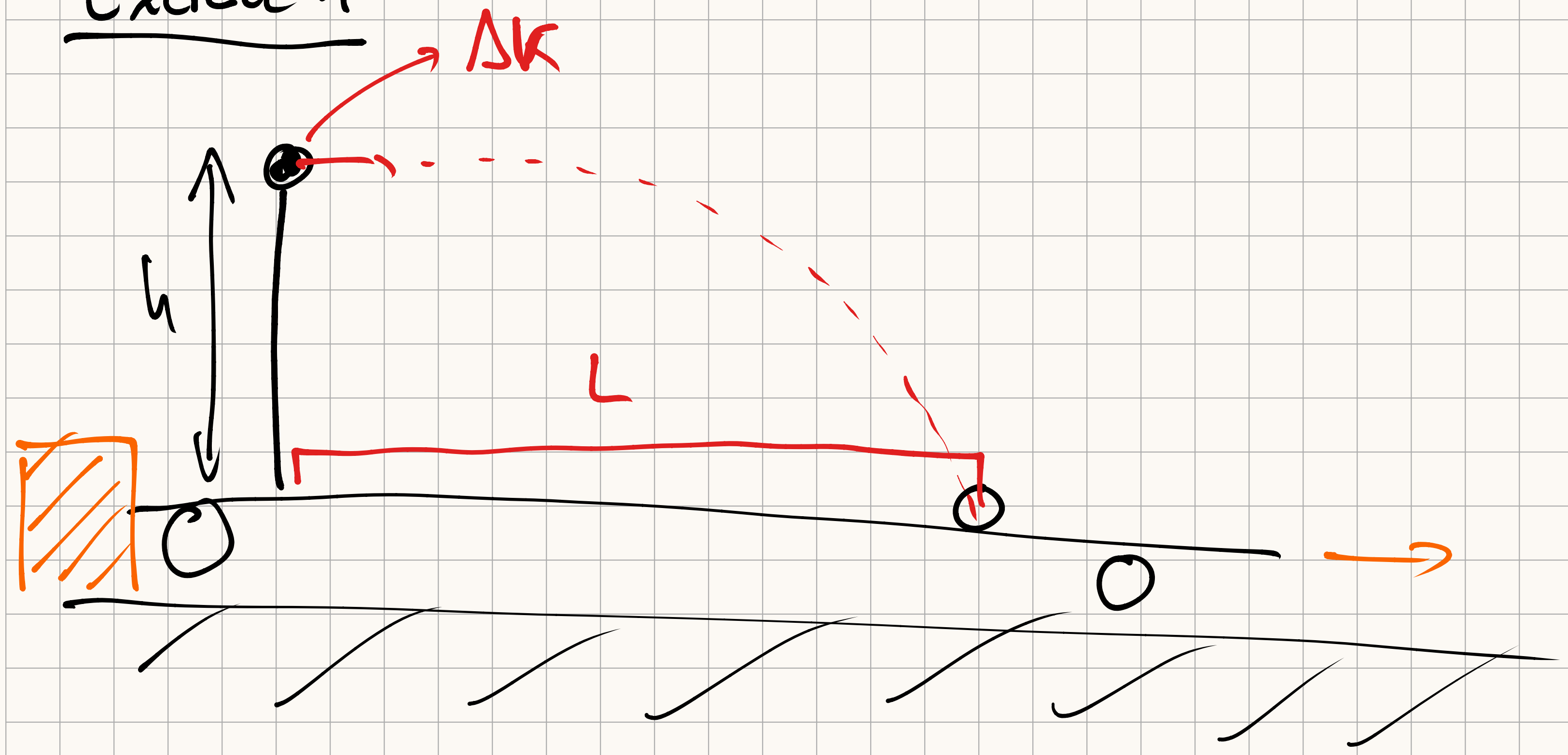
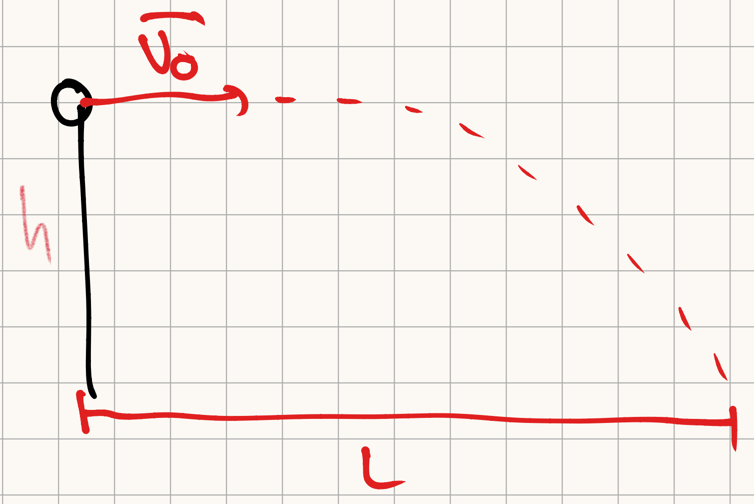
EXAMEN GRANDJEAN

