General Physics: Mechanics

Swiss Plasma Center

PHYS-101(en)

Lecture 10a: Collisions

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Today's agenda (Serway 9, MIT 15)

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1. Collisions
2. Elastic collisions
3. Inelastic collisions
4. More collisions

Last few weeks - conserved quantities

- Conservation of momentum:

There is no net external force and no mass exchange

In a given inertial reference frame,
the total momentum of an <u>isolated system</u> stays constant.

Last few weeks - conserved quantities

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the total momentum of an <u>isolated system</u> stays constant.

At time t_i At time t_f

$$\sum \vec{p}_i = \sum \vec{p}_f$$

Last few weeks - conserved quantities

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- Conservation of momentum:

In a given inertial reference frame,
the total momentum of an isolated system stays constant.

$$\sum \vec{p}_i = \sum \vec{p}_f \qquad \Rightarrow \qquad \sum m\vec{v}_i = \sum m\vec{v}_f$$

Last few weeks - conserved quantities

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- Conservation of energy:

The total energy of a system stays constant,
as long as energy doesn't leave or enter the system.

Last few weeks - conserved quantities

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- Conservation of mechanical energy:

If all forces doing work on a system are conservative, then its mechanical energy is conserved.

Last few weeks - conserved quantities

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$$\sum E_{mi} = \sum E_{mf}$$

Last few weeks - conserved quantities

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The total energy of a system stays constant,
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$$\sum E_i = \sum E_f$$

- Conservation of mechanical energy:

If all forces doing work on a system are conservative, then its mechanical energy is conserved.

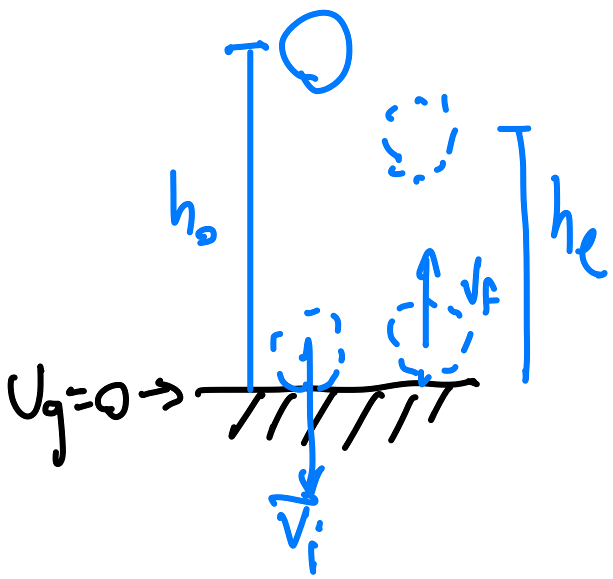
At time t_i At time t_f

$$\sum E_{mi} = \sum E_{mf} \Rightarrow \sum K_i + \sum U_i = \sum K_f + \sum U_f$$

DEMO (82)

Drop, Collision, Final height

$$t_0 < t_i < t_f < t_e$$



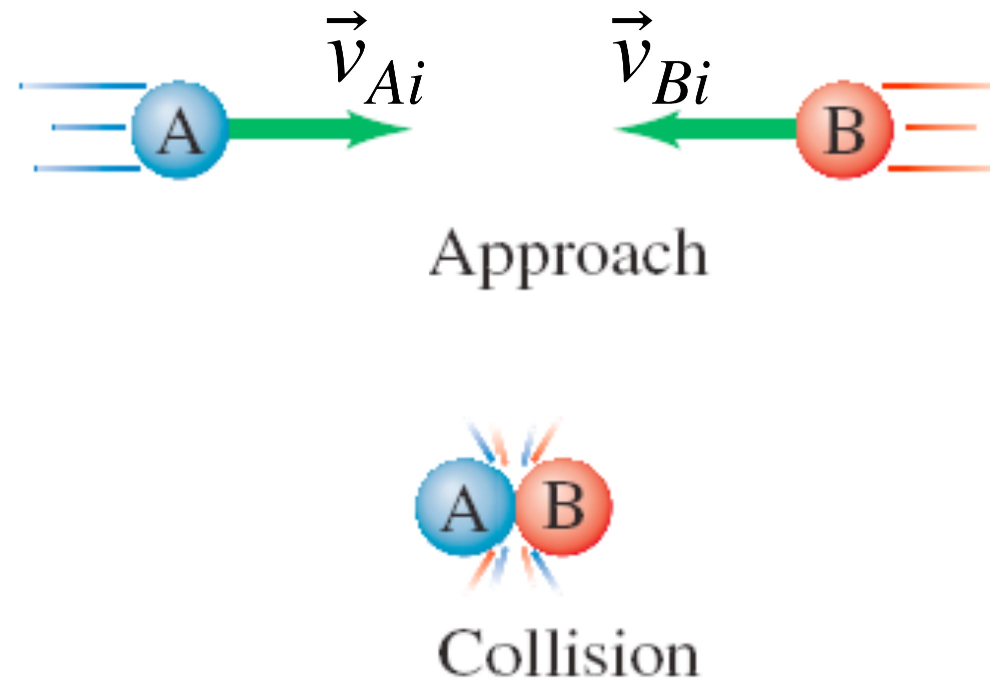
$$K_0 + U_{g0} = K_i + U_{gi} \Rightarrow mgh_0 = \frac{1}{2}mv_i^2 \Rightarrow v_i = \sqrt{2gh_0}$$

$$K_f + U_{gF} = K_e + U_{ge} \Rightarrow \frac{1}{2}mv_f^2 = mgh_e \Rightarrow v_f = \sqrt{2gh_e}$$

Dropping things on an anvil

Elastic versus inelastic collisions

- Throughout a collision:
    - **Momentum** is always conserved (when the net external force is zero or when using the impulse approximation)



Elastic versus inelastic collisions

- Throughout a collision:
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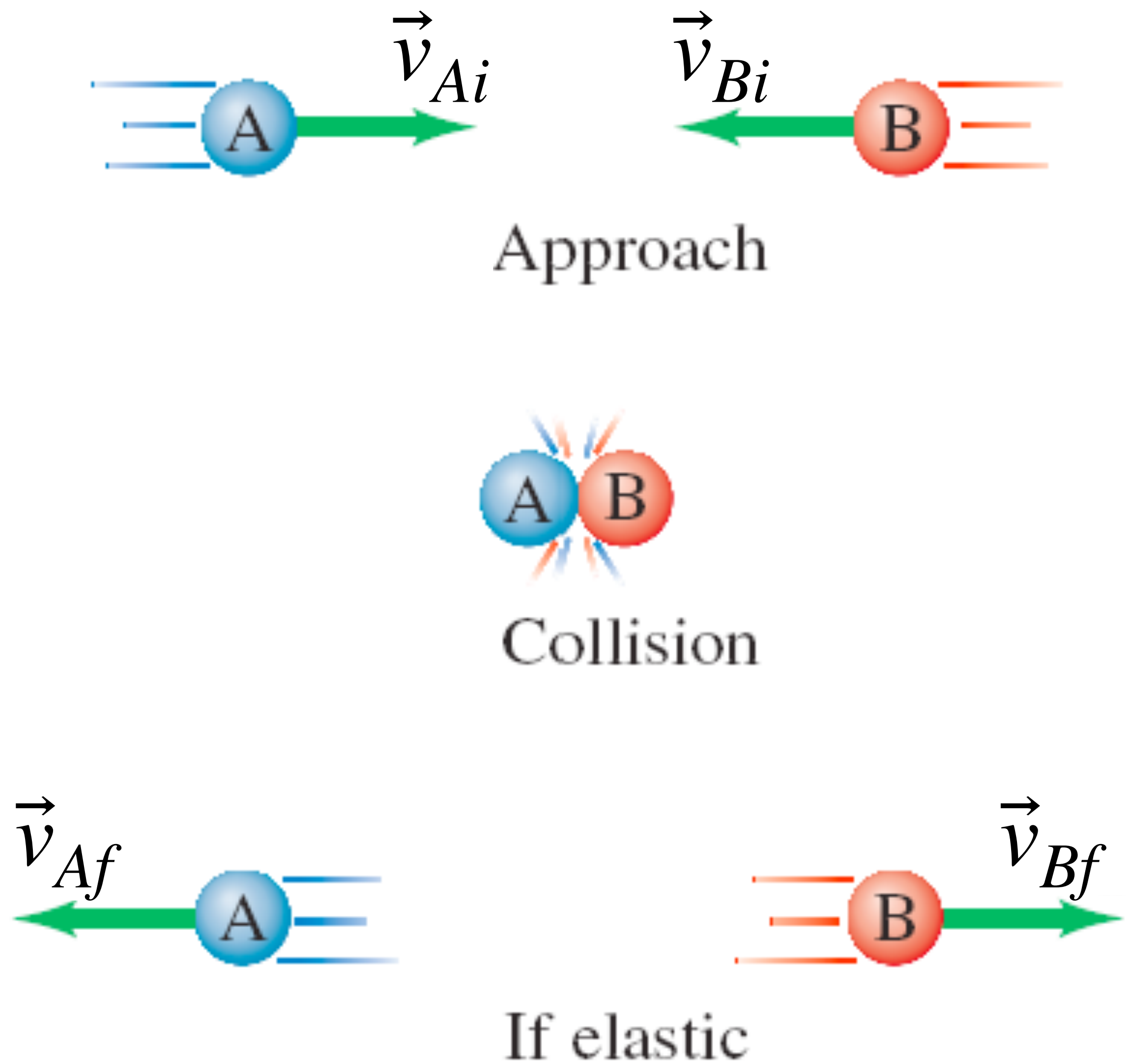
  - **Kinetic energy** is conserved when the collision is **elastic** (i.e. no nonconservative work, no change in potential energy)



