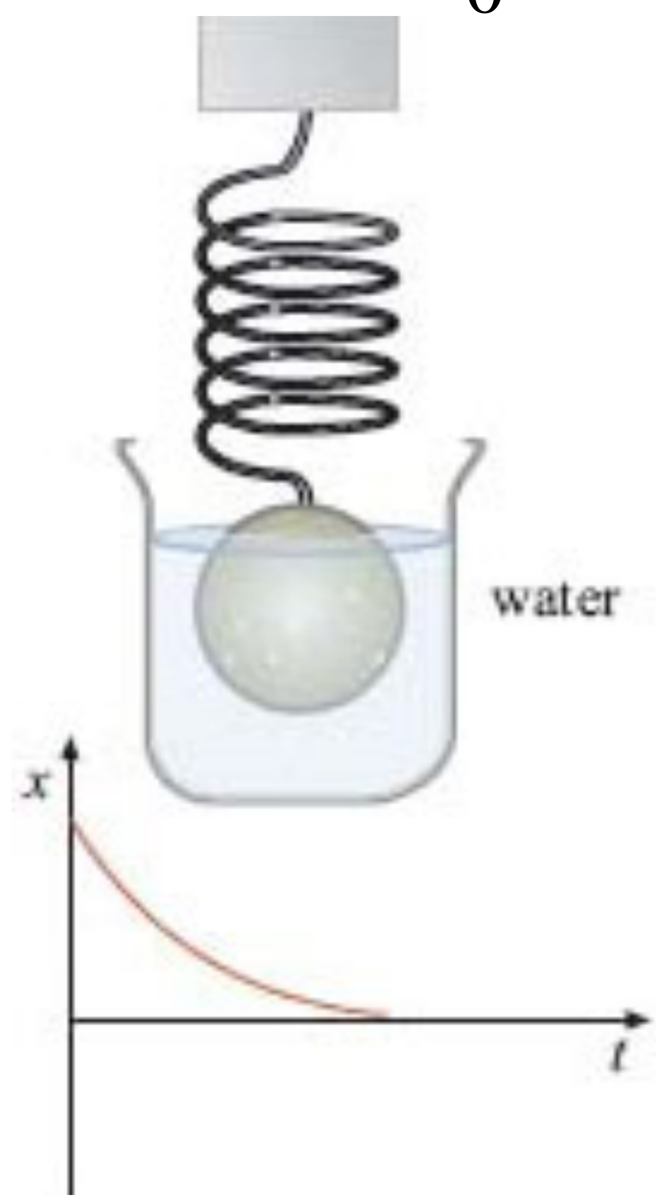
Weak damping

$$\lambda^2 < \omega_0^2$$



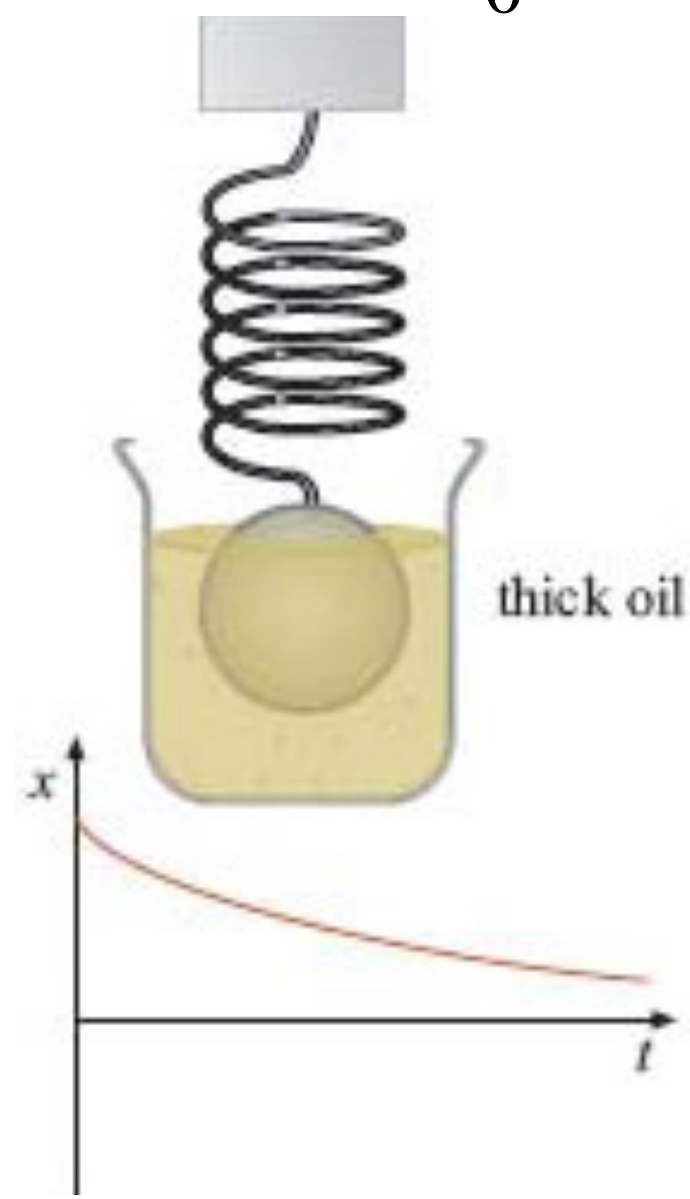
Critical damping

$$\lambda^2 = \omega_0^2$$



Strong damping

$$\lambda^2 > \omega_0^2$$



Forced oscillation

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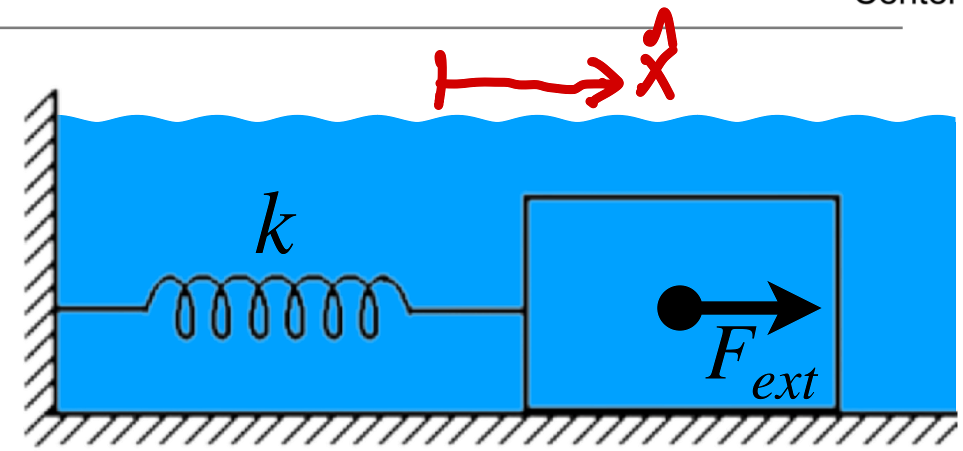
- Now let's add yet another term, an external driving force

$$F_{ext}(t) = F_d \cos(\omega_d t)$$



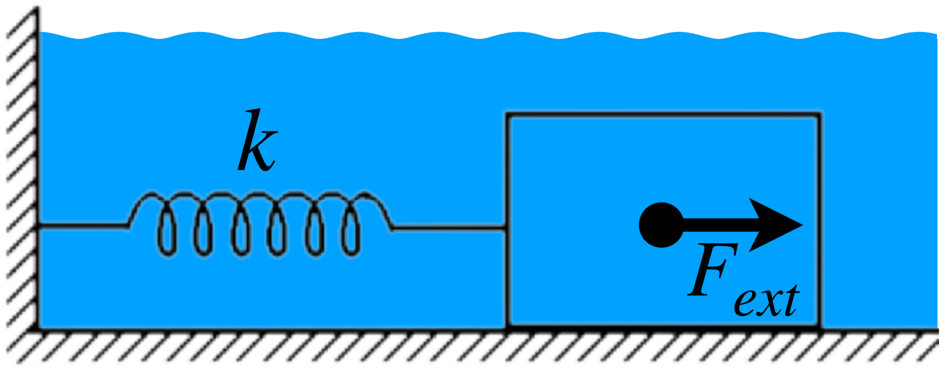
- This represents the influence of an externally applied force with amplitude F_d and angular frequency ω_d

$$\sum F_x: \; F_s + F_d + F_{ext} = -kx - \beta v + F_{ext} = ma \qquad \left(v = \frac{dx}{dt},\; a = \frac{d^2x}{dt^2}\right)$$

$$\Rightarrow m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F_{ext}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \underbrace{\frac{\beta}{m}}_{2\lambda}\frac{dx}{dt} + \underbrace{\frac{k}{m}}_{\omega_0^2}x = \frac{1}{m}F_{ext} = \frac{1}{m}F_d\cos(\omega_d t)$$

