

General Physics: Mechanics

PHYS-101(en)
Lecture 12a:
Angular momentum

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Announcements

- We'll hold another *mock exam* tomorrow
 - You can bring a “cheat” sheet containing formulas or all of your notes, as you wish
 - Turn in at the end if you want exam to be graded (optional). Graded exams will be returned on Monday December 15th
 - Solutions will be published on the Moodle
 - Does **not** matter at all for your final grade
 - Exam will contain some Multiple choice questions to practice for the final

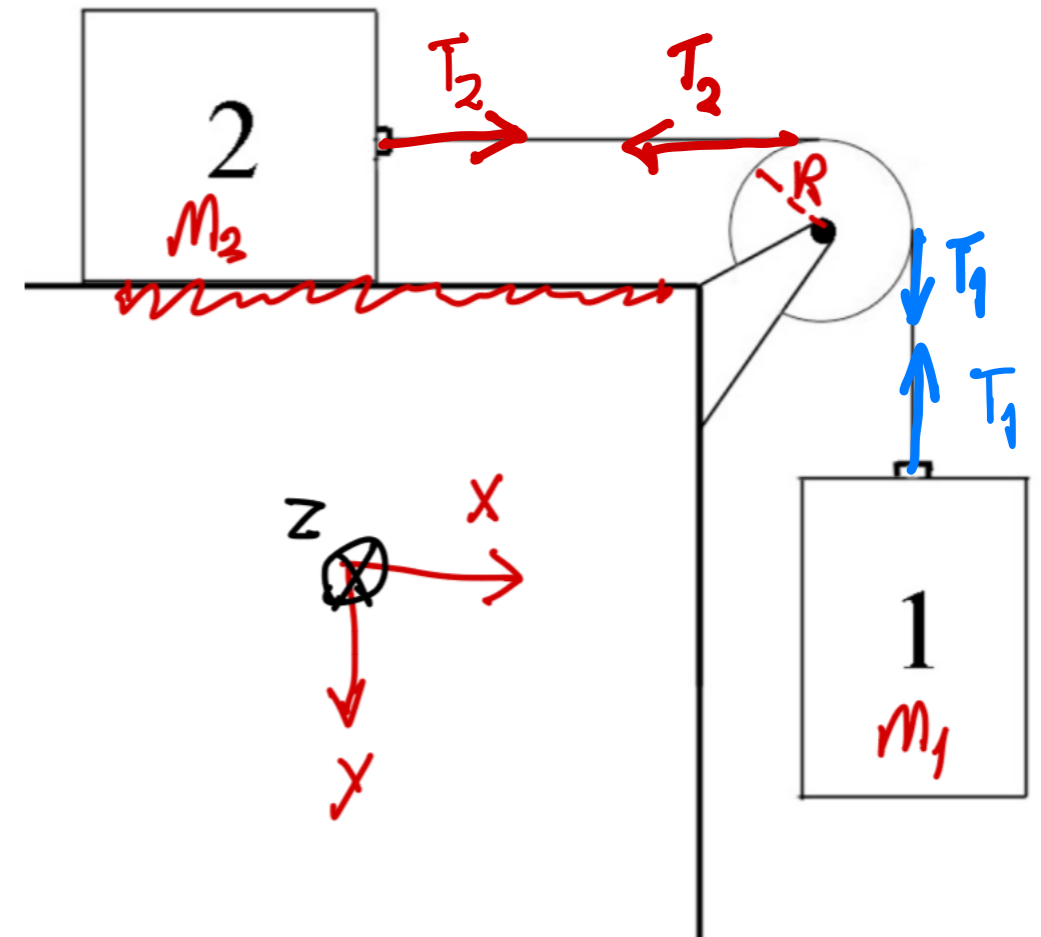
Today's agenda (Serway 11,13, MIT 19)

Conclusion of rotation of rigid objects about a fixed axis

1. Work and power for rotation
2. Work-kinetic energy theorem for rotation
3. Angular momentum and its conservation

Example (continued): Massive pulley

A pulley (with radius R and moment of inertia about its center of mass I) is attached to the edge of a table. A massless string connects two blocks as shown. Block 1 has mass m_1 and hangs off the edge of the table. Block 2 has mass m_2 and can slide along a table with a coefficient of kinetic friction of μ . Note that $m_1 > \mu m_2$. The blocks are released from rest and the string does not slip around the pulley.



Find the magnitude of the acceleration of each block. Express your answer in terms of R , I , m_1 , m_2 , and μ as needed.

Example (continued): Massive pulley

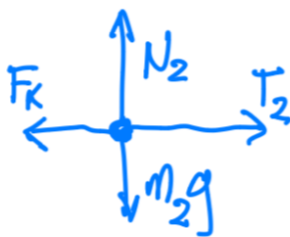
Block 1



$$\Sigma F_y: m_1 g - T_1 = m_1 a_{1y}$$

$$\Rightarrow T_1 = m_1 g - m_1 a_{1y} \quad (1)$$

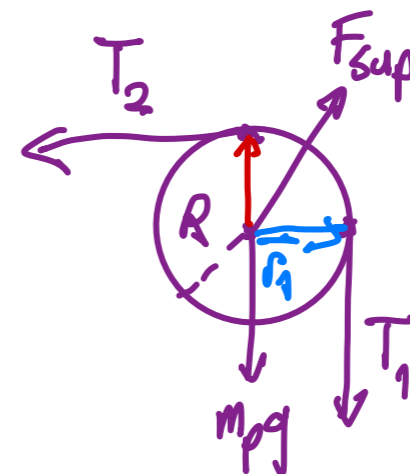
Block 2



$$\Sigma F_y: N_2 = m_2 g$$

$$\Sigma F_x: T_2 - \mu m_2 g = m_2 a_{2x} \quad (2)$$

Pulley:



$$\vec{\tau}_{T_1} = \vec{r}_1 \times (T_1 \hat{y})$$

$$= (R \hat{x}) \times (T_1 \hat{y})$$

$$= RT_1 (\hat{x} \times \hat{y})$$

$$= RT_1 \hat{z}$$

$$\vec{\tau}_{T_2} = \vec{r}_2 \times (-T_2 \hat{x}) = (-R \hat{y}) \times (-T_2 \hat{x})$$

$$= RT_2 (-\hat{z}) = -RT_2 \hat{z}$$

$$\vec{\tau}_{mg} = \vec{0} \times (m g \hat{y}) = \vec{0} \quad \vec{\tau}_{F_{sup}} = \vec{0}$$

$$\Sigma \vec{\tau} = \vec{\tau}_{T_1} + \vec{\tau}_{T_2} = R(T_1 - T_2) \hat{z}$$

$$= I \vec{\alpha} = I \alpha \hat{z} \quad (3)$$

Notice that: $a_{1y} = a_{2x} = a$

Also, from no slipping cond.

$$\alpha = \frac{a}{R} \Rightarrow a = \alpha R$$

Example (continued): Massive pulley

$$\text{From ① : } T_1 = m_1 g - m_1 \overset{a}{a_{1y}} = m_1 g - m_1 a$$

$$\text{From ② : } T_2 = m_2 a + \mu m_2 g$$

$$\text{From ③ : } R(T_1 - T_2) = I\alpha = I \frac{a}{R}$$

$$\begin{aligned} \Rightarrow \frac{I}{R^2} a &= T_1 - T_2 = m_1 g - \underline{m_1 a} - \underline{m_2 a} - \mu m_2 g \\ &= -(m_1 + m_2) a + (m_1 - \mu m_2) g \end{aligned}$$

$$\Rightarrow (m_1 - \mu m_2) g = \frac{I}{R^2} a + (m_1 + m_2) a = \left(\frac{I}{R^2} + m_1 + m_2 \right) a$$

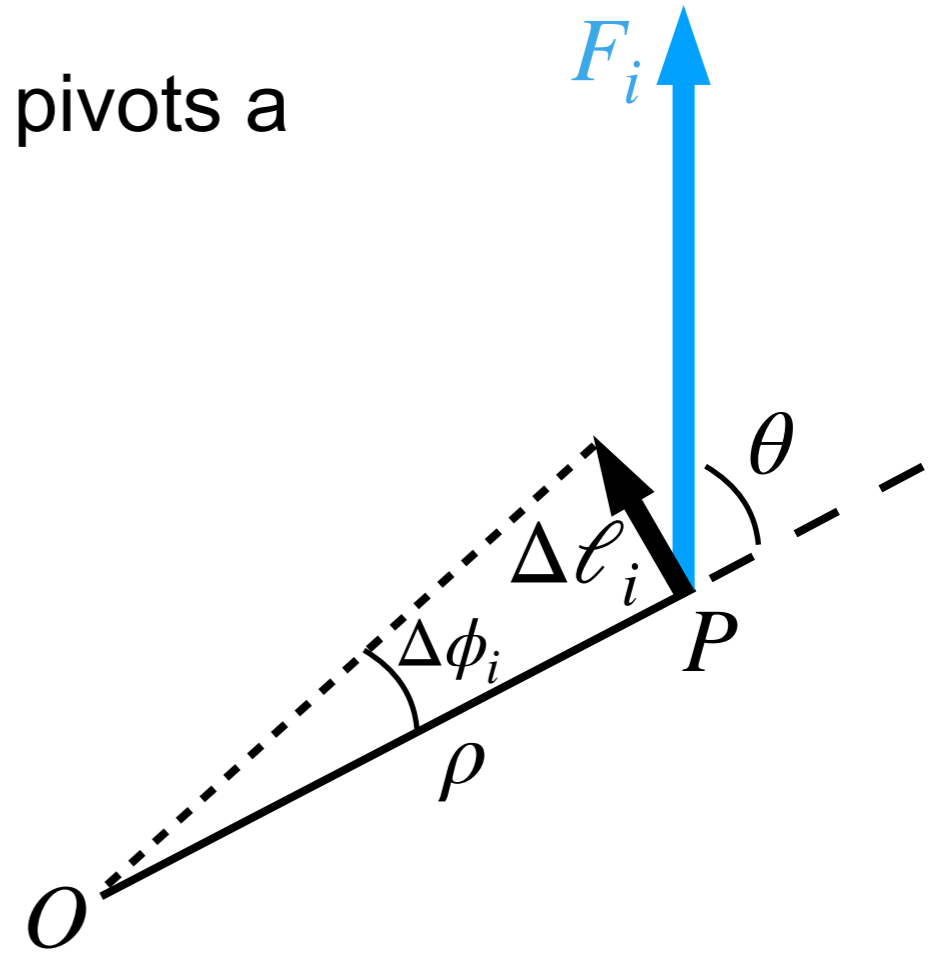
$$\Rightarrow \boxed{a = \frac{m_1 - \mu m_2}{\frac{I}{R^2} + m_1 + m_2} g}$$