Elastic versus inelastic collisions

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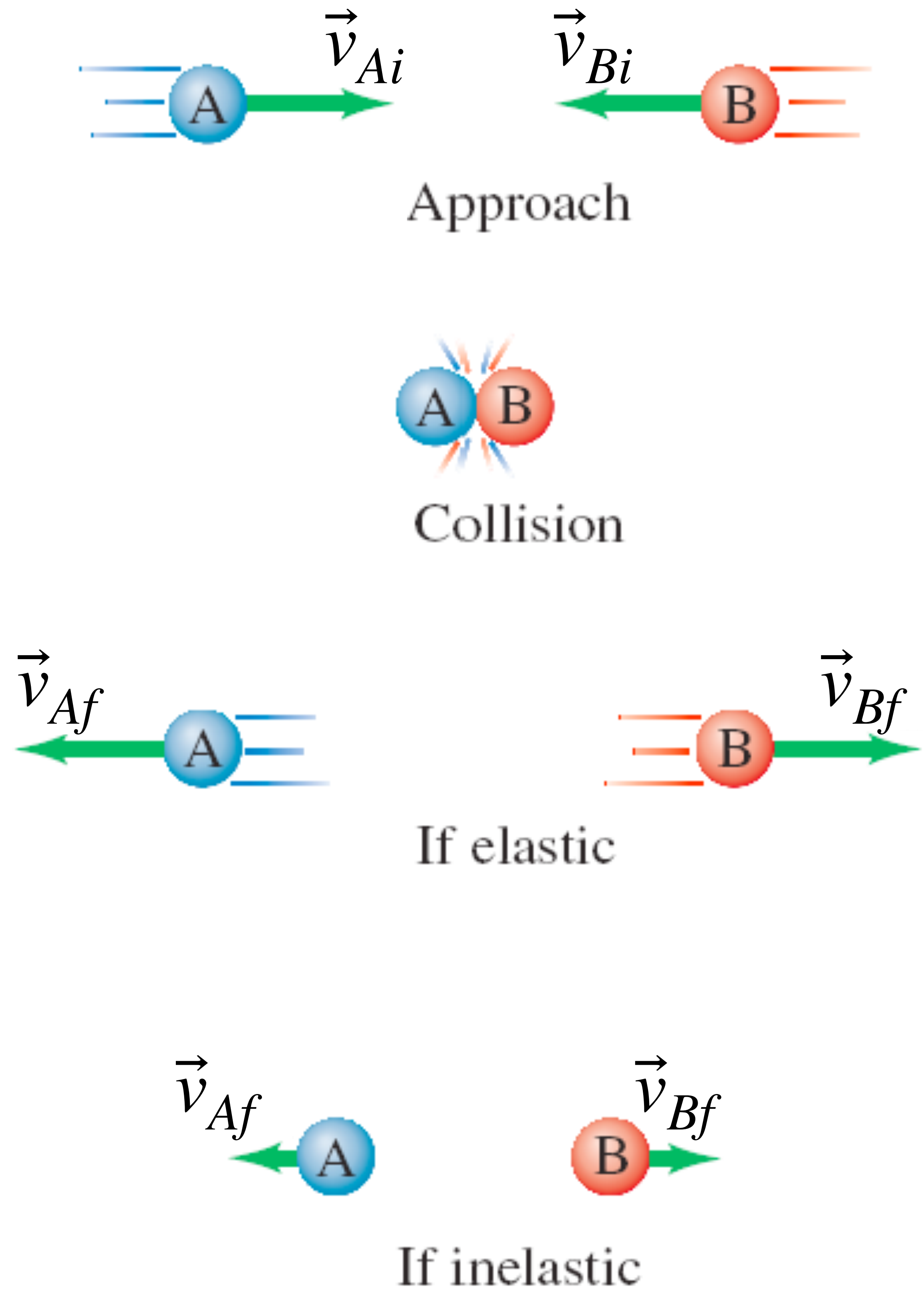
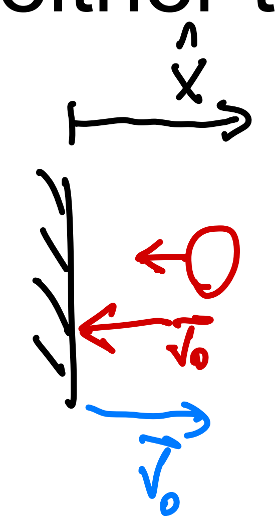
  - **Momentum** is always conserved (when the net external force is zero or when using the impulse approximation)
  - **Kinetic energy** is conserved when the collision is **elastic** (i.e. no nonconservative work, no change in potential energy)
  - **Kinetic energy** is <u>not</u> conserved when the collision is **inelastic**



Conceptual question

A ball is thrown at a wall. The ball bounces off and returns with a speed equal to the speed it had before colliding with the wall. Which of the following quantities are the same after the collision as they were before the collision?

A. The kinetic energy of the ball.
B. The momentum of the ball.
C. Both the kinetic energy and the momentum of the ball.
D. Neither the kinetic energy nor the momentum of the ball.



Before coll:

$\vec{p}_i = -mv_0\hat{x}$

$K_i = \frac{1}{2}m(-v_0)^2 = \frac{1}{2}mv_0^2$

$\neq$

$=$

After:

$\vec{p}_f = mv_0\hat{x}$

$K_f = \frac{1}{2}m(v_0)^2 = \frac{1}{2}mv_0^2$

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 2mv_0\hat{x}$

Elastic collision in one dimension

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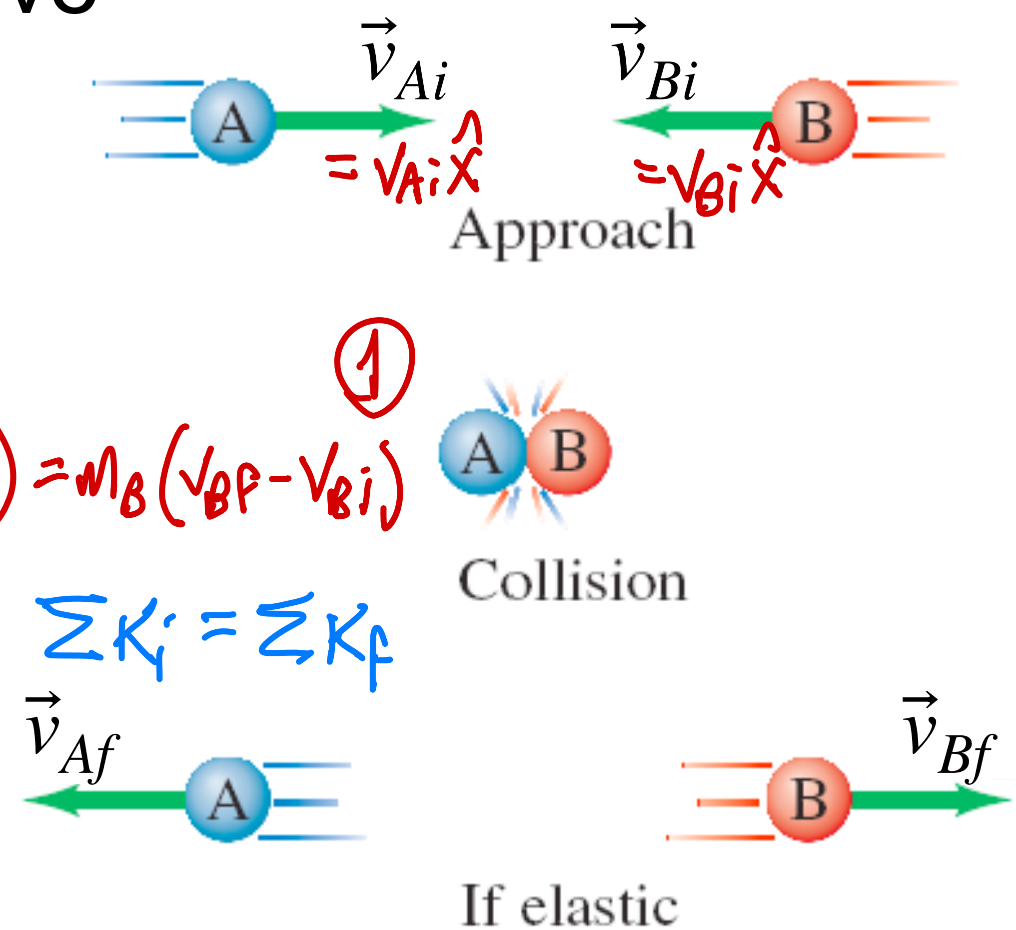
- Remember *elastic collisions* conserve both momentum and kinetic energy