Exercice 1



① Distance L



$$\Delta K = \frac{1}{2} m v_0^2 + \frac{1}{2} m \cancel{V_p^2}^{=0}$$

$$= \frac{1}{2} m V_0^2$$

$$\Rightarrow V_0^2 = \frac{2\Delta K}{m}$$

Pour décrire T, on utilise les eqts de la balistique

$$\vec{r}(t) = \begin{cases} x(t) = V_0 \cdot t + \cancel{x_0}^{=0} \\ y(t) = \underbrace{y_0}_{h} - \frac{1}{2} g t^2 \end{cases} \Rightarrow \begin{cases} x(t) = V_0 \cdot t \\ y(t) = h - \frac{1}{2} g t^2 \end{cases}$$

On s'intéresse à t_c tel que $\vec{r}(t_c) = (L, 0) \Rightarrow$

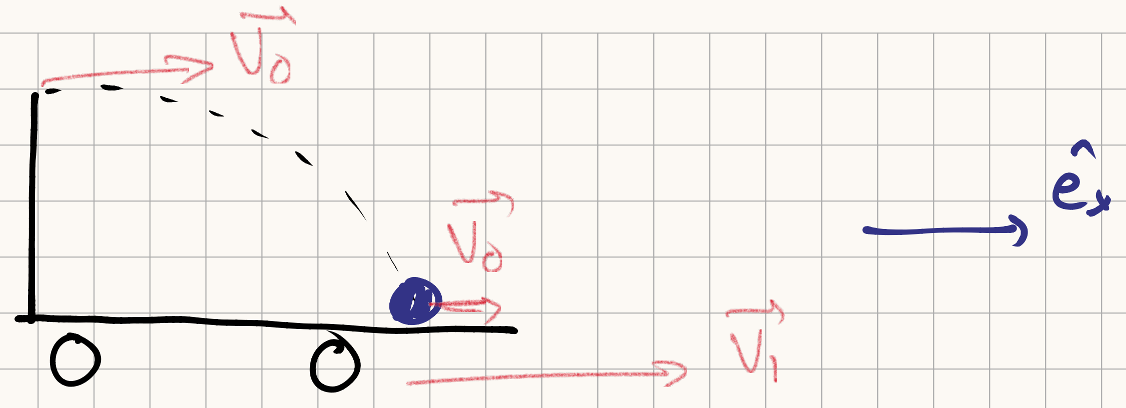
$$\boxed{\begin{aligned} x(t_c) &= V_0 \cdot t_c = L \\ y(t_c) &= h - \frac{1}{2} g t_c^2 = 0 \end{aligned}}$$