Summary of rotation and translation

Rotational motion (about a fixed axis)		Translational motion (in one dimension)	
Angular position	ϕ	Position	x
Angular speed	$\omega = d\phi/dt$	Speed	$v = dx/dt$
Angular acceleration	$\alpha = d\omega/dt$	Acceleration	$a = dv/dt$
Moment of inertia	$I = \int \rho^2 dm$	Mass	m
Net torque	$\Sigma \tau_{ext} = I\alpha$	Net force	$\Sigma F_{ext} = ma$
Rotational kinetic energy	$K^{rot} = I\omega^2/2$	Translational kinetic energy	$K^{trans} = mv^2/2$
Work	?	Work	$W = \int_{x_a}^{x_b} F dx$
Power	?	Power	$P = Fv$
Angular momentum	?	Momentum	$p = mv$
Net torque	?	Net force	$\Sigma F_{ext} = dp/dt$

Work and power for rotation

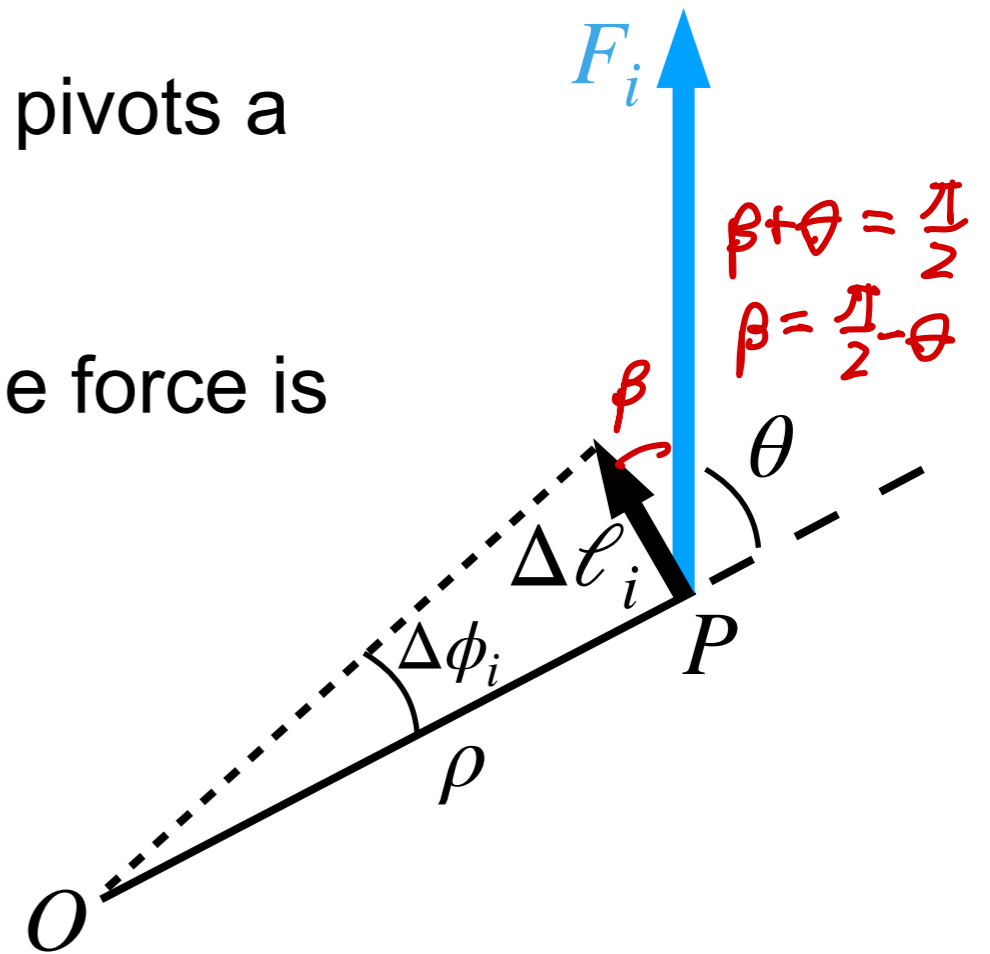
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Work and power for rotation

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- As in week 8, the work done by a variable force is

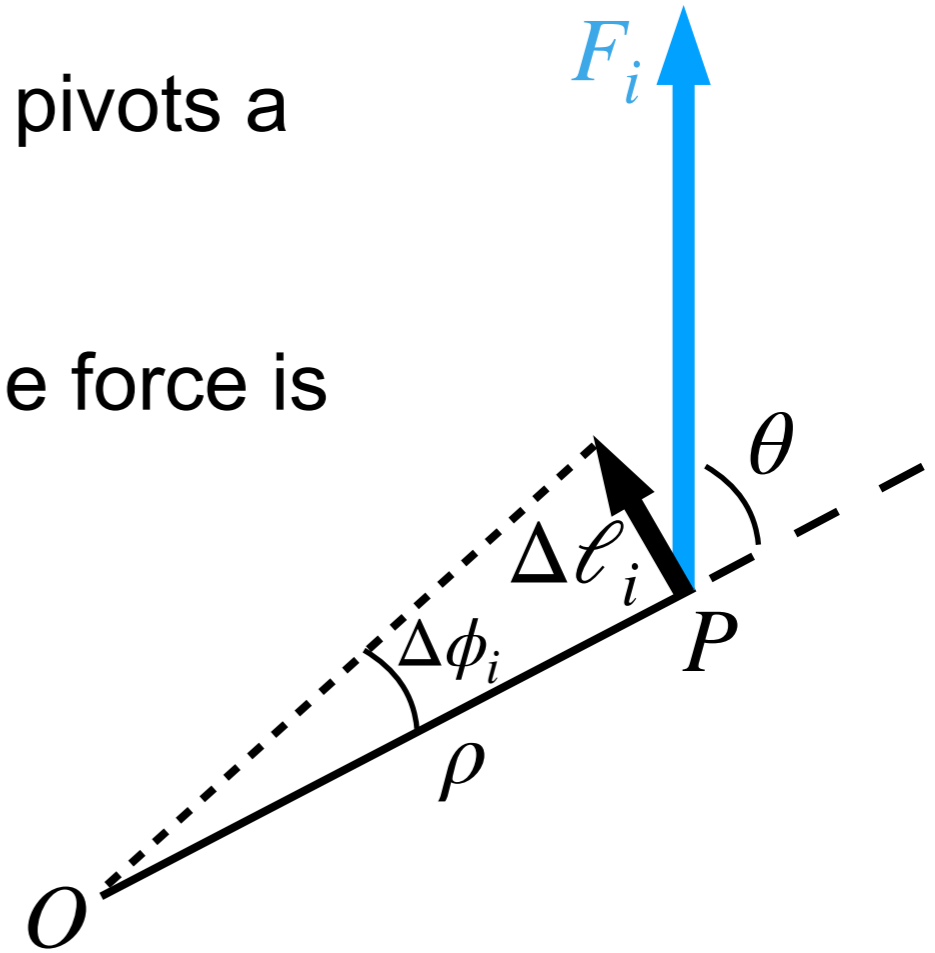
$$\begin{aligned} \Delta W_i &= \vec{F}_i \cdot \Delta \vec{\ell}_i = F_i \Delta \ell_i \cos(\beta) \\ &= F_i \Delta \ell_i \cos\left(\frac{\pi}{2} - \theta\right) \\ &= F_i \Delta \ell_i \sin(\theta) \end{aligned}$$



Work and power for rotation

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$$\begin{aligned} \Delta W_i &= \vec{F}_i \cdot \Delta \vec{\ell}_i = (F_i \sin \theta) \Delta \ell_i \\ &= F_i \sin \theta (\rho \Delta \phi_i) = \tau_i \Delta \phi_i \\ &\quad \text{with } F_i \rho \sin(\theta) = |\vec{r} \times \vec{F}| \end{aligned}$$