Cons. of momentum: $\sum \vec{p}_i = \sum \vec{p}_f$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$$

$$\Rightarrow m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf} \Rightarrow m_A (v_{Ai} - v_{Af}) = m_B (v_{Bf} - v_{Bi}) \quad (1)$$

Since "elastic", kin. energy is conserved: $\sum K_i = \sum K_f$

$$\tfrac{1}{2} m_A v_{Ai}^2 + \tfrac{1}{2} m_B v_{Bi}^2 = \tfrac{1}{2} m_A v_{Af}^2 + \tfrac{1}{2} m_B v_{Bf}^2$$

$$\Rightarrow m_A v_{Ai}^2 - m_A v_{Af}^2 = m_B v_{Bf}^2 - m_B v_{Bi}^2$$

$$\Rightarrow m_A (v_{Ai} - v_{Af})(v_{Ai} + v_{Af}) = m_B (v_{Bf} - v_{Bi})(v_{Bf} + v_{Bi}) \quad (2)$$

I divide (2) by (1) to obtain: $v_{Ai} + v_{Af} = v_{Bf} + v_{Bi}$ (3)

I multiply (3) by m_B: $m_B v_{Ai} + m_B v_{Af} = m_B v_{Bi} + m_B v_{Bf}$ (4)

Elastic collision in one dimension

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Finally I subtract ④ from ①

$$m_A v_{Ai} - m_A v_{Af} - m_B v_{Ai} - m_B v_{Af} = m_B v_{Bf} - m_B v_{Bi} - m_B v_{Bi} - m_B v_{Bf}$$

$$= -2 m_B v_{Bi}$$

$$\Rightarrow v_{Ai}(m_A - m_B) + v_{Af}(-m_A - m_B) = -2 m_B v_{Bi}$$

$$\Rightarrow v_{Af}(m_A + m_B) = v_{Ai}(m_A - m_B) + 2 m_B v_{Bi}$$

$$\Rightarrow \boxed{v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} + \frac{2 m_B}{m_A + m_B} v_{Bi}}$$

$$v_{Bf} = \frac{m_B - m_A}{m_A + m_B} v_{Bi} + \frac{2 m_A}{m_A + m_B} v_{Ai}$$

DEMO (766): Elastic collision, same mass

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Cart A, with mass m moving with speed v_{Ai}, collides head-on with cart B of equal mass. What are the speeds of the two carts after the collision, assuming it is elastic?

A. Cart B is initially at rest ($v_{Bi} = 0$)

$m_A = m_B = m$

$$v_{Af} = \frac{m-m}{m+m} v_{Ai} + \frac{2m}{m+m} v_{Bi} = 0$$

$$v_{Bf} = \frac{m-m}{m+m} v_{Bi} + \frac{2m}{m+m} v_{Ai} = v_{Ai}$$

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$$v_{Bf} = \frac{m-m}{m+m} v_{Bi} + \frac{2m}{m+m} v_{Ai} = v_{Ai}$$

B. Cart B is also moving with an initial velocity v_{Bi}

$$v_{Af} = \frac{m-m}{m+m} v_{Ai} + \frac{2m}{m+m} v_{Bi} = v_{Bi}$$

$$v_{Bf} = v_{Ai}$$

DEMO (766): Elastic collision, same mass

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Cart A, with mass m moving with speed v_{Ai}, collides head-on with cart B of equal mass. What are the speeds of the two carts after the collision, assuming it is elastic?

- When objects with **equal mass** collide elastically in 1D, they simply <u>swap velocities</u>

DEMO (89): Newton's cradle

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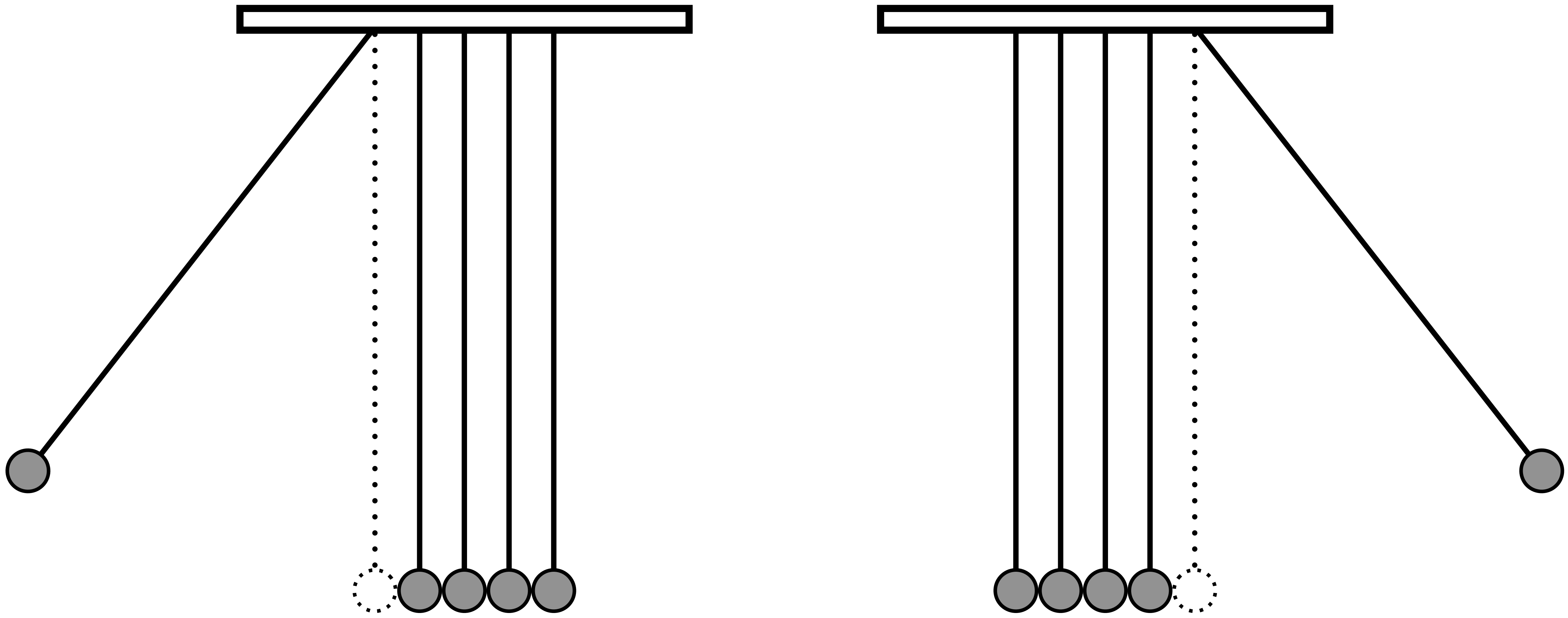

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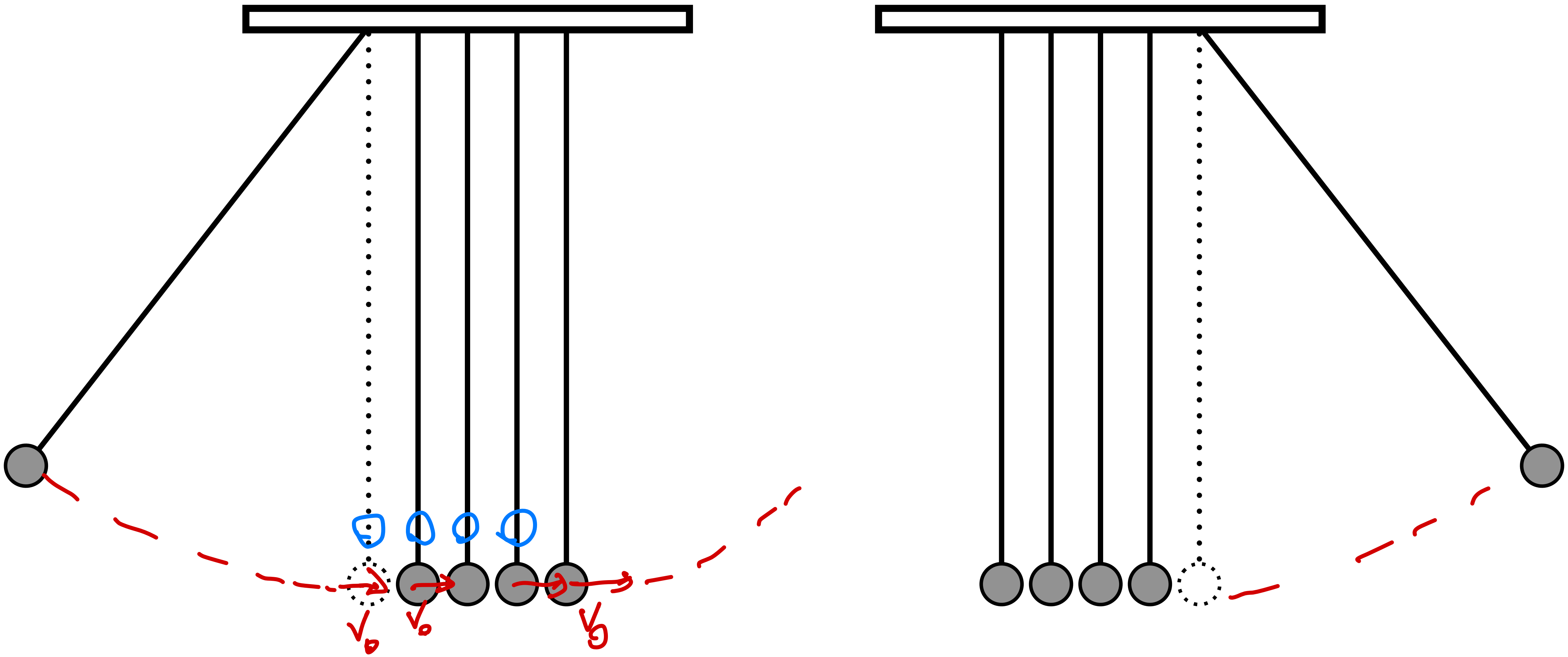

