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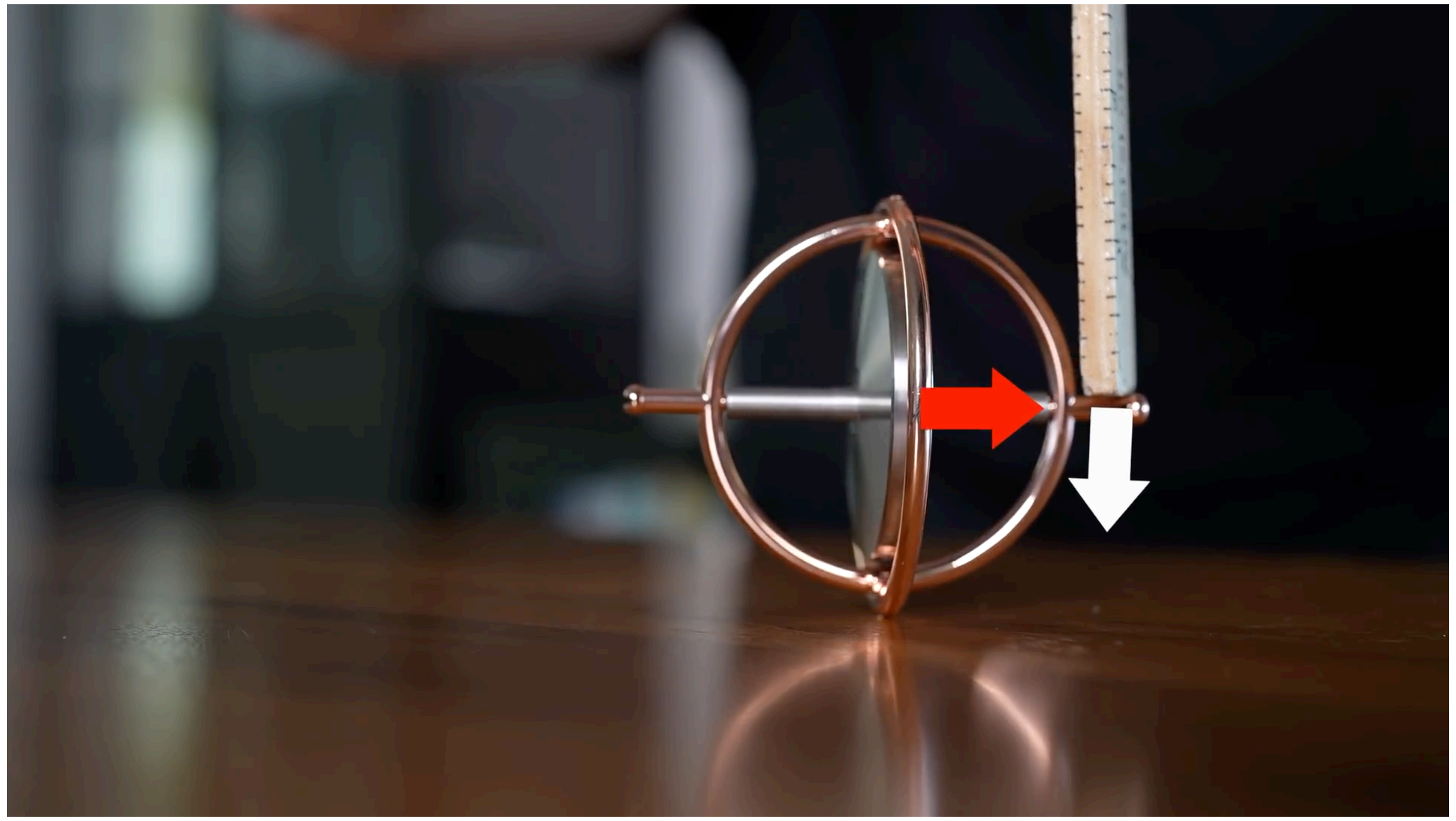
General Physics: Mechanics



PHYS-101(en)

Lecture 13b: Gyroscopes and harmonic motion

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Example: Motorcycle steering

