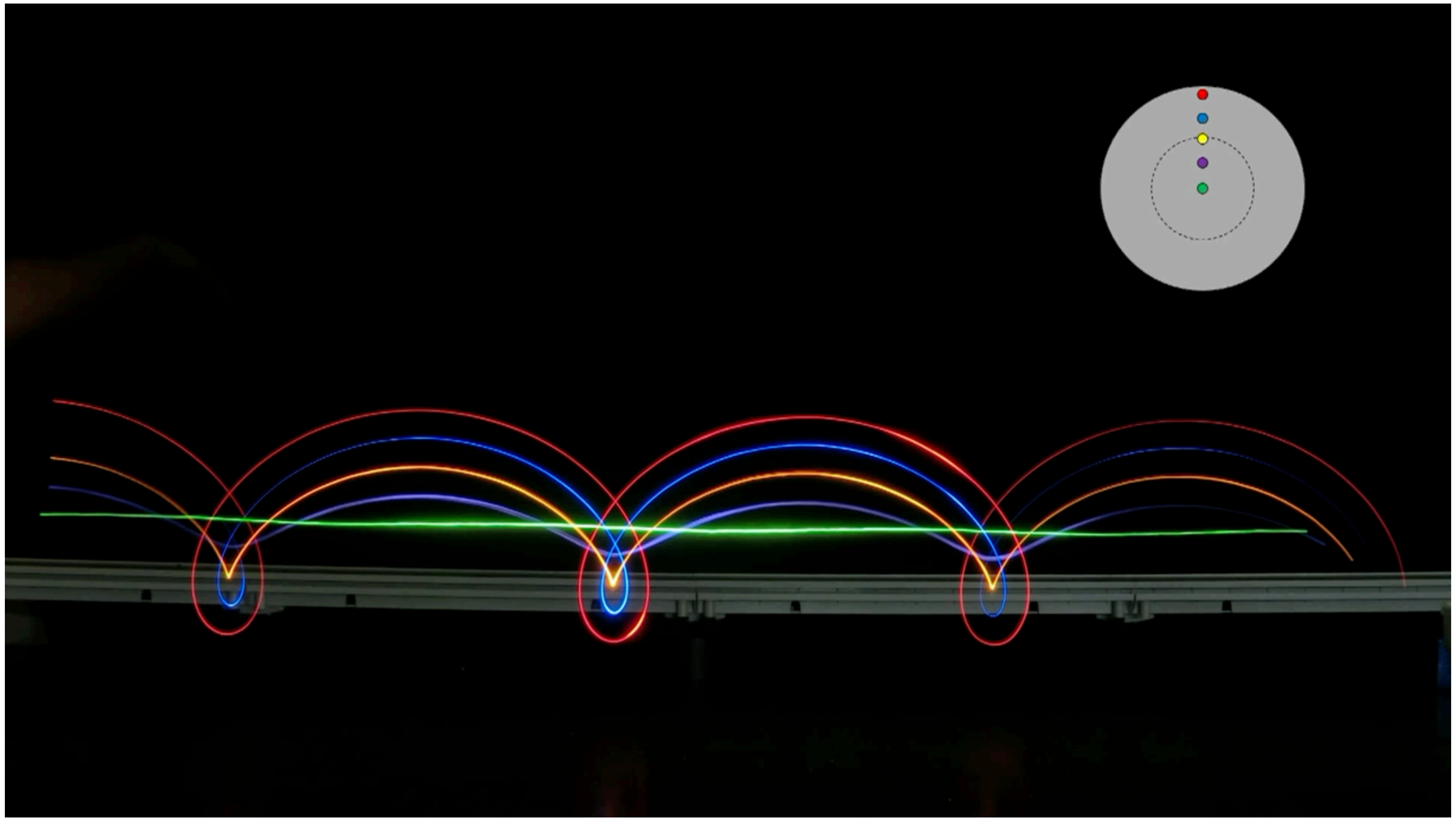
General Physics: Mechanics

Swiss Plasma Center

PHYS-101(en)

Lecture 11a: Rotational motion and static equilibrium



Dr. Marcelo Baquero
marcelo.baquero@epfl.ch
November 24th, 2025

Announcement

EPFL Swiss Plasma Center

- We'll hold another ***mini mock exam*** next Tuesday December 2nd
    - In-class (SG1) during normal lecture hours (10:15-11:00)
    - You can bring a "cheat" sheet containing formulas or all of your notes, as you wish
    - Hand exam to me at the end if you want to have it graded (optional)
    - Does **not** matter at all for your final grade
    - Exam solutions will be published

Today's agenda (Serway 10,12; MIT 16-18)

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1. Review of circular motion
2. Rotation of rigid objects about a <u>fixed axis</u>
   - Rotational kinetic energy
   - Moment of inertia
   - Torque
3. Static equilibrium

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Week 4: Cylindrical coordinates

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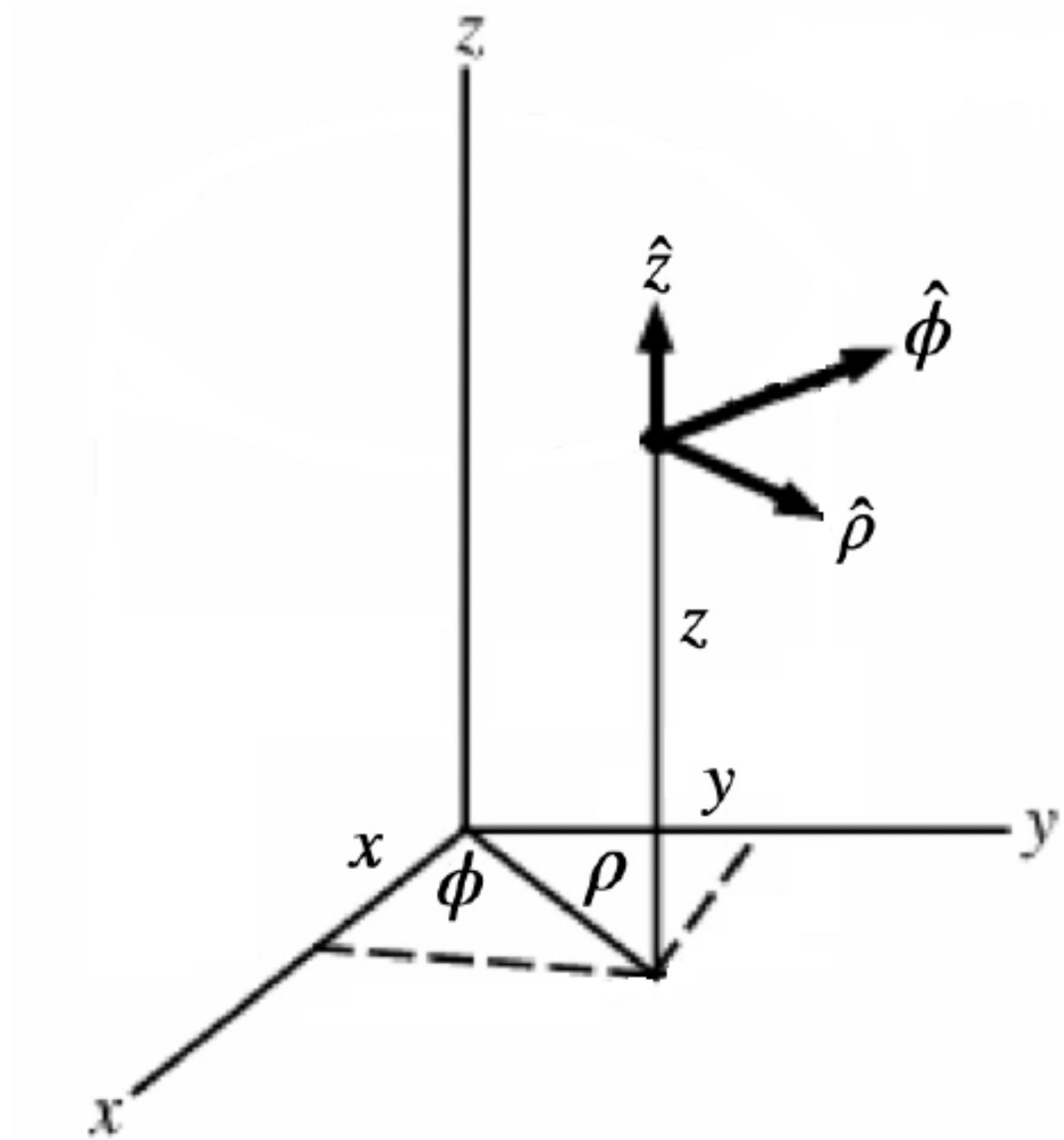
- Position vector:
  $\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$



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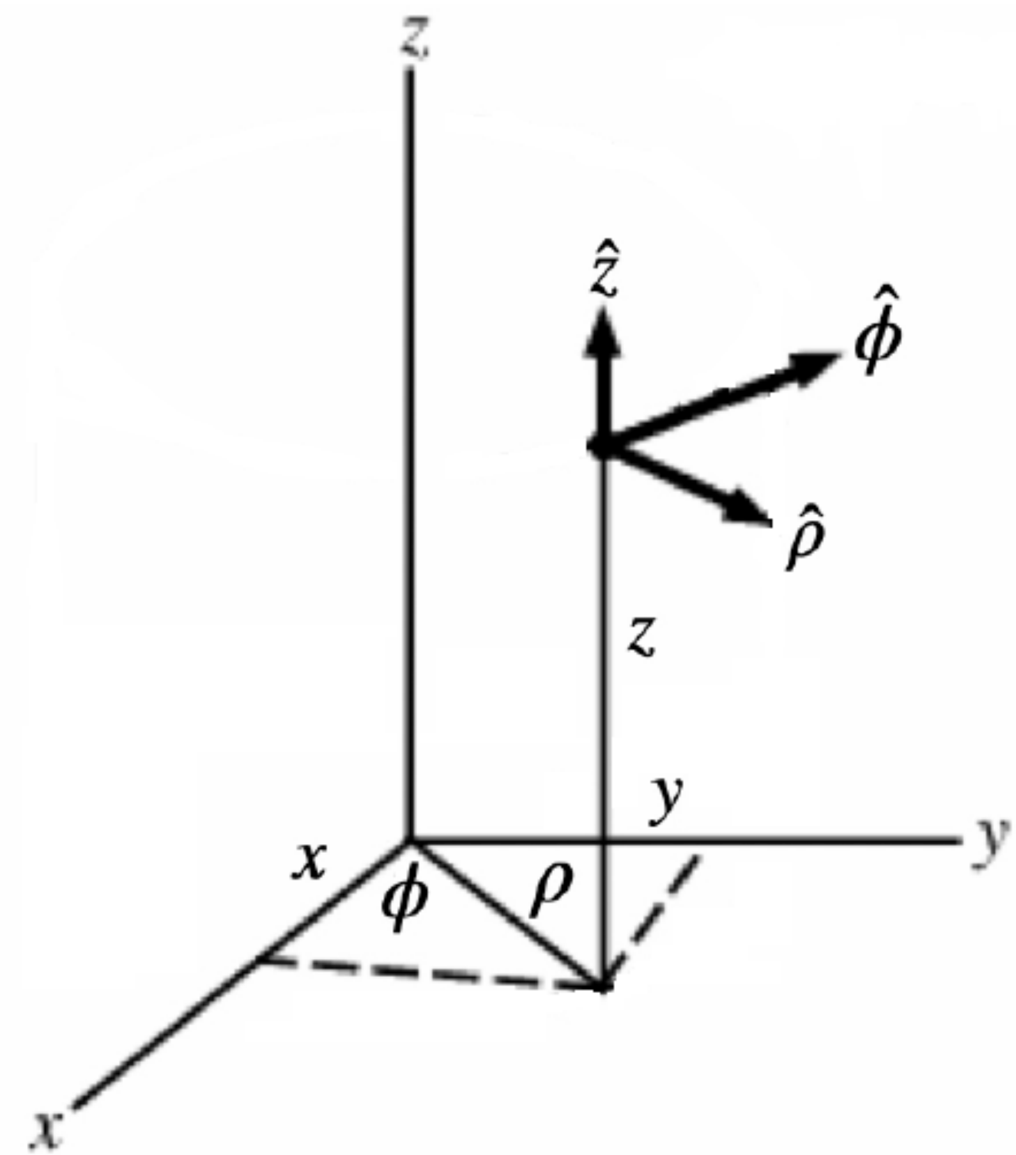
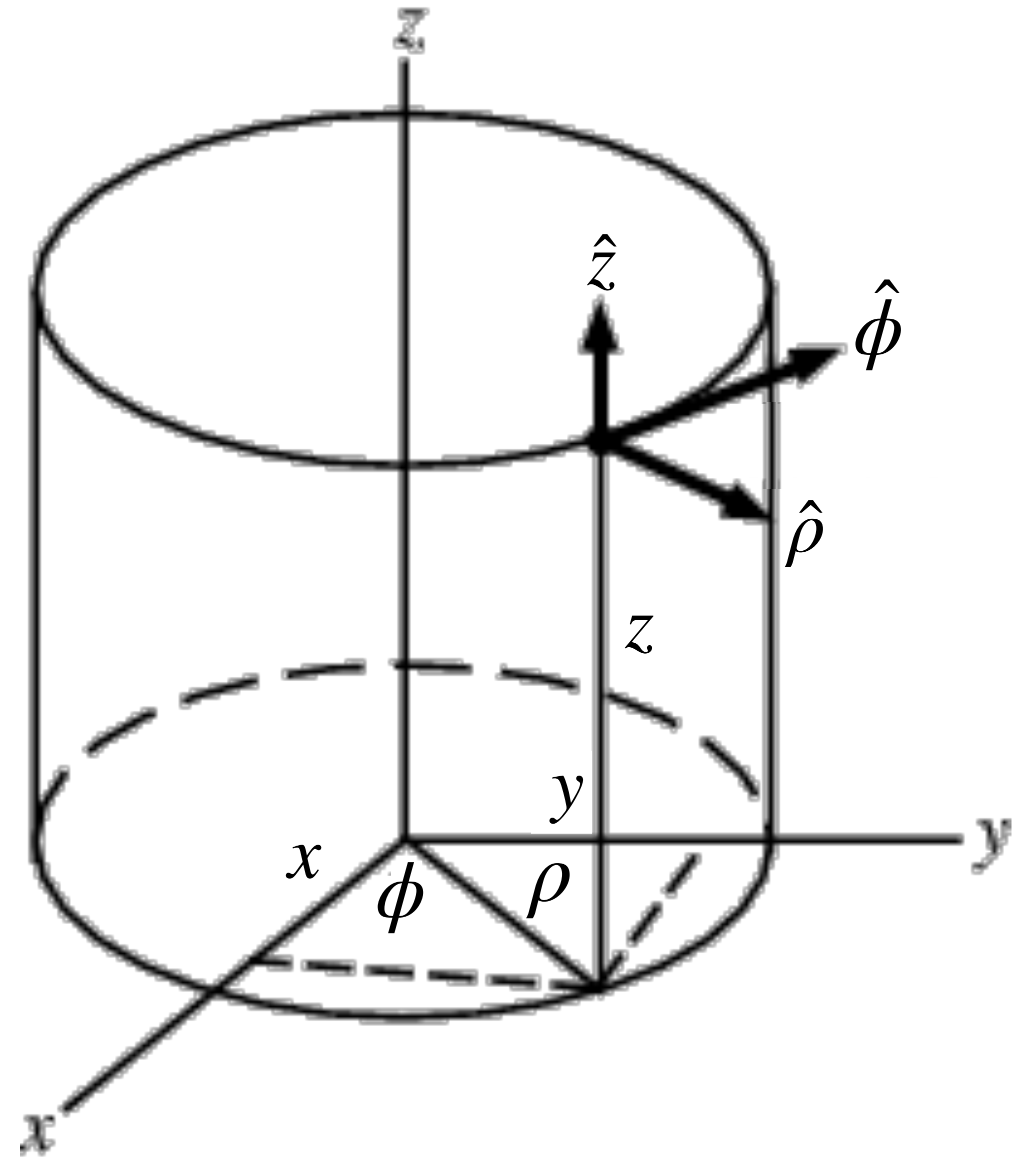


- Position vector:
  $\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$
- *Linear* velocity:
  $\vec{v}(t) = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$
- *Linear* acceleration:
  $\vec{a}(t) = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right)\hat{\phi} + \ddot{z}\hat{z}$