DEMO (89): Newton's cradle

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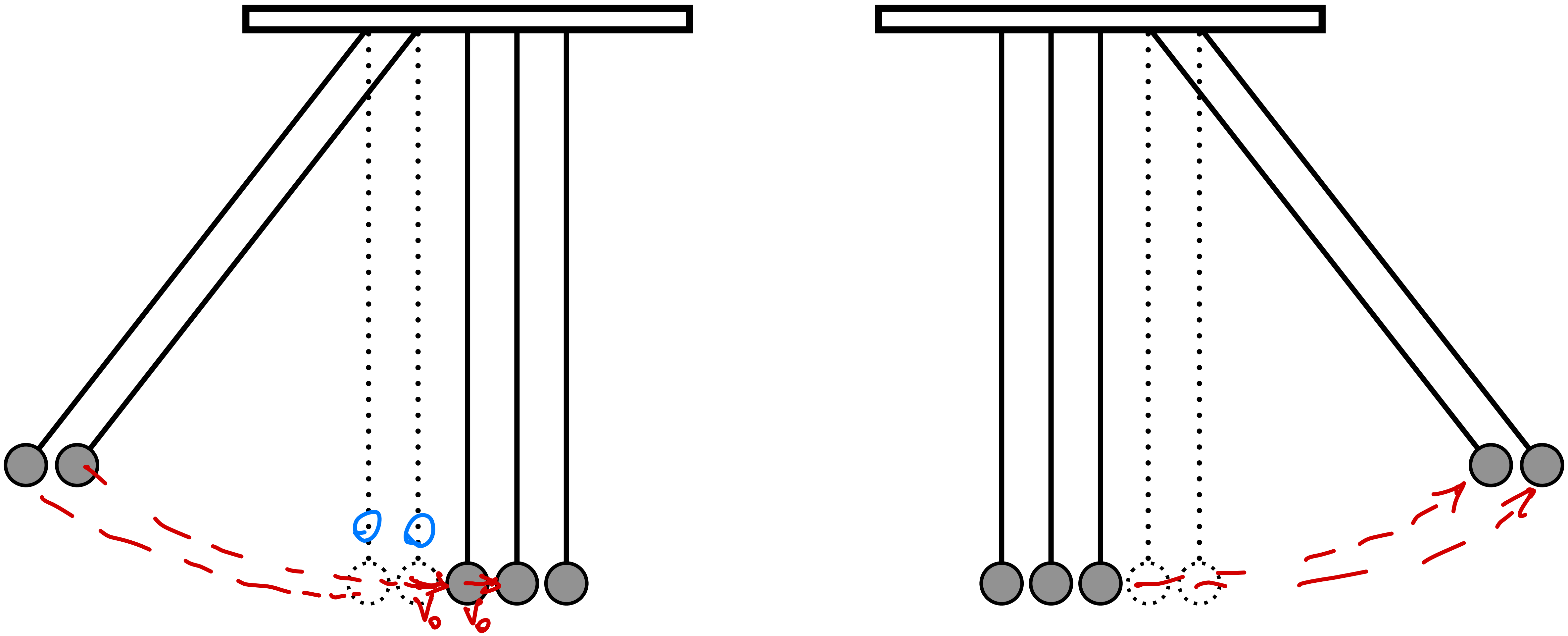

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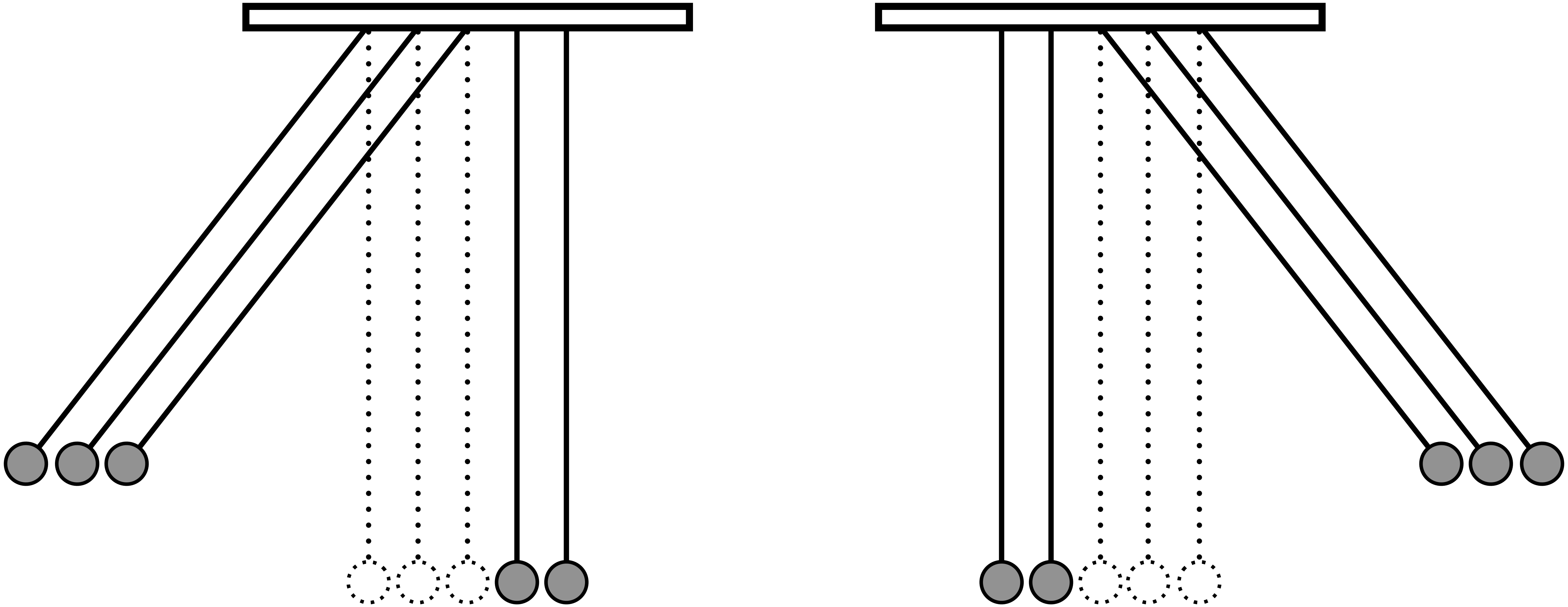

DEMO (766): Elastic collision, different mass

Cart A, with mass m_A moving with speed v_{Ai}, experiences a head-on elastic collision with cart B, which has mass m_B and is at rest.

$v_{Bi} = 0$

A. What are the final velocities of the carts?

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai}$$

$$v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai}$$

DEMO (766): Elastic collision, different mass

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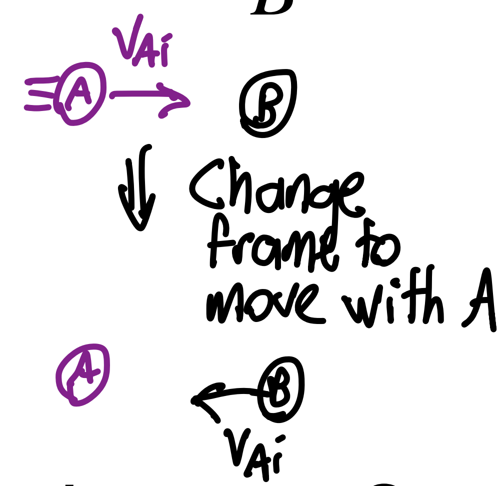
Cart A, with mass m_A moving with speed v_{Ai}, experiences a head-on elastic collision with cart B, which has mass m_B and is at rest. $v_{Bi} = 0$

B. What if m_A is much *larger* than m_B?

$m_A \gg m_B$ means that
$m_A + m_B \approx m_A$
$m_A - m_B \approx m_A$

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} \approx \frac{m_A}{m_A} v_{Ai} = v_{Ai}$$

$$v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai} \approx \frac{2m_A}{m_A} v_{Ai} = 2v_{Ai}$$