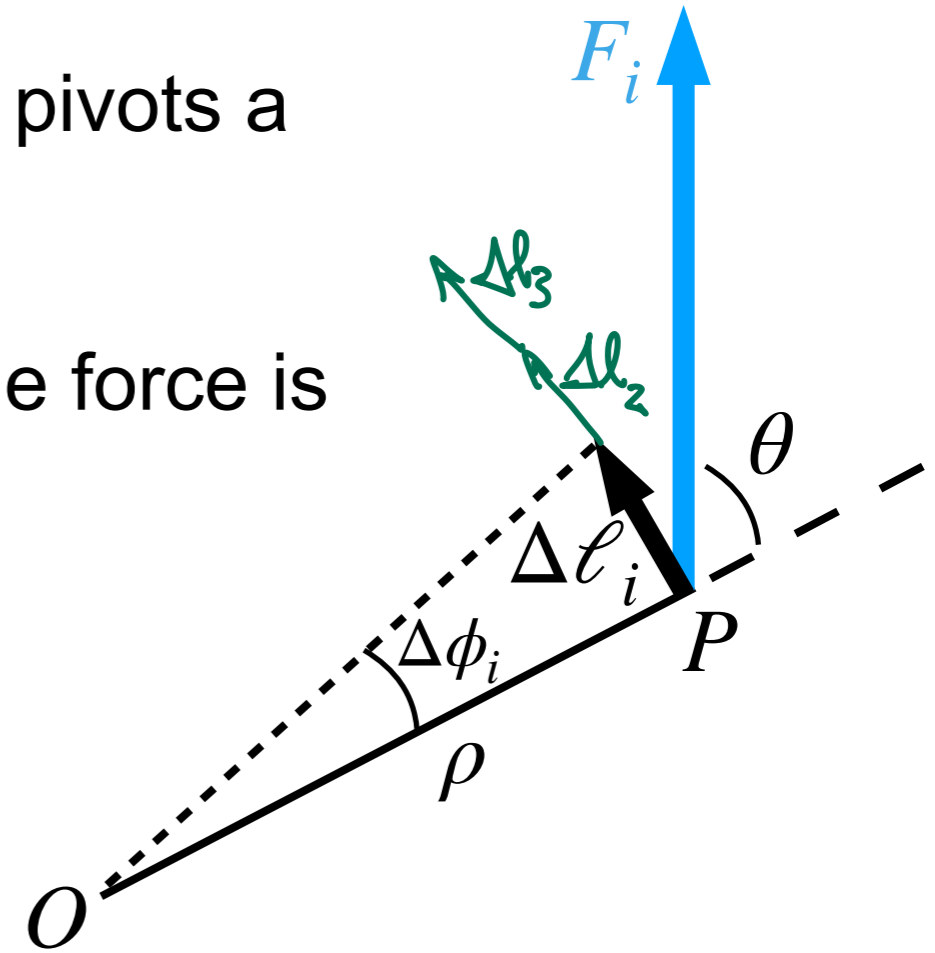
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- Total work is the sum over differential changes in angle

$$W = \lim_{\Delta \phi_i \rightarrow 0} \sum_i \tau_i \Delta \phi_i = \int_{\phi_a}^{\phi_b} \tau d\phi = \int_{t_a}^{t_b} \tau \frac{d\phi}{dt} dt = \int_{t_a}^{t_b} \tau \omega dt$$



Work and power for rotation

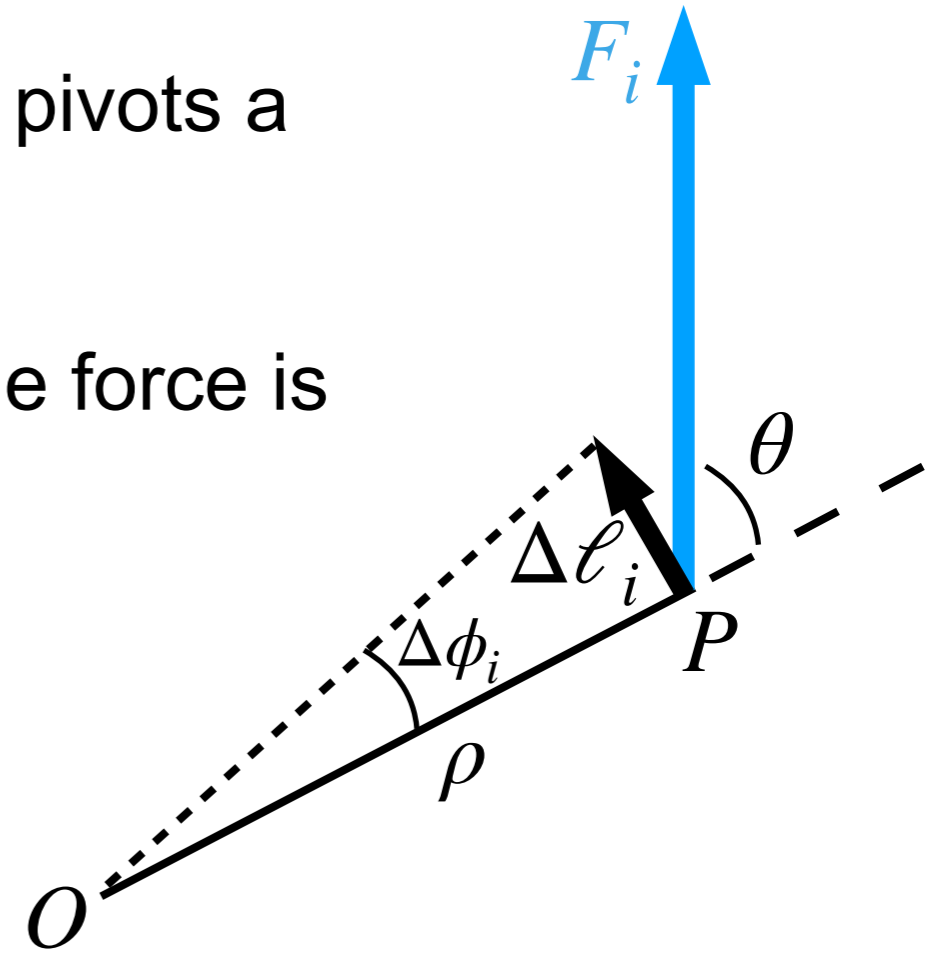
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- Total work is the sum over differential changes in angle

$$W = \lim_{\Delta\phi_i \rightarrow 0} \sum_i \tau_i \Delta\phi_i = \int_{\phi_a}^{\phi_b} \tau d\phi$$

- Thus, the power is $P = \frac{dW}{dt} = \tau \omega$



Today's agenda (Serway 11,13, MIT 19)

Conclusion of rotation of rigid objects about a fixed axis

2. Work-kinetic energy theorem for rotation

Work-kinetic energy theorem for rotation

(For constant I)

- Reminder: This tells us how the kinetic energy of an object changes due to the work performed on it

$$K^{\text{rot}} = \frac{1}{2} I \omega^2$$

$$\begin{aligned} \frac{dK^{\text{rot}}}{dt} &= \frac{d}{dt} \left[\frac{1}{2} I \omega^2 \right] = \frac{1}{2} I \left(2\omega \frac{d\omega}{dt} \right) = I \omega \alpha \\ &= (I \alpha) \omega = \tau_{\text{net}} \omega = \tau_{\text{net}} \frac{d\phi}{dt} \end{aligned}$$

$$\tau_{\text{net}} = I \alpha$$

$$\Rightarrow \int \frac{dK^{\text{rot}}}{dt} dt = \int \tau_{\text{net}} \frac{d\phi}{dt} dt = \int_{\phi_a}^{\phi_b} \tau_{\text{net}} d\phi = W^{\text{rot}}$$

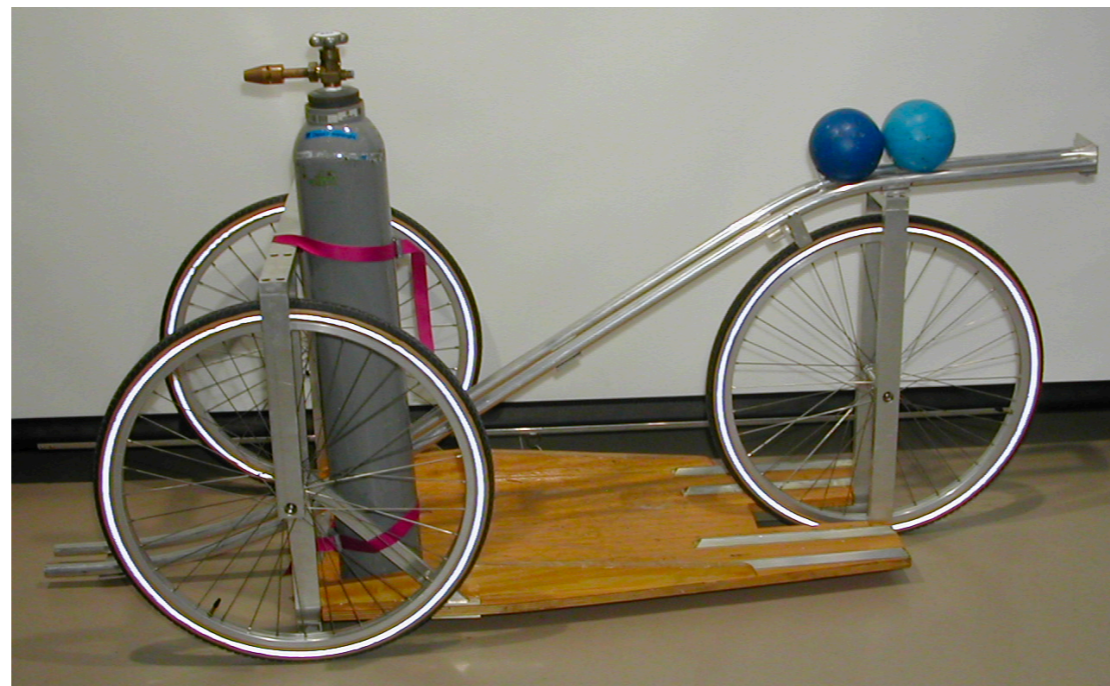
$$\underbrace{K^{\text{rot}}(t_b) - K^{\text{rot}}(t_a)}_{= \Delta K^{\text{rot}}}$$

\Rightarrow

$$\Delta K^{\text{rot}} = W^{\text{rot}}$$

DEMO (15)

Action-reaction **disk**



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Conclusion of rotation of rigid objects about a fixed axis

