



**Final exam
PHYS-101(en)**

Problem booklet

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Important guidelines

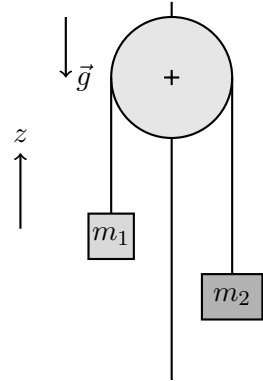
- **Do NOT open this booklet before the start of the exam!**
- **Provide your answers in the designated spaces in your Ans booklet only.** Anything written outside these spaces will be ignored by the scanner.
- **Justify your answers to the open questions!** Answers without a justification will not be counted.
- You may use sheets of scrap paper to prepare your answers. However, only the answers in the Ans booklet will be scanned and graded.
- **Do not unstaple** the pages of the Ans booklet.
- **Write in black or blue ink only.** Red ink, friction pens (such as Frixion Ball) and pencils are not allowed because they cannot be properly scanned.
- Calculators, smartphones, tablets, headphones, or any other electronic devices are **not** allowed.
- Assistants will check your student ID card and have you sign an attendance sheet. Please have your Camipro card in front of you on your table, ready to be checked.
- Assistants are **not** allowed to answer any questions, except in cases of doubt about the meaning of a word or a phrase.
- **Formula sheet:** One handwritten single-sided A4 sheet. No other material is permitted.

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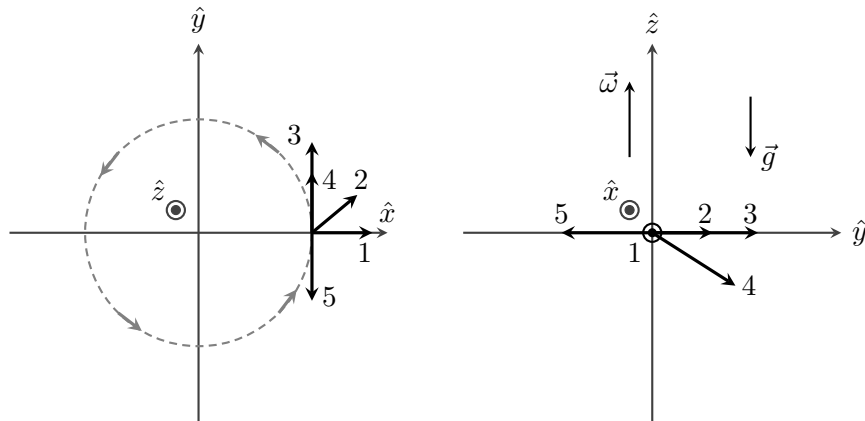
Multiple choice questions (2 points each)

Q1. Consider the setup shown in the figure, which consists of an inextensible rope and a fixed pulley, both massless. Assume that the rope does not slip on the pulley. The system is placed in a uniform gravitational field \vec{g} . It is then observed that the acceleration of mass m_1 is $\ddot{z}_1 = a_1 > 0$. If the massless pulley is replaced by a uniform cylindrical pulley of mass $M > 0$, what can be said about the magnitude a'_1 of the acceleration of m_1 ?

- A. It is larger than in the case of the massless pulley
- B. It is smaller than in the case of the massless pulley
- C. It does not change
- D. The answer depends on the initial conditions
- E. It decreases with time



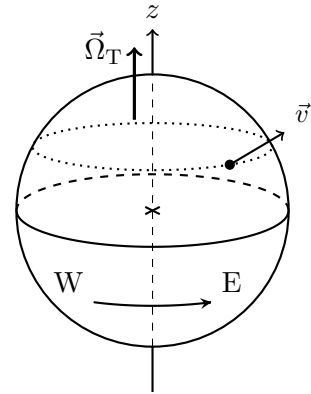
Q2. A point mass m is attached to a massless string of fixed length and rotates in a horizontal plane with constant angular velocity $\vec{\omega} = \omega \hat{z}$. Suddenly, the string breaks. The figure shows the situation at that instant, as seen from above (left figure) and from the side (right figure). What is the correct direction of the velocity vector of mass m immediately after the string breaks?



- A. Vector 1
- B. Vector 2
- C. Vector 3
- D. Vector 4
- E. Vector 5

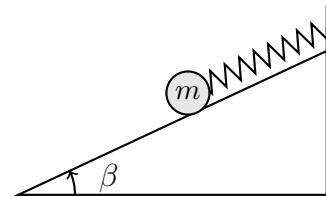
Q3. A rocket takes off vertically with velocity \vec{v} from the Cape Canaveral launch site in Florida, indicated by a point in the figure. What is the effect of Earth's rotation, $\vec{\Omega}_T$, on the rocket's trajectory, as observed from a ground-based reference frame?