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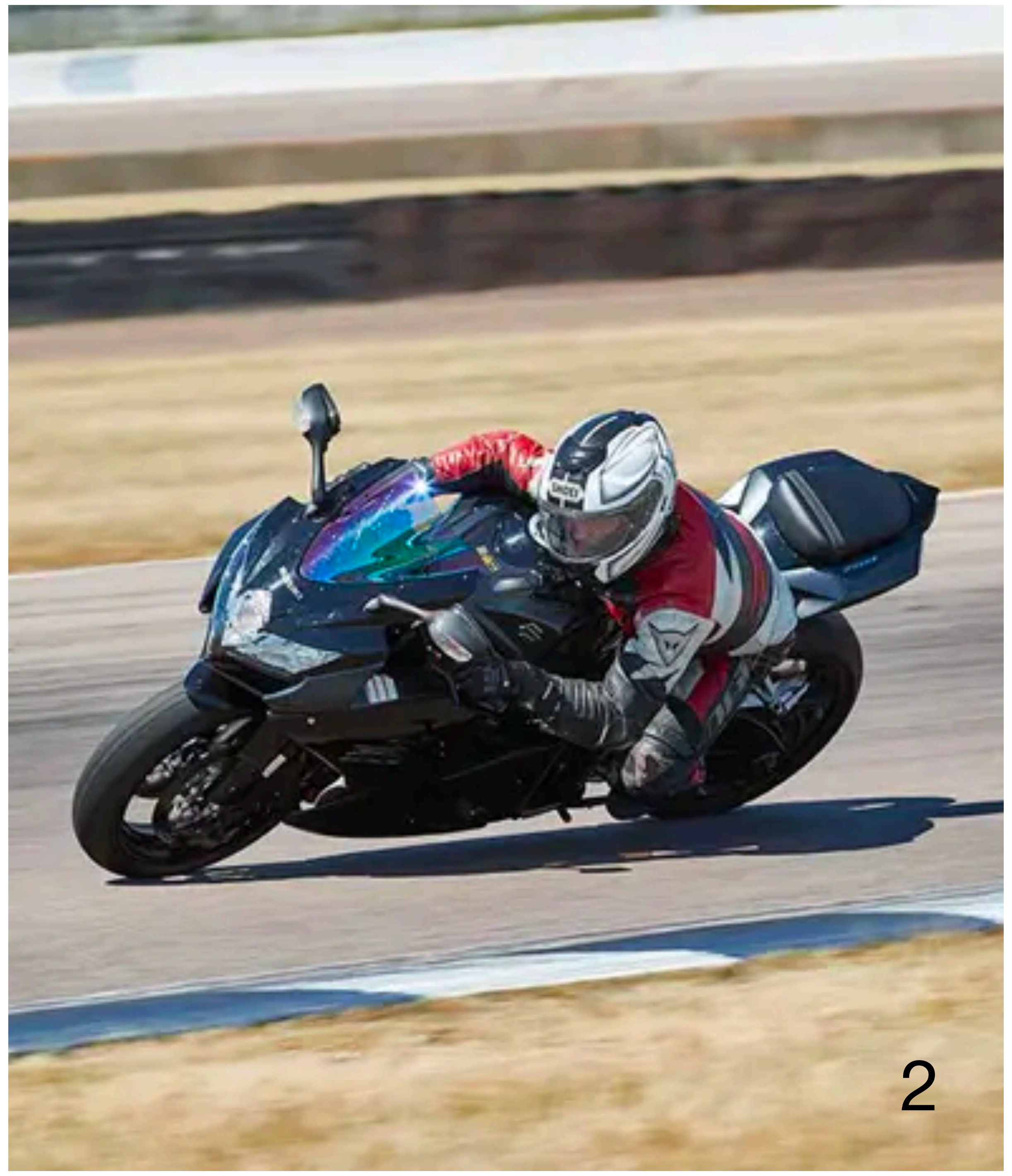
You're racing on motorcycle at a constant speed v and are entering a turn. To stay on the road, you must turn the front wheel by an angle $\Delta\phi$ within a distance d. What torque must you apply to the axle of the wheel to make the turn? Treat the motorcycle wheel as a solid cylinder of radius R and mass m. Ignore friction/drag and assume rolling without slipping.