Week 4: Circular motion

- In circular motion $\rho(t) = \rho_0$ where ρ_0 is constant



Week 4: Circular motion

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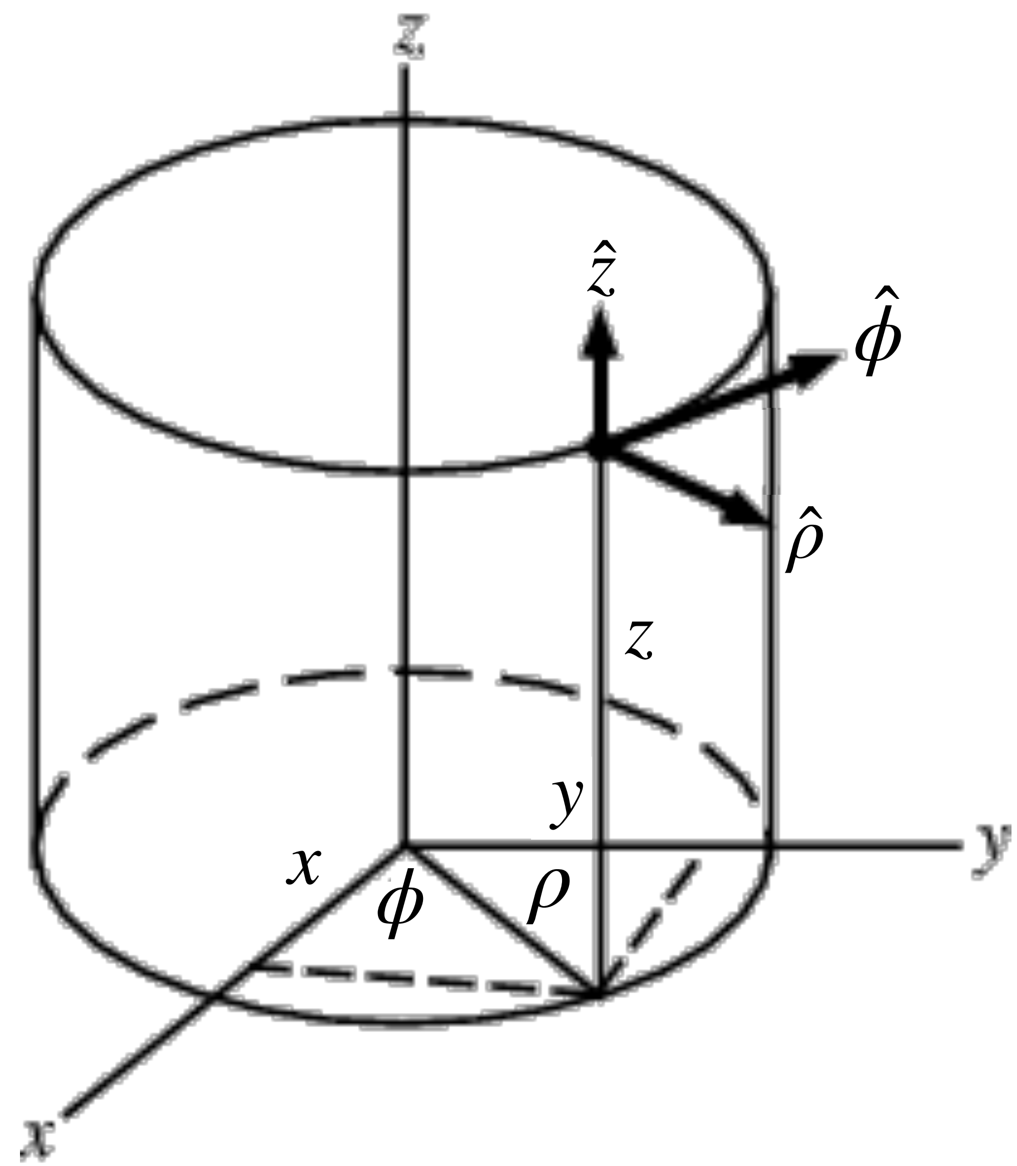
- Also $z(t) = z_0$ where z_0 is a constant (which we typically choose to be 0 with an appropriate reference frame)



Week 4: Circular motion

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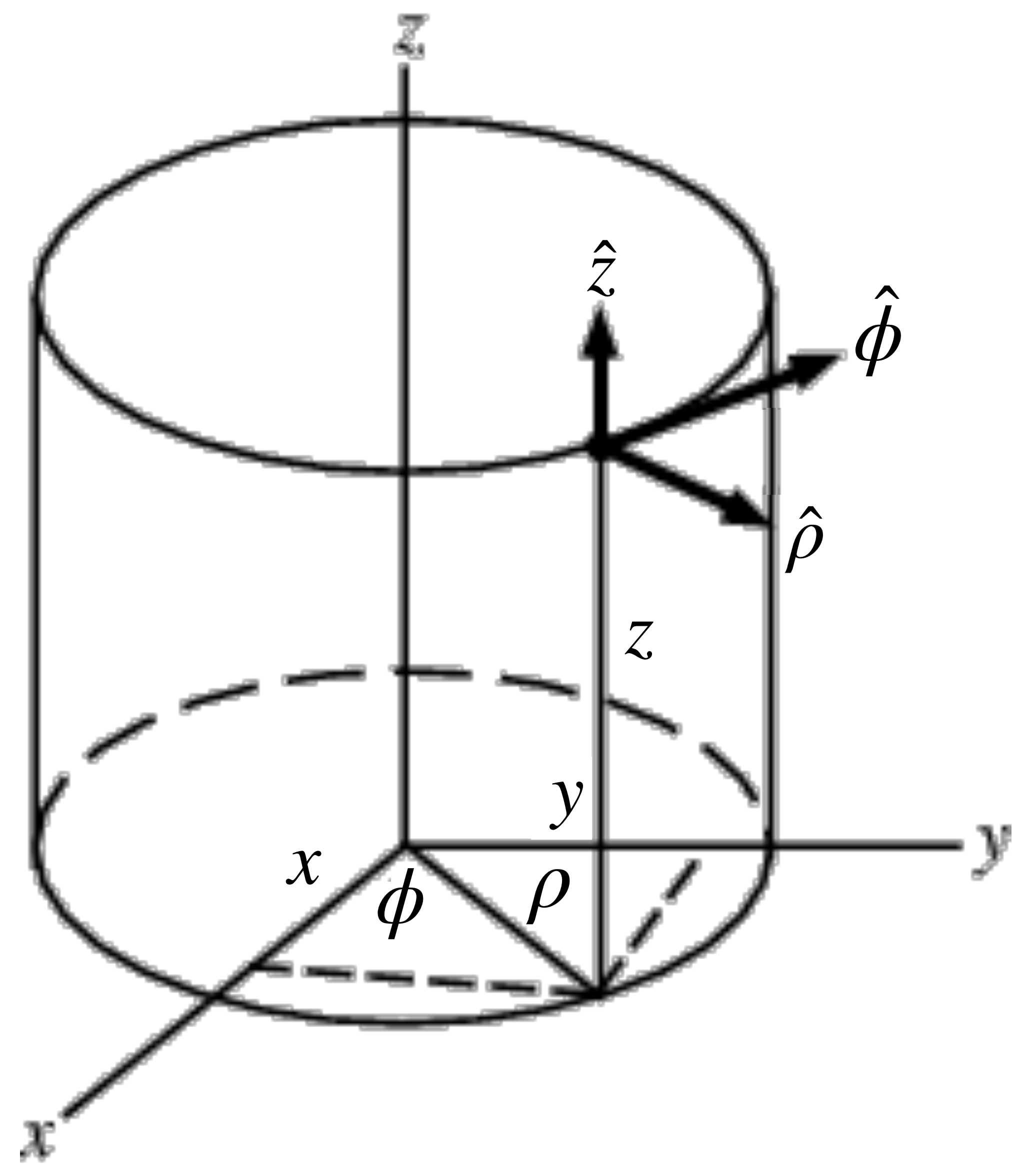
$\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$

- *Linear* velocity:
$\vec{v}(t) = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$

- *Linear* acceleration:
$\vec{a}(t) = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right)\hat{\phi} + \ddot{z}\hat{z}$



Week 4: Circular motion

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- Angular speed: $\omega(t) = \frac{d\phi}{dt} = \dot{\phi}$
- Angular acceleration: $\alpha(t) = \dot{\omega} = \ddot{\phi}$

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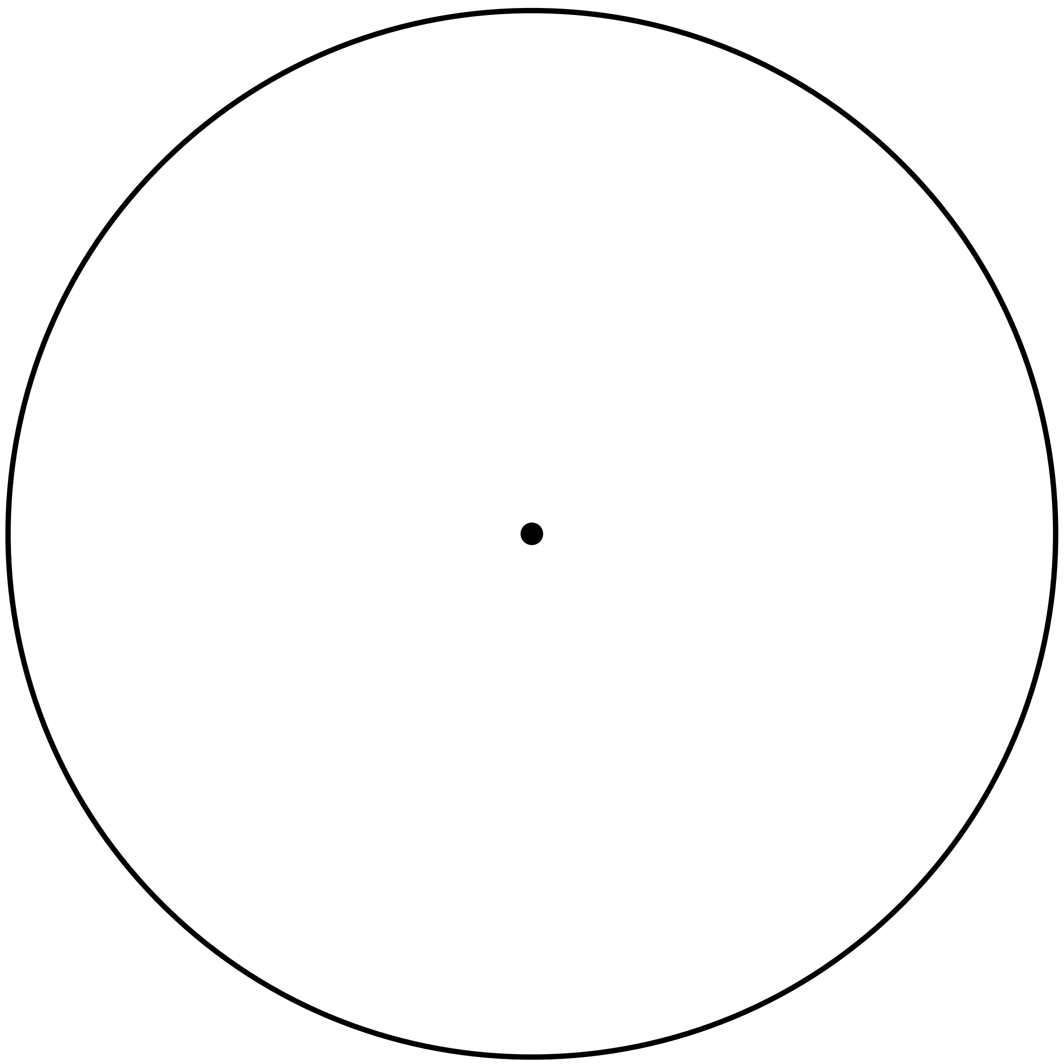
- Angular acceleration: $\alpha(t) = \dot{\omega} = \ddot{\phi}$

- Position: $\vec{r}(t) = \rho_0 \hat{\rho}(t)$
- Velocity: $\vec{v}(t) = \rho_0 \omega(t) \hat{\phi}(t)$
- Acceleration:
  $\vec{a}(t) = -\rho_0[\omega(t)]^2\,\hat{\rho} + \rho_0 \alpha(t)\,\hat{\phi}$



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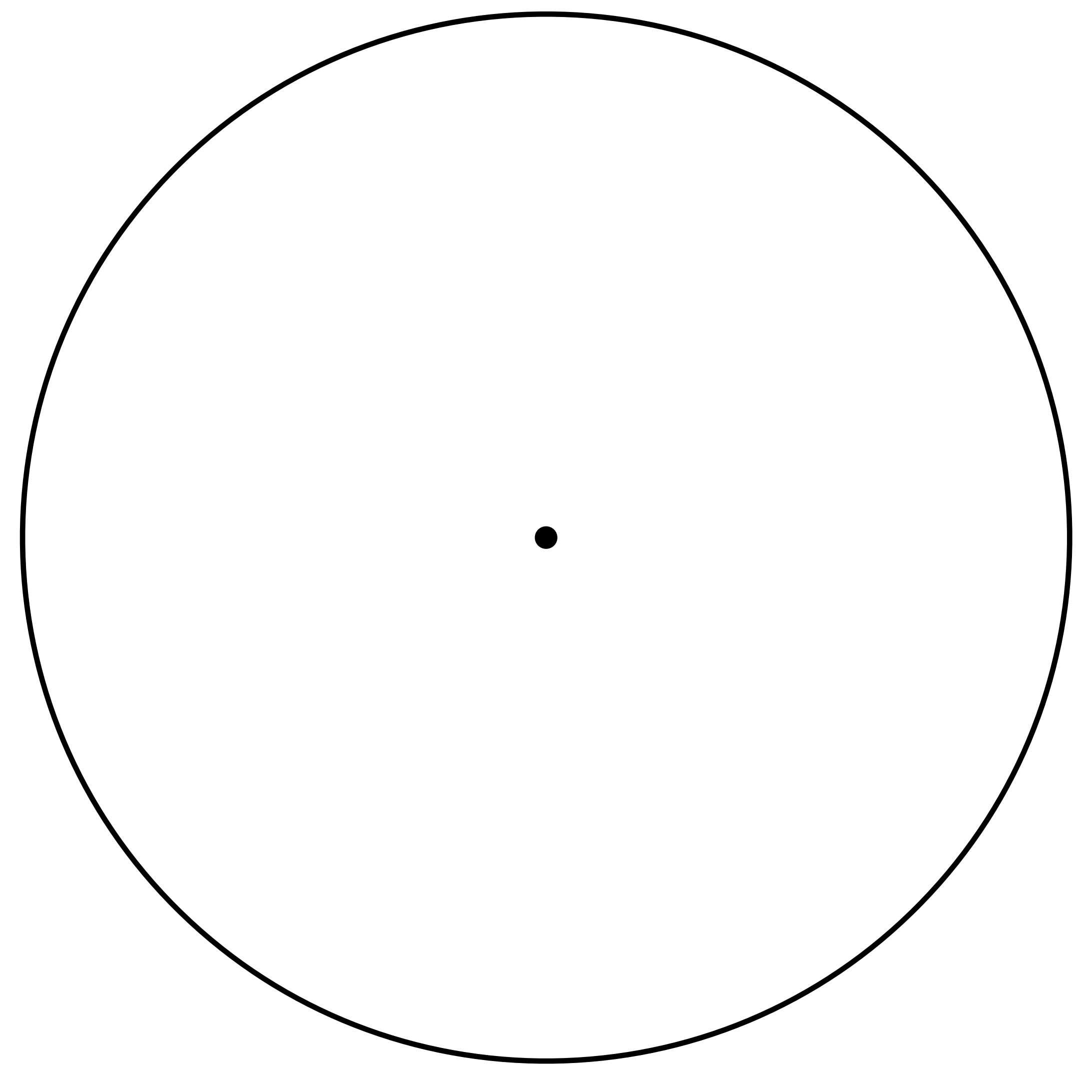
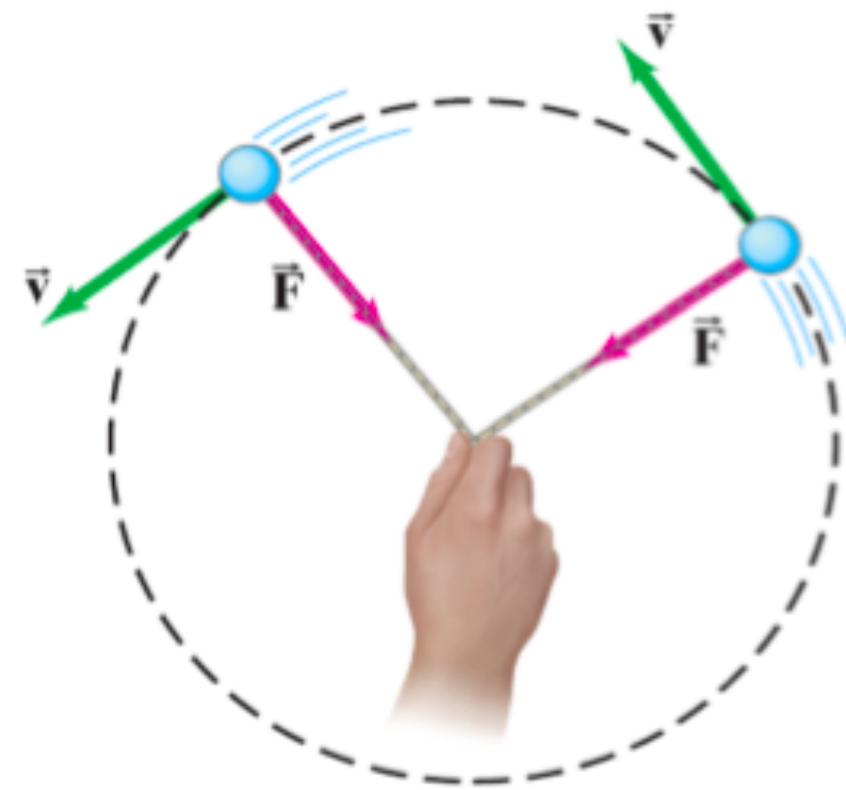


- Position: $\vec{r}(t) = \rho_0 \hat{\rho}(t)$
- Velocity: $\vec{v}(t) = \rho_0 \omega(t) \hat{\phi}(t)$
- Acceleration:
  $\vec{a}(t) = -\rho_0 [\omega(t)]^2 \, \hat{\rho} + \rho_0 \alpha(t) \, \hat{\phi}$
- Angular displacement: $\Delta\phi\hat{\phi}$
- Distance traveled (arc length): $\ell = \rho_0 \Delta\phi$

