Exercise 1 - Neural Membranes and Electrodes

Exercise 1. Nernst potential

The Nernst equilibrium potential of bicarbonate (HCO3-) is -12.55mV. This means that the concentration of that molecule is _______ outside the cellular membrane compared to inside.

A) Higher
B) Lower
C) Equal

Answer: A) Higher

Exercise 2. Membrane phenomena

a. What is the Nernst equation? Give the names of all the quantities that are present and their respective units. What phenomena does it explain?

b. A special single-cell organism that lives in a natural mineral water spring has permeabilities of 0.09, 1.00 and 0.04 for Cl^-, K^+ and Na^+, respectively. The following concentrations (in $mmol/mm^3$):$[Cl^-]_i$=178, $[Cl^-]_o$=0.47, $[K^+]_i$=135, $[K^+]_o$=83, $[Na^+]_i$=0.05, $[Na^+]_o$=118.
Which direction (inwards/outwards of the cell membrane) do the Cl^- ions move passively (not actively, i.e., via pumps)? T = 37 °C.

Solution to Exercise 2. Membrane phenomena

a. Nernst Equation: This equation defines the relation between the concentrations of an ion on either side of a membrane that is perfectly selective for that ion and the potential difference (voltage, E) that will be measured across that membrane under equilibrium conditions.

$$E_{ion} = \frac{RT}{zF}.\ ln\frac{[ion]_{out}}{[ion]_{in}}$$

[ ] = ionic concentration (inside or outside of the cell)

R = ideal gas constant = 8.314 J/K°/Mole

T = absolute temperature in K (37°C = 310K)

z = ion valence e.g. (Cl^- = 1, Ca^{2+} = +2, K^+ = +1 etc…)

F = Faraday's constant 96'500 C/Mole

b. Given: Permeabilities(p) Cl^- = 0.09, K^+ = 1.00 and Na^+ = 0.04

Ionic concentrations $[Cl^-]_i$=178, $[Cl^-]_o$=0.47, $[K^+]_i$=135, $[K^+]_o$=83, $[Na^+]_i$=0.05, $[Na^+]_o$=118; all in mmol/mm^3

T = 37 °C = 310 K

Gas constant R = 8.314 J/mol*K, Faraday constant: F = 9.649x104 C/mol]

Derived from Nernst equation and electrical model for neuronal membrane

$$V_m = \frac{g_{Na}E_{Na}+g_K E_K+g_{Cl}E_{Cl}}{g_{Na}+g_K+g_{Cl}}$$

We obtain Goldman-Hodgkin-Katz equation as:

$$V_m = \frac{RT}{F}\frac{p_K[K^+]_o+p_{Na}[Na^+]_o+p_{Cl}[Cl^-]_i}{p_K[K^+]_i+p_{Na}[Na^+]_i+p_{Cl}[Cl^-]_o}$$

Entering corresponding values, we obtain V_m = - 7.044 mV

And from Nernst equation, we obtain equilibrium potential of Cl^- ions E_{Cl-} = 158.59 mV

Thus, the electrochemical driving force acting on Cl^- would be $\Delta V_{Cl-} = V_m - E_{Cl-}$ = -165.63 mV

Thus, the driving force acting on Cl^- leads to Cl^- efflux from the cell.

Exercise 3. Equivalent model of an electrode

Consider three electrode systems: A, B, and C.

- System A has an ideally polarizable electrode with no charge transfer, and its impedance components at 1 kHz are as follows: Rtrack = 8’000 Ω, Rspread = 1’000 Ω and |Z_Ci| = 2’000 Ω
- For system B, the surface area of the electrode is increased by a factor of 10 compared to system A through roughening.
- For system C, instead of roughening the electrodes, the track width of system A is increased by a factor of x.

a) Determine x such that the impedance at 1 kHz of systems B and C are equal.

b) Can we say that systems B and C are equivalent when used for stimulation? Justify your answer.