3. Angular momentum and its conservation

Angular momentum

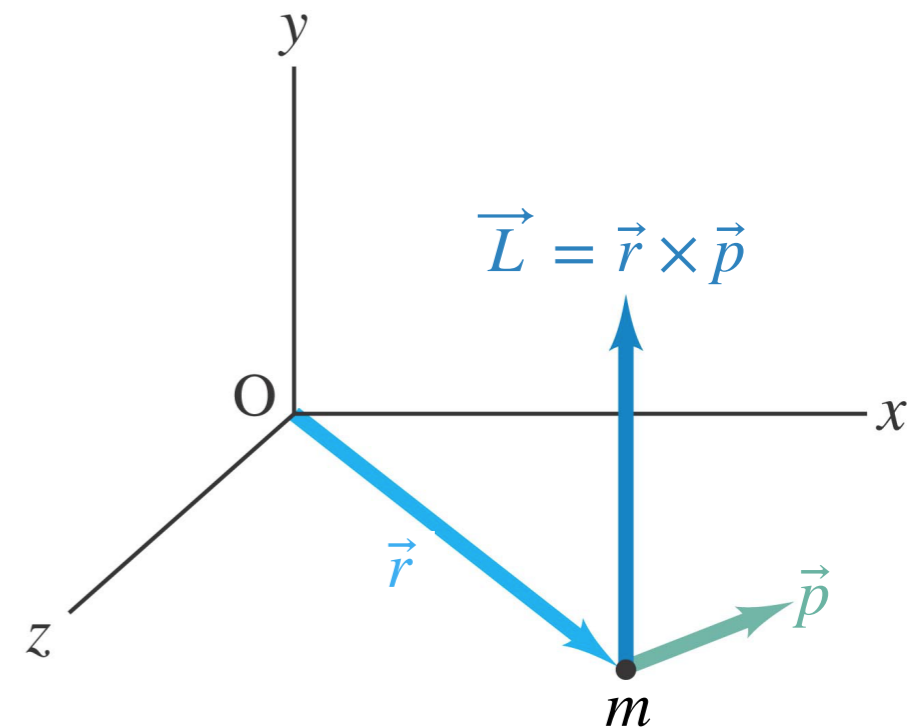
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Angular momentum

- As you may suspect, **angular momentum** \vec{L} is the rotational analogue of *linear* momentum $\vec{p} = m\vec{v}$ (lecture 6)
- It is the *moment* of momentum, defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

where \vec{r} is the position vector from a pivot point to the object



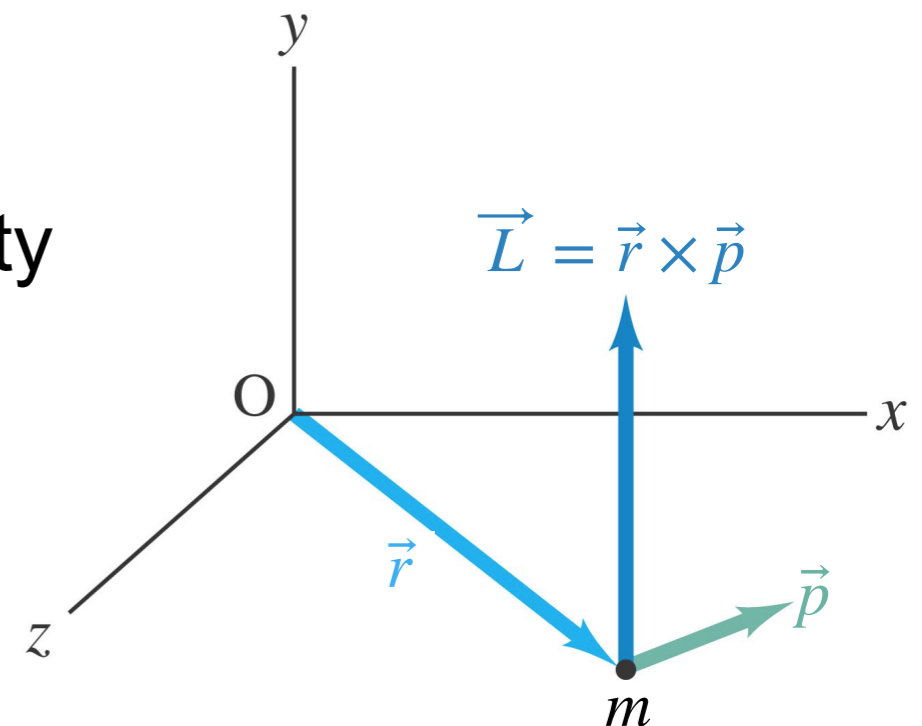
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- Like linear momentum, it is a vector quantity and will be conserved under certain conditions



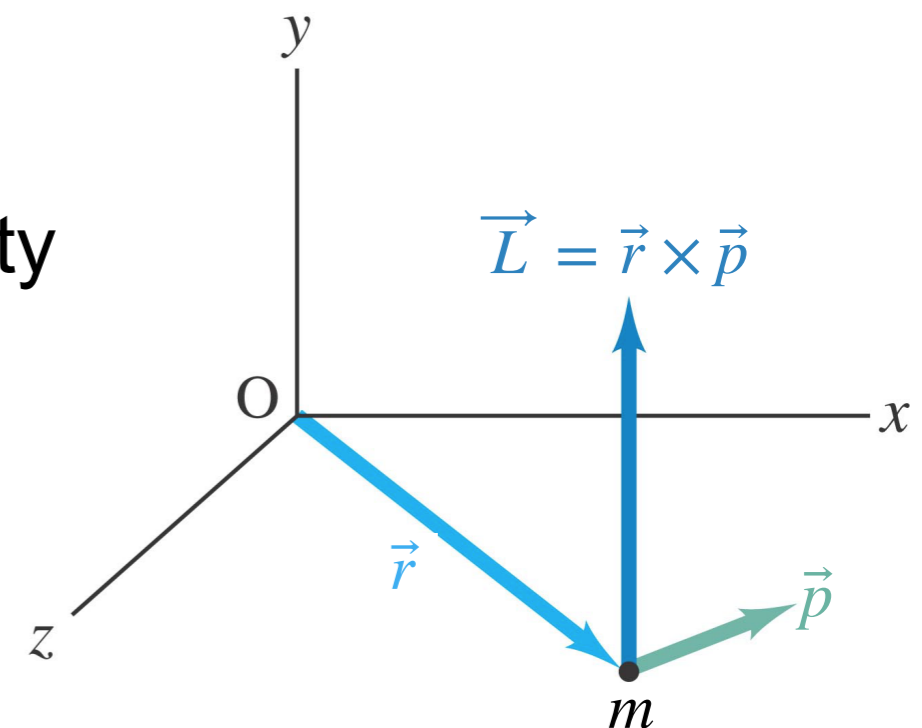
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- It has units of [kg·m²/s]
- Like linear momentum, it is a vector quantity and will be conserved under certain conditions
- Unlike linear momentum, it depends on where the pivot is chosen

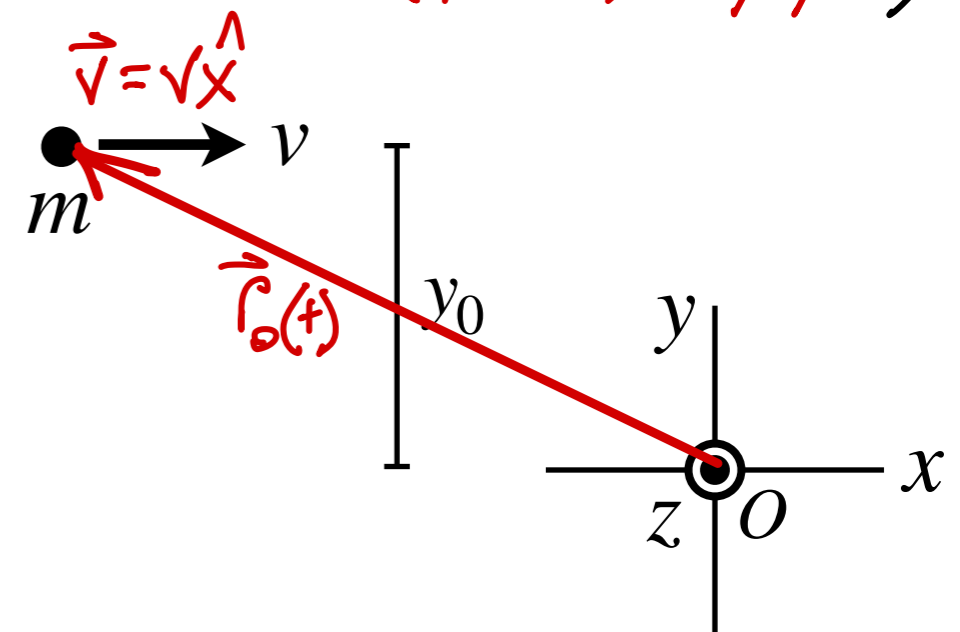


Conceptual question

A particle is moving in the x - y plane with a constant velocity and constant height y_0 (as shown below). The magnitude of the angular momentum L_O about the origin...

- A. is zero because this is not circular motion.
 B. decreases, then increases.
 C. increases, then decreases.
 D. is constant.