Week 4: Circular motion

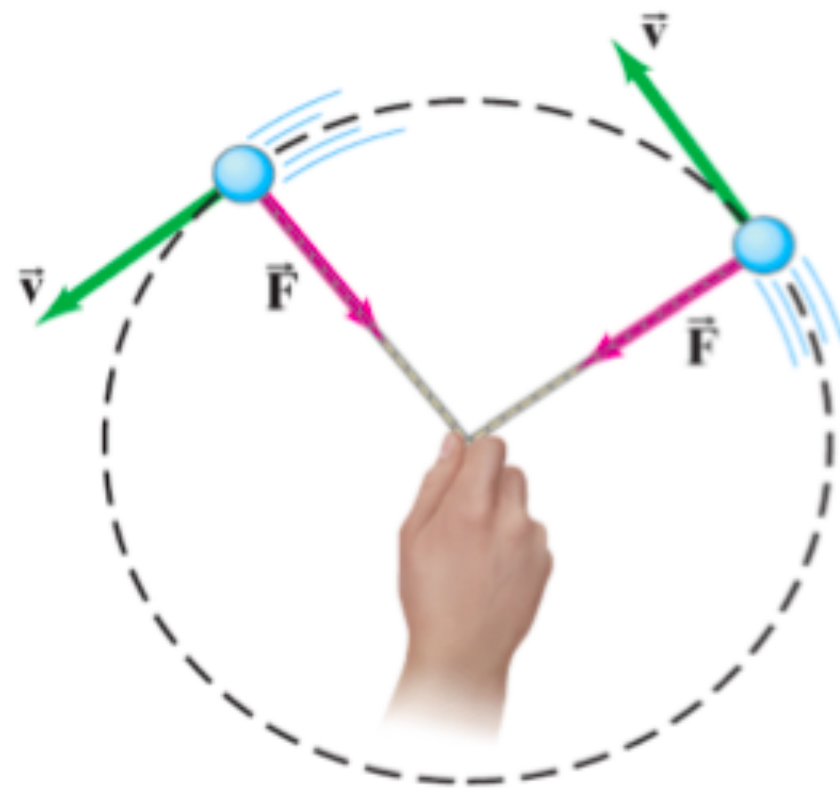
- Due to Newton's 1st law, an object in circular motion must be experiencing a net radial force (called the centripetal force)



Week 4: Circular motion

- Due to Newton's 1st law, an object in circular motion must be experiencing a net radial force (called the centripetal force)

- Determining the identity of this force requires further investigation

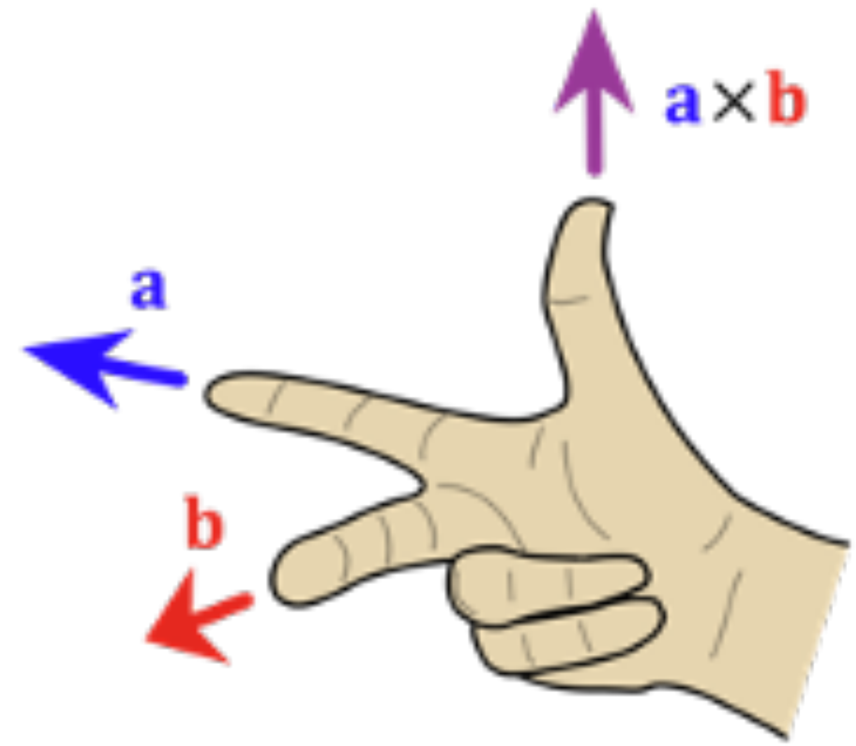


- For the case of a ball on a string, this force is provided by tension in the string (possibly in combination with gravity)

- Using $\vec{a} = a_\rho \hat{\rho} + a_\phi \hat{\phi} = -\rho_0 \omega^2 \hat{\rho} + \rho_0 \alpha \hat{\phi}$, we know that

$$\overrightarrow{F}_{net} = m\vec{a} = -m\rho_0\omega^2\hat{\rho} + m\rho_0\alpha\hat{\phi}$$

Week 4: Cross (or vector) product



- Two vectors are multiplied in a cross product to produce another vector

$$\vec{a} \times \vec{b} = \vec{c}$$

- Magnitude: $|\vec{c}| = c = ab\sin\theta$
- Direction: Use right hand rule
- If $\vec{a}\,||\,\vec{b}$, then $\vec{a} \times \vec{b} = 0$ or if $\vec{a} \perp \vec{b}$, then $|\vec{a} \times \vec{b}| = ab$
- Not commutative: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- Distributive: $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- Derivative product rule: $\frac{d}{dt}(\vec{a} \times \vec{b}) = \frac{d\vec{a}}{dt} \times \vec{b} + \vec{a} \times \frac{d\vec{b}}{dt}$

Conceptual question

responseware.eu
Session ID: epflphys101en

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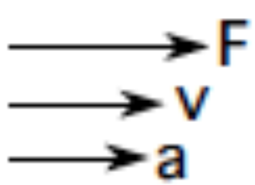
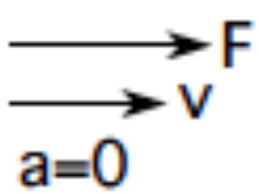
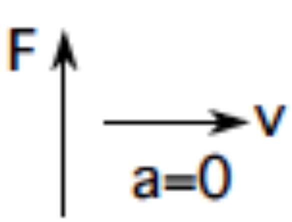
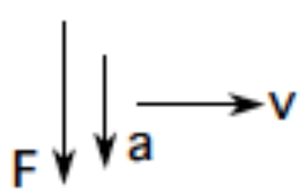
A piece of sushi rests on a circular turntable, rotating about a vertical axis at a constant angular speed ω as illustrated in the diagram below.

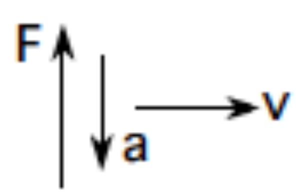
The sushi rotates with the turntable and fortunately does not slip. What are the directions of the velocity, acceleration and net force vectors acting on the sushi at the moment shown in the diagram?

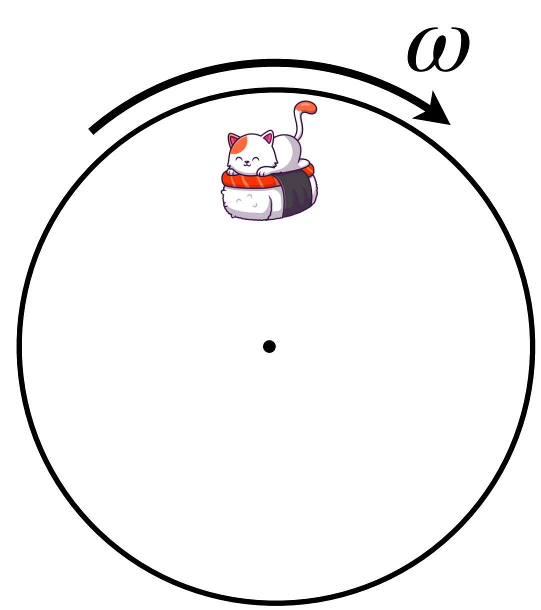
A. 

B. 

C. 

D. 

E. 



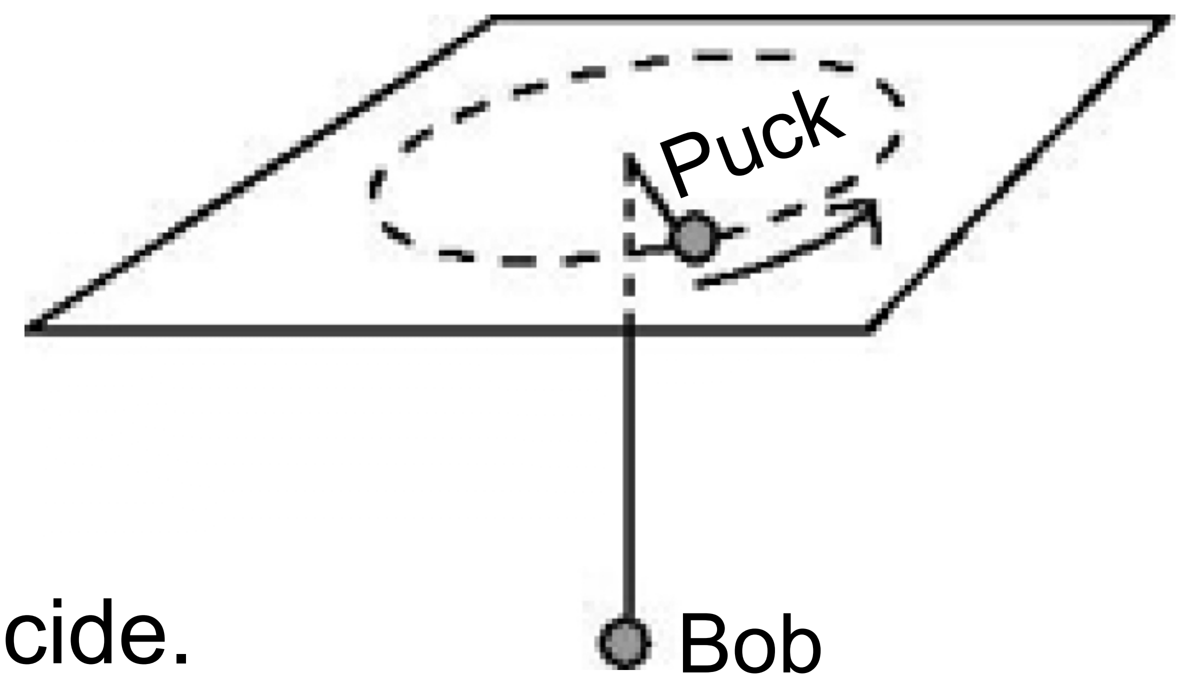
View of turntable from above

Conceptual question

responseware.eu
Session ID: epflphys101en

A puck of mass m is moving in a circle at constant speed on a frictionless table. The puck is connected by a massless string to a suspended bob, also of mass m, which is at rest below the table. What is the magnitude of the centripetal acceleration of the moving puck? Ignore friction.

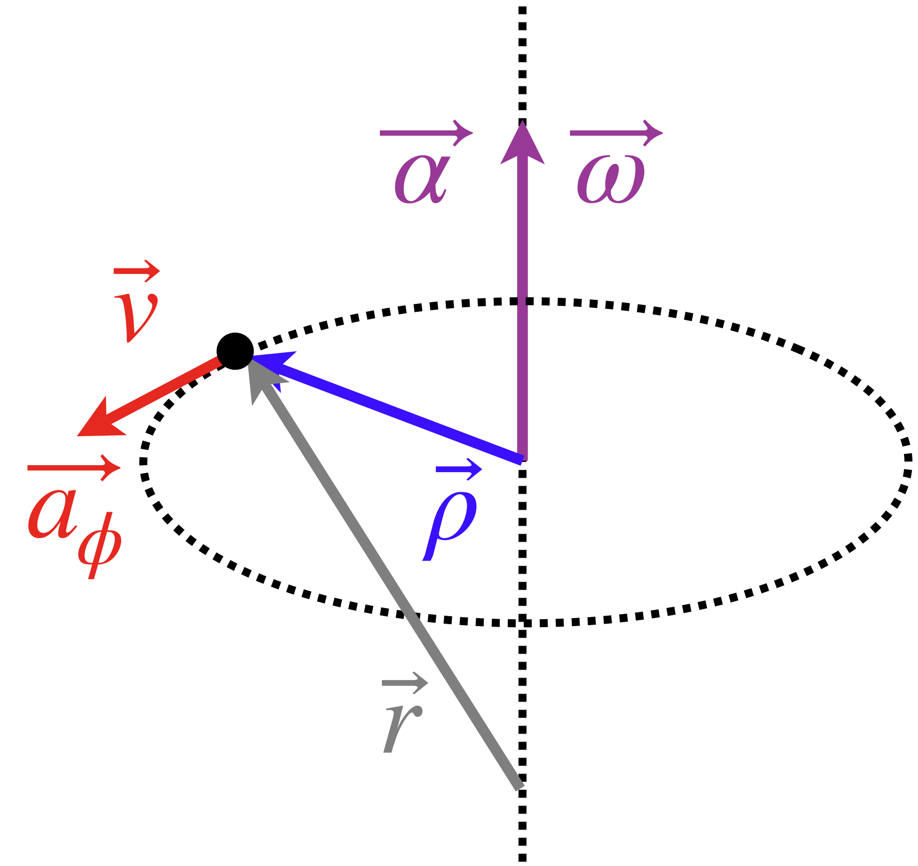
A. Greater than g
B. Equal to g
C. Less than g
D. 0
E. Not enough information to decide.

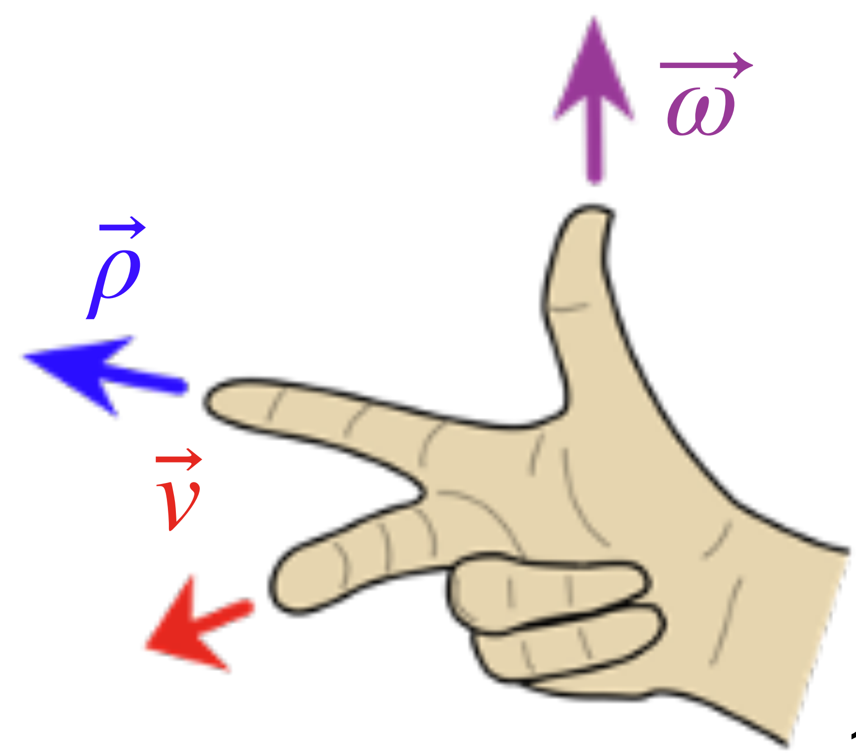


Angular velocity vector in circular motion

- Angular velocity **vector** points along the axis of rotation according to the right-hand rule

$$\vec{v} = \vec{\omega} \times \vec{\rho} \quad \text{and} \quad \vec{\omega} = \frac{\vec{\rho} \times \vec{v}}{\rho^2}$$