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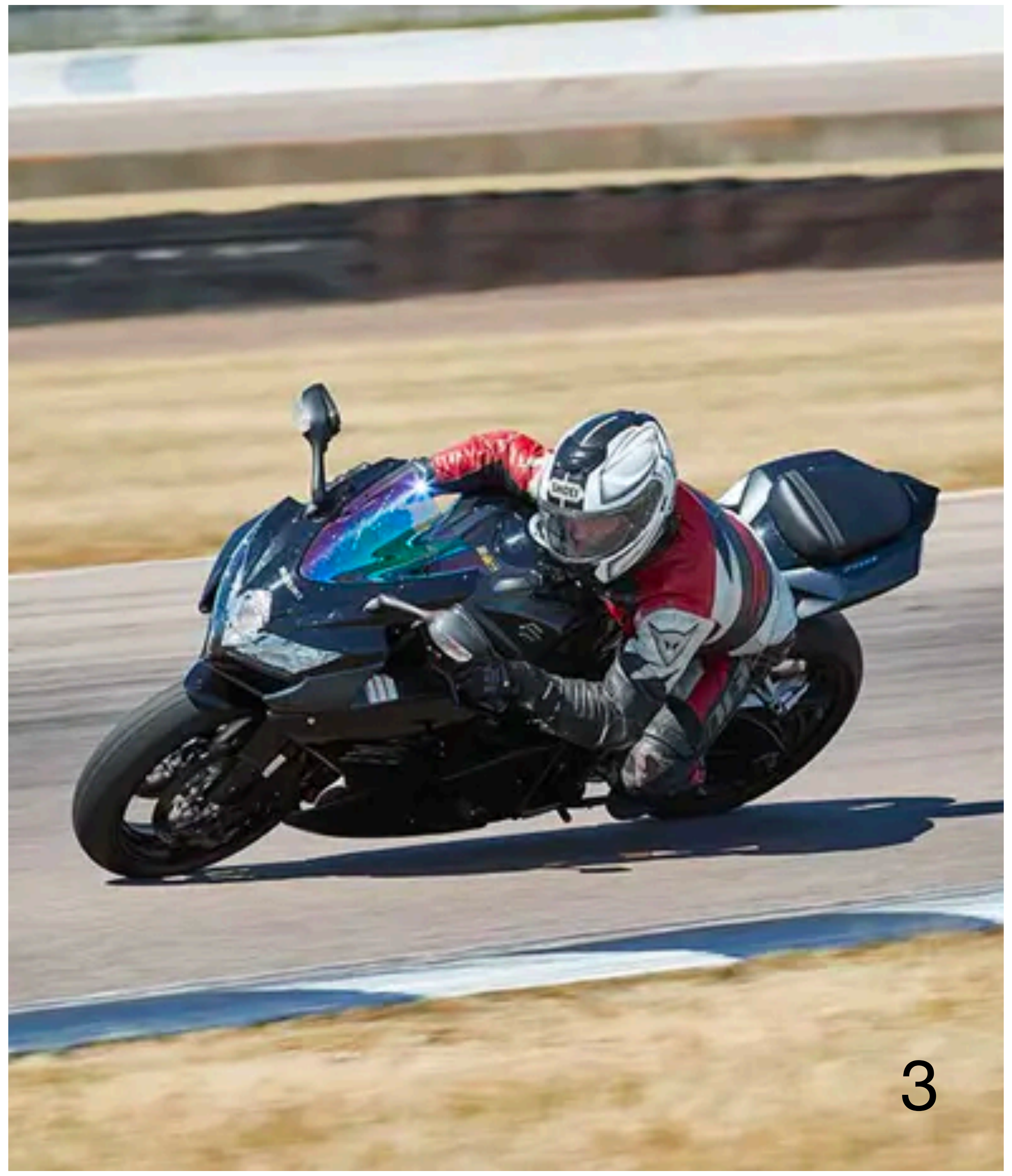
A. Find an expression for the angular speed of the wheel ω_w.

B. Find an expression for the constant angular speed ω_t with which you need to turn the wheel in order to stay on the road.

C. Find the total angular momentum of the wheel in terms of the moments of inertia, I_w and I_t, for the two types of rotation. (Let the center of the **axle's rotation** be the origin of a cylindrical coordinate system.)