$$t_c^2 = \frac{2h}{g}$$

$$V_0 \cdot \sqrt{\frac{2h}{g}} = L$$

$$\Rightarrow L = \sqrt{\frac{2\Delta K}{m} \cdot \frac{2h}{g}}$$

2



Choc mou $\Rightarrow \Delta E_{mec} \neq 0$

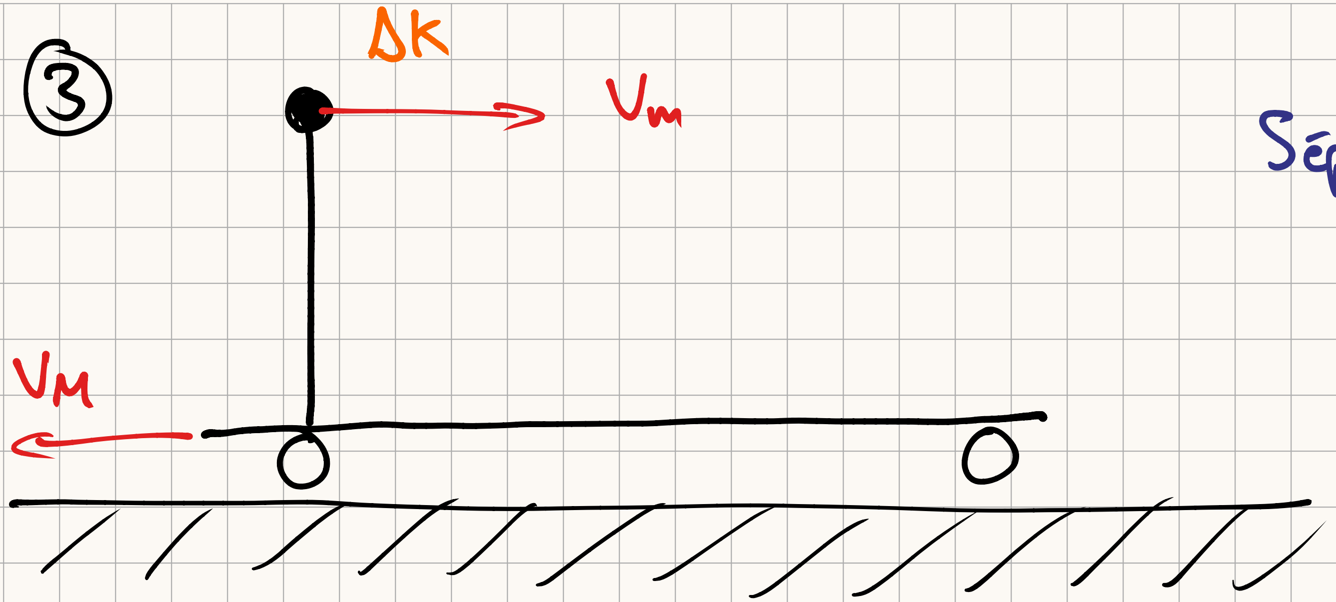
↳ Qté de mvt conservée.

Le long de l'axe x : $p_{x,\text{avant}} = p_{x,\text{après}}$

$$m \cdot V_0 = (m+M) \cdot V_1 \Rightarrow V_1 = \frac{m}{m+M} \cdot V_0$$

$$V_1 = \frac{m}{m+M} V_0 = \frac{m}{m+M} \cdot \sqrt{\frac{2\Delta K}{m}} = \frac{\sqrt{2\Delta K \cdot m}}{m+M}$$