Forced oscillation



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- Equation of motion becomes

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = \frac{F_d}{m}\cos(\omega_d t)$$

Forced oscillation

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- The solution is

$$x(t) = \begin{pmatrix} \text{homogeneous} \\ \text{solution} \end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

Forced oscillation

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$$x(t) = \begin{pmatrix}\text{homogeneous} \\ \text{solution}\end{pmatrix} + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

$$A_d = A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

Forced oscillation

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- The solution is

Decays to zero after waiting long enough

$$x(t) = \left(\begin{array}{c}\text{homogeneous}\\\text{solution}\end{array}\right) + A_d \cos(\omega_d t + \varphi_d)$$

where A_d (the *forced* amplitude) is

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and the *forced* phase is

$$\varphi_d = \varphi_d(\omega_d) = \tan^{-1}\left(\frac{2\lambda\omega_d}{\omega_d^2 - \omega_0^2}\right)$$

Resonance

- When the driving frequency and natural frequency are very close, $\omega_d \approx \omega_0$, weird stuff can happen

$$A_d(\omega_d, F_d) = \frac{F_d/m}{\sqrt{(2\lambda\omega_d)^2 + (\omega_0^2 - \omega_d^2)^2}}$$

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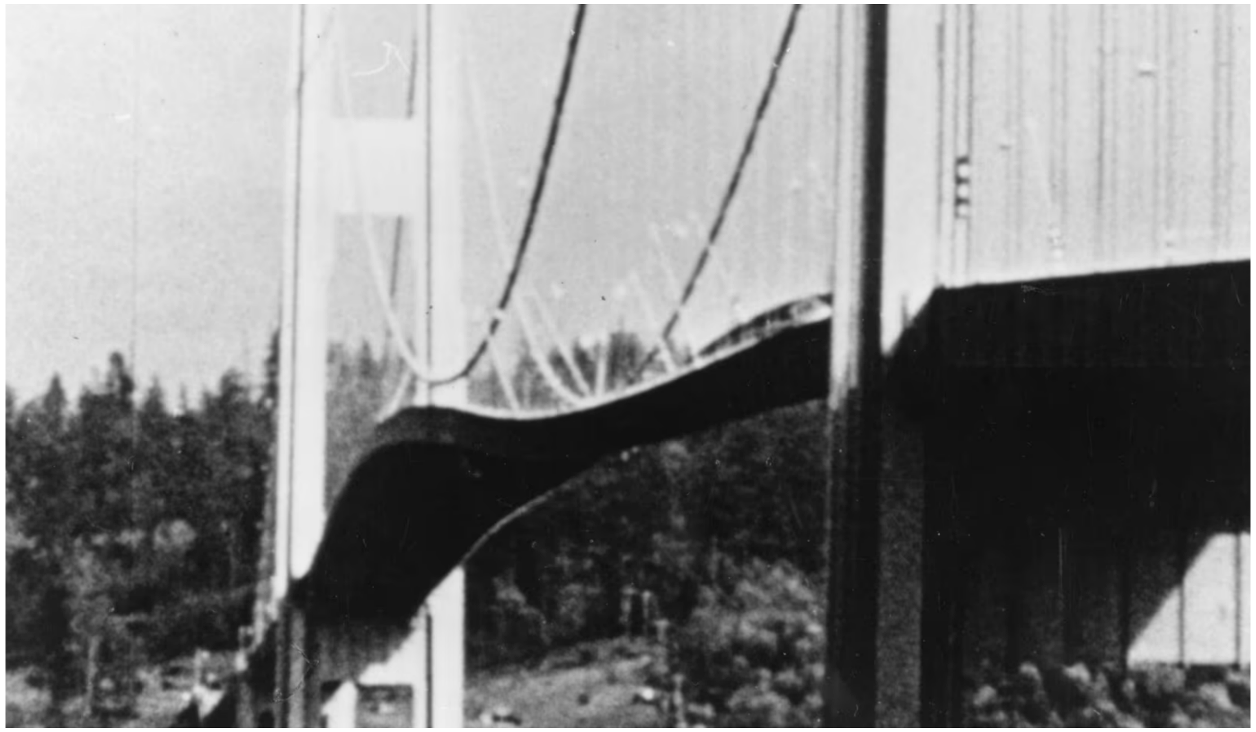