C. What if m_A is much *smaller* than m_B?

$m_A \ll m_B$ means that
$m_A + m_B \approx m_B$
$m_A - m_B \approx -m_B$
$\frac{m_A}{m_B} \approx 0$

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} \approx \frac{-m_B}{m_B} v_{Ai} = -v_{Ai}$$

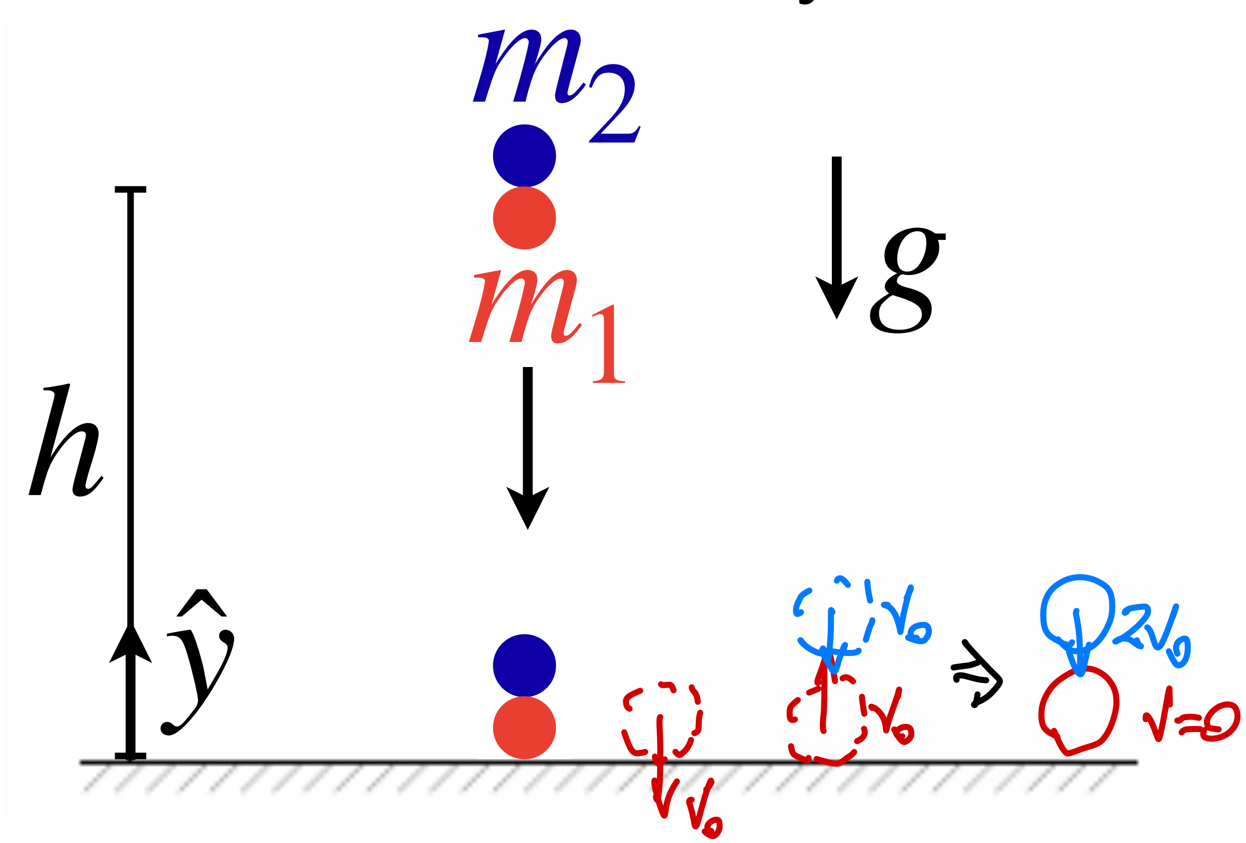
$$v_{Bf} = \frac{2m_A}{m_A + m_B} \approx 2\frac{m_A}{m_B} \approx 0$$

Example

Two small balls are dropped from the same height h, one on top of the other. Ball 2 is on top, while ball 1 is below and is much more massive with $m_1 \gg m_2$. First, ball 1 collides with the ground at speed v_0 and rebounds elastically. Then, as ball 1 starts to move upward, it collides elastically with ball 2 which is still moving downwards also with speed v_0. What is the **relative** speed between the two balls after they collide?

It is $2v_0$

Notice that in the fixed, the speed of the small ball is $3v_0$



DEMO (492)

Seismic accelerator

Conceptual question

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A small spacecraft with speed v_i approaches Saturn, which is moving in the opposite direction at speed v_S. Due to gravitational interactions with Saturn, the spacecraft swings around Saturn and heads off in the direction opposite to its approach. After it is far enough away to be effectively free of Saturn's gravity, the final speed of the spacecraft v_f is…

$m \ll m_S$

If $v_S > 0$ $|v_S| = v_S$

A. $v_i - v_S$.
B. $v_i + 2v_S$.
C. $v_i - 2v_S$.
D. $v_i + v_S$.
E. $2v_i - v_S$.

In fixed frame

$\vec{v}_S = -|v_S|\hat{x}$

$\vec{v}_i = v_i\hat{x}$

In frame of Saturn

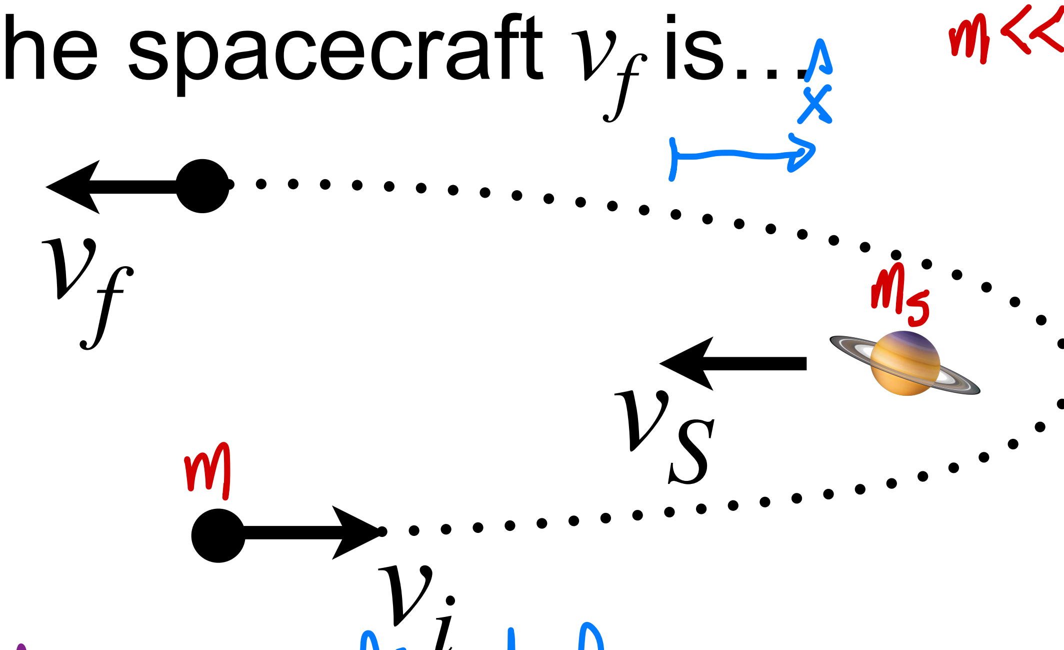
$\vec{v}_S{}' = \vec{v}_S - \vec{v}_S = 0$

$\vec{v}_i{}' = \vec{v}_i - \vec{v}_S = v_i\hat{x} - (-|v_S|\hat{x})$

$= (v_i + |v_S|)\hat{x}$

Elastic collision, $m \ll m_S$

$\vec{v}_f{}' = -\vec{v}_i = -(v_i + |v_S|)\hat{x}$



In fixed frame

$\vec{v}_f = \vec{v}_f{}' + \vec{v}_S$

$= -(v_i + |v_S|)\hat{x} - |v_S|\hat{x}$

$= -(v_i + 2|v_S|)\hat{x}$

Inelastic collisions

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- *Inelastic collisions* do not conserve kinetic energy, but still conserve momentum (as long as the system is isolated)

Inelastic collisions

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- Energy can be lost to potential energy or thermal energy (i.e. heat) due to friction

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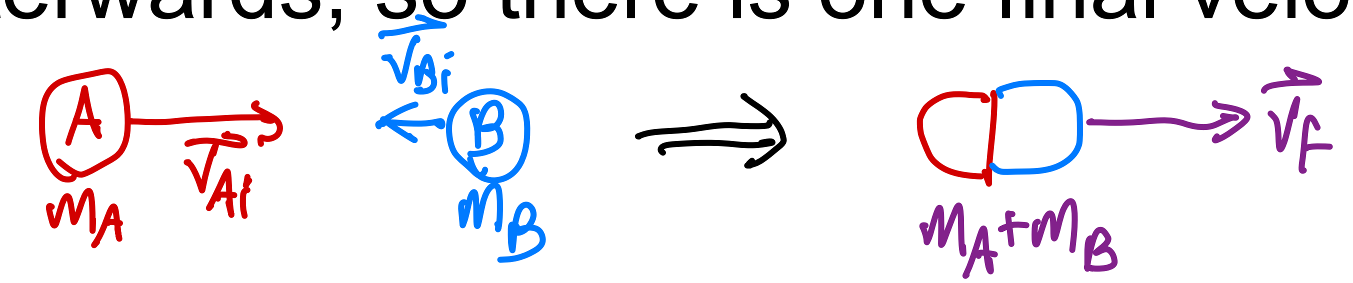
- Energy can be lost to potential energy or thermal energy (i.e. heat) due to friction
- A “perfectly inelastic” collision is when the objects stick together afterwards, so there is one final velocity