D. Find the torque. How do you create such a torque?



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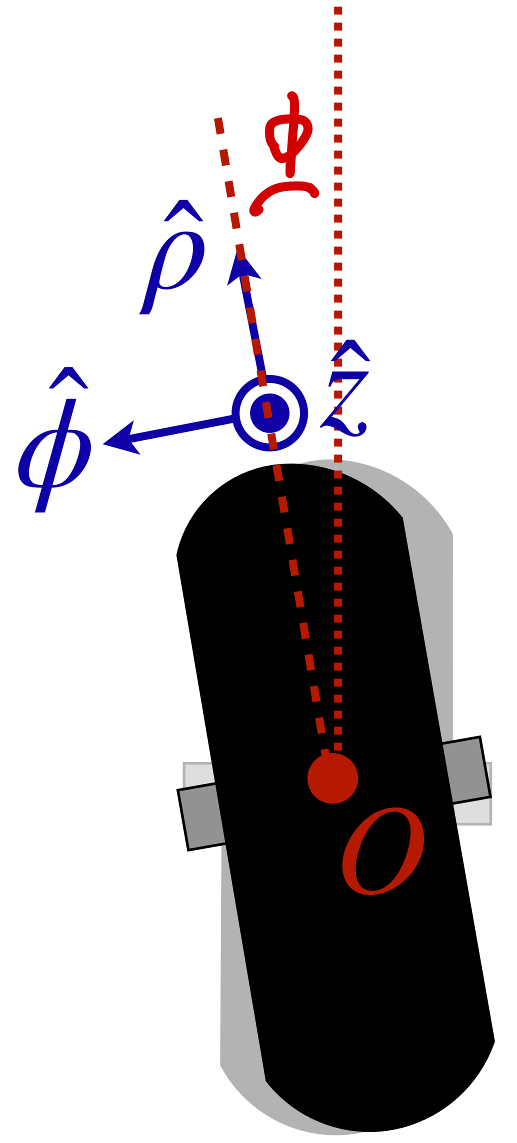
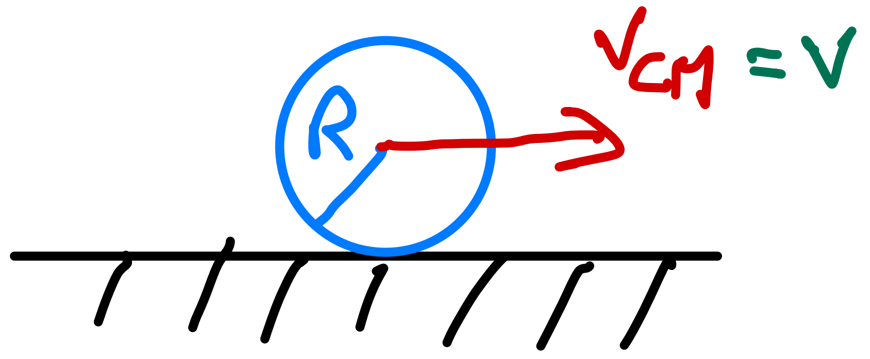


A.



No slipping means

$$v = v_{CM} = R\omega_w \Rightarrow \omega_w = \frac{v}{R} \quad \text{(constant)}$$

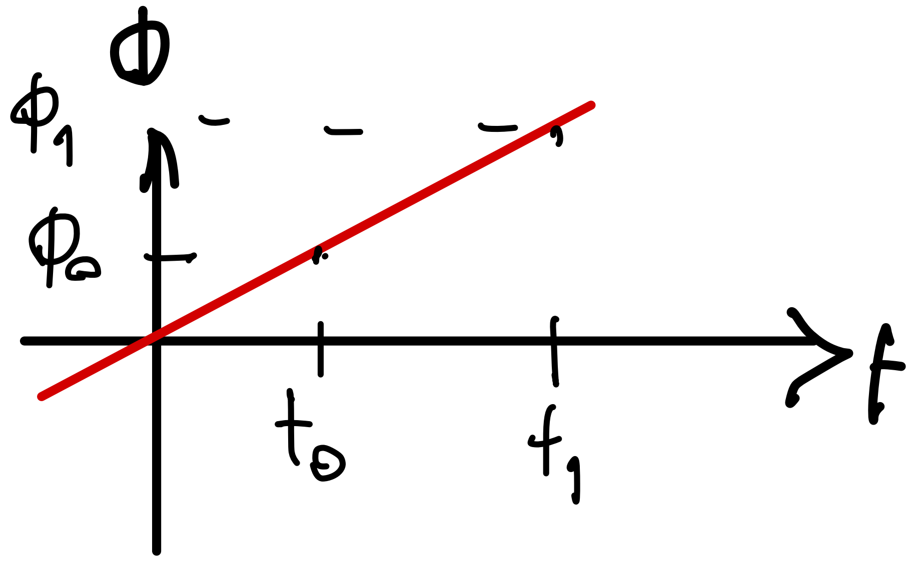
B.