Solution to Exercise 3. Equivalent model of an electrode

Part a)

For ideally polarizable electrodes the charge transfer resistance $R_{CT} \to \infty$, thus the equivalent circuit model consists of a series connection of R_{track}, C_i and R_{spread}.
The total impedance of this circuit is

$$Z = Z_{track} + Z_{C_i} + Z_{spread}.$$

The impedance modulus of the circuit is

$$|Z| = \sqrt{Re(Z)^2 + Im(Z)^2},$$

where $Re(Z)$ refers to the real part of Z and $Im(Z)$ to the imaginary part.
Since the track impedance Z_{track} and spreading impedance Z_{spread} only have a real part (purely resistive) and the impedance due to the capacitor Z_{C_i} has only an imaginary part, for the three systems A, B, and C:

$$Z = R_{track} + \frac{1}{jwC_i} + R_{spread},$$

$$|Z| = \sqrt{\left(R_{track} + R_{spread}\right)^2 + \left(\frac{1}{j\omega C_i}\right)^2}.$$

System A:

$$Re\left(Z_{System\ A}\right) = \left|Z_{System\ A,track}\right| + \left|Z_{System\ A,spread}\right| = 8000\ \Omega\ +\ 1000\ \Omega = 9000\ \Omega$$

$$Im\left(Z_{System\ A}\right) = \left|Z_{System\ A,C_i}\right| =\ 2000\ \Omega$$

$$\left|Z_{System\ A}\right| = \sqrt{Re\left(Z_{System\ A}\right)^2 + Im\left(Z_{System\ A}\right)^2} =\ \sqrt{9000^2 + 2000^2} = 9219.5\ \Omega$$

For system B, increasing the electrochemical surface area of the electrode by a factor 10 compared to system A is equivalent to increasing the interface capacitance by a factor 10. A tenfold increase in capacitance yields a tenfold reduction in impedance, therefore

$$Z_{System\ B,C_i} = \frac{Z_{System\ A,C_i}}{10}.$$

Moreover, the spreading resistance R_{spread} is not affected by surface roughening, because the geometrical surface area remains the same.

$$Re\left(Z_{System\ B}\right) = \left|Z_{System\ B,track}\right| + \left|Z_{System\ B,spread}\right| = 8000\ \Omega\ +\ 1000\ \Omega = 9000\ \Omega$$

$$Im\left(Z_{System\ B}\right) = \left|Z_{System\ B,C_i}\right| =\ 200\ \Omega$$

For system C, increasing the track width w by a factor x compared to system A yields

$$Z_{System\ C,track} = \frac{Z_{System\ A,track}}{x},$$

since $R_{track} = R_S \frac{L}{w}$.

To have $|Z_{System\ B}| = |Z_{System\ C}|$:

$$|Z_{System\ B}| = \sqrt{Re(Z_{System\ C})^2 + Im(Z_{System\ C})^2}$$

$$|Z_{System\ B}|^2 = Re(Z_{System\ C})^2 + Im(Z_{System\ C})^2$$

$$|Z_{System\ B}|^2 = \left(\frac{|Z_{System\ A,track}|}{x} + |Z_{System\ C,spread}|\right)^2 + |Z_{System\ C,C_i}|^2$$

$$\sqrt{|Z_{System\ B}|^2 - |Z_{System\ C,C_i}|^2} = \frac{|Z_{SystemA,track}|}{x} + |Z_{System\ C,spread}|$$

$$x = \frac{|Z_{System\ A,track}|}{\sqrt{|Z_{System\ B}|^2 - |Z_{System\ C,C_i}|^2} - |Z_{System\ C,spread}|}$$

$$x = \frac{8000}{\sqrt{(9002.2)^2 - (2000)^2} - 1000} = 1.028$$

The track width in system C needs to be increased by a factor 1.028.

Part b)

Even if systems B and C can be tuned to have the same impedance magnitude at 1 kHz, they are not equivalent for stimulation.

- System B: roughening increases the effective surface area with a corresponding decrease of the capacitive impedance Z_{Ci}, lowering current density and increasing the charge capacity. In EIS, increasing the electrochemical electrode area appears as a decrease of the low-frequency impedance.

- System C: a wider track only decreases the series resistance of the lead and does not change the current density at the electrode-tissue interface. In EIS, the high-frequency intercept shifts slightly lower, but the low-frequency capacitive region stays the same as system A.

Looking at the voltage transient, the difference is also clear: in system B the access resistance drop (Va) is unchanged but the polarization term (Ep) is reduced thanks to the larger capacitance, leading to a smaller voltage excursion. In system C, Va is slightly lower due to the wider track, while Ep remains the same as in system A. Thus, the voltage transients in B and C behave very differently even if the impedance magnitudes at 1 kHz can match.

In short:

If the main issue of an electrode stimulation is voltage drop along leads or heating, widening the track (C) is beneficial.
If the limitation is charge density, electrode polarization, or electrochemical safety, increasing effective surface area (B) is the right solution.

Exercise 4. Electrode characterization

Which of the following statements correctly describes the Cathodic Charge Storage Capacity (CSC) of an electrode?