③



Séparation → choc élastique

→ $\Delta E_{mec} = 0$

2 propriétés : ① $\Delta p_x = 0$

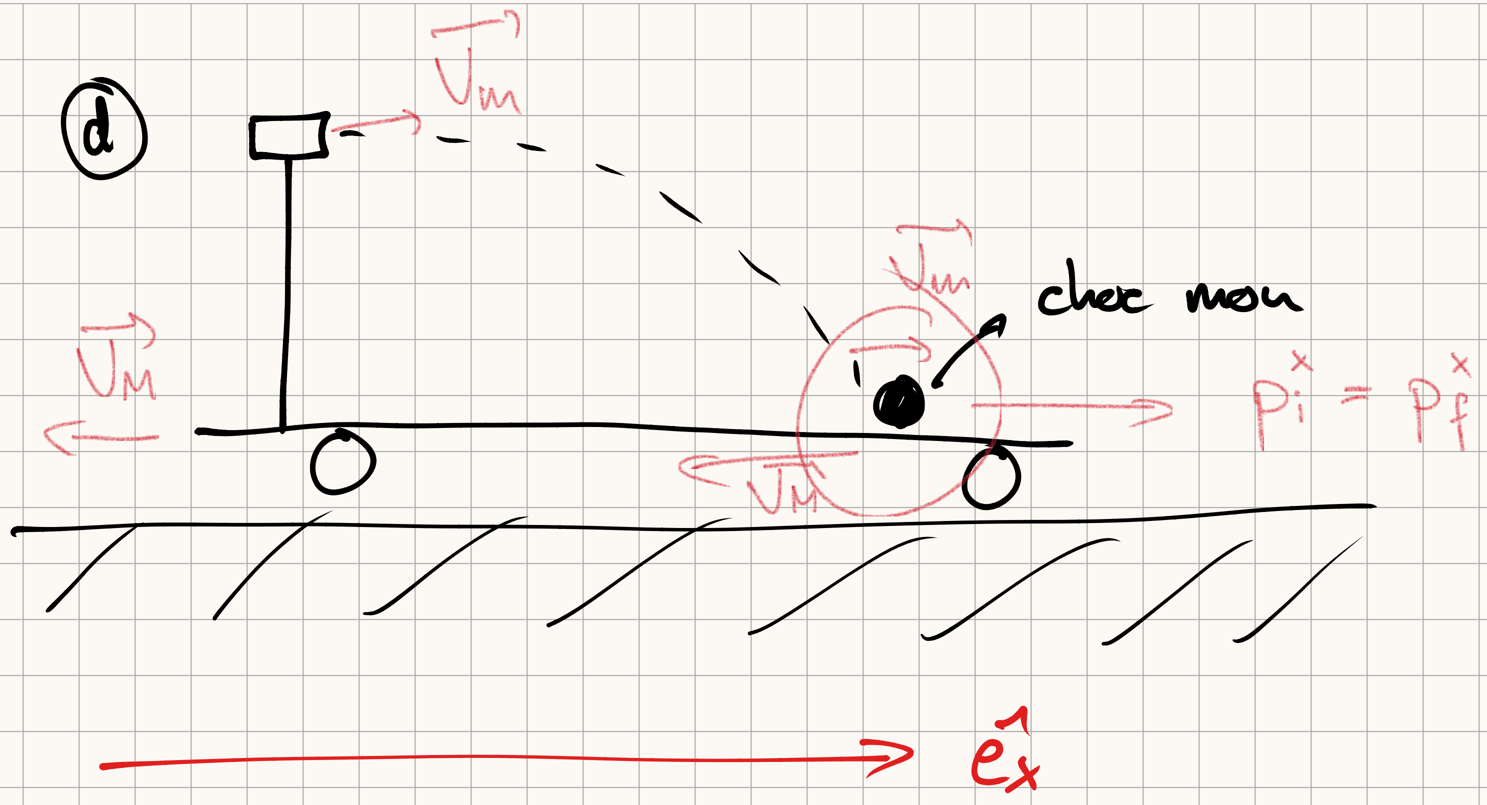
② $\Delta E_{mec} = 0 \rightarrow \Delta E_{cin} = 0$

① $p_i = \vec{0}$, $\vec{p_f} = m \cdot \vec{v_m} + M \cdot \vec{v_M} \Rightarrow m\vec{v_m} = -M\vec{v_M} \Rightarrow \boxed{\vec{v_m} = -\frac{M}{m}\vec{v_M}}$