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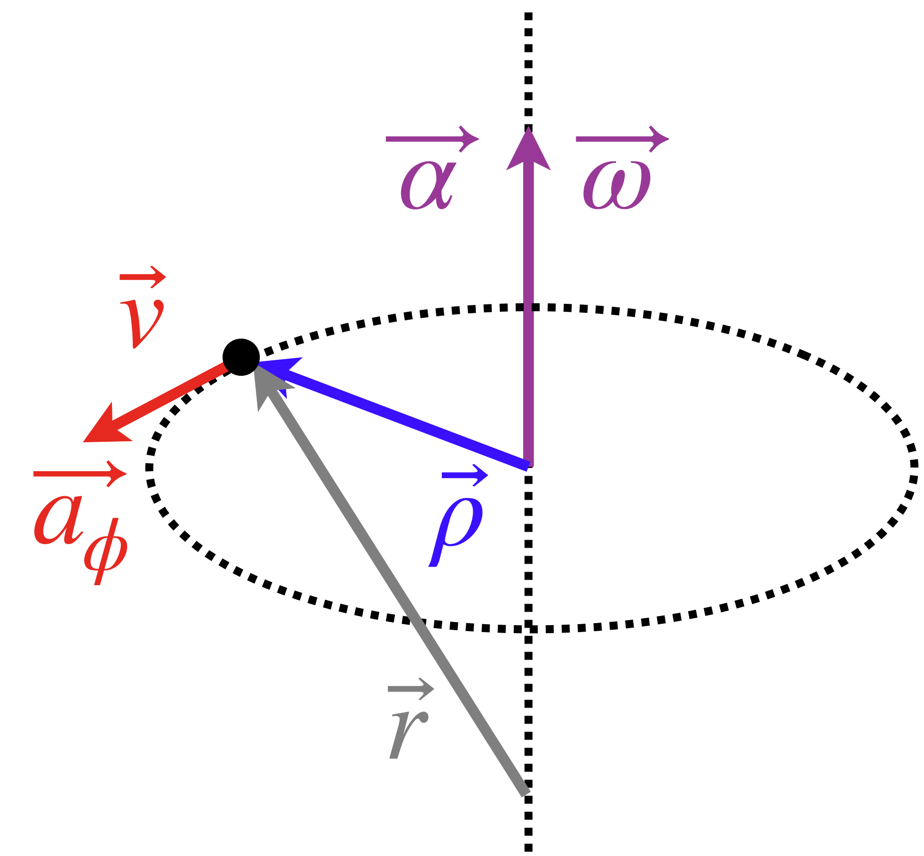
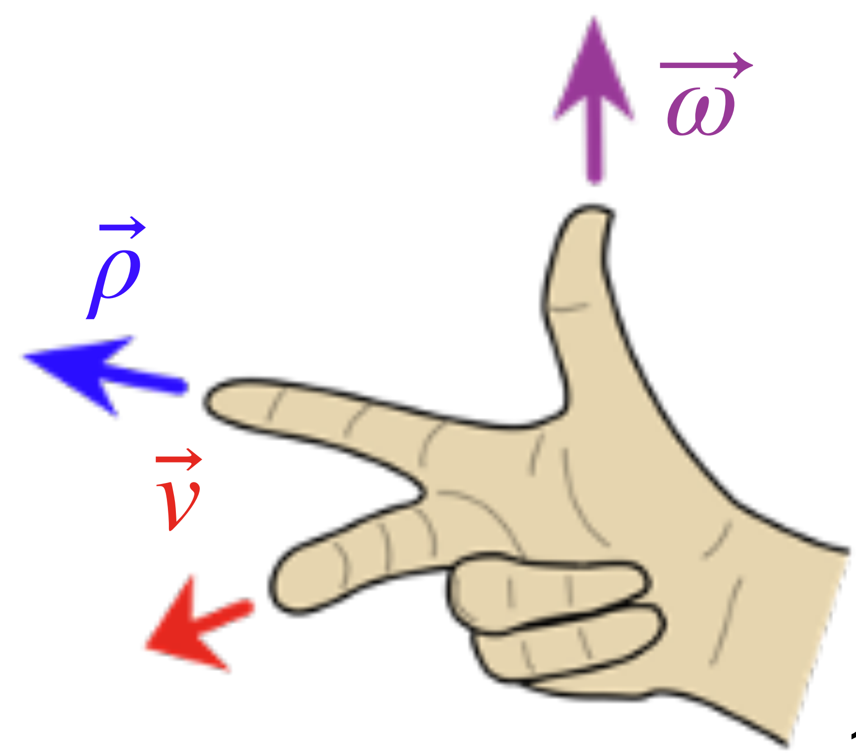


- The angular acceleration vector is

$$\vec{a_\phi} = \vec{\alpha} \times \vec{\rho} \quad \text{and} \quad \vec{\alpha} = \frac{\vec{\rho} \times \vec{a_\phi}}{\rho^2}$$

- If the direction of the rotation axis does not change, the angular acceleration vector points along it



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Rigid body

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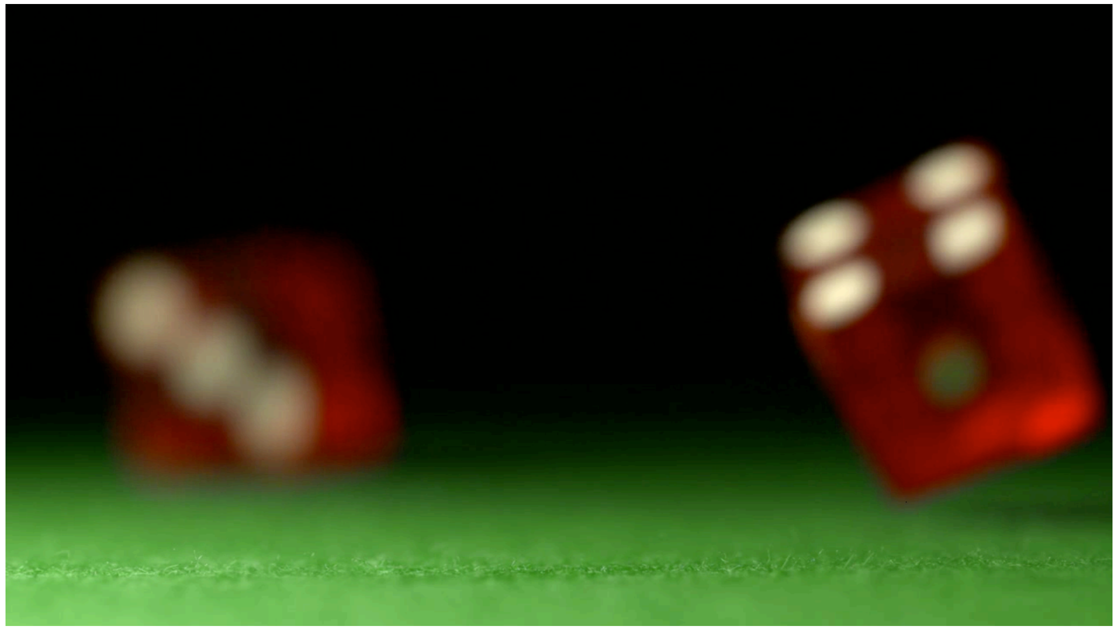
- In a **rigid body** all points move together without deformation, i.e. the distance between any two points is fixed in time



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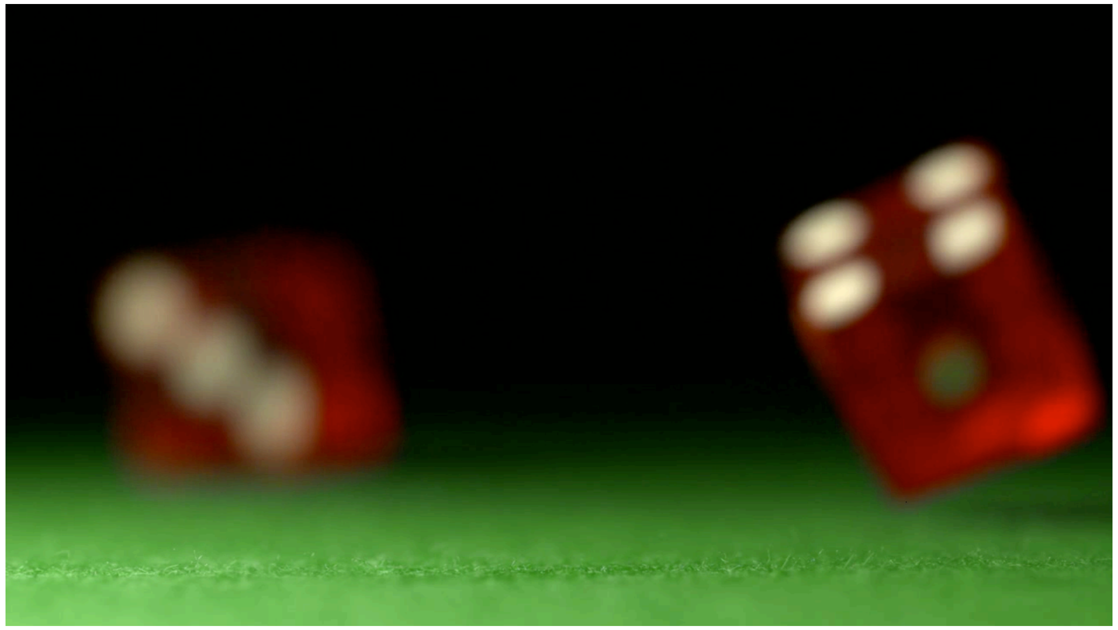
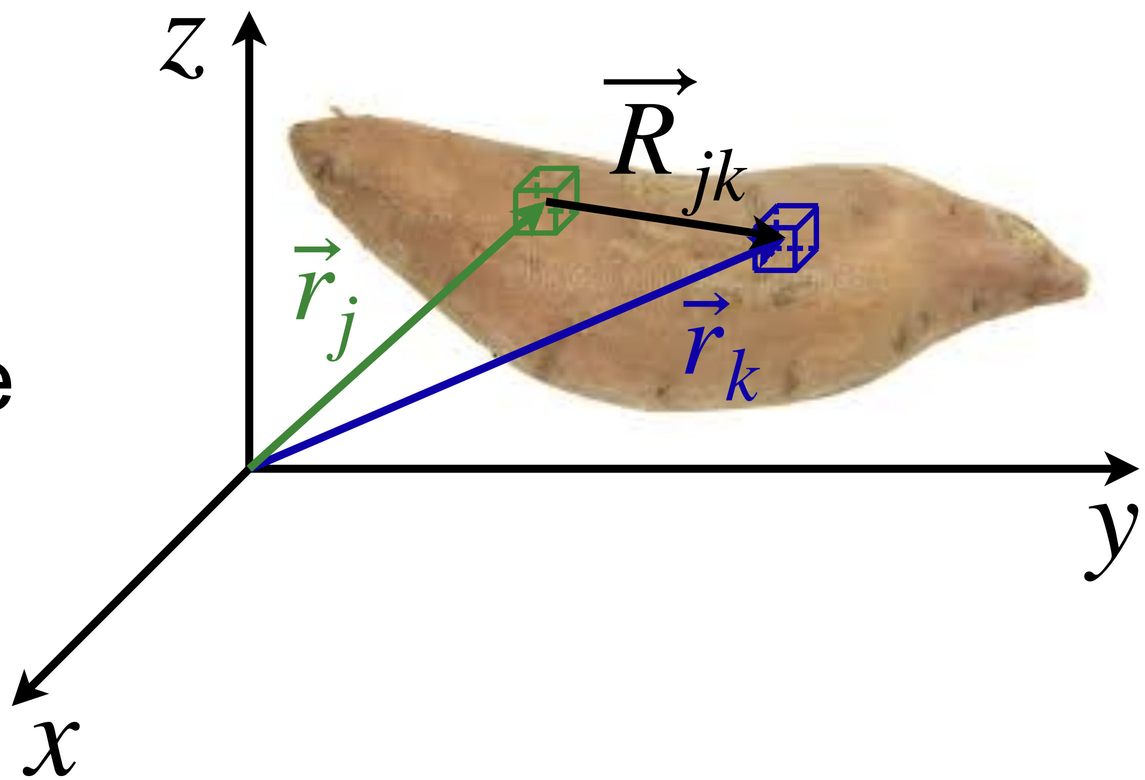


- Imagine an object is composed of a huge number of tiny differential elements, labeled $i = 1, 2, 3, \ldots$ with positions $\vec{r}_i$
- A body is rigid if, for all pairs of differential elements j and k, the distance $R_{jk} = |\vec{r}_k - \vec{r}_j|$ is constant in time



Pure translation of rigid bodies

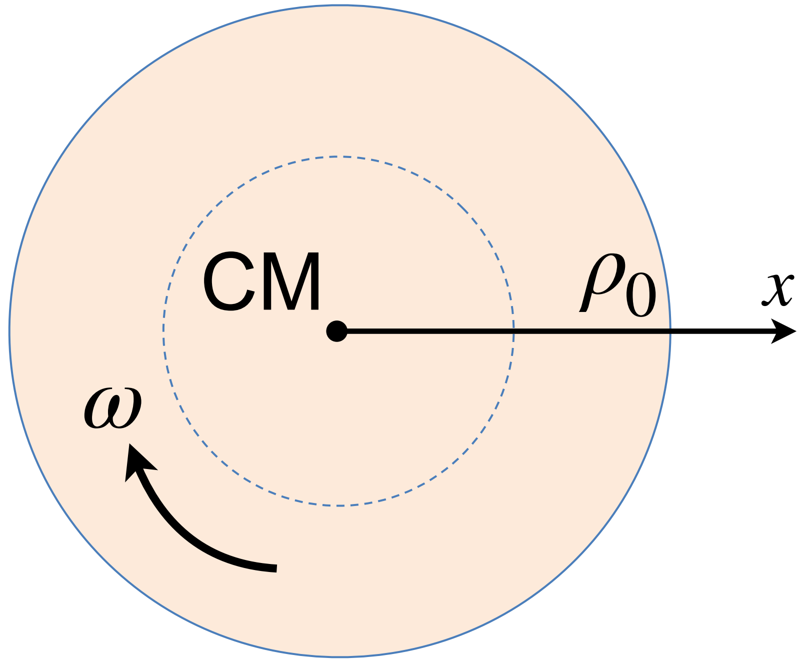
- We already know how to handle pure translation of a rigid body
  - Represent the entire object as a point at the object's Center of Mass (CM), as discussed in lecture 6
  - Then apply all net external forces to the CM and calculate its motion

Pure translation of rigid bodies

- We already know how to handle pure translation of a rigid body
  - Represent the entire object as a point at the object's Center of Mass (CM), as discussed in lecture 6
  - Then apply all net external forces to the CM and calculate its motion
- What about if the object is also rotating?
  - Can decompose motion into pure translation of CM (treated as above) and pure rotation around CM (will study now)

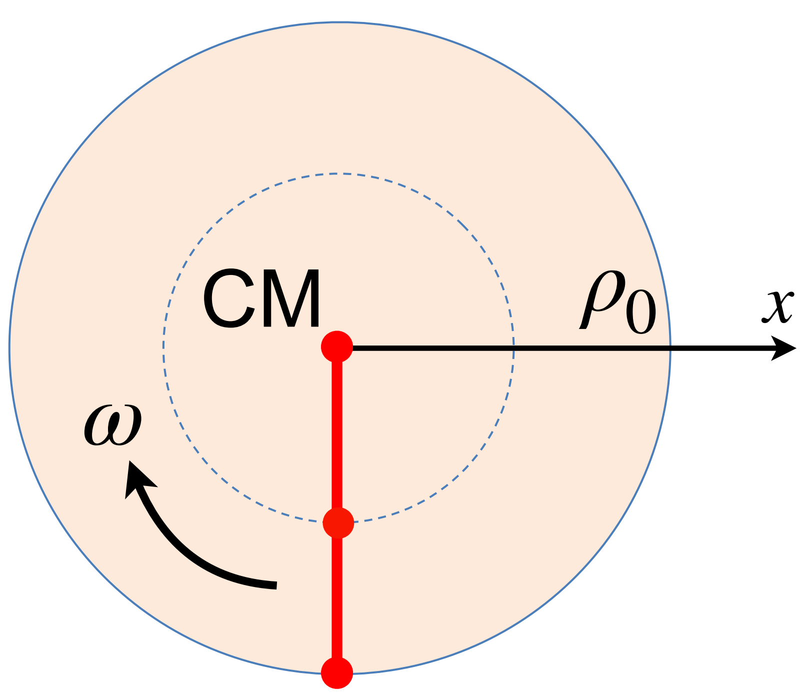
Pure rotation of a rigid body

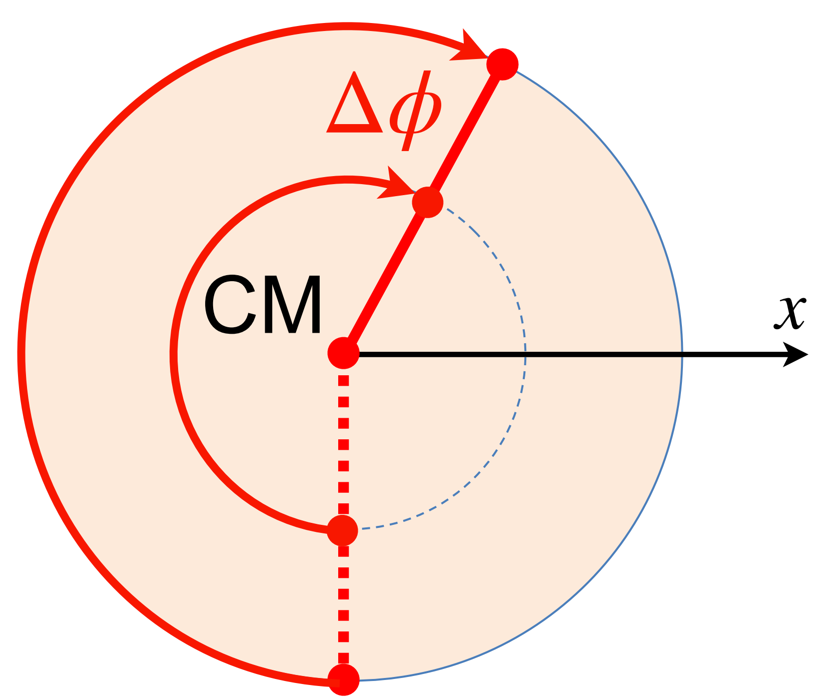
- In pure rotational motion, all points in the object move in circles around an **axis of rotation** through the CM



Pure rotation of a rigid body

- In pure rotational motion, all points in the object move in circles around an **axis of rotation** through the CM
- All points on a straight line drawn through the axis move through the same angle in the same time