Inelastic collisions

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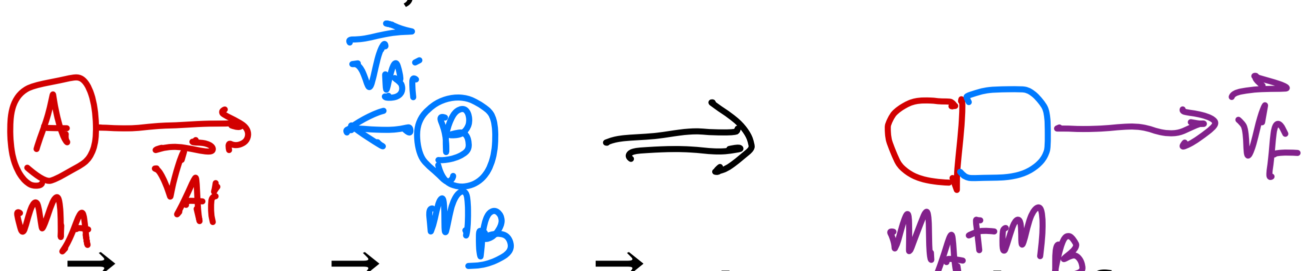

  - Impose $\vec{v}_{Af} = \vec{v}_{Bf} = \vec{v}_f$ instead of energy conservation

Inelastic collisions

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- A "perfectly inelastic" collision is when the objects stick together afterwards, so there is one final velocity
  - Impose $\vec{v}_{Af} = \vec{v}_{Bf} = \vec{v}_f$ instead of energy conservation
- Kinetic energy can sometimes even be gained through an inelastic collision!
- "How?!", you ask…

Fusion!

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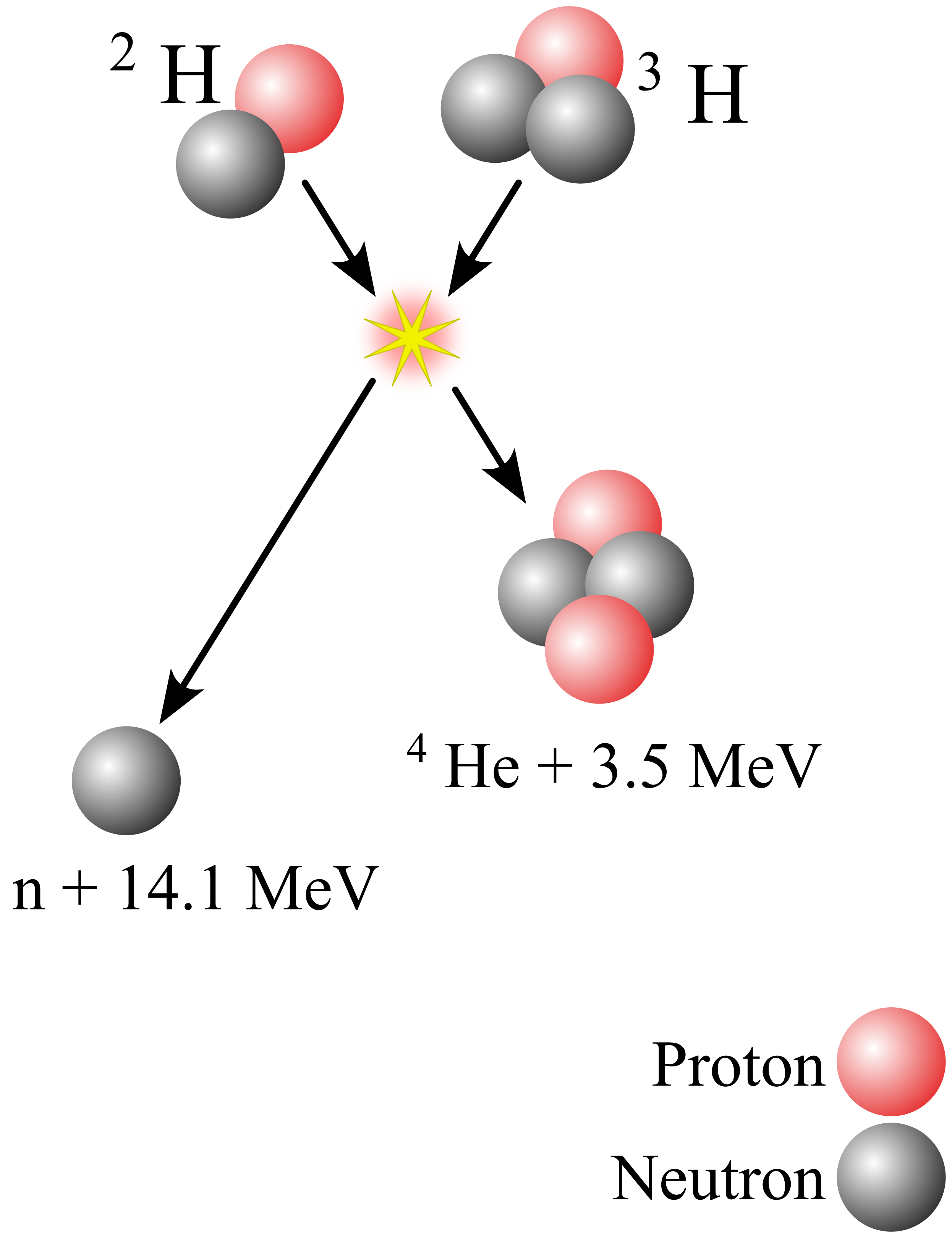
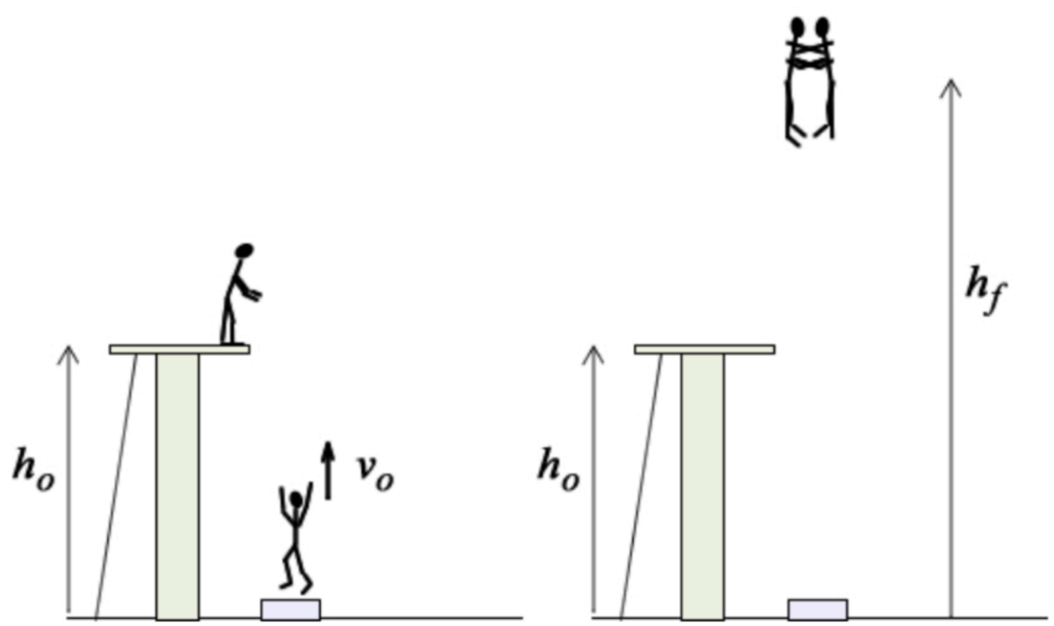