② $E_{cin,i} = \Delta K$, $E_{cin,f} = \frac{1}{2} m v_m^2 + \frac{1}{2} M v_M^2 = \frac{1}{2}\cancel{m}\frac{M^2}{m^{\cancel{2}}} v_M^2 + \frac{1}{2} M v_M^2$

$\Rightarrow \Delta K = \frac{1}{2} v_M^2 \left(\frac{M^2}{m} + M\right) = \frac{1}{2m}\left(M(M+m)\right) v_M^2 \Rightarrow \boxed{v_M = \sqrt{\frac{2m\Delta K}{M(m+M)}}}$

d

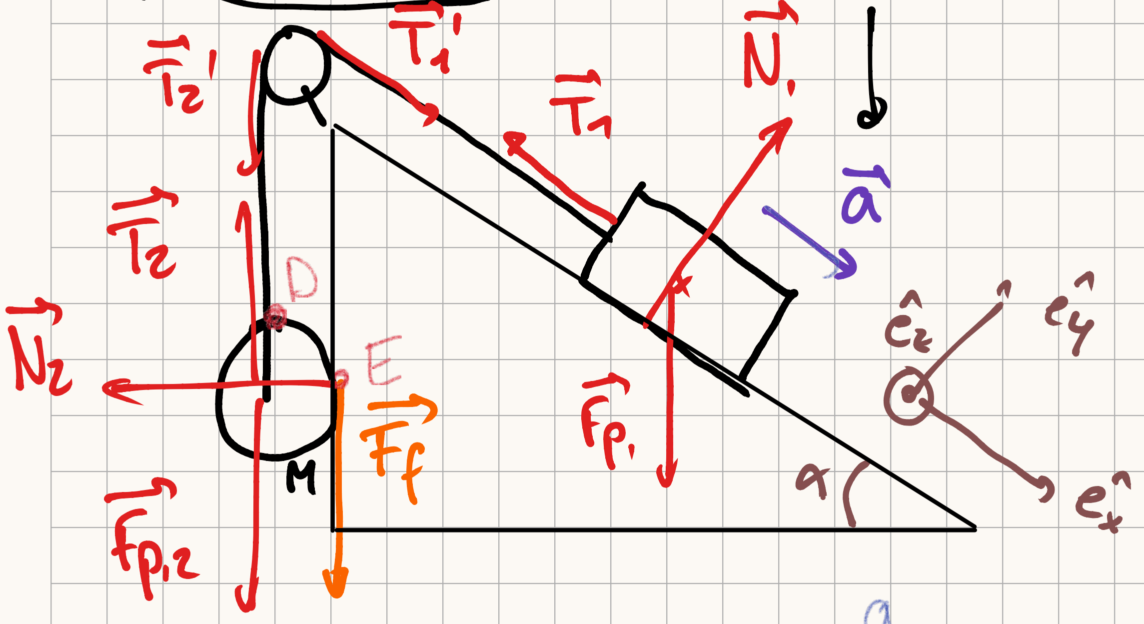


Pas de force externe selon $\hat{e_x}$ $\Leftrightarrow$ p_x est toujours conservée

$\Rightarrow$ on compare p_x Ⓐ avant l'éjection Ⓑ après le choc mou.