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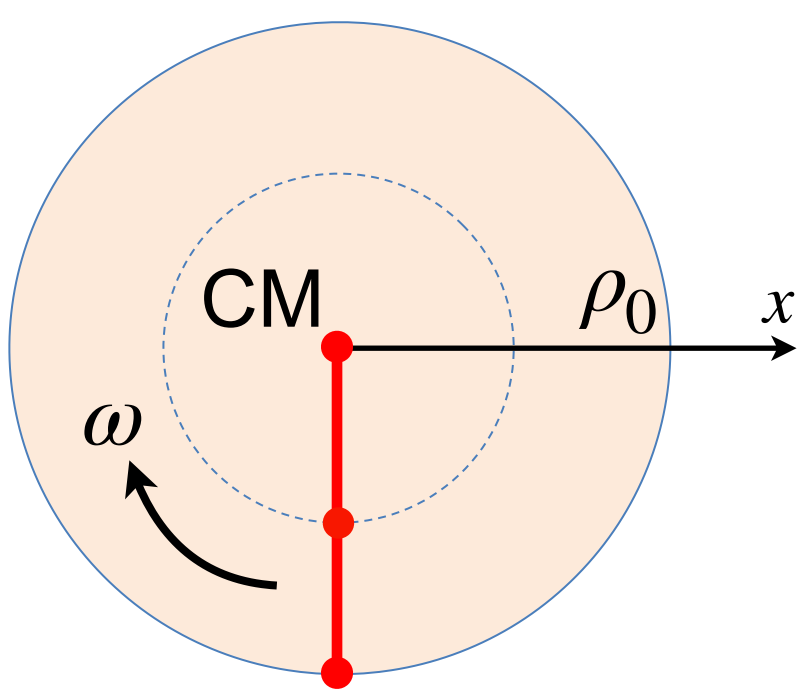
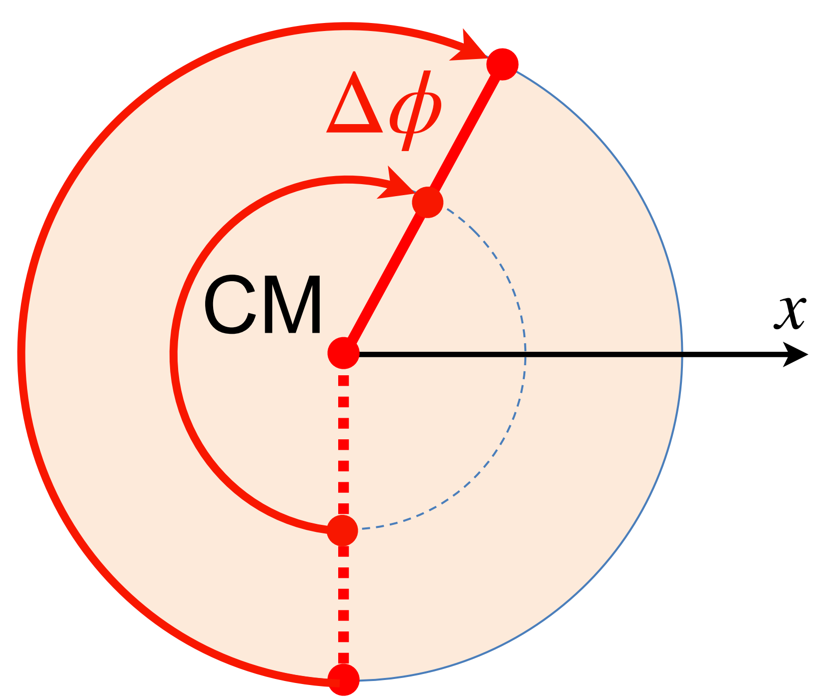
- All points on a straight line drawn through the axis move through the same angle in the same time

- Therefore, every point has the same value of $\omega = \dfrac{\Delta\phi}{\Delta t}$ (and α)





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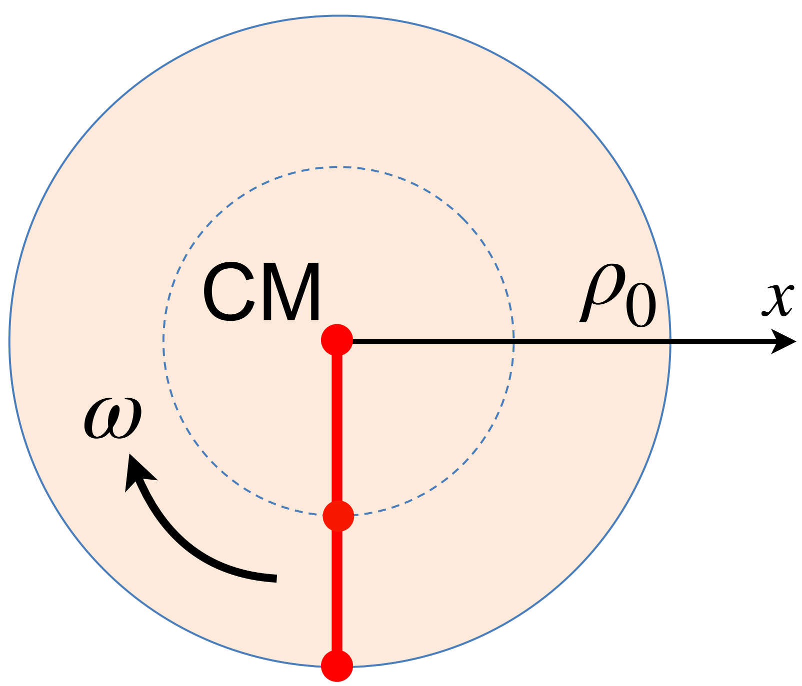
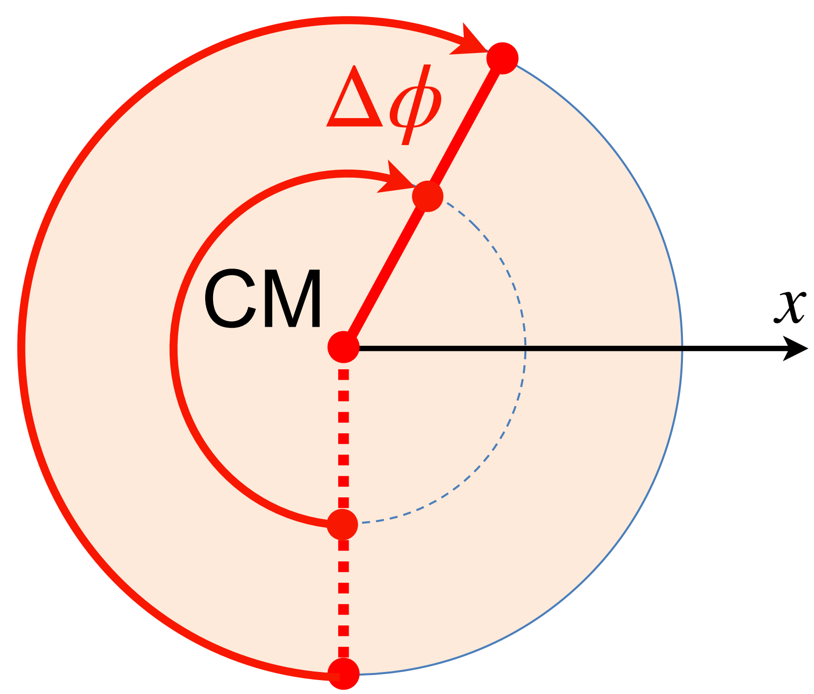
- Therefore, every point has the same value of $\omega = \dfrac{\Delta\phi}{\Delta t}$ (and α)

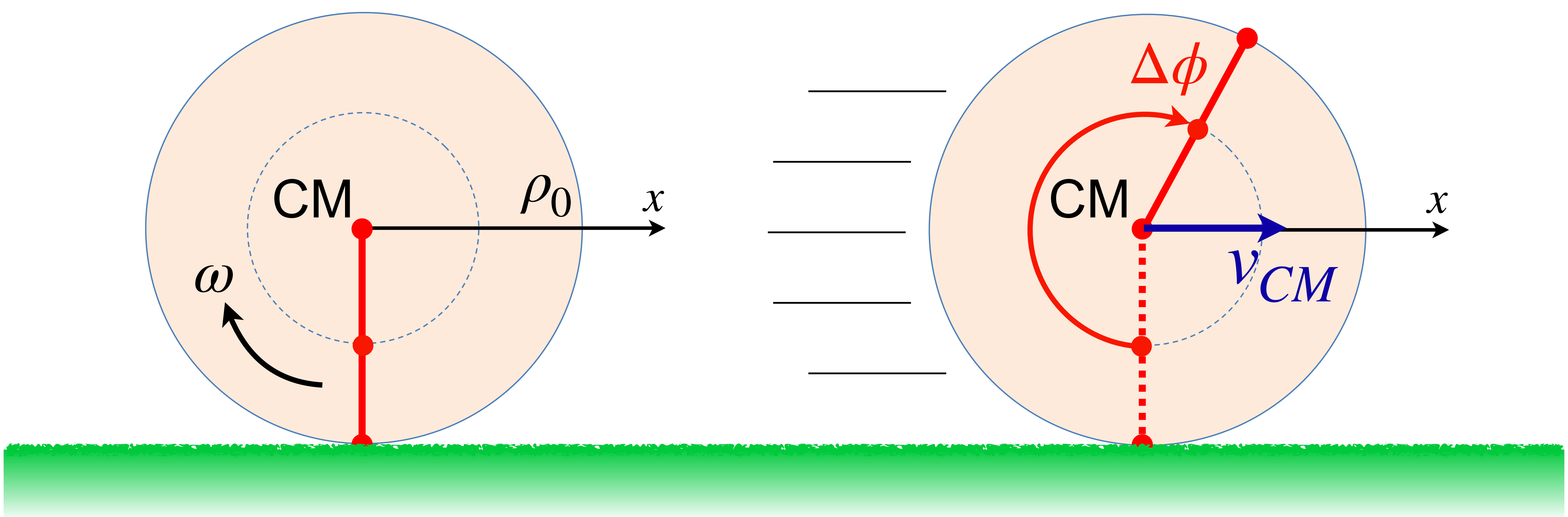
- The distance they move is the arc length $\ell(\rho) = \rho\Delta\phi$





Rolling without slipping

- If an object rolling on a surface does not slip, then the distance traveled along the ground must be equal to the distance traveled by the rim of the wheel



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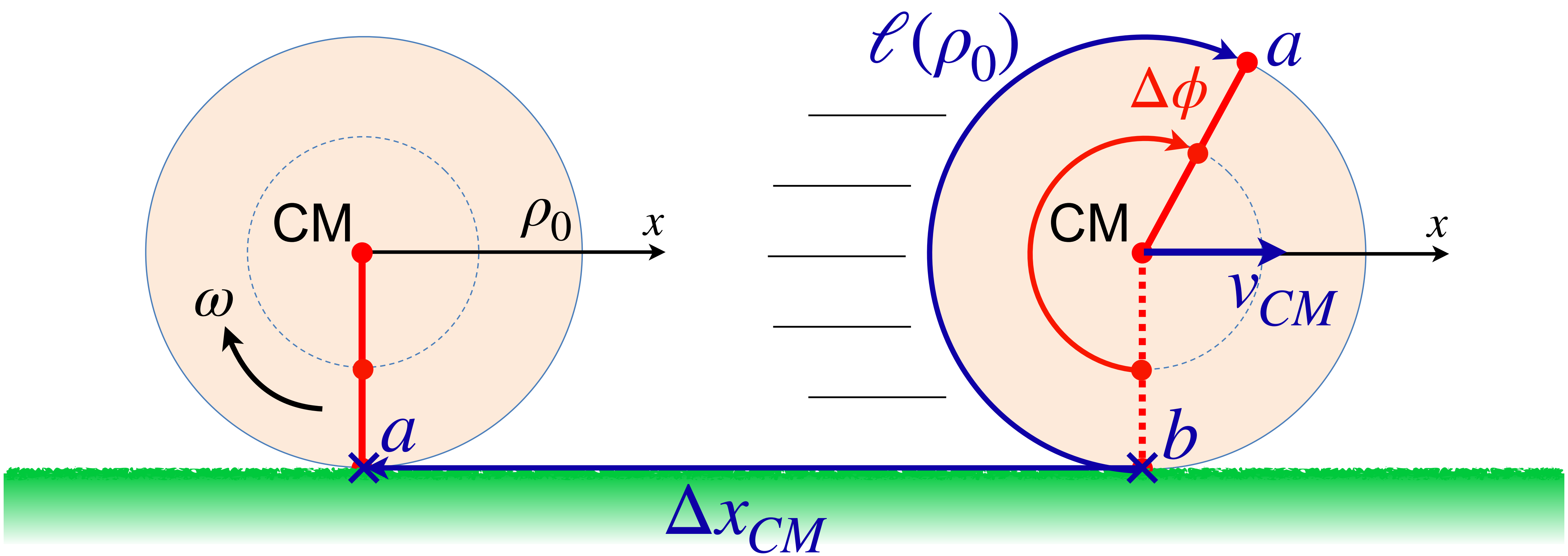

Rolling without slipping

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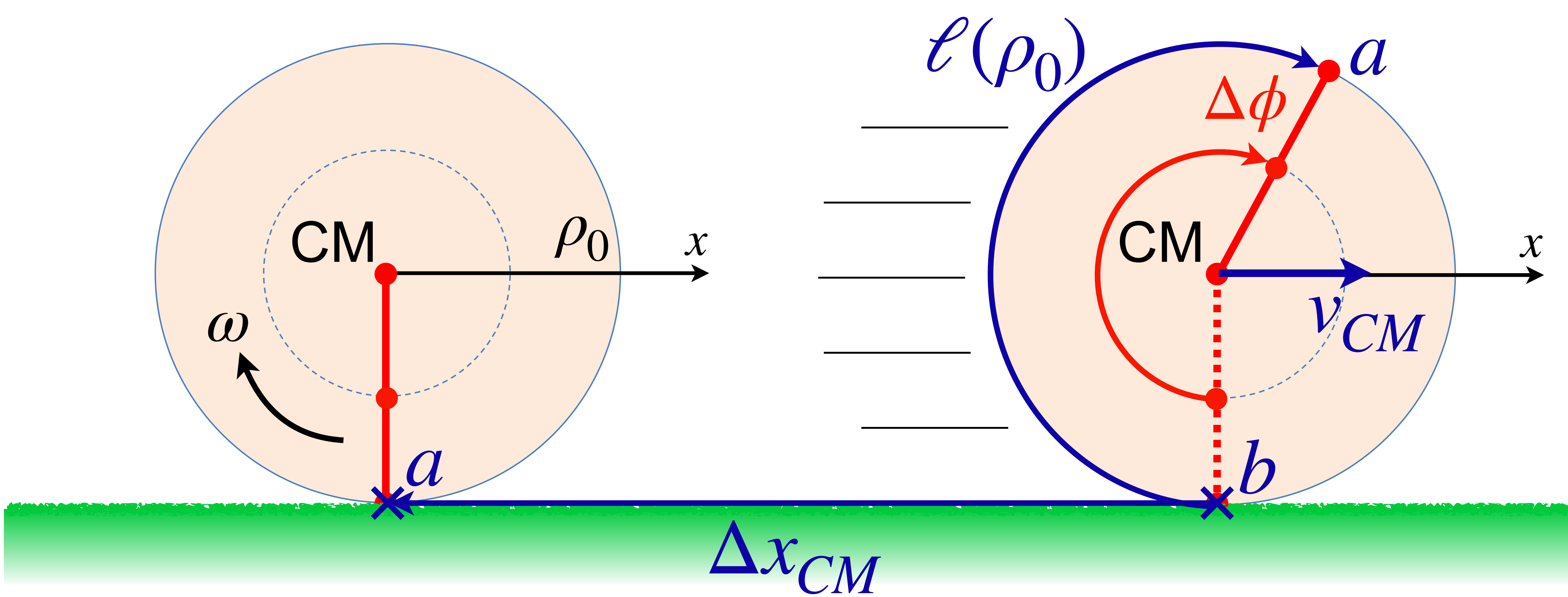
$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi$$



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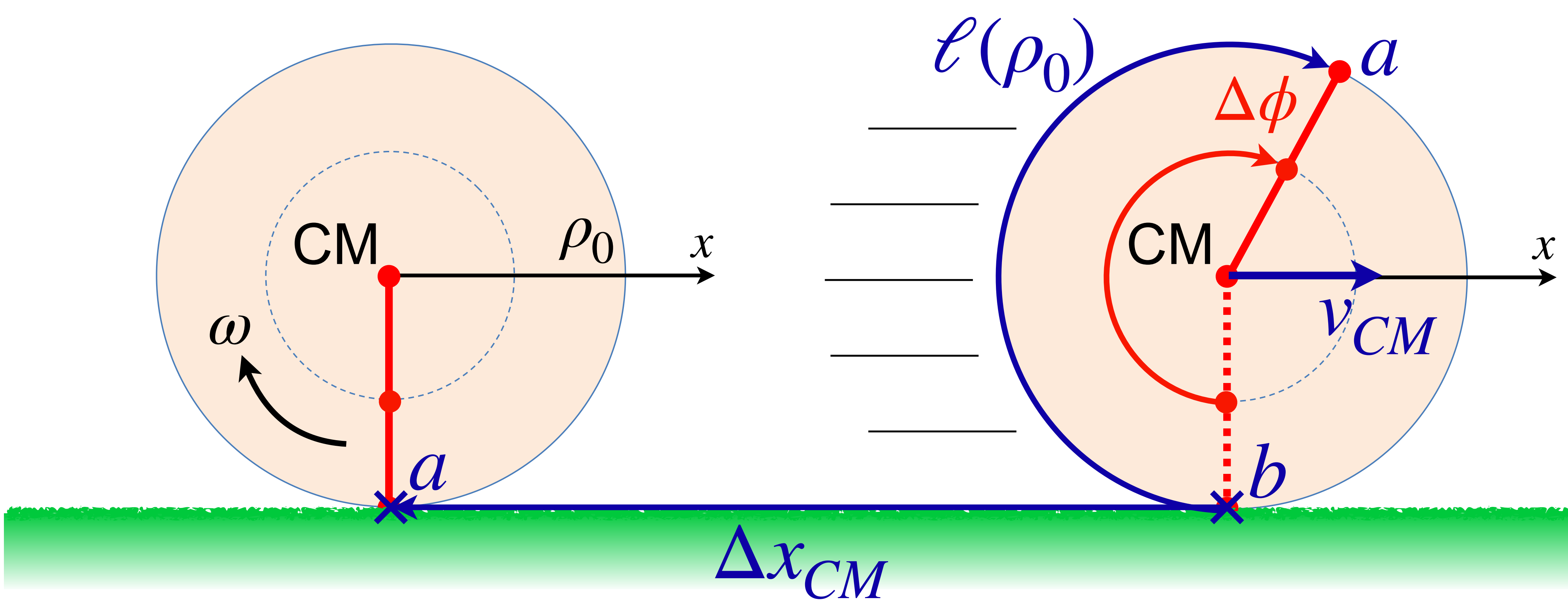
$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi \quad \Rightarrow \quad \frac{\Delta x_{CM}}{\Delta t} = \rho_0 \frac{\Delta\phi}{\Delta t}$$