We've already seen inelastic collisions

Problem set 7

1. Acrobat and clown

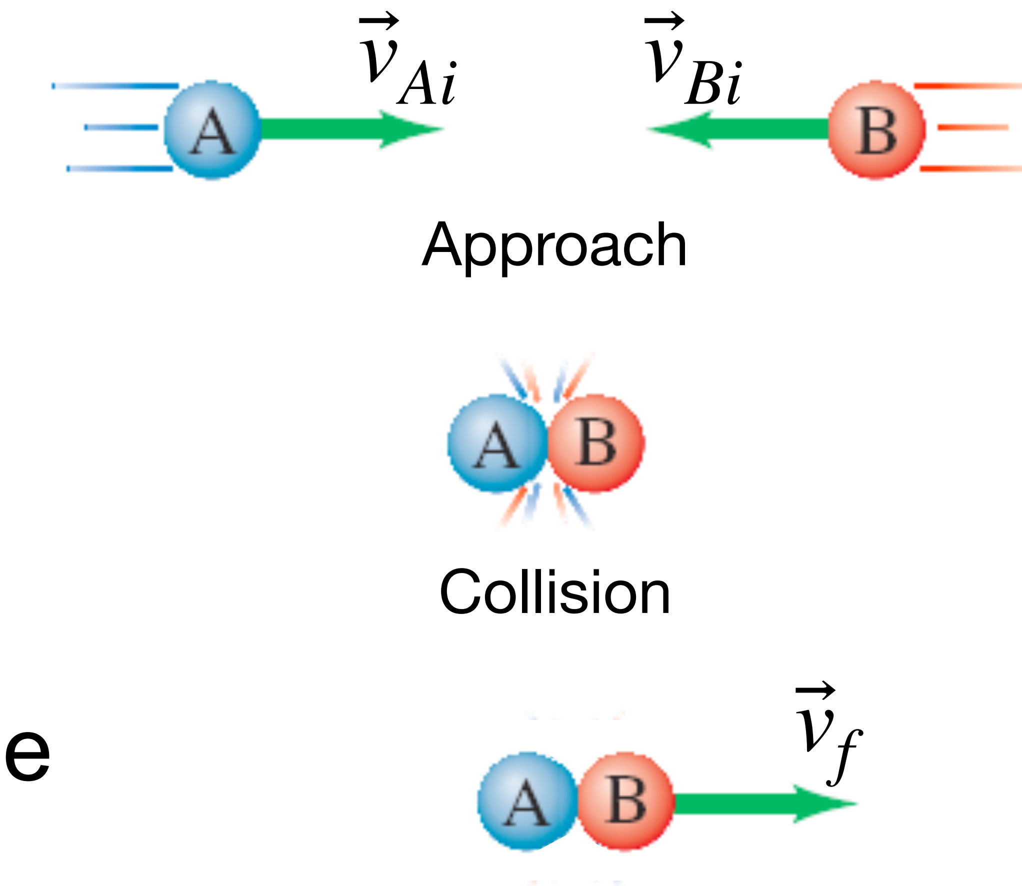
An acrobat of mass m_A jumps upwards off a trampoline with an initial speed v_0. At a … t grabs a clown of mass m_C, who is standing stationary at the edge of a platform. They the… together. Assume that the time it takes for the acrobat to grab the clown is very short.



What is the maximum height h_f reached by the acrobat and clown? Write your answer in terms of some or all of the following: m_A, m_C, g, h_0, and v_0.

Perfectly inelastic collision in one dimension

- Inelastic collisions conserve momentum, but **not** kinetic energy (so we need an additional constraint)
- In this case, we can find a completely general solution for the final velocities in terms of the initial velocities



Cons. of momentum: $\sum \vec{p}_i = \sum \vec{p}_f$

$$m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$$

In perfectly inelastic case, $\vec{v}_{Af} = \vec{v}_{Bf} = \vec{v}_f$

$$\Rightarrow m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_f + m_B \vec{v}_f = (m_A + m_B)\vec{v}_f \Rightarrow \boxed{\vec{v}_f = \frac{m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi}}{m_A + m_B}}$$

DEMO (766): Perfectly inelastic collision

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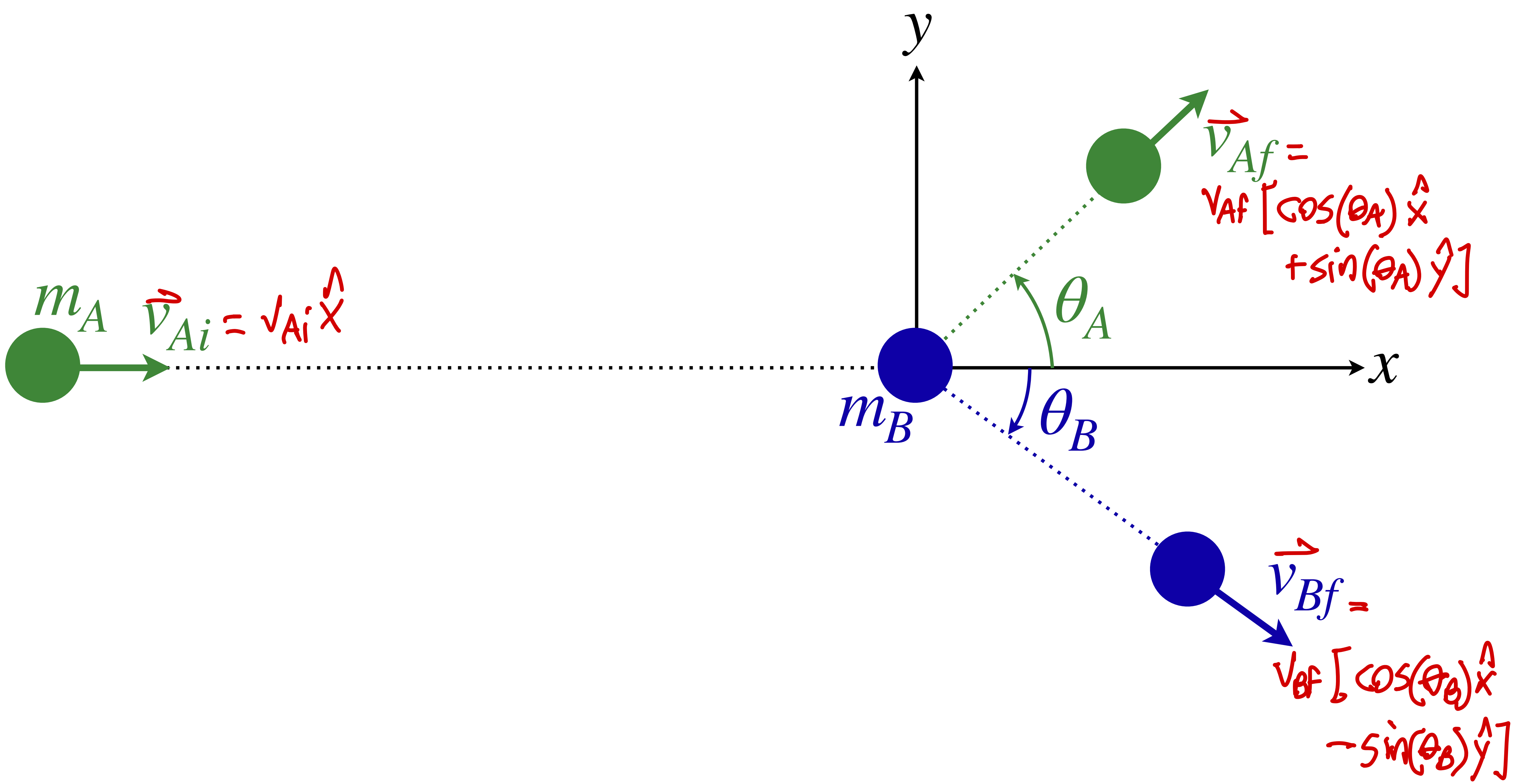
Cart A, with mass m_A moving with speed v_{Ai}, collides head-on with cart B, which has mass m_B and is at rest. What are the speeds of the two carts after the collision, assuming it is perfectly inelastic?