$$\vec{r}_O(t) = x(t)\hat{x} + y_0\hat{y} + 0\hat{z}$$



$$\begin{aligned}\vec{L}_O &= \vec{r}_O \times \vec{p} = \vec{r}_O \times (m\vec{v}) = m\vec{r}_O \times \vec{v} \\ &= m(x(t)\hat{x} + y_0\hat{y}) \times (v\hat{x}) \\ &= m[x(t)v(\hat{x} \times \hat{x}) + y_0v(\hat{y} \times \hat{x})] \\ &= -my_0v\hat{z}\end{aligned}$$

$$\Rightarrow |\vec{L}_O| = my_0v$$

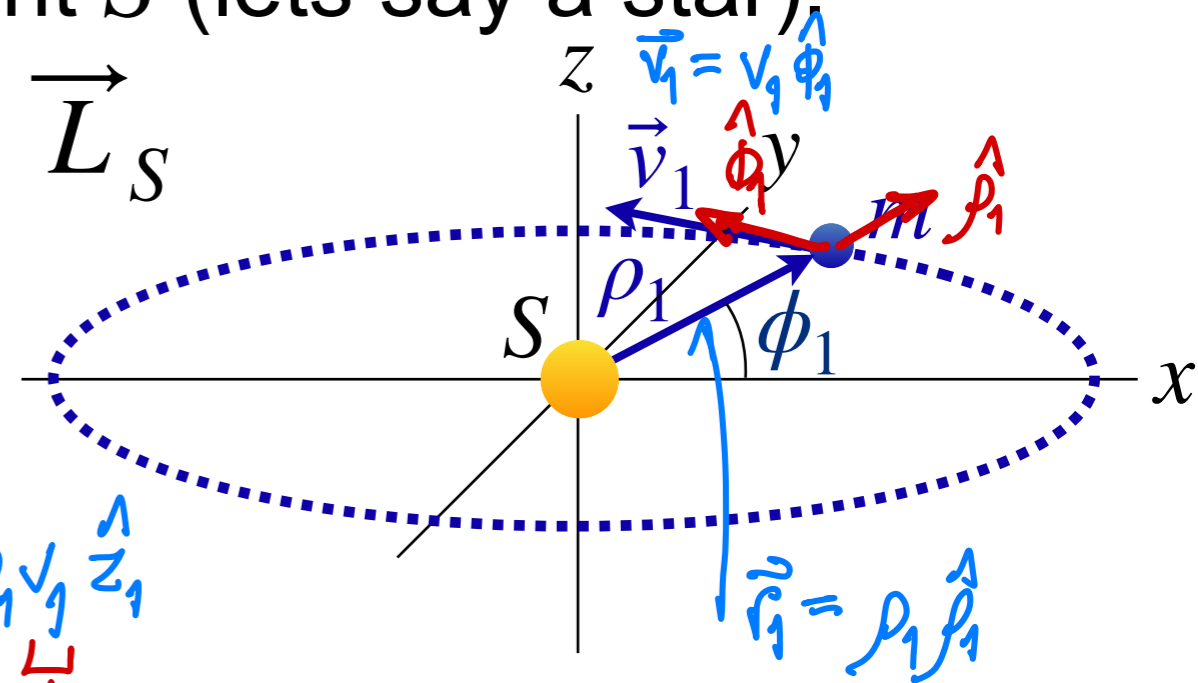
Example: Point particle angular momentum

A point of mass m (lets say a planet) is executing uniform circular motion with $\vec{\omega}$ around point S (lets say a star).

A. What is its angular momentum \vec{L}_S about S ?

$$\begin{aligned}
 \vec{L}_S &= \vec{r}_1 \times \vec{p}_1 = \vec{r}_1 \times (m\vec{v}_1) = m\vec{r}_1 \times \vec{v}_1 \\
 &= m(\rho_1 \hat{\rho}_1) \times (v_1 \hat{\phi}_1) = m\rho_1 v_1 (\hat{\rho}_1 \times \hat{\phi}_1) = m\rho_1 v_1 \hat{z}_1 \\
 &= m\rho_1 \omega \rho_1 \hat{z}_1 = \underbrace{m\rho_1^2}_{I_{1z}} \underbrace{\omega_1}_{\vec{\omega}_1} \hat{z}_1
 \end{aligned}$$

$$\vec{L}_S = I_{1z} \vec{\omega}_1$$

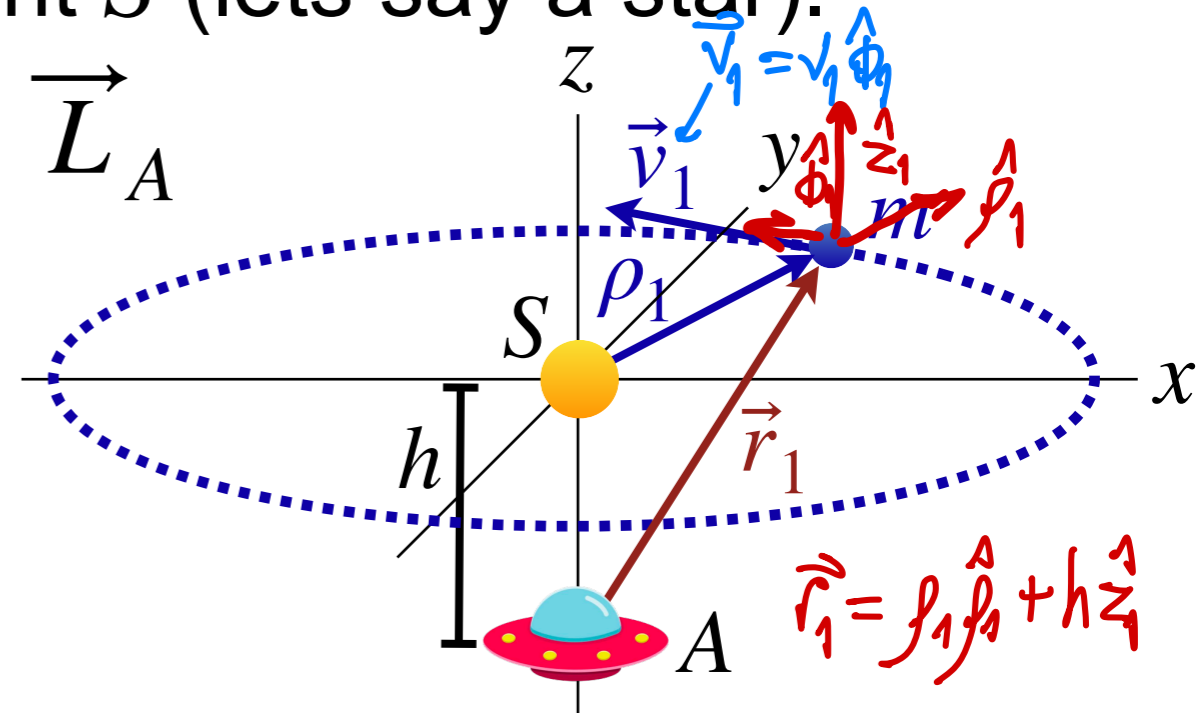


Example: Point particle angular momentum

A point of mass m (lets say a planet) is executing uniform circular motion with $\vec{\omega}$ around point S (lets say a star).

B. What is its angular momentum \vec{L}_A about a lower point A ?

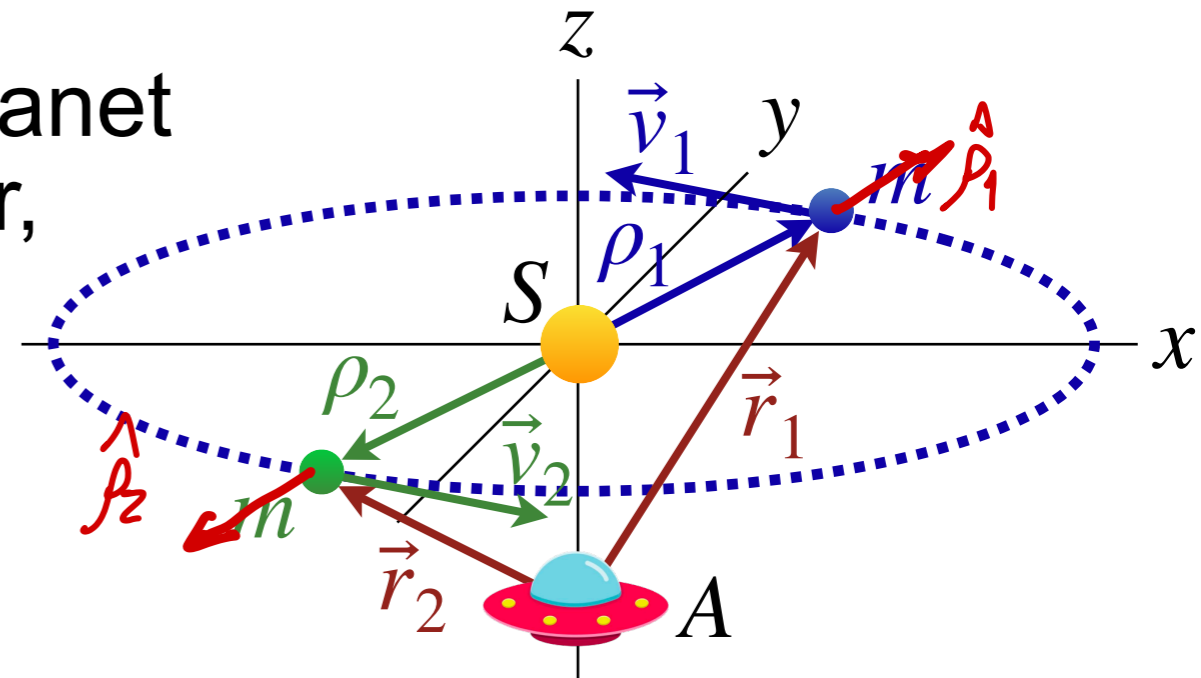
$$\begin{aligned}
 \vec{L}_A &= \vec{r}_1 \times \vec{p}_1 = m \vec{r}_1 \times \vec{v}_1 \\
 &= m (\rho_1 \hat{\rho}_1 + h \hat{z}_1) \times (v_1 \hat{\phi}_1) \\
 &= \underbrace{m \rho_1 v_1 (\hat{\rho}_1 \times \hat{\phi}_1)}_{\vec{L}_S} + \underbrace{mh v_1 (\hat{z}_1 \times \hat{\phi}_1)}_{-\hat{\rho}_1} \\
 &= \vec{L}_S - mh v_1 \hat{\rho}_1 \\
 &= \boxed{\vec{L}_S - mh \rho_1 \omega_1 \hat{\rho}_1} \leftarrow \hat{\rho}_1 = \hat{\rho}_1(t)
 \end{aligned}$$



Example: Point particle angular momentum

A point of mass m (lets say a planet) is executing uniform circular motion with $\vec{\omega}$ around point S (lets say a star).