- A. It is deflected toward the east
- B. It is deflected toward the west
- C. It is deflected toward the north
- D. It is deflected toward the south
- E. It is not deflected



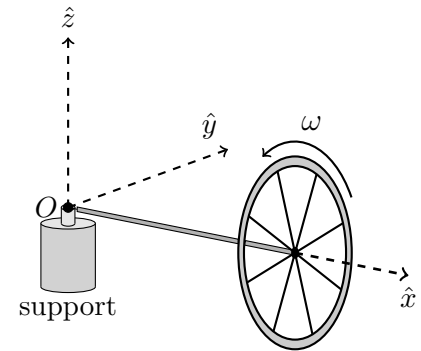
Q4. Consider a mass attached to a spring (whose other end is fixed) that can slide without friction on an inclined plane. Which statement is correct?

- A. There is no equilibrium point
- B. The equilibrium point corresponds to a minimum of gravitational potential energy
- C. The equilibrium point corresponds to a minimum of the sum of the spring's elastic potential energy and gravitational potential energy
- D. There are two equilibrium points: the first corresponds to a minimum of gravitational potential energy, and the second to a minimum of the spring's elastic potential energy
- E. The equilibrium point corresponds to a minimum of mechanical energy



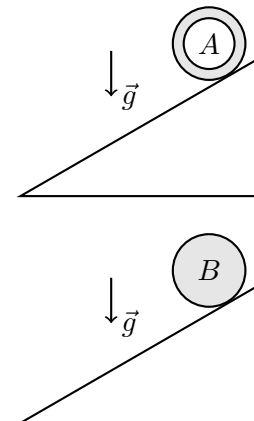
Q5. A wheel is spinning rapidly with angular velocity $\vec{\omega} = \omega \hat{x}$, about a horizontal axis. This axis is supported at its far end (away from the wheel) by a vertical support. Friction is neglected. What motion is expected?

- A. The wheel undergoes a precession motion about axis \hat{z} in the counterclockwise direction (top view), with angular velocity $\vec{\Omega}_p = \Omega \hat{z}$
- B. The wheel undergoes a precession motion about axis \hat{z} in the clockwise direction (top view), with $\vec{\Omega}_p = -\Omega \hat{z}$
- C. The wheel tips over due to gravity and oscillates like a physical pendulum about axis \hat{y} around a stable vertical equilibrium position
- D. The wheel responds to gravity by slowly rotating about axis \hat{y} in order to conserve its angular momentum. Its center of mass rises until stabilizing at a height $h > 0$
- E. The change in the wheel's angular momentum can be neglected compared to its initial angular momentum. The wheel keeps rotating about \hat{x} , but does not experience any other type of motion



Q6. Two cylinders of identical shape and equal mass are released from rest at the top of an inclined plane. They roll without slipping. Cylinder A is hollow, with a cylindrical hole along its axis of rotation. Its mass is otherwise uniformly distributed. Cylinder B is solid and homogeneous. Which cylinder reaches the bottom of the incline first?

- A. It depends on the coefficient of kinetic friction, μ_k , between each cylinder and the ground
- B. It depends on the size of the cylindrical hole in cylinder A
- C. They arrive at the same time
- D. Cylinder A
- E. Cylinder B



1. Inclined plane, blocks, and pulley (13 points)

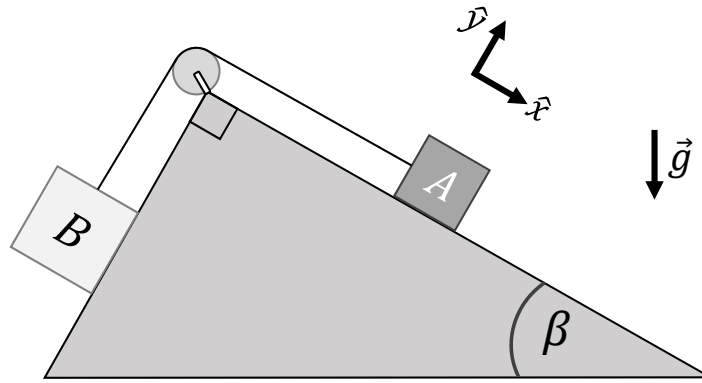


Figure 1: Blocks, rope, and pulley system.

In Fig. 1, block A with mass m_A lies on a flat inclined surface that makes an angle β with the horizontal. Block B with mass m_B lies on another flat inclined surface that is perpendicular to the surface on which block A lies. Both blocks are connected via an inextensible, massless rope that passes over a pulley of radius R . The two blocks are initially at rest.

Block A has a coefficient of static friction μ_s with the inclined surface, and a coefficient of kinetic friction μ_k . There is no friction between block B and the surface on which it can move. All other friction forces, as well as air drag, are negligible.