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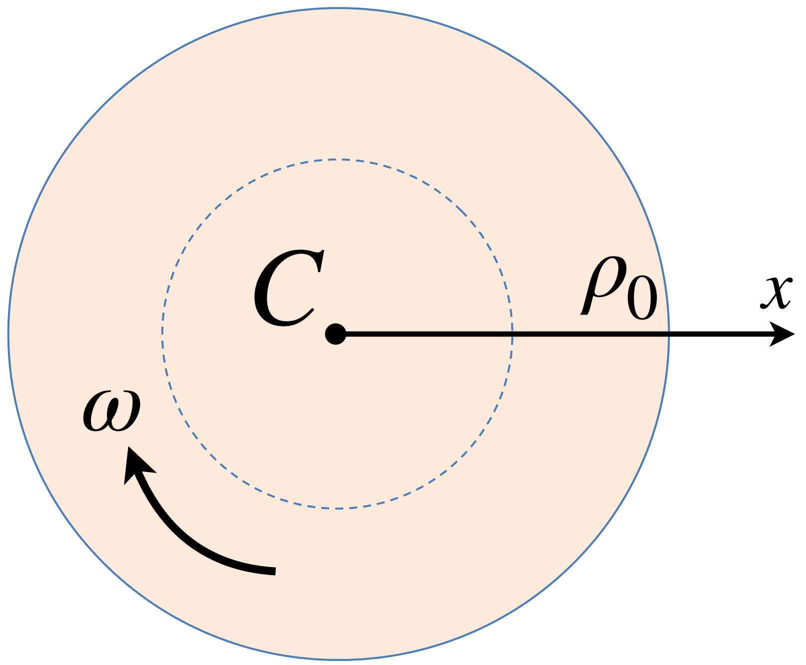
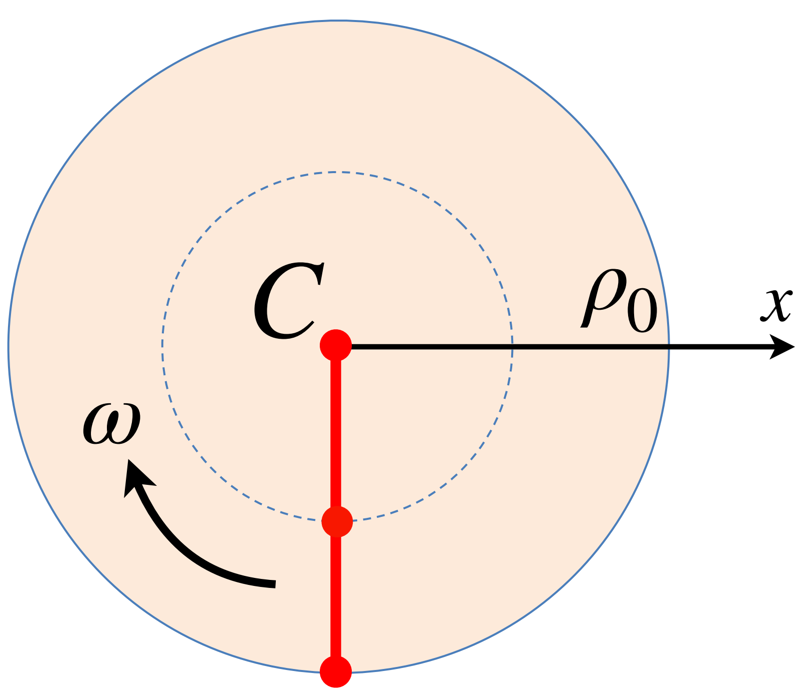
Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C



Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt



Pure rotation of a rigid body

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Pure rotation of a rigid body

- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α

$\frac{d\phi}{dt}$ $\frac{d^2\phi}{dt^2}$

Pure rotation of a rigid body

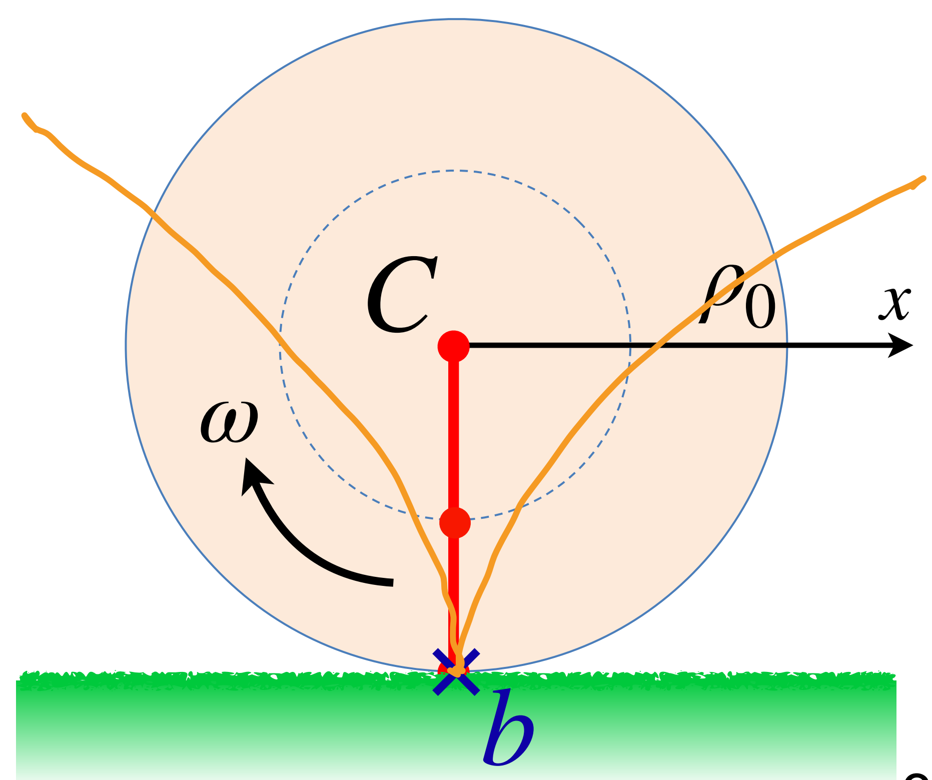
- All points exhibit circular motion about the Center of Mass C
- All points on a straight line drawn through the axis move through the same angle $\Delta\phi$ in the same time Δt
- Therefore, every point has the same value of ω and α
- The distance they move is the arc length $\Delta\ell(\rho) = \rho\Delta\phi$, so any point p has $\vec{v}_{Cp} = \frac{\Delta\ell}{\Delta t}\hat{\phi} = \frac{\rho\Delta\phi}{\Delta t}\hat{\phi} = \rho\omega\,\hat{\phi}$ in the CM frame

Rolling without slipping

- If an object is rolling without slipping, then at the point of contact with the ground b, the wheel has

$$\vec{v}_{gb} = 0$$

<u>in the ground frame of reference</u>



Rolling without slipping

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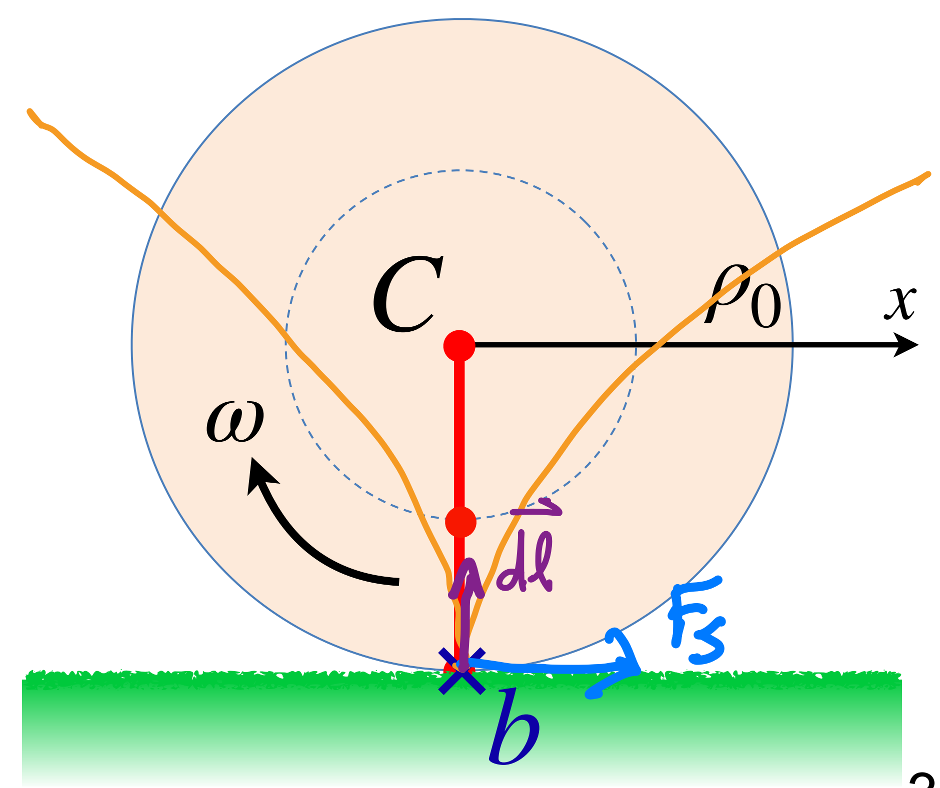
- This means the friction is static, points in the opposite direction to translational acceleration, and does no work