C. If we add a second identical planet on the opposite side of the star, what is \vec{L}_{sys} of the system?



We have here :

$$\rho_1 = \rho_2 \quad \hat{\rho}_2 = -\hat{\rho}_1$$

$$\phi_2 = \phi_1 + \pi \quad \hat{\phi}_2 = -\hat{\phi}_1$$

$$\omega_2 = \omega_1 \quad v_2 = v_1 \quad \hat{z}_1 = \hat{z}_2$$

$$\vec{L}_{1A} = m\rho_1^2\omega_1\hat{z}_1 - m\rho_1\omega_1\hat{\rho}_1$$

$$\begin{aligned} \vec{L}_{2A} &= m\rho_2^2\omega_2\hat{z}_2 - m\rho_2\omega_2\hat{\rho}_2 \\ &= m\rho_1^2\omega_1\hat{z}_1 - m\rho_1\omega_1(-\hat{\rho}_1) \end{aligned}$$

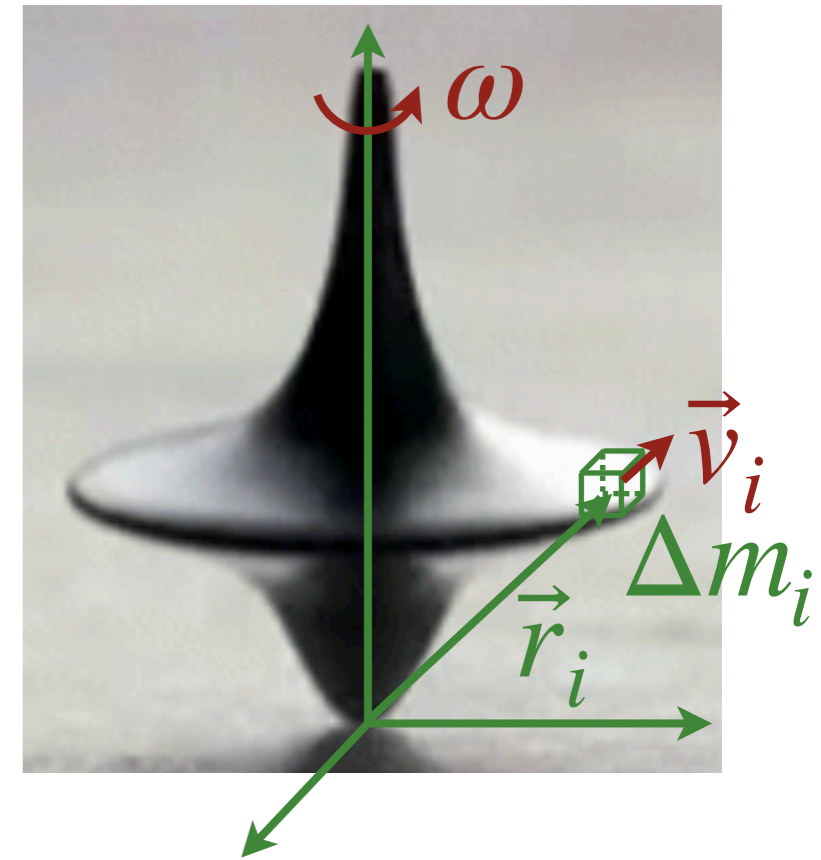
$$\vec{L}_{\text{sys}} = \vec{L}_{1A} + \vec{L}_{2A} = \underbrace{2m\rho_1^2\omega_1\hat{z}_1}_{\vec{I}_{\text{tot},A}} + \vec{0} = \vec{I}_{\text{tot},A}\vec{\omega}$$

$$\begin{aligned} \vec{I}_{\text{tot},A} &= \sum_i m_i \rho_i^2 = \frac{m}{\rho_1} \rho_1^2 + \frac{m}{\rho_2} \rho_2^2 \\ &= 2m\rho_1^2 \end{aligned}$$

Angular momentum of rotating rigid bodies

- Imagine the object is composed of many differential elements, labeled $i = 1, 2, 3, \dots$, with positions \vec{r}_i

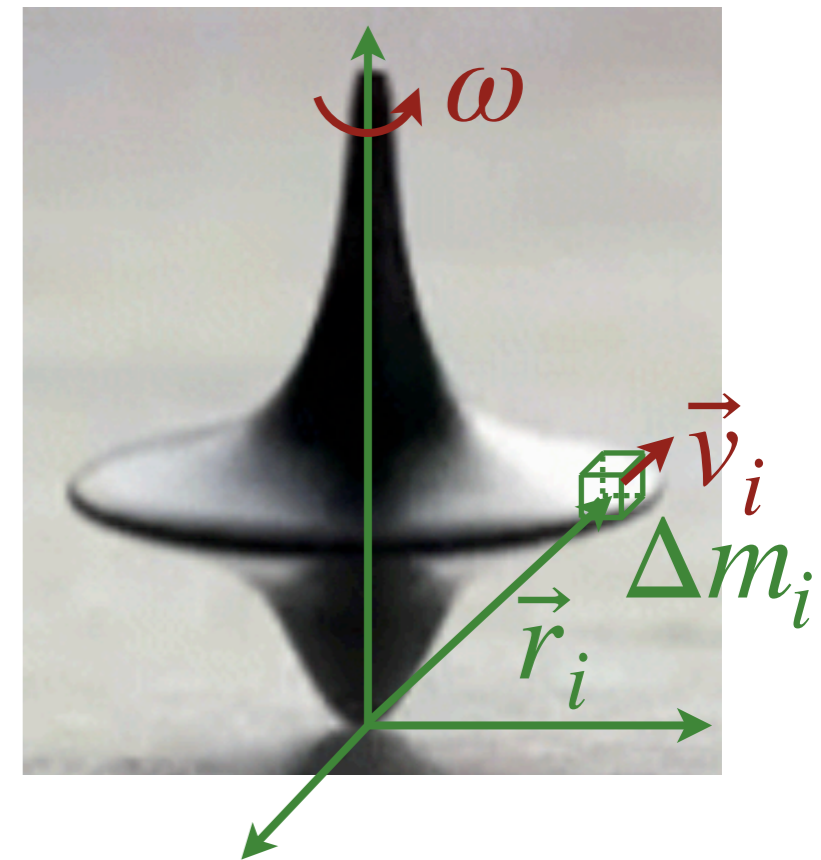
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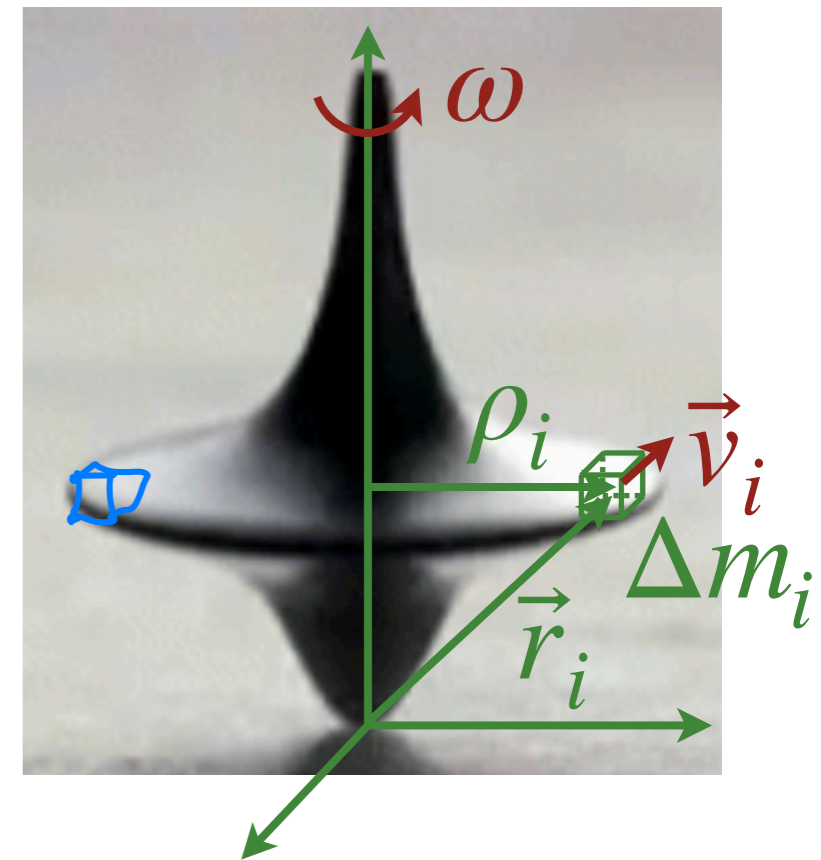
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- For pure rotation of a symmetric object, the $\hat{\rho}$ component of \vec{L} cancels

$$\begin{aligned} \vec{L} &= \sum_i \vec{L}_i = \sum_i \Delta m_i (\rho_i \hat{\rho}_i + \cancel{z_i \hat{z}_i}) \times (v_i \hat{\phi}_i) \\ &= \sum_i \Delta m_i \rho_i v_i \hat{z}_i \quad \hat{z}_i = \hat{z} \\ &= \sum_i \Delta m_i \rho_i v_i \hat{z} = \sum_i \Delta m_i \rho_i^2 \underbrace{v_i}_{\rho_i \omega} \hat{z} = \sum_i \Delta m_i \rho_i^2 \vec{\omega} \end{aligned}$$