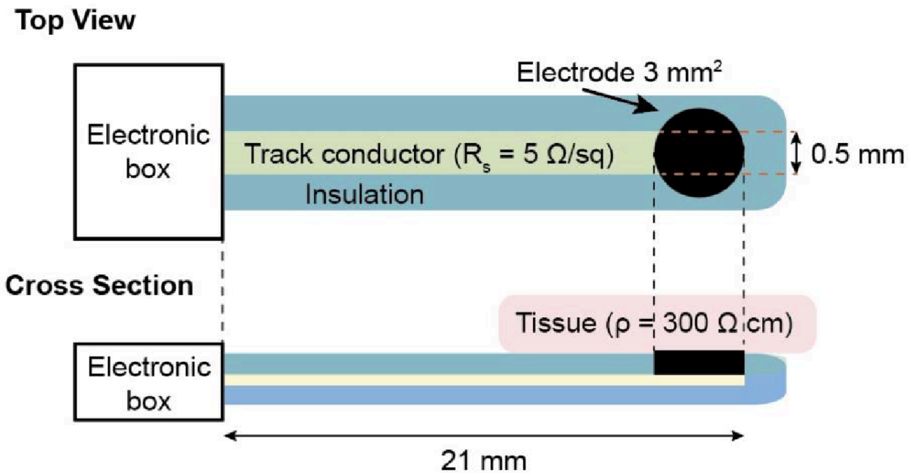
A) CSC refers to the maximum voltage an electrode can sustain without damage.
B) CSC measures the resistance of an electrode to electrical currents.
C) CSC represents the maximum charge an electrode can store during cathodic polarization.
D) CSC indicates the speed at which an electrode can discharge stored energy.

Solution To Exercise 4 Electrode characterization

By definition: CSC represents the maximum charge an electrode can store during cathodic polarization. **Answer C)**.

Exercise 5. Electrical stimulation waveforms

Consider the electrode system illustrated below. The electrode is coated such that it is ideally polarizable (i.e., there is no charge transfer across the electrode-tissue interface), and you would like to use this electrode to deliver charge over an area A = 3 mm^2. You choose a cathodic-first, biphasic, symmetric stimulation waveform that delivers a charge density q = 25 μC.cm^{-2}, with a pulse-width PW = 0.6 ms and inter-phase delay of 0.01 ms.



Draw qualitative overlaid diagrams of the current and voltage waveforms during brain stimulation. Use the quantities illustrated in the figure above to provide an approximate estimation of the access voltage Va in the voltage transient curve. Ignore the concentration overpotential.

Solution To Exercise 5. Electrical stimulation waveforms

The waveform is cathodic-first, biphasic, symmetric pulse with pulse width (PW) = 0.6 ms and t_{delay}=0.01 ms.

$$\text{Charge } Q = q.A = 25\ \mu Ccm^{-2} * 0.03\ cm^2 = 0.75\ \mu C$$

$$Current\ Amplitude\ (I) = \frac{Q}{PW} = \frac{0.75\ \mu C}{0.6\ ms} = 1.25\ mA$$

$$Va = I \bullet R_{series} + \eta_c$$

Ignoring η_c:

$$R_{series} = R_{track} + R_{spread}$$

$$R_{track} = R_{sheet} \bullet \frac{L}{W} = 5 \bullet \frac{21\,mm}{0.5\,mm} = 210\,\Omega$$

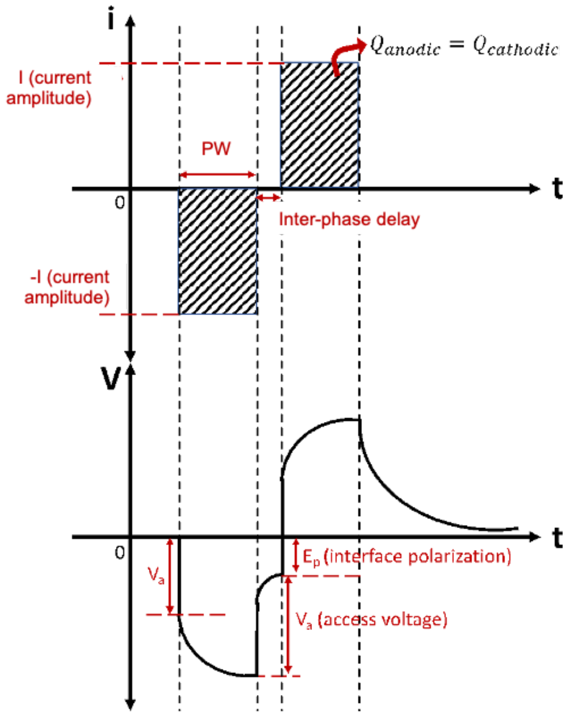
$$R_{spread} = \frac{\rho}{4\cdot r} = \frac{\rho}{4\cdot\sqrt{\frac{A}{\pi}}} = \frac{300\,\Omega\cdot cm}{4\cdot\sqrt{\frac{0.03\,cm^2}{\pi}}}$$