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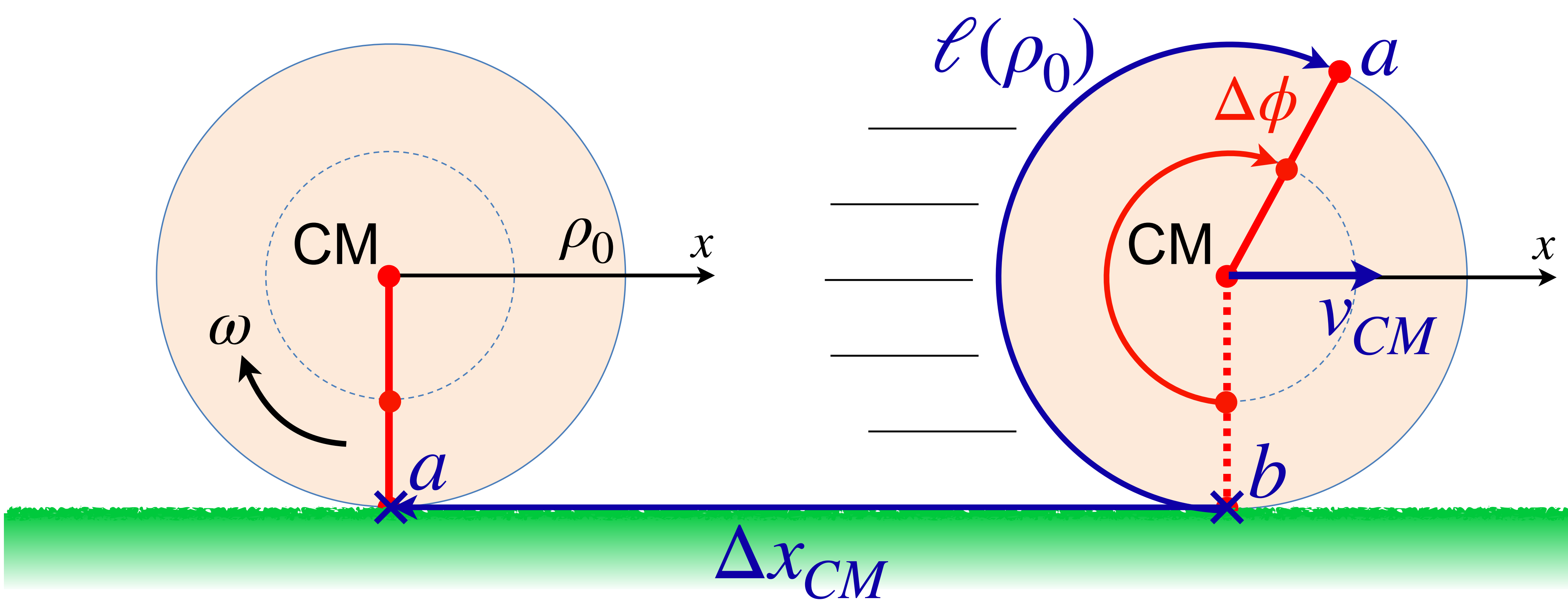
$$\Delta x_{CM} = \ell(\rho_0) = \rho_0 \Delta\phi \quad \Rightarrow \quad \frac{\Delta x_{CM}}{\Delta t} = \rho_0 \frac{\Delta\phi}{\Delta t} \quad \Rightarrow \quad v_{CM} = \rho_0 \omega$$



Rolling without slipping

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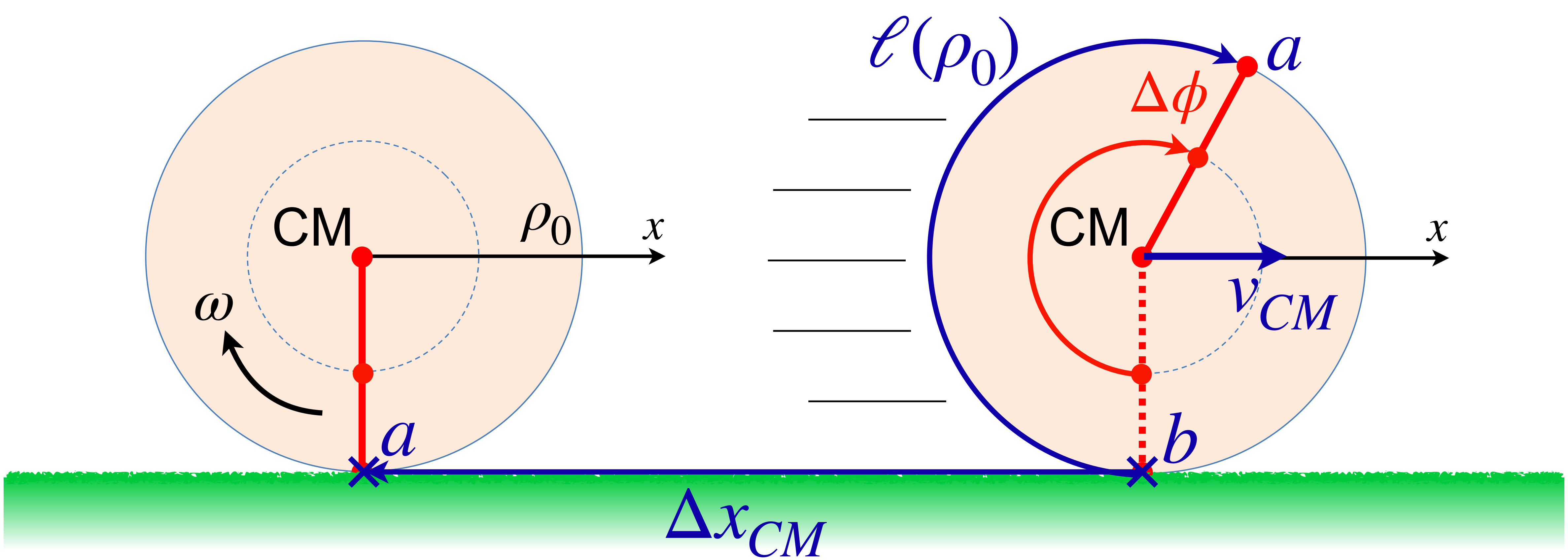
- At point of contact b, $\vec{v}_{ground} = \vec{v}_{rim}$ (i.e. friction is static)



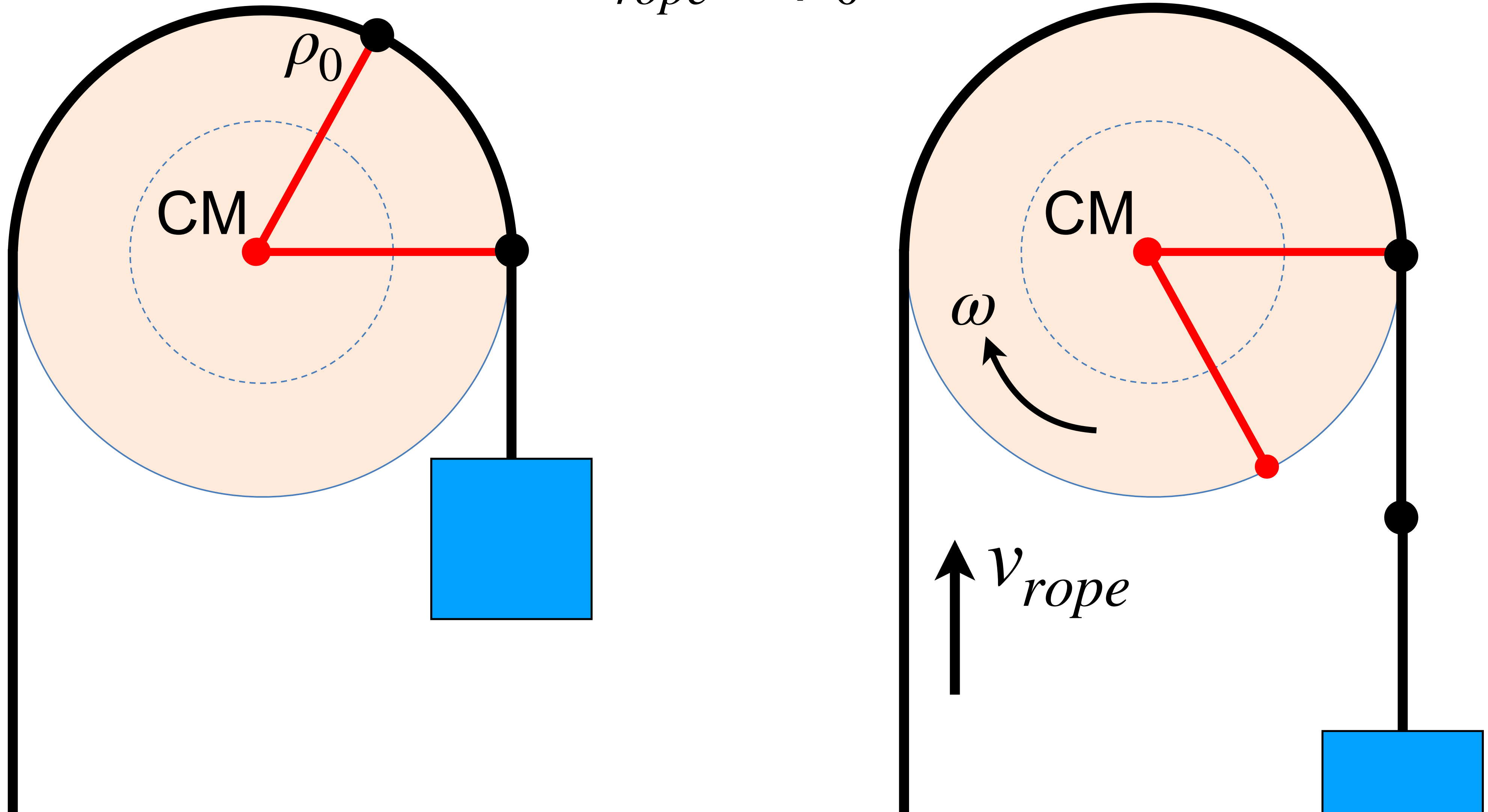
No-slip pulleys

- If a rope rotates a pulley without slipping, then at the points of contact

$$v_{rope} = v_{rim} = \rho_0 \omega$$

- Taking a derivative in time shows

$$a_{rope} = \rho_0 \alpha$$



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Rotational kinetic energy

- An object with no translational motion still has kinetic energy, if it is rotating

Rotational kinetic energy

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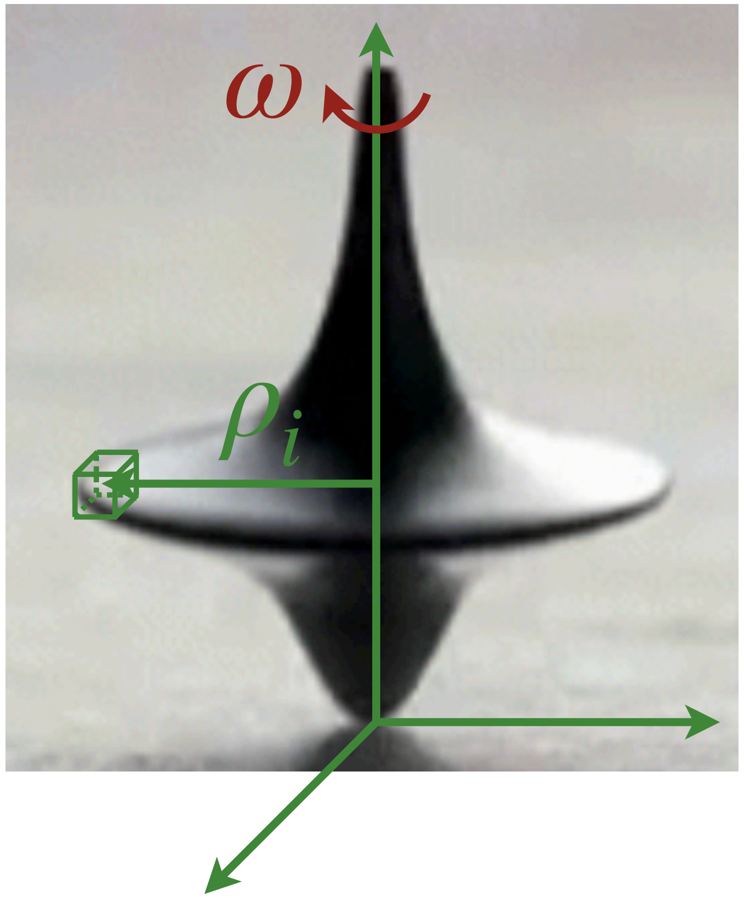
- An object with no translational motion still has kinetic energy, if it is rotating

- Rotational kinetic energy must be considered in conservation of energy

$$K = K^{trans} + K^{rot}$$

Rotational kinetic energy

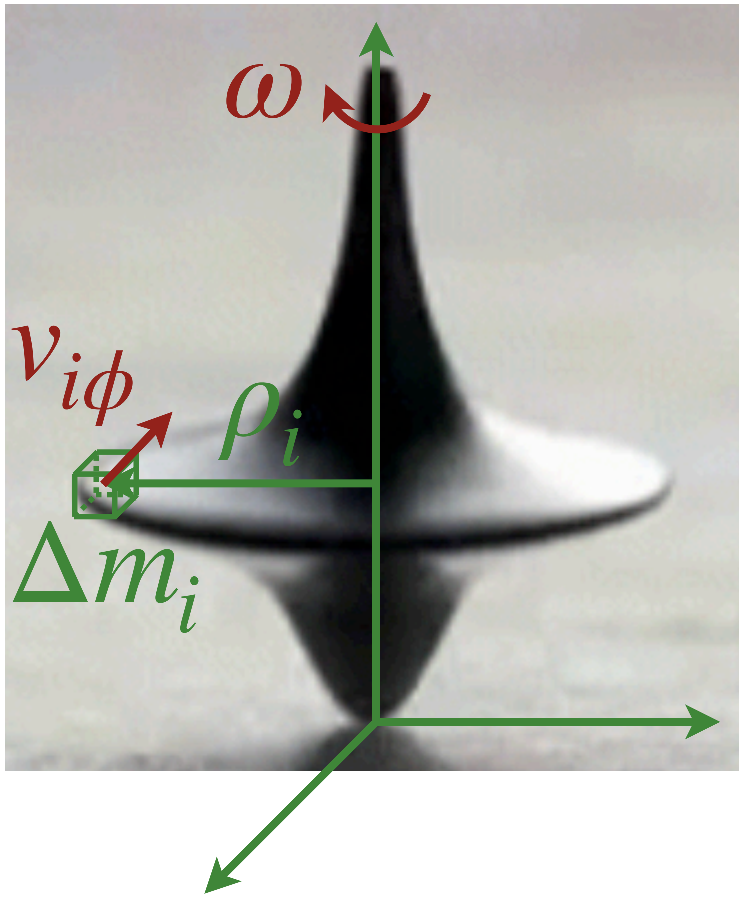
- Again imagine an object is composed of many differential elements, labeled $i = 1, 2, 3, \ldots$, at a distance ρ_i from the axis



Rotational kinetic energy

- Again imagine an object is composed of many differential elements, labeled $i = 1, 2, 3, \ldots$, at a distance ρ_i from the axis

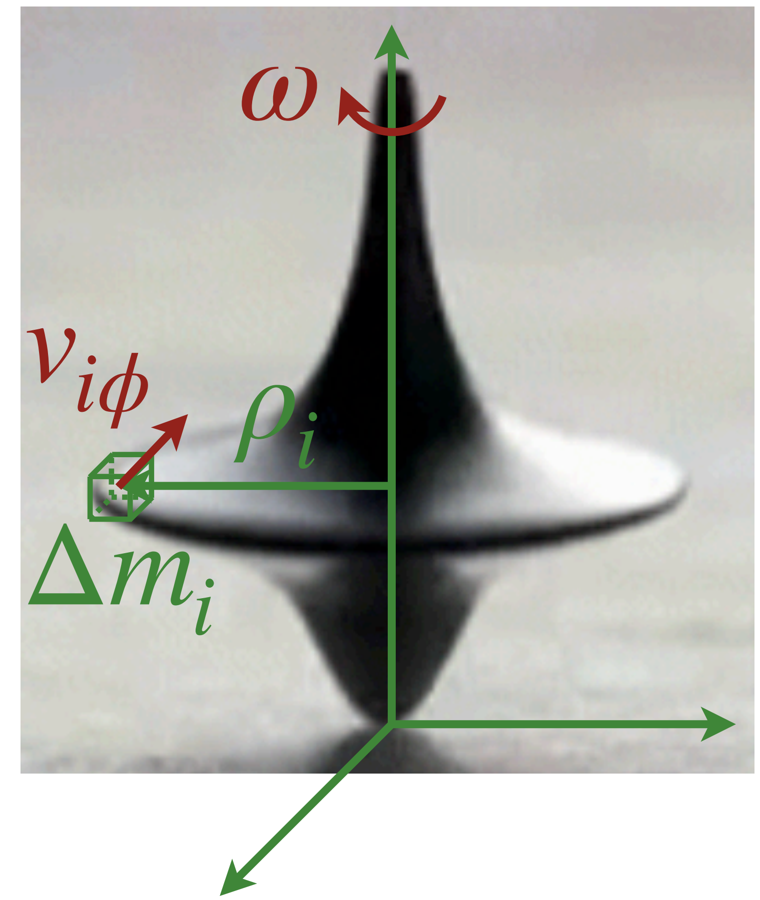
$$K^{rot} = \sum_i K_i^{trans} = \sum_i \frac{\Delta m_i}{2} v_{i\phi}^2$$



