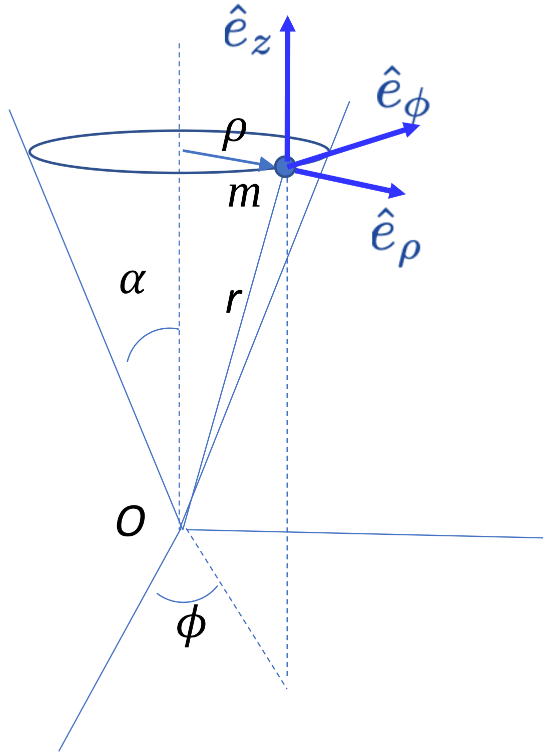
Coordonnées cylindriques: $\vec{r} = \rho\hat{e}_\rho + z\hat{e}_z$

$$\dot{\vec{r}} = \dot{\rho}\hat{e}_\rho + \rho\dot{\phi}\hat{e}_\phi + \dot{z}\hat{e}_z$$

$$\ddot{\vec{r}} = (\ddot{\rho} - \rho\dot{\phi}^2)\,\hat{e}_\rho + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{e}_\phi + \ddot{z}\hat{e}_z$$




Forces: pesanteur $mg\hat{e}_z$ et reaction $\vec{N} = -N\sin\alpha\,\hat{e}_\rho + N\cos\alpha\,\hat{e}_z$

Equations du movement: $m(\ddot{\rho} - \rho\dot{\phi}^2) = -N\sin\alpha$

$$m(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}) = 0$$

$$m\ddot{z} = N\cos\alpha - mg$$

$$\vec{L}_O = \vec{r} \wedge m\vec{v} = m(\rho\hat{e}_\rho + z\hat{e}_z) \wedge (\dot{\rho}\hat{e}_\rho + \rho\dot{\phi}\hat{e}_\phi + \dot{z}\hat{e}_z) = m(\rho^2\dot{\phi}\hat{e}_z - \rho\dot{z}\hat{e}_\phi + z\dot{\rho}\hat{e}_\phi - z\rho\dot{\phi}\hat{e}_\rho)$$

$$L_z = \vec{L}_O \cdot \hat{e}_z = m\left(\rho^2\dot{\phi}\hat{e}_z - \rho\dot{z}\hat{e}_\phi + z\dot{\rho}\hat{e}_\phi - z\rho\dot{\phi}\hat{e}_\rho\right) \cdot \hat{e}_z = m\rho^2\dot{\phi} = m\dot{\phi}(r\sin\alpha)^2$$

$$\frac{dL_z}{dt} = 2m\rho\dot{\rho}\dot{\phi} + m\rho^2\ddot{\phi} = m\rho\left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right) = 0 \quad \text{(voir deuxieme equation du mouvement)}$$

$$E = \frac{1}{2}mv^2 + mgz = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2\right) + mgz = \frac{1}{2}m\left((\dot{r}\sin\alpha)^2 + (\dot{r}\cos\alpha)^2 + (r\sin\alpha)^2\dot{\phi}^2\right) + mgr\cos\alpha$$

$$= \frac{1}{2}m\left(\dot{r}^2 + (r\sin\alpha)^2\dot{\phi}^2\right) + mgr\cos\alpha$$