$$p_{x,A} = 0, \quad p_{x,B} = (m+M) \cdot v_f \Rightarrow \boxed{v_f = 0}$$

Exercice 2



BLOC $\sum \vec{F} = m\vec{a}$

$\vec{a} = (a, 0)$

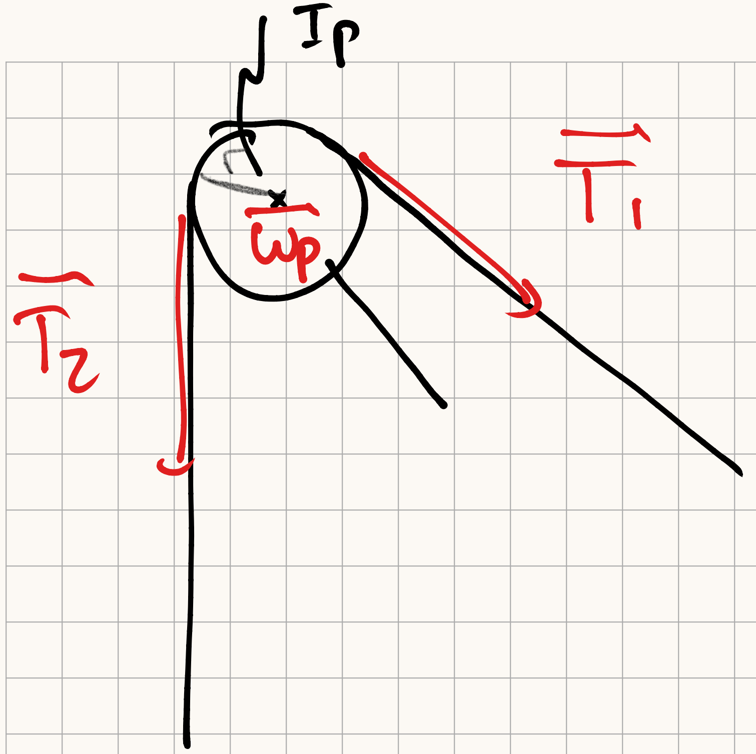
$$\Rightarrow \begin{cases} \boxed{mg\sin\alpha - T_1 = ma} & (1) \\ -mg\cos\alpha + N_1 = 0 \end{cases}$$

ROUE translation : $\boxed{M \cdot a_r = \sum F^y = -Mg + T_2 - F_f}$ (2)

Pour m'assurer que la roue tourne

rotation : $\frac{dL_c}{dt} = I_r \cdot \dot{\omega} = \sum_i \vec{CP_i} \wedge \vec{F_i} = \underbrace{\vec{M_c}(\vec{N_2})}_{=0} + \underbrace{\vec{M_c}(\vec{T_2})}_{=0} + \cancel{\vec{M_c}(\vec{F_{p_c}})}^{=0} + \underbrace{\vec{M_c}(\vec{F_f})}_{(\hat{e_n}) \wedge F_f \cdot (-\hat{e_v})}$

$\Rightarrow \boxed{I_r \cdot \dot{\omega}_r \, \hat{e_z} = -R \cdot F_f \, \hat{e_z}}$ (3)



POULIE

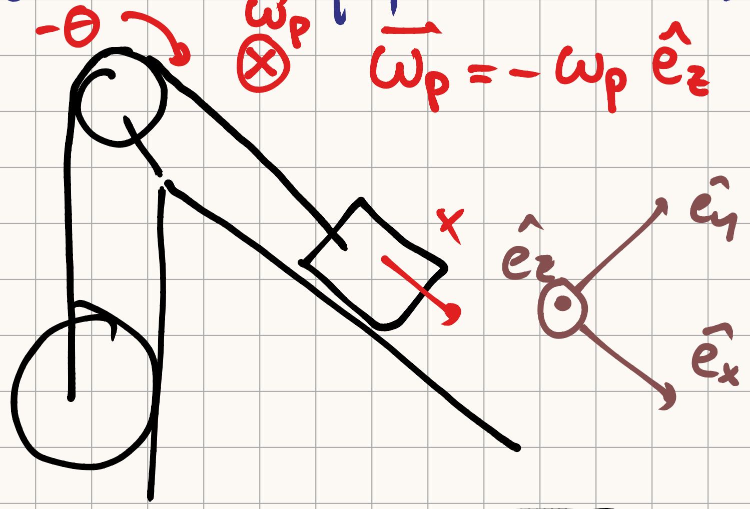
$$\frac{d\vec{L_p}}{dt} = I\dot{\vec{\omega}}_p = \sum \vec{M_p}$$