Rotational kinetic energy

- Again imagine an object is composed of many differential elements, labeled $i = 1, 2, 3, \ldots$, at a distance ρ_i from the axis

$$K^{rot} = \sum_i K_i^{trans} = \sum_i \frac{\Delta m_i}{2} v_{i\phi}^2$$



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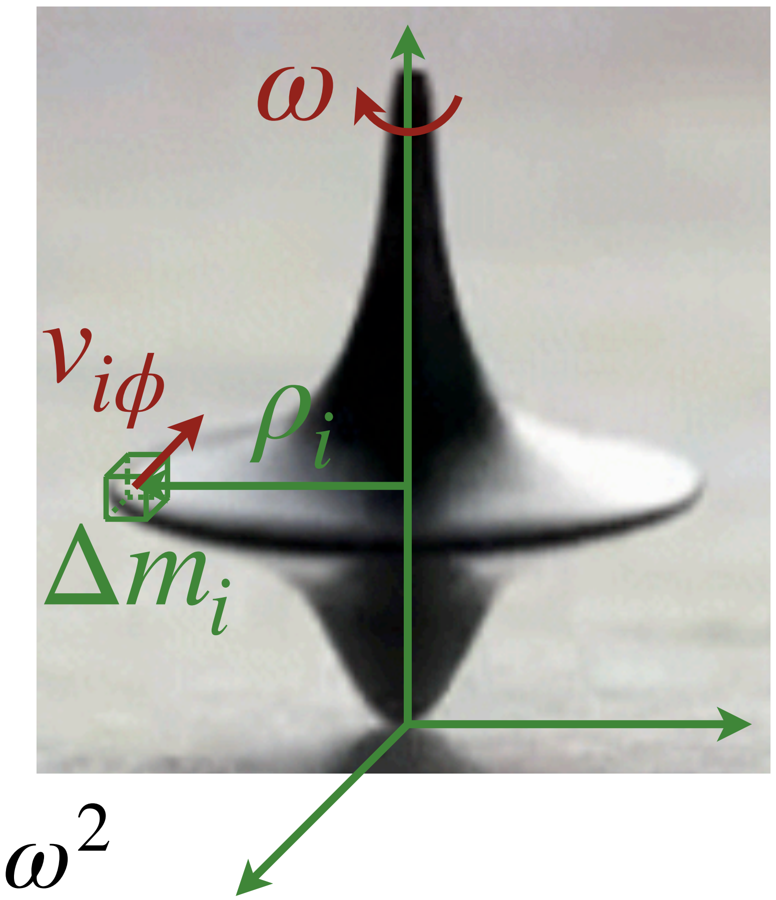

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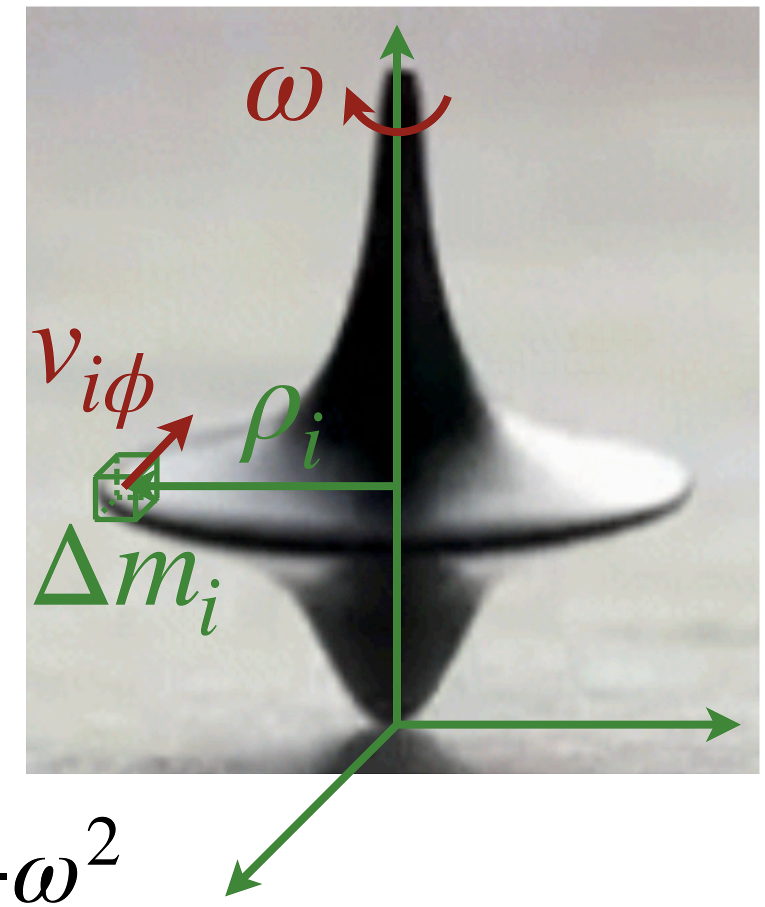

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- Thus, total kinetic energy is $K = \frac{m}{2}v^2 + \frac{I_{CM}}{2}\omega^2$

Today's agenda (Serway 10,12; MIT 16-18)

1. Review of circular motion
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   - Rotational kinetic energy
   - **Moment of inertia**
   - Torque
3. Static equilibrium

Moment of inertia

- The moment of inertia I is analogous to the mass m
- Quantifies the rotational inertia of an object **about a given axis of rotation**, i.e. its resistance to changing its rotation

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- Defined for discrete objects as $I = \sum_i m_i \rho_i^2$, where ρ_i is the object's distance from the axis of rotation
- In the limit of infinitesimally small differential elements

$$I = \int_M \rho^2 dm$$

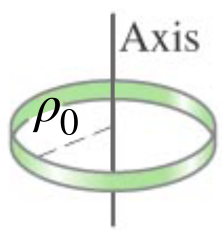
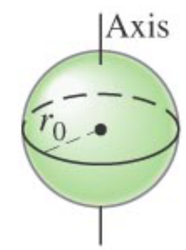
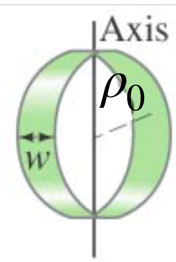
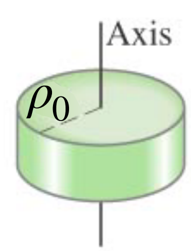
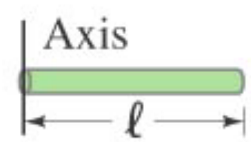
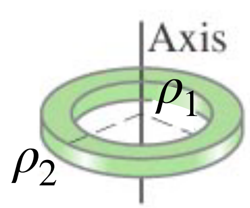
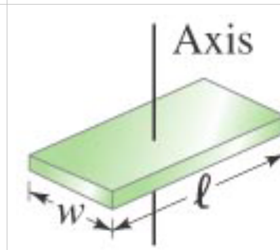
- Units of [kg·m^2]

Example: Uniform disk

What is the moment of inertia of a uniform disk with mass M, radius ρ_0, and height h, rotating about its axis of symmetry $\hat{z}$ at an angular velocity $\omega\hat{z}$?

Moment of inertia for various uniform objects

- Moment of inertia depends on shape and mass distribution
- Also depends on the axis of rotation

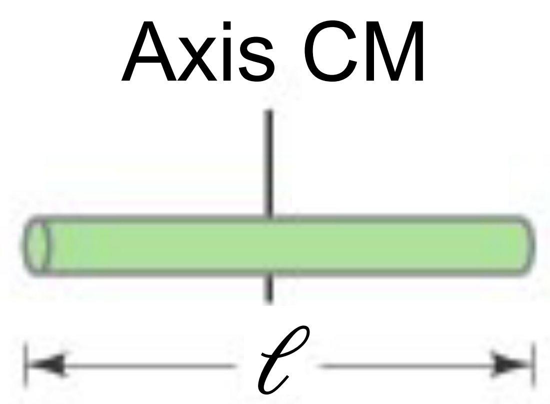
Object (rotation axis)	Geometry	Moment of inertia	Object (rotation axis)	Geometry	Moment of inertia
Thin hoop (about center)		$M\rho_0^2$	Uniform sphere (about center)		$\frac{2}{5}Mr_0^2$
Thin hoop (about diameter)		$\frac{M}{2}\rho_0^2 + \frac{M}{12}w^2$	Thin rod (about center)		$\frac{M}{12}\ell^2$
Solid cylinder (about center)		$\frac{M}{2}\rho_0^2$	Thin rod (about end)		$\frac{M}{3}\ell^2$
Hollow cylinder (about center)		$\frac{M}{2}\left(\rho_1^2 + \rho_2^2\right)$	Thin plate (about center)		$\frac{M}{12}\left(\ell^2 + w^2\right)$

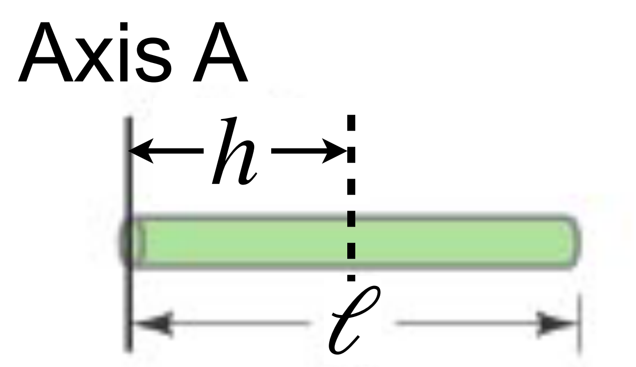
Parallel axis (or Steiner) theorem

- The moment of inertia about any axis parallel to an axis that goes through the center of mass is given by

$$I = I_{CM} + Mh^2$$

- For example:





Conceptual question

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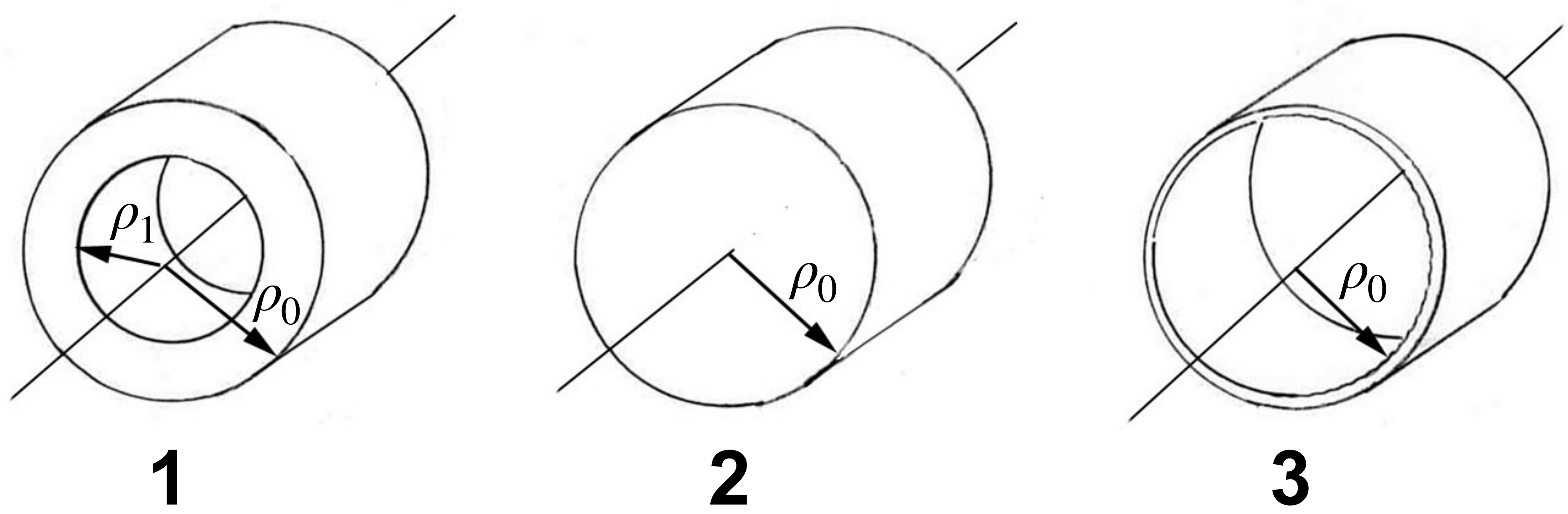
Session ID: epflphys101en



All of the objects above have the same **mass**, the same **radius**, and are made of materials with different but **uniform density**. How are their moments of inertia about the axis related?

A. $I_3 > I_2 > I_1$

B. $I_1 > I_2 > I_3$

C. $I_3 > I_1 > I_2$

D. $I_2 > I_1 > I_3$

DEMO (60): Racing cylinders

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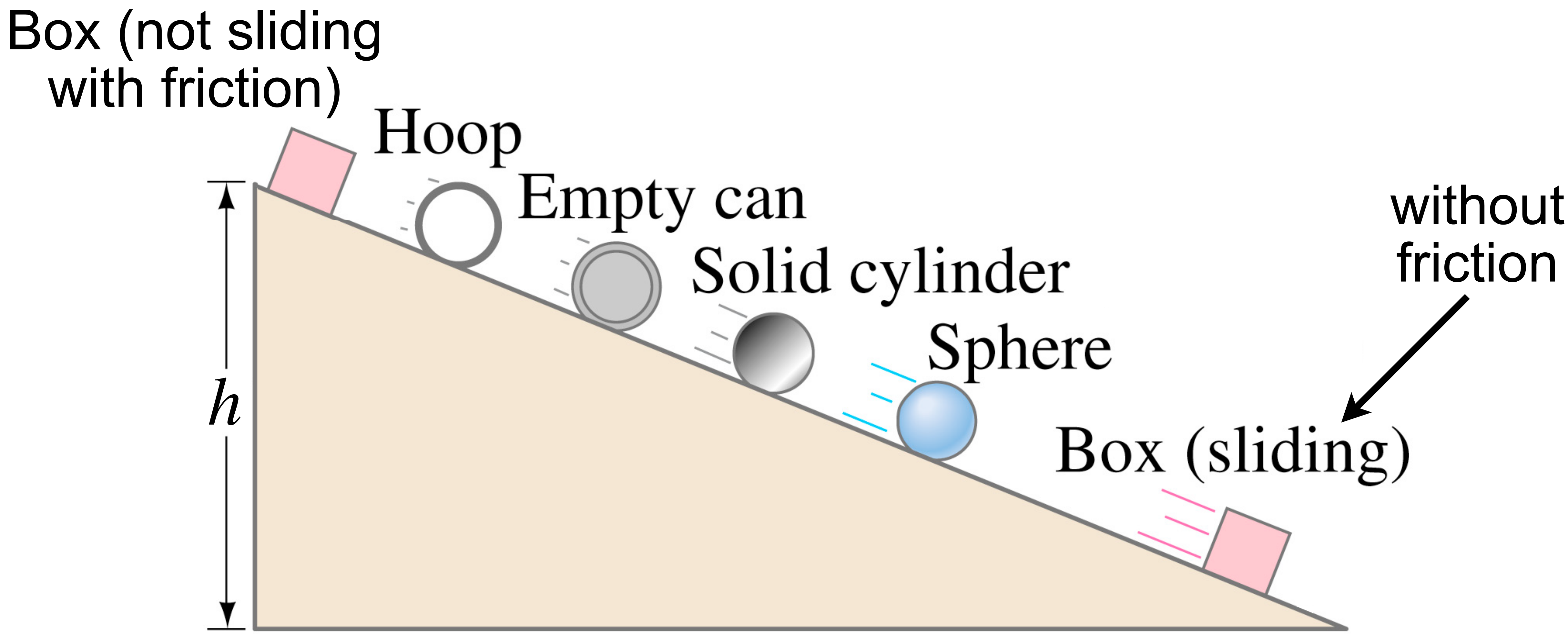
A cylinder with moment of inertia I, radius ρ_0, and mass m is initially at rest on an inclined plane. It rolls without slipping, descending a vertical distance h. What is its *translational* speed at the bottom?