constant ω_t

$$\omega_t = \frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t}$$

$$\Delta t = \frac{d}{v}$$

$$\Rightarrow \omega_t = \frac{\Delta\phi}{d/v} = \frac{v}{d}\Delta\phi$$



$$\Delta\phi = \phi_1 - \phi_0$$
$$\Delta t = t_1 - t_0$$
$$\frac{\Delta\phi}{\Delta t} = \frac{d\phi}{dt}$$

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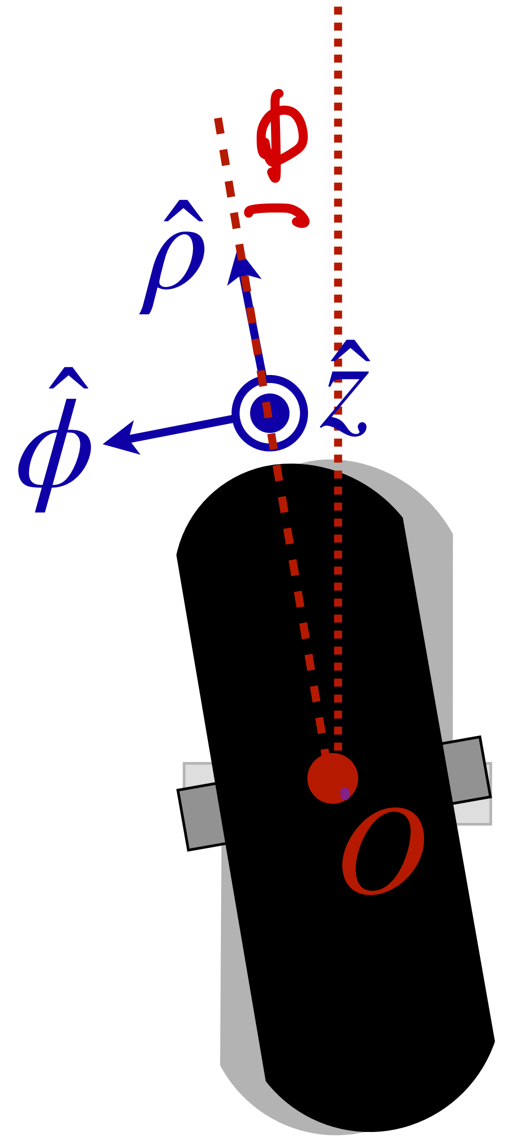
C. Find the total angular momentum of the wheel in terms of the moments of inertia, I_w and I_t, for the two types of rotation. (Let the center of the **axle's rotation** be the origin of a cylindrical coordinate system.)



$$\vec{L}_{tot} = \vec{L}_w + \vec{L}_t$$

$$\vec{L}_w = I_w \vec{\omega}_w = I_w \omega_w \hat{\phi}$$

$$\vec{L}_t = I_t \vec{\omega}_t = I_t \omega_t \hat{z}$$

$$\Rightarrow \vec{L}_{tot} = I_w \omega_w \hat{\phi} + I_t \omega_t \hat{z}$$

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D. Find the torque. How do you create such a torque?