$$\Rightarrow I_p \cdot \dot{\omega}_p \hat{e}_z = -rT_1 \hat{e}_z + rT_2 \hat{e}_z$$

$$\Rightarrow \boxed{I_p \dot{\omega}_p = r(T_2 - T_1)} \quad (4)$$

On a 4 eqts, 6 inconnues $(a, T_1, T_2, \dot{\omega}_p, \dot{\omega}_r, F_f)$

$\vec{\omega_p} = -\omega_p \hat{e}_z$

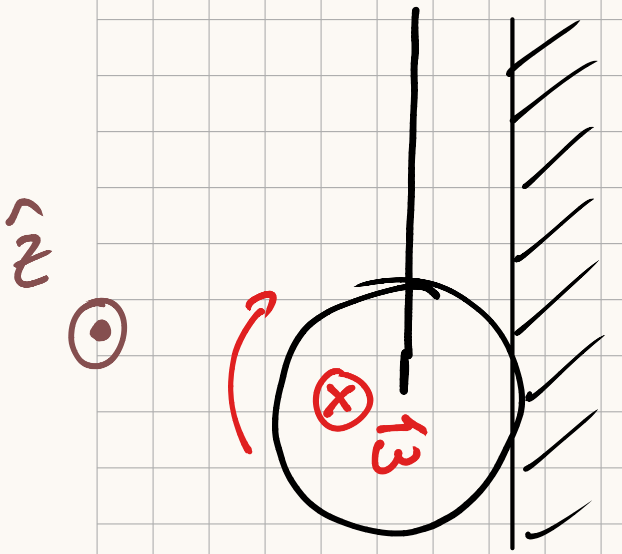


Si le bloc se déplace d'une distance x, ma poulie aura tourné d'un angle θ, donc la distance parcourue le long de la poulie vaudra $d = \theta \cdot r \Rightarrow V_x = \omega_p \cdot r$