Rotation steals energy from translation



- More rotational inertia means rotation takes more energy
- But rolling without slipping enables an object to avoid friction

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   - **Torque**
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Torque

- Defined to be the *moment* of a force about a pivot point

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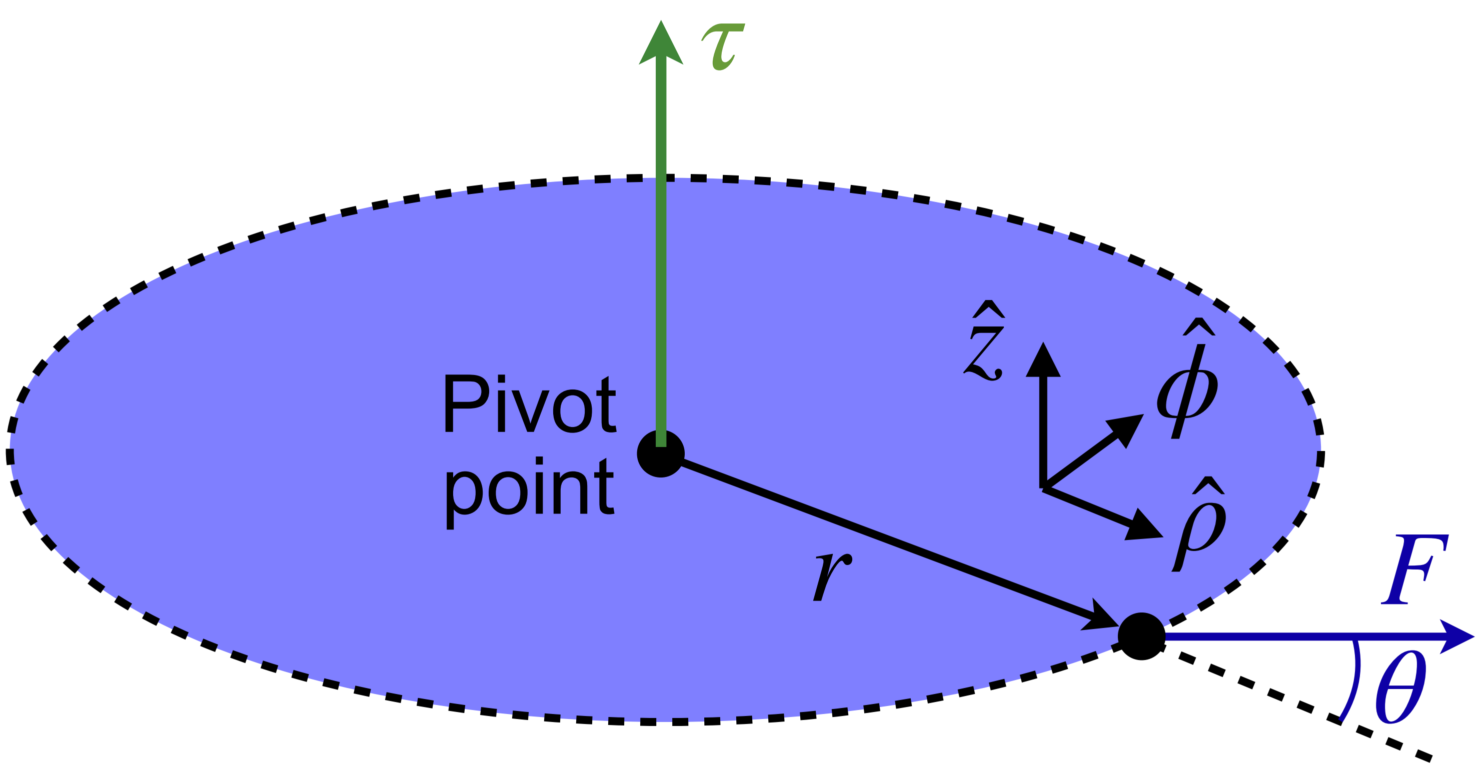

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$$\vec{\tau} = \vec{r} \times \overrightarrow{F} = rF \sin\theta \hat{z}$$

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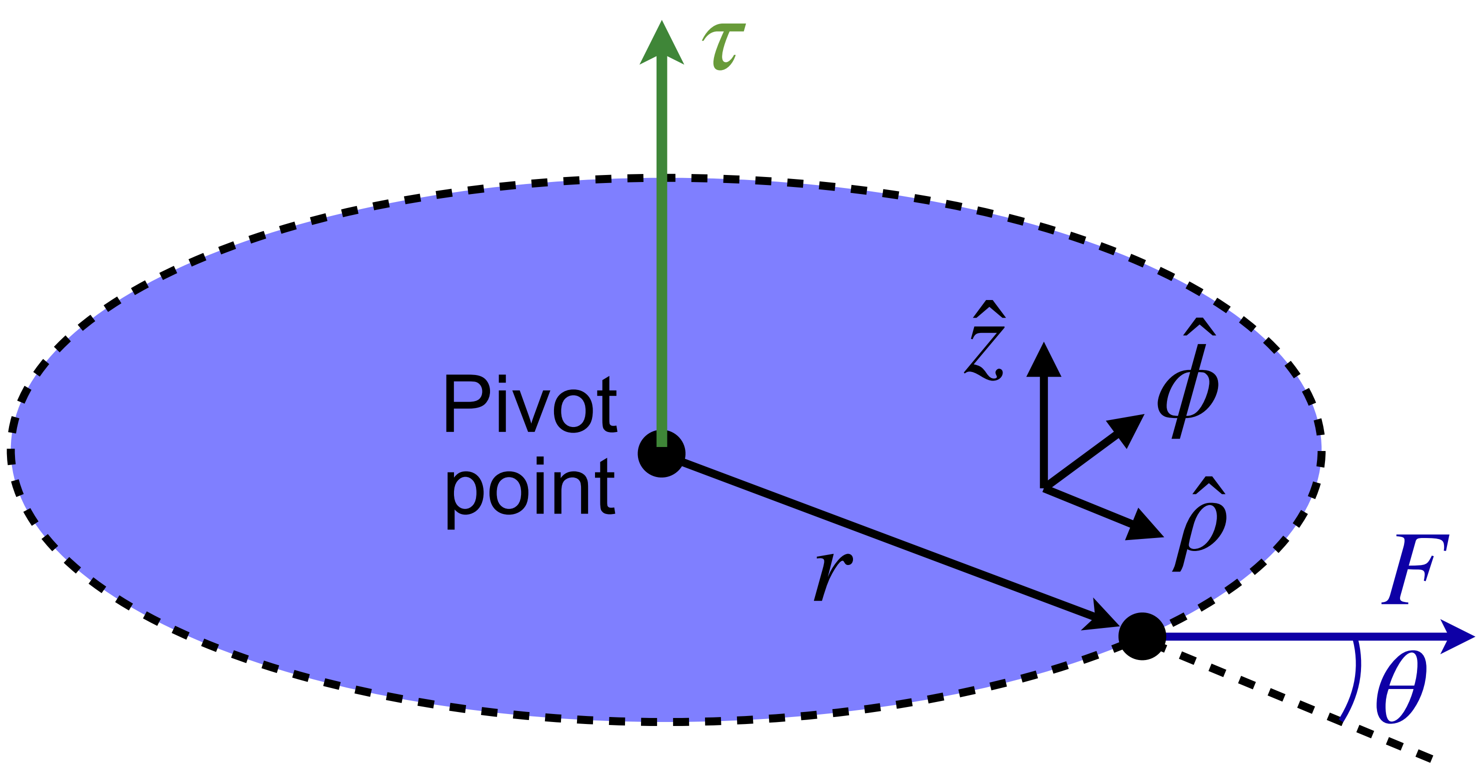

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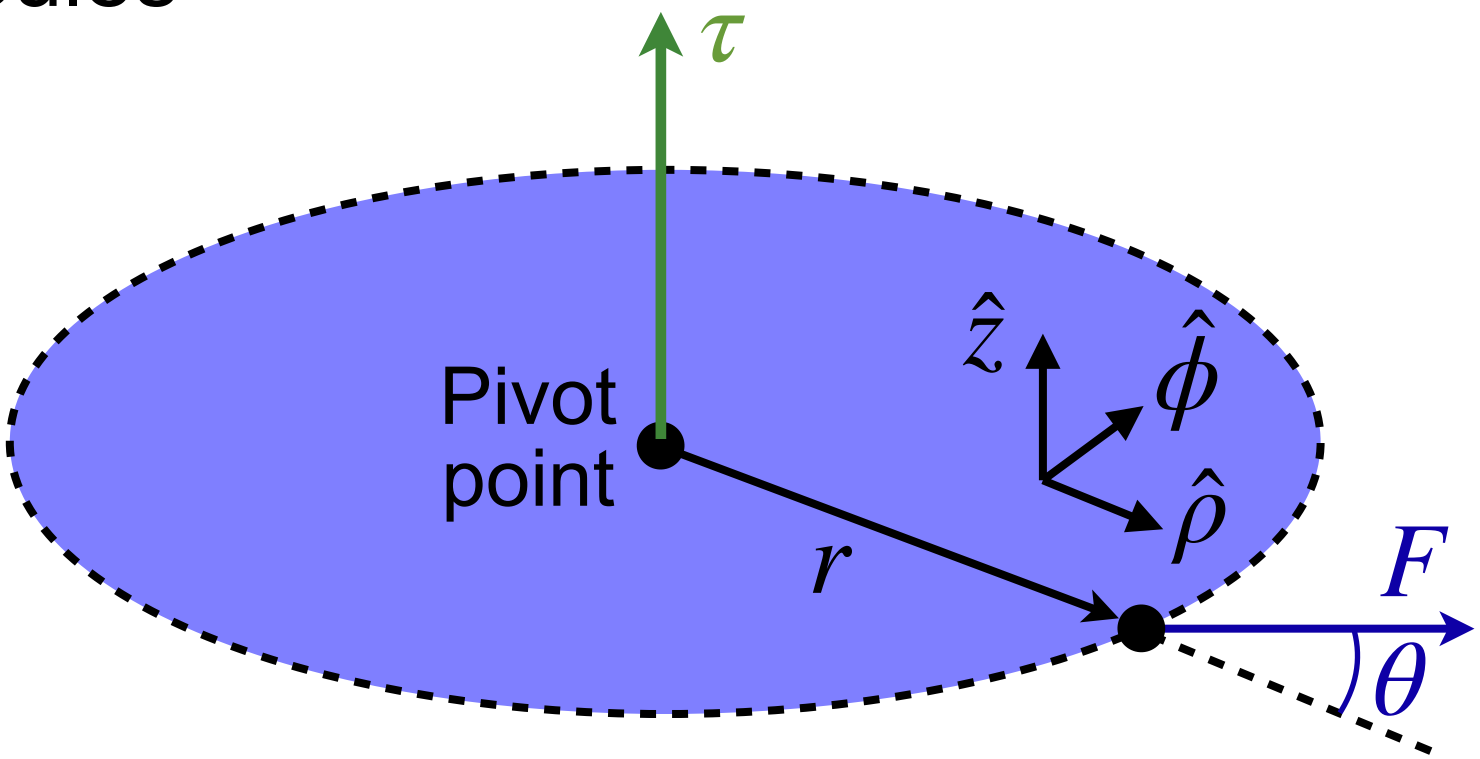
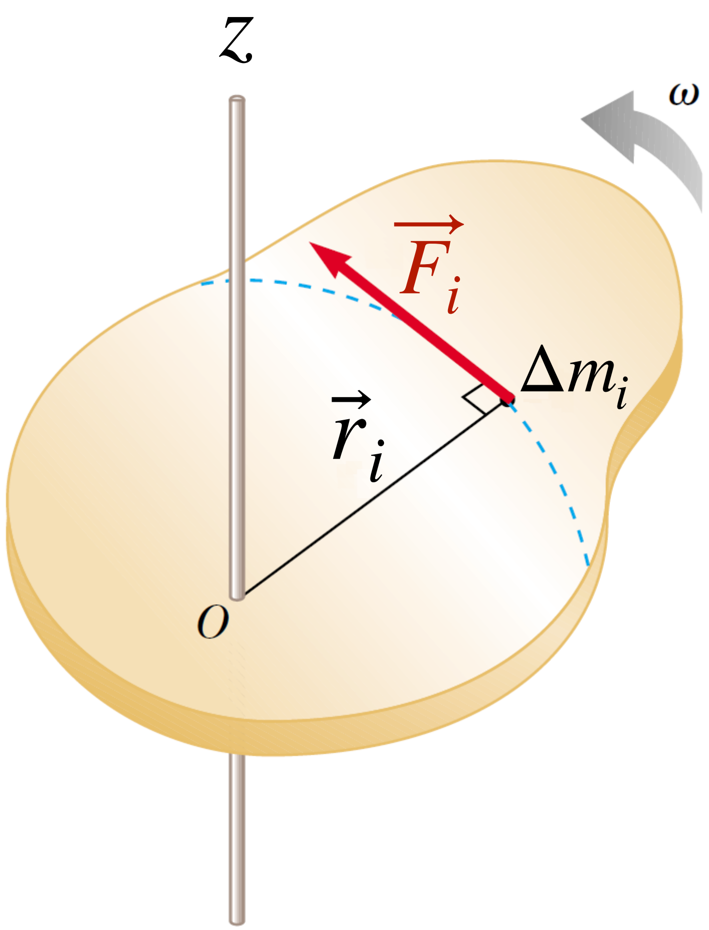
- It has units of [N·m], which is similar to [J], but torques are never expressed in Joules

- It is the analogue of a force for rotation



Newton's laws for rotation about a fixed axis

- Consider rigid body rotation about a fixed axis



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- Fictitious forces act at the center of mass

DEMO (22)

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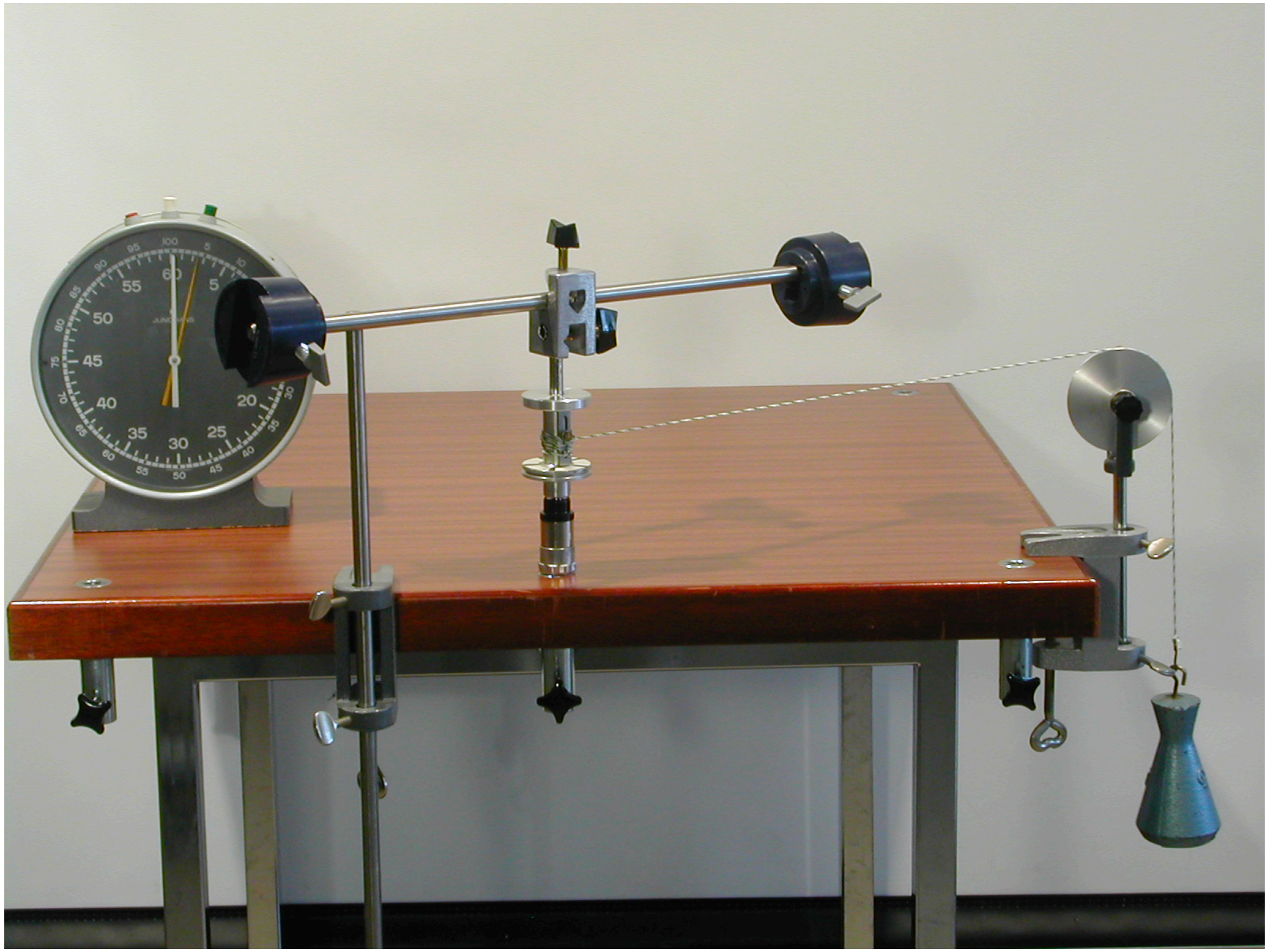
Torque and static equilibrium

DEMO (30): Conceptual question

A fixed torque is applied to rotate the shaft of a beam. If the two weights on the beam are slid out, the angular acceleration of the wheel will…

A. increase.
B. decrease.
C. remain the same.
D. Not enough information.



Summary of rotation and translation

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Rotational motion (about a fixed axis)		Translational motion (in one dimension)	
Angular position	ϕ	Position	x
Angular speed	$\omega = d\phi/dt$	Speed	$v = dx/dt$
Angular acceleration	$\alpha = d\omega/dt$	Acceleration	$a = dv/dt$
Moment of inertia	$I = \int \rho^2 dm$	Mass	m
Net torque	$\Sigma\tau_{ext} = I\alpha$	Net force	$\Sigma F_{ext} = ma$
Rotational kinetic energy	$K^{rot} = I\omega^2/2$	Translational kinetic energy	$K^{trans} = mv^2/2$
Work	$W = \int_{\phi_a}^{\phi_b} \tau d\phi$	Work	$W = \int_{x_a}^{x_b} F dx$
Power	$P = \tau\omega$	Power	$P = Fv$
Angular momentum	$L = I\omega$	Momentum	$p = mv$
Net torque	$\Sigma\tau_{ext} = dL/dt$	Net torque	$\Sigma F_{ext} = dp/dt$