$$R_{spread} = \frac{300\,\cdot cm}{4\cdot 0.098\,cm} = 765\,\Omega$$

$$R_{series} = R_{track} + R_{spread} \cong 975\,\Omega$$

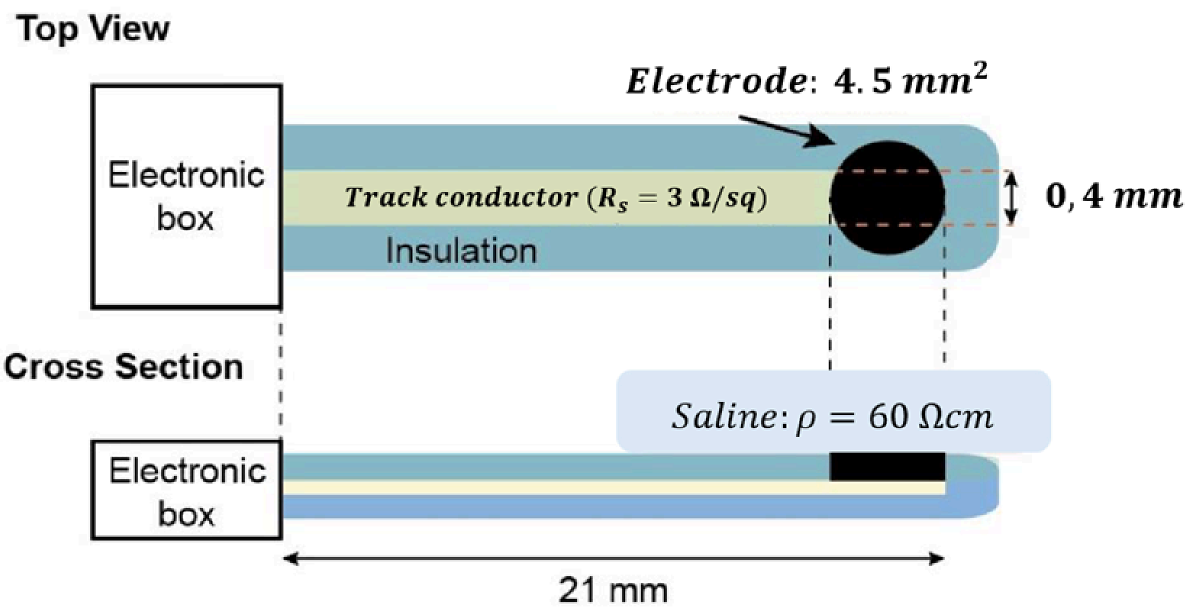
$$V_a = I \bullet R_{series} + \eta_c \qquad (ignore\ c)$$

$$V_a = 1.25\,mA\ *\ 975\,\Omega$$
$$V_a = 1.219\,V$$



Exercise 6. Electrochemical impedance

Electrochemical impedance spectroscopy is an important characterization technique that enables the electrical behavior of an electrode to be measured and modeled. Consider the electrode system illustrated below; it is immersed in a saline solution (**ρ = 60 Ωcm**). The electrode coating determines a capacitance of **10 μF/cm²** and it is electrochemically inert.



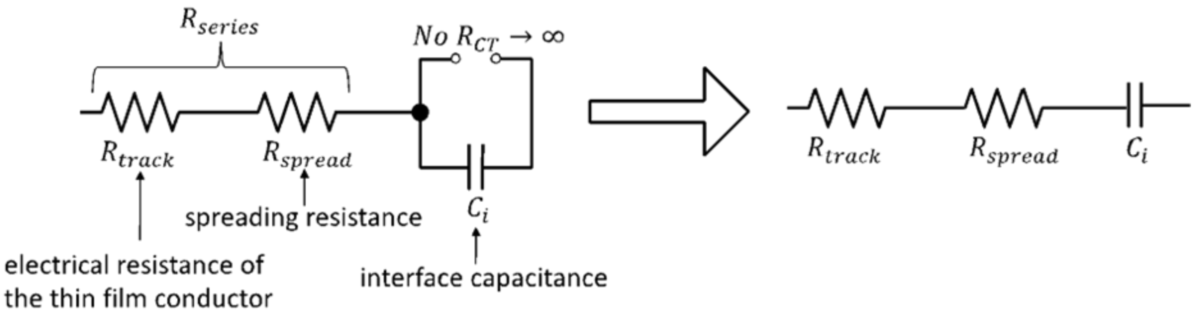
a. Draw an equivalent circuit model of the electrode system based on the Randles cell. Identify the electrical components and their physical meaning.