$$W_s = \int \vec{F}_s \cdot \vec{dl} = 0$$

because $\vec{F}_s \perp \vec{dl}$



Rolling velocity and switching reference frames

- In the CM frame, all points on the rim have $\vec{v}_{Cp} = \rho_0\omega\,\hat{\phi}$
- In the frame of reference of the ground, the point touching the ground b has $\vec{v}_{gb} = 0$

$$\vec r_{gb} = \vec r_{gc} + \vec r_{cb}$$

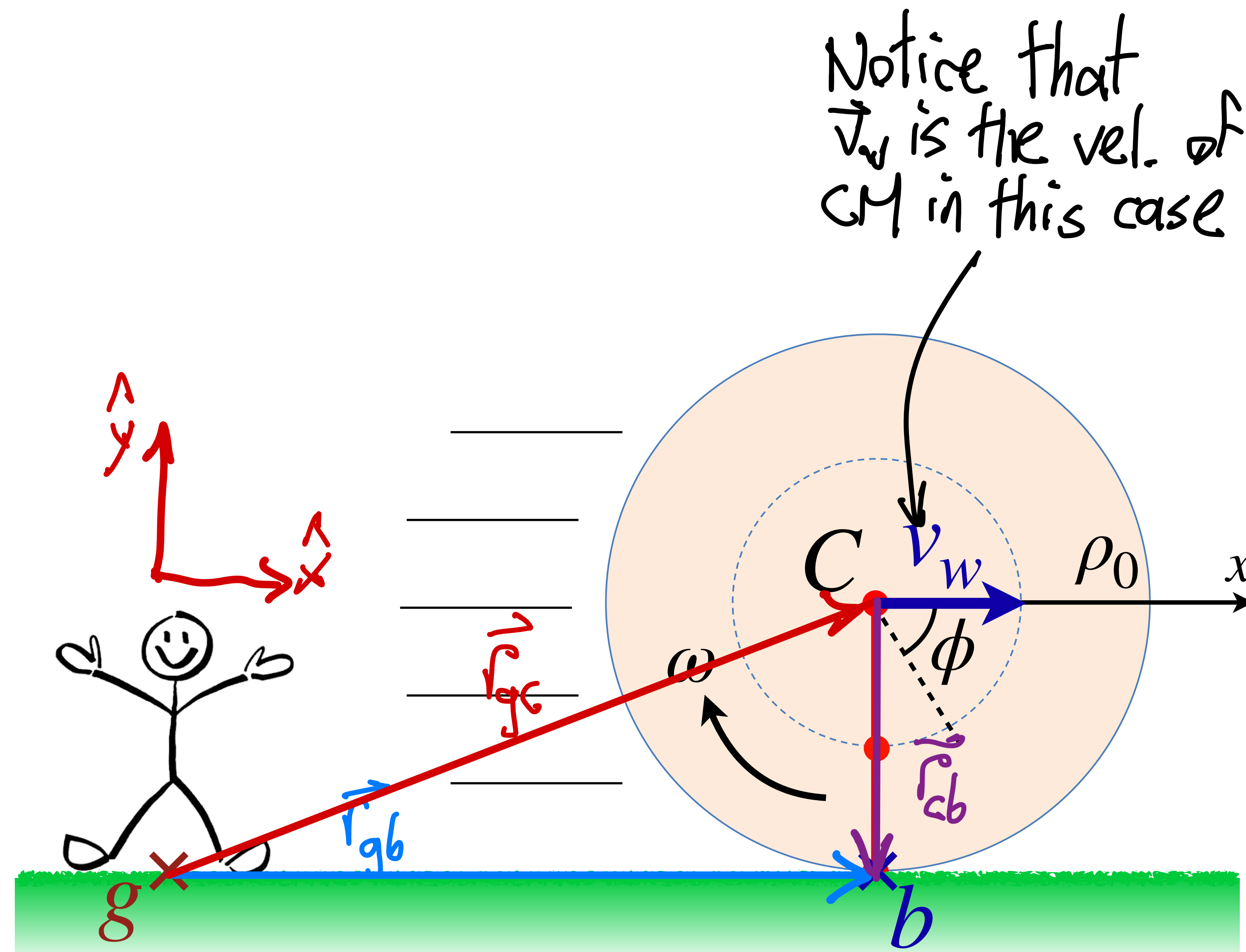
$$\Rightarrow \frac{d}{dt}(\vec r_{gb}) = \frac{d}{dt}(\vec r_{gc} + \vec r_{cb})$$