First, consider the situation in which the pulley is massless.

- 1a) Draw free-body diagrams for blocks A and B including all the forces acting on each object. Assume that in the absence of friction block A slides down the inclined plane.
- 1b) Find the minimum value of m_A for block A to start sliding *down* the inclined plane. Call this value m_A^{\min} .
- 1c) Assuming that the above condition is met so that block A starts moving (i.e. $m_A > m_A^{\min}$), find the acceleration of block A in terms of the parameters of the problem.
Hint: The final expression for \vec{a}_A should be in terms of g , m_A , m_B , μ_k , $\sin(\beta)$, and $\cos(\beta)$ only.
- 1d) For parts (d)–(f), consider the situation in which the pulley has mass M_p and moment of inertia I_p . The rope does not slip on the pulley.
Draw a diagram showing the forces acting on the pulley and their point of application.
- 1e) If T_A is the tension of the rope in the portion attached to block A and T_B is the tension in the portion attached to block B , show that the magnitude of the angular acceleration of the pulley is $|\vec{\alpha}| = (R/I_p) |T_A - T_B|$.
- 1f) Assuming that the pulley is a solid cylinder and that $m_A > m_A^{\min}$, find the acceleration of block A in terms of only g , m_A , m_B , μ_k , $\sin(\beta)$, $\cos(\beta)$, and M_p .

2. Hole-in-one (12 points)

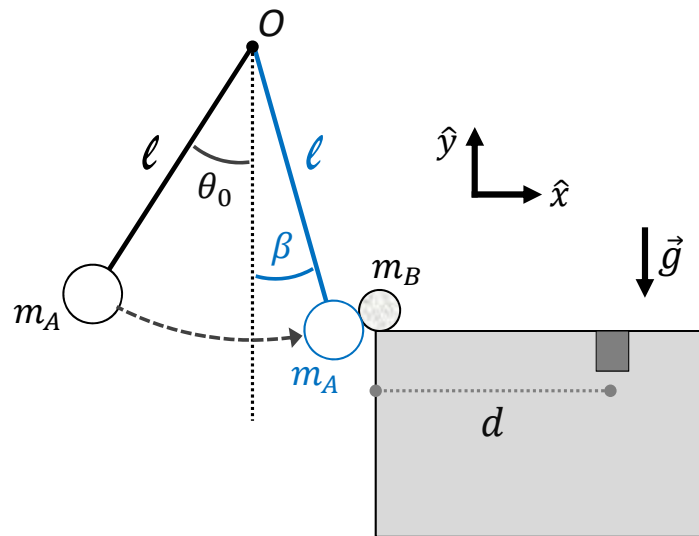


Figure 2: Table-top golf.

You are playing a game of table-top golf. As shown in Fig. 2, it consists of a small mass m_A attached to a massless string of length ℓ whose other end is attached to (and pivots around) a fixed point O .

Initially, the mass m_A , together with the string, are displaced from the vertical by an angle θ_0 and are let go from rest. The system then evolves under the influence of gravity until, at an angle β , m_A undergoes a head-on elastic collision with a small ball of mass $m_B = m_A/2$, which is at rest.

The goal of the game is to make the ball arrive at the hole located a distance d from its starting point (and at the same height).

In this problem, all friction forces and air drag can be neglected.

- 2a) Given an initial release angle of θ_0 , find the speed v_A of m_A right before the collision with the ball.
- 2b) Show that the velocity of the ball right after the collision is given by:

$$\vec{v}_B = (4\sqrt{2}/3)\sqrt{g\ell(\cos(\beta) - \cos(\theta_0))}(\cos(\beta)\hat{x} + \sin(\beta)\hat{y}).$$
 Assume that the interaction time between m_A and the ball is short enough for the impulse approximation to hold. Why is this assumption necessary?
- 2c) From the mechanical constraints of the game, θ_0 must always be chosen such that $0 \leq \theta_0 \leq \pi/2$.

Find the maximum range L_{\max} of the ball given the allowable values of θ_0 . Express your answer as a function of only β and ℓ . The *range* is understood here as the horizontal distance covered by the ball during its flight.

Hint: First show that, as a function of v_B and β , the range of the ball can be expressed as $L = 2(v_B^2/g)\sin(\beta)\cos(\beta) = (v_B^2/g)\sin(2\beta)$.
- 2d) Assuming that the hole is within range (i.e., $d \leq L_{\max}$), find the value of θ_0 required for the ball to land directly in the hole. This is the value of θ_0 required to achieve a hole-in-one!
- 2e) You prepare your shot according to the preceding calculations and are convinced that you are set for a hole-in-one. However, while you are not watching, a friend who is a prankster replaces the ball with a much lighter one such that now $m_B \ll m_A$.

How far away from the hole will the ball land? Express the answer in terms of d only.