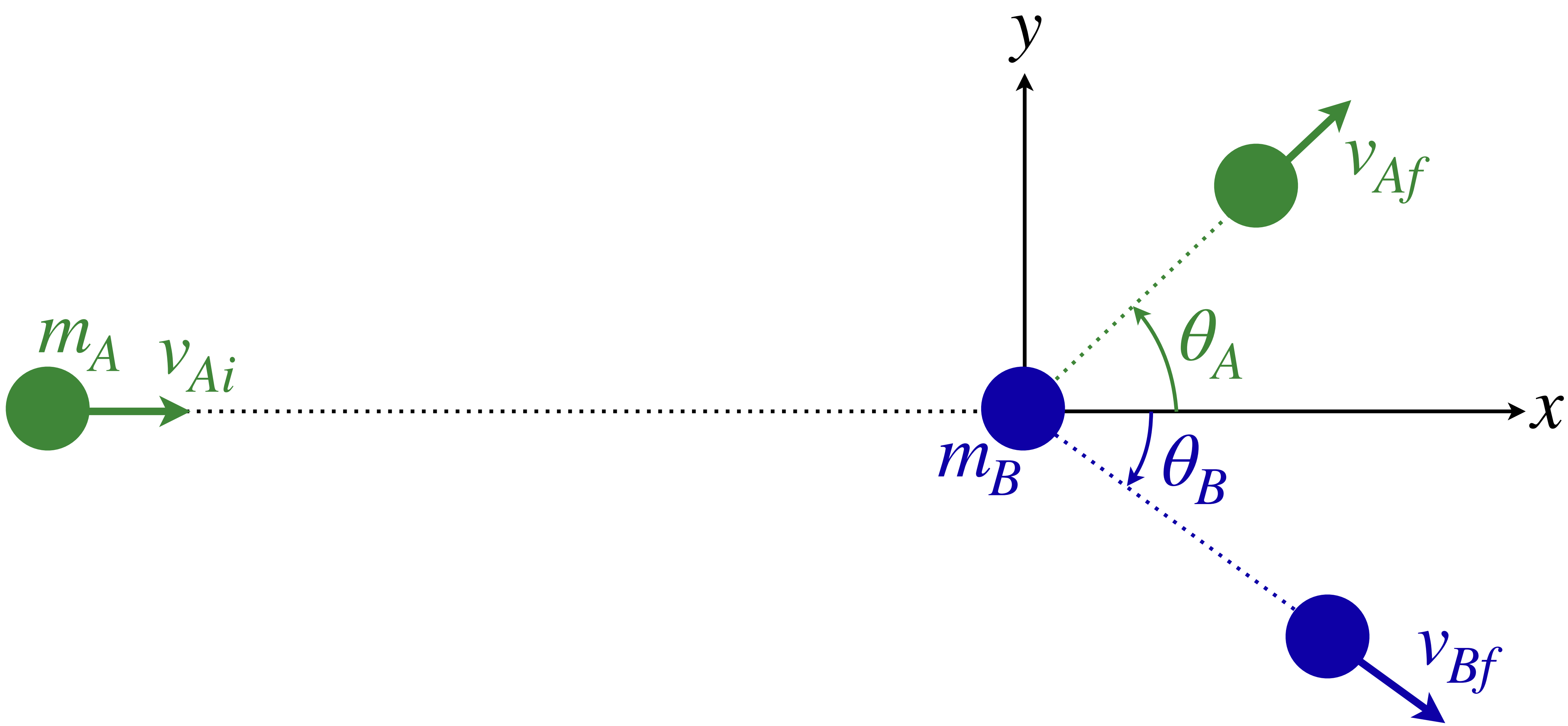
$v_{Bi} = 0$

$$\vec{v}_f = \frac{m_A \vec{v}_{Ai} + m_B (0)}{m_A + m_B} = \frac{m_A}{m_A + m_B} \vec{v}_{Ai}$$

Collisions in two or three dimensions



Collisions in two or three dimensions



- Still apply conservation of momentum and kinetic energy (or alternative condition), but significantly more math

2D elastic collision, **same** mass

A ball of mass m moving with speed v_{Ai} without friction on a horizontal surface collides elastically with another ball of an equal mass, at rest. After the collision, what is the angle between their velocities $\theta_A + \theta_B$?

$m_A = m_B = m \qquad v_{Bi} = 0$

Cons. of momentum: $m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf} \Rightarrow \vec{v}_{Ai} = \vec{v}_{Af} + \vec{v}_{Bf}$

In $\hat{x}$: $v_{Ai} = v_{Af}\cos(\theta_A) + v_{Bf}\cos(\theta_B)$

In $\hat{y}$: $0 = v_{Af}\sin(\theta_A) - v_{Bf}\sin(\theta_B)$

Now I square both sides and add them:

$$v_{Ai}^2 + 0 = [v_{Af}\cos(\theta_A) + v_{Bf}\cos(\theta_B)]^2 + [v_{Af}\sin(\theta_A) - v_{Bf}\sin(\theta_B)]^2$$

$$= v_{Af}^2\cos^2(\theta_A) + v_{Bf}^2\cos^2(\theta_B) + 2v_{Af}v_{Bf}\cos(\theta_A)\cos(\theta_B)$$

$$+ v_{Af}^2\sin^2(\theta_A) + v_{Bf}^2\sin^2(\theta_B) - 2v_{Af}v_{Bf}\sin(\theta_A)\sin(\theta_B)$$

$$= v_{Af}^2 + v_{Bf}^2 + 2v_{Af}v_{Bf}[\cos(\theta_A)\cos(\theta_B) - \sin(\theta_A)\sin(\theta_B)] \rightarrow \cos(\theta_A+\theta_B)$$

$$= v_{Af}^2 + v_{Bf}^2 + 2v_{Af}v_{Bf}\cos(\theta_A+\theta_B) \quad (1)$$

2D elastic collision, **same** mass

Elastic collision: $\sum K_i = \sum K_f$

$\frac{1}{2} m v_{Ai}^2 = \frac{1}{2} m v_{Af}^2 + \frac{1}{2} m v_{Bf}^2 \Rightarrow v_{Ai}^2 = v_{Af}^2 + v_{Bf}^2$ ②

Now I subtract ② from ①:

$$0 = 2 v_{Af} v_{Bf} \cos(\theta_A + \theta_B)$$

This condition is satisfied when:

1) $v_{Af} = 0$ Head-on collision

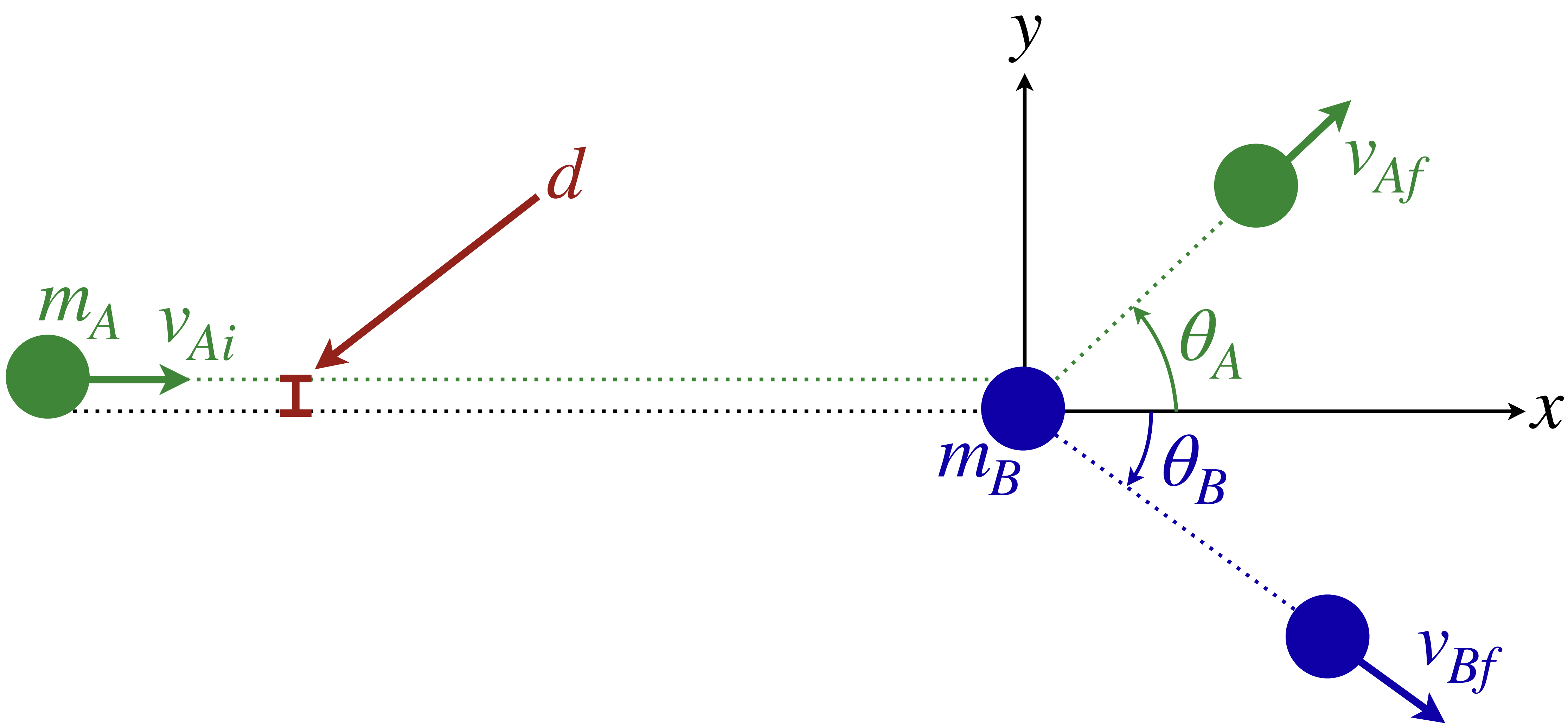
or 2) $v_{Bf} = 0$ No collision

or 3) $\cos(\theta_A + \theta_B) = 0 \Rightarrow \theta_A + \theta_B = \pm \frac{\pi}{2}$

DEMO (763)

Billiard table

2D elastic collision, **same** mass



- To solve for the 4 unknowns (v_{Af}, v_{Bf}, θ_A, θ_B), we need a fourth equation
- Knowing the "impact parameter" d, allows you to directly determine θ_B

Summary

- As long as the system is *isolated*, momentum is conserved in any collision
- If the collision is *elastic*, kinetic energy is also conserved
  - *Elastic* means that the nonconservative work is zero and there is no change in the potential energy
- If the collision is *inelastic*, kinetic energy is not conserved, so you need to find some other condition
- If the collision is *perfectly inelastic*, the objects stick together, so the other condition is to set the final velocities of the objects to be equal

See you tomorrow for fusion reactions

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