$$\vec v_{gb} \to 0 = \underbrace{\vec v_{gc}}_{\vec v_w} + \underbrace{\vec v_{cb}}_{=\rho_0\omega\hat\phi_{cb}}$$

$$0 = \vec v_w + \rho_0\omega\hat\phi_{cb}$$

$$\Rightarrow \vec v_w = -\rho_0\omega\hat\phi_{cb} = -\rho_0\omega(-\hat x)$$

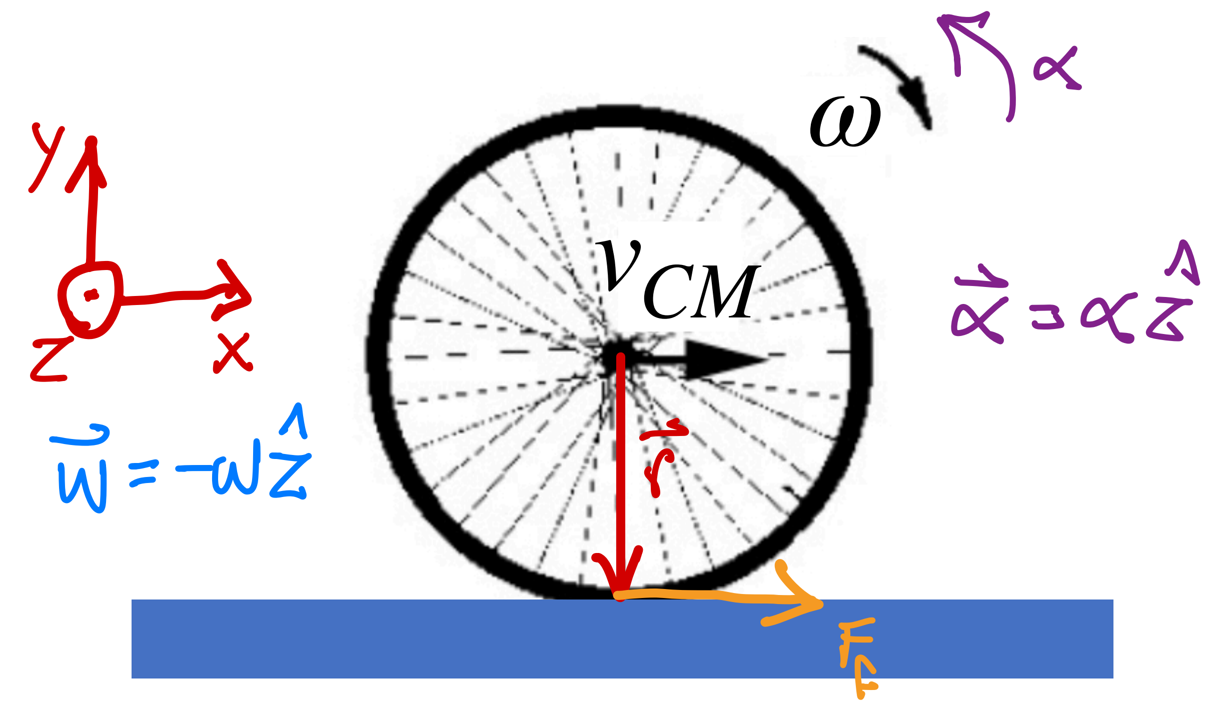
$$= \rho_0\omega\hat x$$



Conceptual question

A bicycle wheel is initially spinning in the air and then is put into contact with a rough surface (see figure). It slips against the surface. What is the direction of the kinetic friction force acting on the wheel?