Angular momentum of rotating rigid bodies

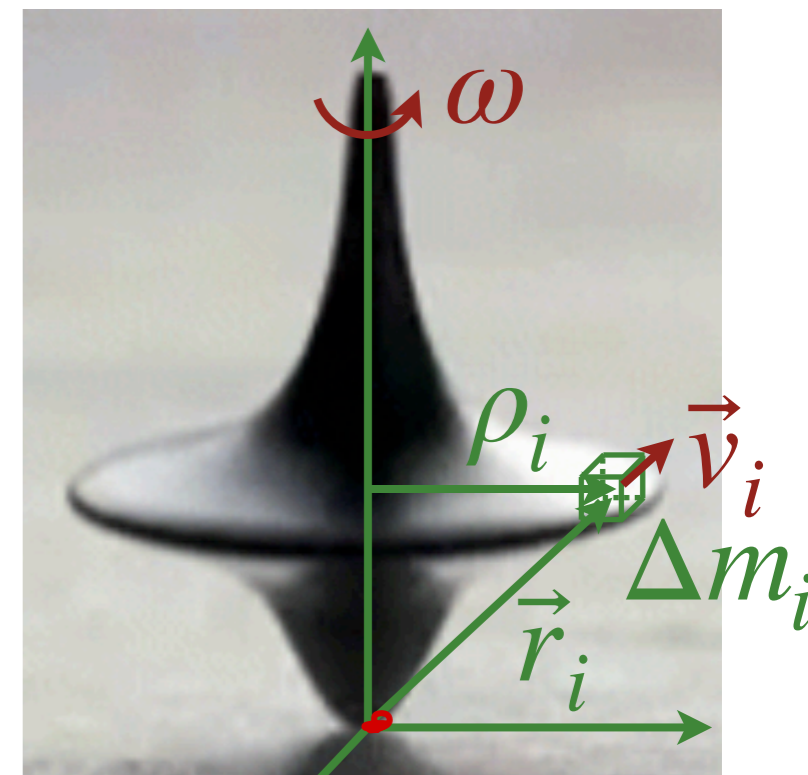
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- Since $v_i = \rho_i \omega$, we see that $\vec{L} = \underbrace{\sum_i \rho_i^2 \Delta m_i}_{I} \vec{\omega} = \vec{\omega} I$



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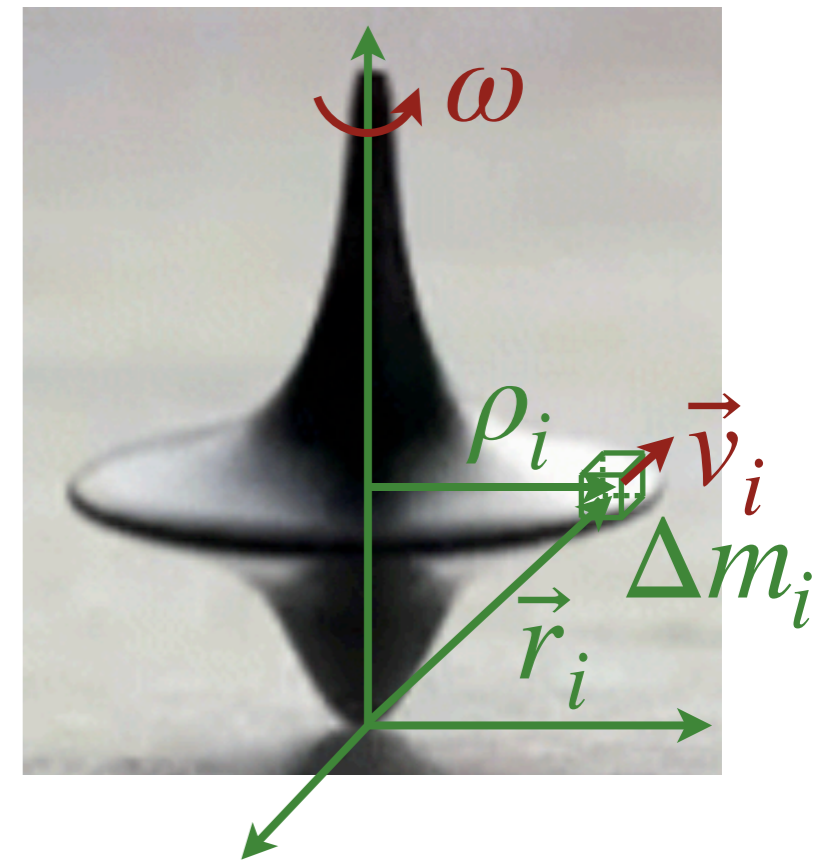
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- Since $v_i = \rho_i \omega$, we see that $\vec{L} = \sum_i \rho_i^2 \Delta m_i \omega \hat{z}$

- In the limit of $\Delta m_i \rightarrow 0$, $\vec{L} = \int_M \rho^2 dm \vec{\omega} \Rightarrow \boxed{\vec{L} = I \vec{\omega}}$



Angular momentum and torque

- If angular momentum \vec{L} is analogous to momentum and torque $\vec{\tau}$ is analogous to force, what is their relationship?

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\frac{d}{dt} \vec{L} = \frac{d}{dt} \left[\sum_i \vec{r}_i \times \vec{p}_i \right] = \sum_i \frac{d}{dt} \left[\vec{r}_i \times \vec{p}_i \right] = \sum_i \left[\frac{d\vec{r}_i}{dt} \times \vec{p}_i + \vec{r}_i \times \frac{d\vec{p}_i}{dt} \right] \quad \vec{F}_i = \frac{d\vec{p}_i}{dt}$$

$$= \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{\tau}_i = \vec{\tau}_{net}$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Conservation of angular momentum

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*If the net torque on a system is **zero**
(and matter is not exchanged),
the total angular momentum does **not** change with time.*

Conservation of angular momentum

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

*If the net torque on a system is **zero** (and matter is not exchanged), the total angular momentum does **not** change with time.*

- In other words, if $\vec{\tau}_{net} = 0$ then the angular momentum is conserved:

$$\vec{L}_i = \vec{L}_f$$

DEMO (17)

$$\left. \begin{array}{l} \vec{L}_i = \vec{L}_f \quad (\text{no net } \vec{\tau}_{\text{ext}}) \\ \vec{L} = I\vec{\omega} \end{array} \right\} I_i \omega_i = I_f \omega_f \Rightarrow \frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$

Swivel stool

Conceptual question

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls her arms in, she reduces her moment of inertia and her angular speed increases. Compared to her initial **rotational kinetic energy**, her **rotational kinetic energy** after she has pulled her arms in must be...

- A. the same.
 B. larger.
 C. smaller.

$$\vec{L} = I\vec{\omega}$$

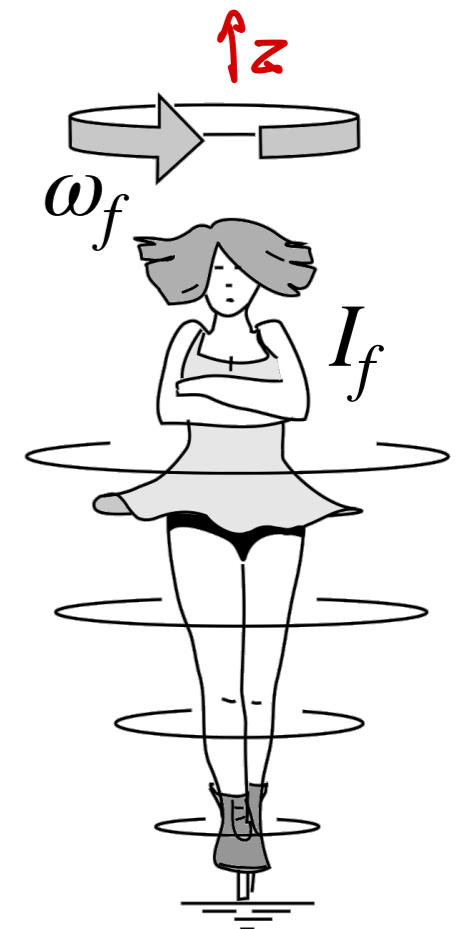
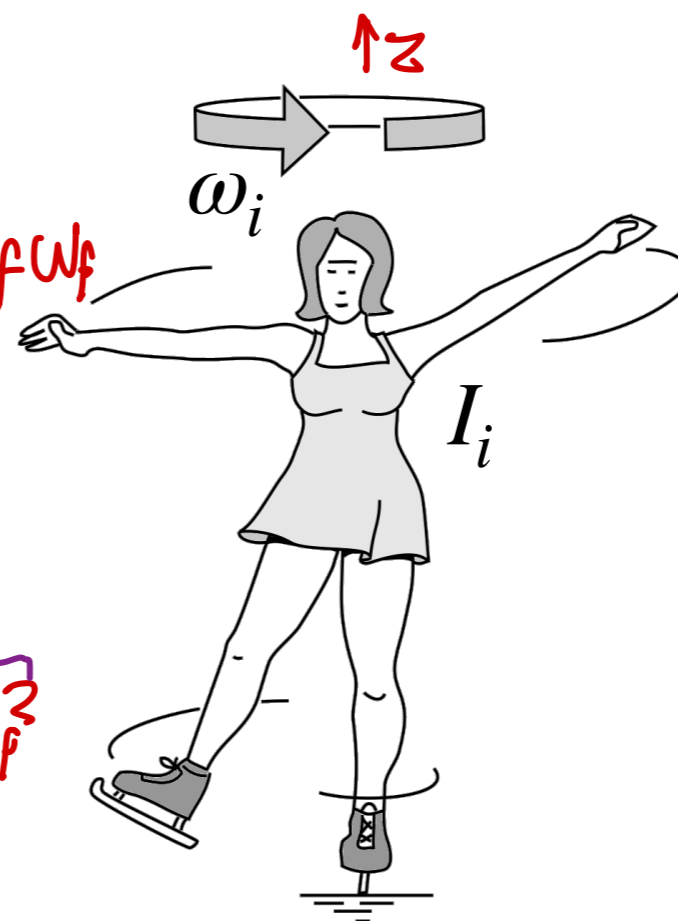
$$\vec{L}_i = \vec{L}_f \Rightarrow I_i\omega_i = I_f\omega_f$$

$$\Rightarrow I_i^2\omega_i^2 = I_f^2\omega_f^2$$

$$\Rightarrow \frac{1}{2}I_i^2\omega_i^2 = \frac{1}{2}I_f^2\omega_f^2$$

$$\Rightarrow \frac{I_i}{I_f} \cdot \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_f\omega_f^2$$

$$\Rightarrow K_f^{\text{rot}} = \frac{I_i}{I_f} K_i^{\text{rot}}$$



Conceptual question

You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is most effective in loosening the nut?