$$\boxed{a_x = \dot{\omega} \cdot r} \quad (5)$$

$\Rightarrow$ Vectoriellement, $\boxed{a = -\dot{\omega} \cdot r}$

2e condition de liaison



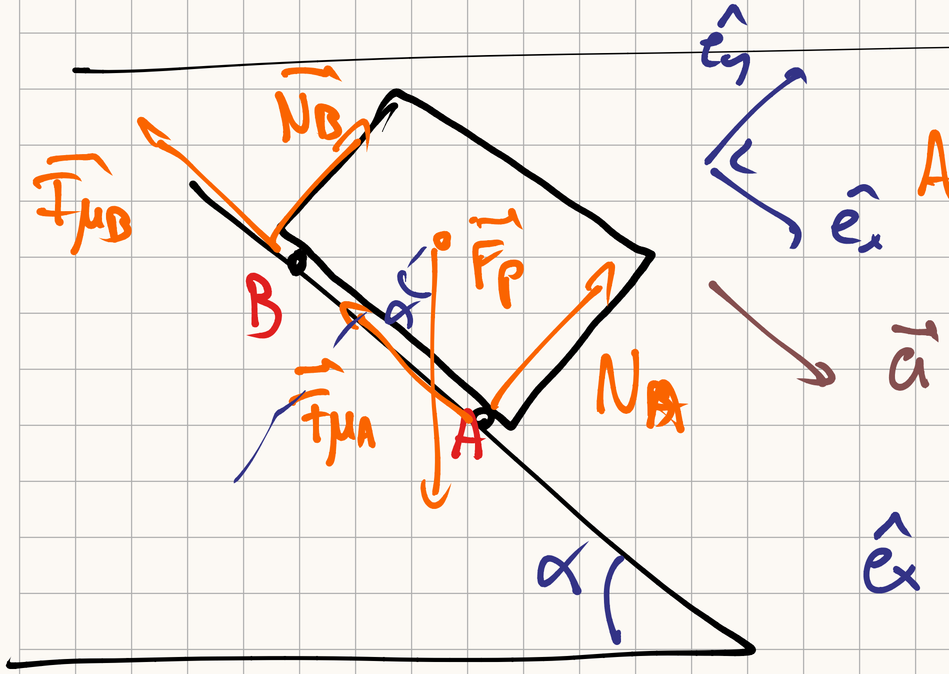
$\vec{\omega}_r = -\omega_r \hat{e}_z$

Avec le même raisonnement,

$$\boxed{-R\dot{\omega}_r = a} \quad (6)$$

quand bloc fait une distance x,
la poulie/roue tourne de $-\theta$

$\Rightarrow$ 6 équations & 6 inconnues. On résoud...



Accélération $\Rightarrow \sum \vec{F} = m\vec{a}$

$$\Rightarrow \vec{N_A} + \vec{N_B} + \vec{F_p} + \vec{F_{\mu A}} + \vec{F_{\mu B}} = m \cdot \vec{a}$$

$$\begin{cases} \hat{e}_x : & mg\sin\alpha - F_{\mu A} - F_{\mu B} = m \cdot a_x \\ \hat{e}_y : & -mg\cos\alpha + N_A + N_B = 0 \end{cases}$$

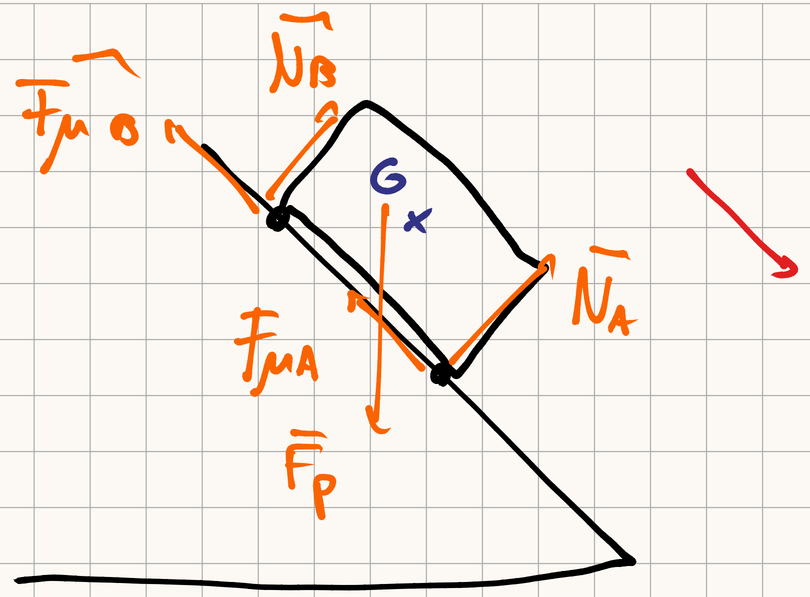
$N_A + N_B = mg\cos\alpha$

Or, $F_{\mu A} = N_A \cdot \mu_c$

$F_{\mu B} = N_B \cdot \mu_c$

$$\Rightarrow e_x : mg\sin\alpha - N_A\mu_c - N_B\mu_c = mg\sin\alpha - \mu_c(N_A + N_B) = ma_x$$

$$\Rightarrow \cancel{m}g\sin\alpha - \mu_c \cancel{m}g\cos\alpha = \cancel{m} \cdot a_x \Rightarrow \boxed{a_x = g(\sin\alpha - \mu_c\cos\alpha)}$$



$\vec{a} = a\,\hat{e}_x$

Que vaut N_B, pour que le bloc ne tourne pas.

Que vaut N_B, pour que $\vec{L}_G = \overrightarrow{cste} = \vec{0}$

$\Rightarrow \sum \vec{M}_G(\vec{F}) = \vec{0}$. Ainsi, $\dfrac{d\vec{L}}{dt} = \vec{0}$, et si $\omega(t=0) = 0$, alors $\omega(t) = 0$.