A. Points to the right
B. Points to the left
C. Points up
D. Points down



$\vec{\alpha}$ must point opposite to $\vec{\omega}$ because wheel slows down

$\vec{\tau}_{net} = I\vec{\alpha} = I\alpha\hat{z}$ but $\vec{\tau}_{net} = \vec{r} \times \vec{F}_f = (-r\hat{y}) \times \vec{F}_f = I\alpha\hat{z}$

Therefore $\vec{F}_f = F_f\hat{x}$

Example: Yo-yo

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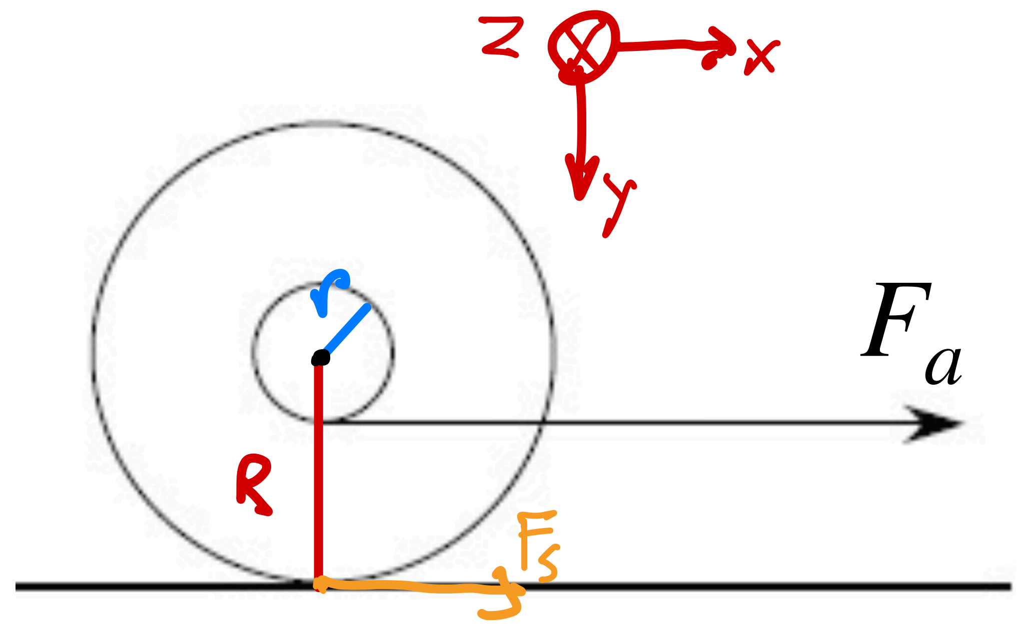
A yo-yo of mass m is placed on a rough surface and rolls without slipping. It is composed of two disks separated by a spindle with a smaller diameter. A string is wound around the spindle and pulled with a force F_a. In which direction does it move? To the right, winding up the string, or to the left, unwinding the string?



ΣF_y: $mg - N = 0 \Rightarrow N = mg$

ΣF_x: $F_a + F_s = m a_{CM}$

$\Rightarrow F_s = m a_{CM} - F_a$

$\vec{\tau}_{net} = I\vec{\alpha}$

$\vec{\tau}_{net} = \Sigma \vec{\tau} = \vec{\tau}_{F_a} + \vec{\tau}_{F_s} = (r\hat{y}) \times (F_a \hat{x}) + (R\hat{y}) \times (F_s \hat{x}) = rF_a(-\hat{z}) - RF_s\hat{z}$

$= -(rF_a + RF_s)\hat{z} = I\vec{\alpha} = I\left(\frac{a_{CM}}{R}\right)\hat{z}$ (because $a_{CM} = \alpha R$ for no slip)

$\Rightarrow I\frac{a_{CM}}{R} = -rF_a - RF_s = -rF_a - R[m a_{CM} - F_a] = -rF_a + RF_a - Rm a_{CM}$

$\Rightarrow \frac{I}{R} a_{CM} + mR a_{CM} = \left(\frac{I}{R} + mR\right) a_{CM} = (R - r)F_a \Rightarrow$ $a_{CM} = (R-r)\left(\frac{I}{R} + mR\right)^{-1} F_a > 0$