$$\vec{\tau}_{net} = \frac{d}{dt}\vec{L}_{tot}$$

$$= \frac{d}{dt}\left[I_w \omega_w \hat{\phi} + I_t \omega_t \hat{z}\right] = I_w \frac{d}{dt}\left[\omega_w \hat{\phi}\right] + I_t \frac{d\omega_t}{dt}\hat{z}$$

$$= I_w\left[\cancel{\frac{d\omega_w}{dt}}\hat{\phi} + \omega_w \frac{d\hat{\phi}}{dt}\right] + I_t \cancel{\frac{d\omega_t}{dt}}\hat{z}$$

$$\approx I_w \omega_w(-\omega_t \hat{\rho}) = -I_w \omega_w \omega_t \hat{\rho}$$

$$= -\frac{1}{2} m R^{\cancel{2}}\left(\frac{v}{\cancel{R}}\right)\left(\frac{v}{d}\Delta\phi\right)\hat{\rho}$$

$$\Rightarrow \boxed{\vec{\tau}_{net} = -\frac{1}{2} m R \frac{v^2}{d}\Delta\phi\,\hat{\rho}}$$

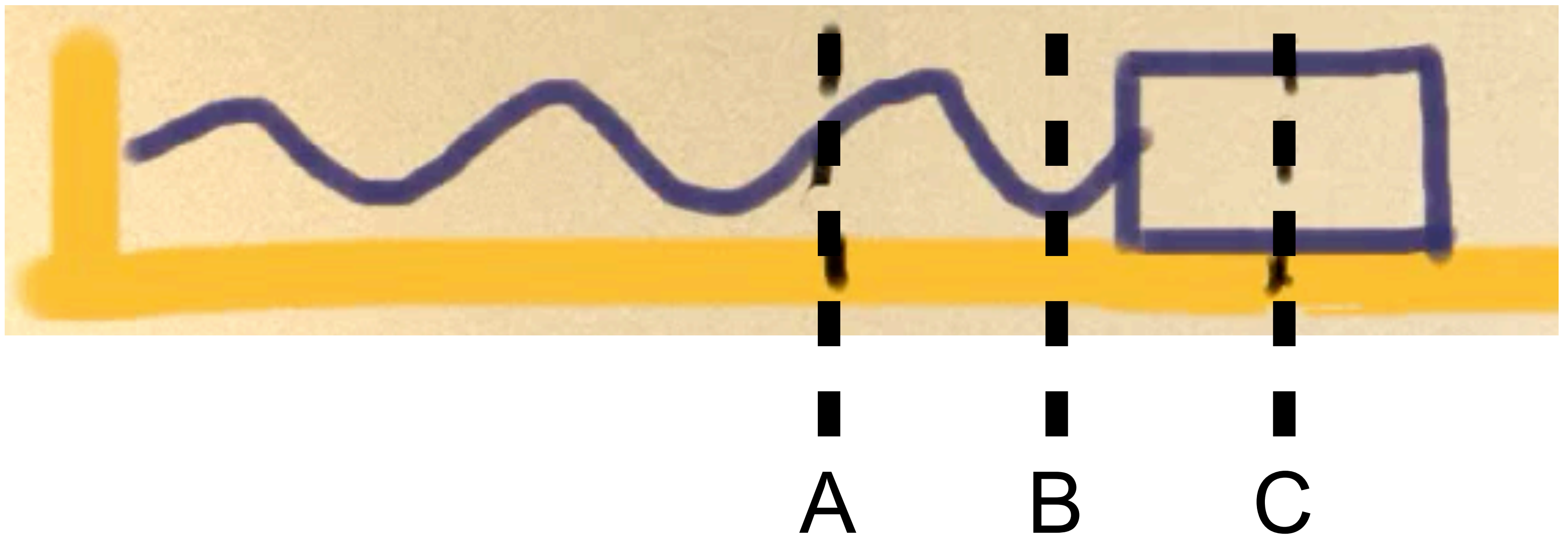
$$\frac{d\hat{\phi}}{dt} = \dot{\hat{\phi}} = -\dot{\phi}\hat{\rho} = -\omega_t \hat{\rho}$$

$$I_w = \frac{1}{2} m R^2 \quad \text{(lecture 11)}$$

Conceptual question

A mass is oscillating back and forth on a spring about point A as shown. Point A is the equilibrium (unstretched) position of the mass. At which position is the magnitude of its acceleration the largest?

A. Point A
B. Point B
C. Point C



$a(t) = \ddot{x}(t) = -\omega_0^2 x(t) \Rightarrow |a(t)| = \omega_0^2 |x(t)|$