b. You would like to decrease the electrode impedance by applying a machining process that can roughen the electrode surface and increase its surface area by a factor of **15**.

1. Quantify the resulting impedance modulus of the whole system at **1 MHz** and at **1 Hz**.
2. Plot the new impedance spectrum after the roughening process and compare it to the previous spectrum.

Solution To Exercise 6 – Electrochemical impedance.

a.Randels model



b1. Calculation of the impedances

$$Z = R_{track} + R_{Spread} + Z_C$$

$$R_{track} = R_{sheet} \cdot \frac{L}{W} = 3\,\Omega \cdot \frac{21\,mm}{0.4\,mm} = 157.5\,\Omega$$

$$R_{spread} = \frac{\rho}{4 \cdot r} = \frac{\rho}{4 \cdot \sqrt{\frac{A}{\pi}}} = \frac{60\,\Omega \cdot cm}{4 \cdot \sqrt{\frac{0.045\,cm^2}{\pi}}} = 125\,\Omega$$

$$R_{series} = R_{track} + R_{spread} = 282.5\,\Omega$$

For the capacitance at 1 Hz:

$$C = C_i * ESA = 10\,\mu F cm^{-2}\ \text{(Area of your electrode)}$$

$$Z_C = \frac{1}{j\,\omega C} = \frac{1}{j\,2\pi f C} \to |Z_{C_i}| = \left|\frac{1}{j\,2\pi f C}\right| = \frac{1}{2\pi \times 1 \times (10\,\mu F cm^{-2} \times 0.045\,cm^2)} = 354\,k\Omega$$

At 1 Hz:

$$|Z_C @\, 1Hz| = \frac{1}{2\pi(1Hz)C} = \frac{1}{2\pi \times 1\,Hz \times (10\,\mu F cm^{-2} \times 0.045\,cm^2)} \cong 353.86\,k\Omega$$

At 1MHz:

$$|Z_C @\, 1MHz| = \frac{1}{2\pi(10^6 Hz)C} = \frac{1}{2\pi \times 10^6 Hz \times (10\,\mu F cm^{-2} \times 0.045\,cm^2)} \cong 0.35\,\Omega$$

$$|Z_{initial@1Hz}| = \sqrt{Re(Z_{initial@1Hz})^2 + Im(Z_{initial@1Hz})^2} \cong 354\ k\Omega$$

$$|Z_{initial@1MHz}| = \sqrt{Re(Z_{initial@1MHz})^2 + Im(Z_{initial@1MHz})^2} \cong 282.5\ \Omega$$

By changing the ESA by a roughening factor of 15

We know that the capacitance is proportional to the area:

$$C = \frac{\boldsymbol{A}\ \epsilon_0\ \epsilon_r}{d} \rightarrow if\ the\ capacitance\ increases\ by\ a\ factor\ \mathbf{15}\ then$$

$Z_{C_{new}} \rightarrow \frac{1}{j2\pi f(15C)}$ the impedance will decrease by a factor of 15

So we can recalculate the impedance values at 1 Hz and 1 MHz and compare the graphs:

$$|Z_{15C@1Hz}| = \frac{1}{2\pi(1Hz)15C} = \frac{Z_{initial@1Hz}}{15} \cong 23.6\ K\Omega$$

$$|Z_{15C@1MHz}| = \frac{1}{2\pi(10^6Hz)15C} = \frac{Z_{initial@1MHz}}{15} \cong\ 0.02\ \Omega$$

Calculate the total impedance at @ 1 Hz and @ 1MHz:

$$|Z_{final@1Hz}| = \sqrt{Re(Z_{final@1Hz})^2 + Im(Z_{final@1Hz})^2} \cong 23.6\ K\Omega$$

$$|Z_{final@1MHz}| = \sqrt{Re(Z_{final@1MHz})^2 + Im(Z_{final@1MHz})^2} \cong 282.5\ \Omega$$