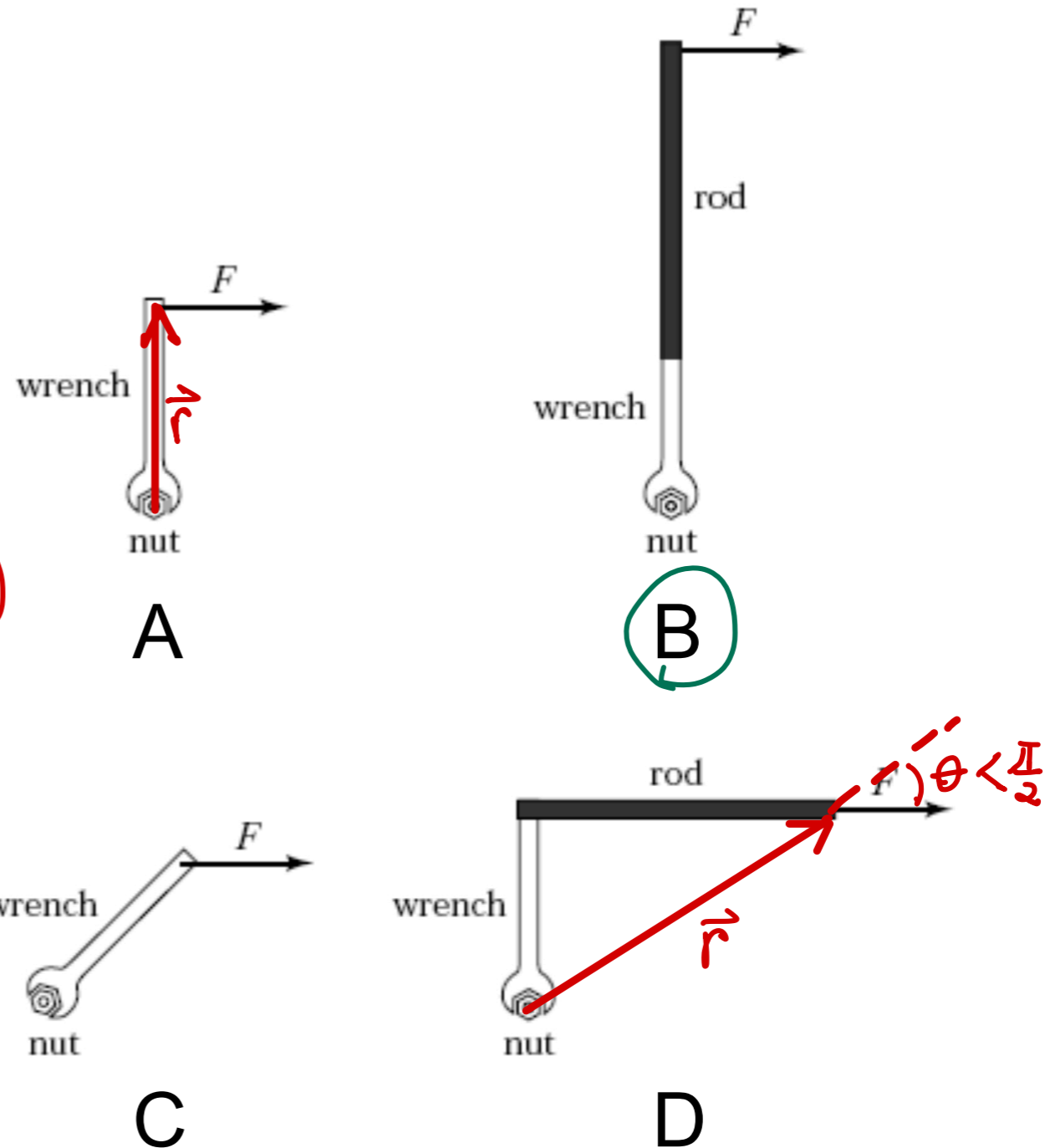
$$\vec{\tau}_{\text{nut}} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}_{\text{nut}}| = |\vec{r} \times \vec{F}| = |\vec{r}| |\vec{F}| \sin(\theta)$$

Large $|\vec{r}|$

~~Large $|\vec{F}|$~~ (\vec{F} equal in all cases)

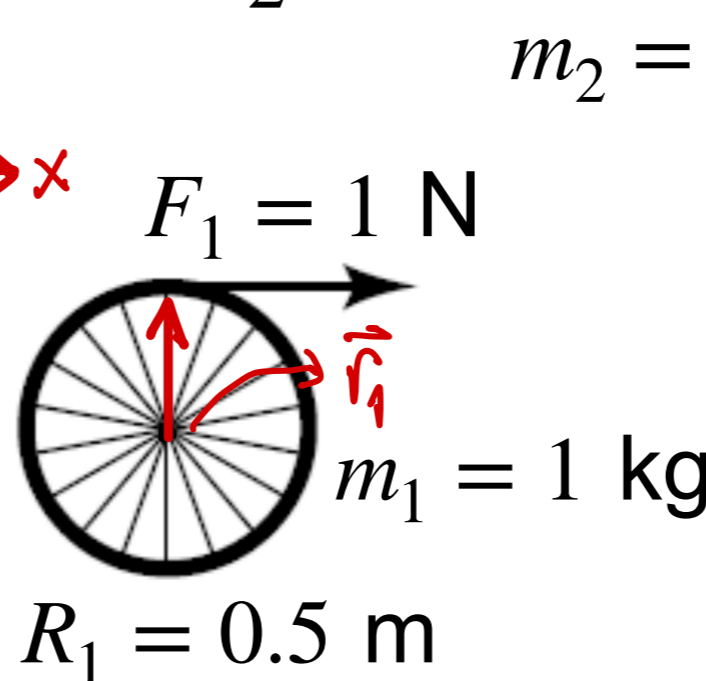
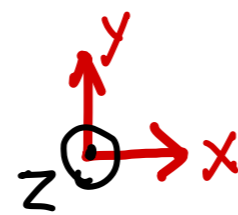
Large $\sin(\theta)$ ($\theta \approx \frac{\pi}{2}$)



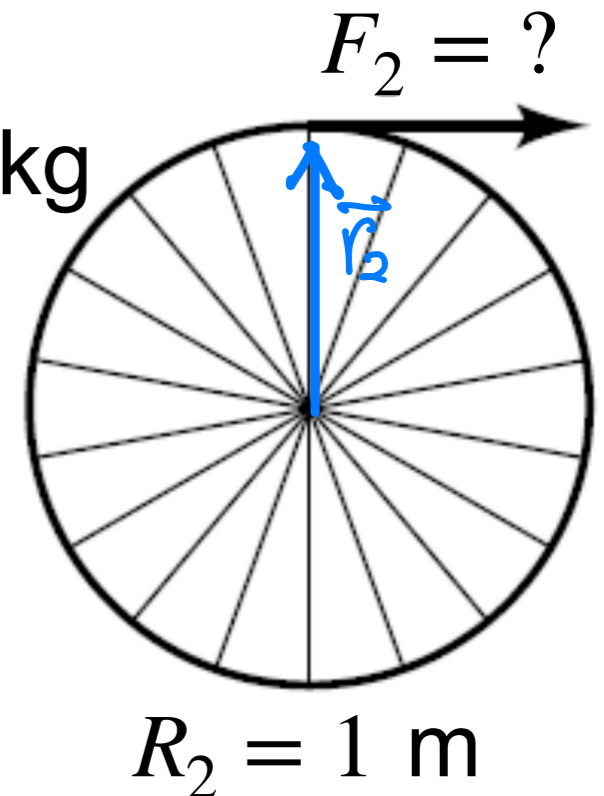
Conceptual question

Two wheels with fixed hubs, each having a mass of 1 kg, start from rest, and forces are applied as shown. Assume the hubs and spokes are massless, so that the moment of inertia is $I = mR^2$. In order to impart identical angular accelerations, how large must F_2 be?

- A. 0.25 N
- B. 0.5 N
- C. 1.0 N
- D. 2.0 N
- E. 4.0 N



$m_2 = 1 \text{ kg}$



$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = (R_1 \hat{y}) \times (F_1 \hat{x}) = R_1 F_1 (\hat{y} \times \hat{x}) = -R_1 F_1 \hat{z} = I_1 \alpha_1 \quad (\alpha_1 \text{ in } -\hat{z} \text{ dir.})$$

$$I_1 \alpha_1 = R_1 F_1 \Rightarrow \alpha_1 = \frac{1}{I_1} R_1 F_1 = \frac{1}{m R_1^2} R_1 F_1 = \frac{1}{m R_1} F_1$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = -R_2 F_2 \hat{z} = I_2 \alpha_2 \Rightarrow \alpha_2 = \frac{1}{m R_2} F_2$$

$$\alpha_1 = \alpha_2 \Rightarrow \frac{F_1}{m R_1} = \frac{F_2}{m R_2} \Rightarrow F_2 = \frac{R_2}{R_1} F_1$$

Summary of rotation and translation

Rotational motion (about a fixed axis)		Translational motion (in one dimension)	
Angular position	ϕ	Position	x
Angular speed	$\omega = d\phi/dt$	Speed	$v = dx/dt$
Angular acceleration	$\alpha = d\omega/dt$	Acceleration	$a = dv/dt$
Moment of inertia	$I = \int \rho^2 dm$	Mass	m
Net torque	$\Sigma \tau_{ext} = I\alpha$	Net force	$\Sigma F_{ext} = ma$
Rotational kinetic energy	$K^{rot} = I\omega^2/2$	Translational kinetic energy	$K^{trans} = mv^2/2$
Work	$W = \int_{\phi_a}^{\phi_b} \tau d\phi$	Work	$W = \int_{x_a}^{x_b} F dx$
Power	$P = \tau\omega$	Power	$P = Fv$
Angular momentum	$L = I\omega$	Momentum	$p = mv$
Net torque	$\Sigma \tau_{ext} = dL/dt$	Net torque	$\Sigma F_{ext} = dp/dt$

Mock exam tomorrow

Bon courage!