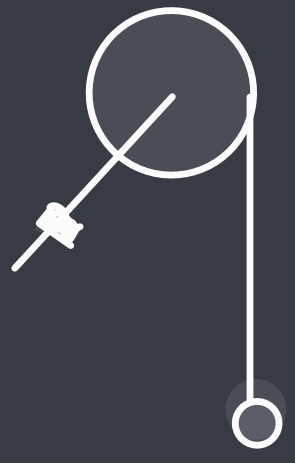
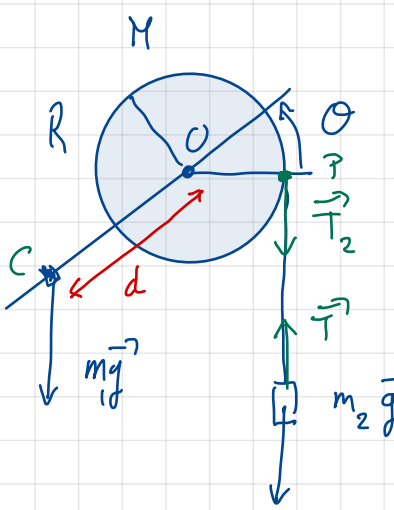
Exercice en classe



Newton & Lagrange

Ph. Müllhaupt



$\vec{T}_2 = -\vec{T}$

$\vec{T} = T\,\hat{y}$

Avec Newton : liaison $y = R\theta - y_0$

$\dot{y} = R\dot{\theta}$

Translation $\vec{y}$ $\quad m_2\ddot{y} = T - m_2 g$

Rotation $(\hat{z})$ $\quad \frac{d}{dt}\vec{L_O} = \vec{M_O} + \vec{M_g}$

$\left(\frac{1}{2}MR^2 + md^2\right)\ddot{\theta} = \sum \vec{M}^{ext} = M_O + M_g$

$[N.m] \quad = -TR + m_1 g d\cos\theta$

$\vec{M_g} = \vec{OC} \wedge m_1\vec{g}$

$\vec{M_O} = \vec{OP} \wedge \vec{T}$

$= -TR$

⚠ éliminer T / exercice

Lagrange

$E_{cin} = T = \frac{1}{2} m_2 \dot{y}^2 + \frac{1}{2}\,\frac{1}{2} MR^2\dot{\theta}^2 + \frac{1}{2} m_1 d^2 \dot{\theta}^2$

liaison $\dot{y} = R\dot{\theta}$ $\quad \frac{1}{2} m_2 \dot{y}^2 = \frac{1}{2} m_2 R^2 \dot{\theta}^2$

1 d° de liberté $\theta = q_1$, $n = 1$

$V = m_2 g (R\theta - y_0) - m_1 g d \sin\theta$

$\mathcal{L} = T - V$

$= \frac{1}{2}\left(m_2 R^2\dot{\theta}^2 + \frac{1}{2}MR^2\dot{\theta}^2 + m_1 d^2\dot{\theta}^2\right) - m_2 g R\theta - m_2 g y_0 + m_1 g d \sin\theta$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \left(m_2 R^2 + \frac{1}{2} m R^2 + m_1 d^2\right) \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = \left(m_2 R^2 + \frac{1}{2} m R^2 + m_1 d^2\right) \ddot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -m_2 g R + m_1 g d \cos\theta$$

$$\left(m_2 R^2 + \frac{1}{2} m R^2 + m_1 d^2\right) \ddot{\theta} = \frac{\partial \mathcal{L}}{\partial \theta}$$

$$= -m_2 g R + m_1 g d \cos\theta$$

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