Solutions to Soft Matter Exercise - Chapter 1: Introduction

1. Viscosity

Water can form hydrogen bonds. Acetone cannot form hydrogen bonds and only undergoes Van-der-Waals interactions.

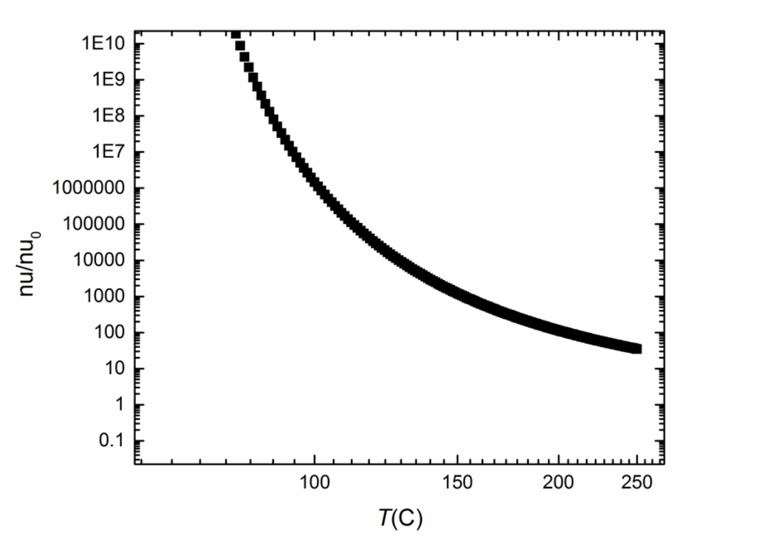
2. Non-Newtonian Fluids

Non-Newtonian fluids are fluids whose viscosity is a function of the shear rate. An example of a fluid that displays shear thickening behavior (where the viscosity increases proportionally to shear rate) is corn starch suspended in water. Examples of fluids that display shear thinning behavior (where viscosity decreases with increasing shear rate) are ketchup, whipped cream, and blood.

A shear-rate dependent change in viscosity can occur if the fluid consists of different components and the interactions between these components (intermolecular or intra-particle forces) change if exposed to shear. This behavior must be taken into account, for example, if these fluids (or melts) are processed through extrusion.

3. Temperature Dependence of the Viscosity of Poly(styrene)

a. Using the Vogel-Fulcher law, $\frac{\eta}{\eta_0} = \exp\left(\frac{B}{T-T_0}\right)$, one finds:



At 80°C:

$$\frac{\eta_{80°C}}{\eta_0} = \exp\left(\frac{710}{353-323}\right) = 1.9 \times 10^{10}$$

At 100°C:

$$\frac{\eta_{100°C}}{\eta_0} = \exp\left(\frac{710}{373-323}\right) = 1.47 \times 10^{6}$$

At 120°C:

$$\frac{\eta_{120°C}}{\eta_0} = \exp\Big(\frac{710}{393-323}\Big) = 2.54 \times 10^4$$

At 140°C:

$$\frac{\eta_{140°C}}{\eta_0} = \exp\Big(\frac{710}{413-323}\Big) = 2.67 \times 10^3$$

Therefore:

$$F_1 = \frac{\eta_{80°C}}{\eta_{100°C}} = 1.3 \times 10^4$$

$$F_2 = \frac{\eta_{120°C}}{\eta_{140°C}} = 9.5$$

At low temperatures, the changes in viscosity are much more pronounced. Poly(styrene) (PS) has a glass transition temperature, T_g, around 100°C. Changes of the viscosity at temperatures below the glass transition temperature, where PS is an undercooled glass are much more pronounced than above T_g, where PS is a liquid. Therefore $F_1 >> F_2$.

b. Factors to consider:

- Glass transition temperature
- Thermal decomposition temperature
- Heating/cooling rate
- Temperature-dependent viscosity
- Feature size
- Production costs

The processing temperature depends on the shape of the part to be processed. Casting very fine structures requires a low viscosity of poly(styrene) so the processing temperature should be significantly above the glass transition temperature, T_g. If the structure to be cast is simple, the processing temperature can be lower.

In general, the processing temperature is a trade-off between minimizing processing costs (and therefore processing temperatures) and reaching a melt viscosity suitable for the process.

4. Temperature Dependence of the Viscosity for Glass-Forming Liquids

a. The glass transition temperature is not a material property because it depends on the rate of heating or cooling. If the temperature-dependent viscosity follows the Volgel-Furcher law, it can be described as:

$$\frac{\eta}{\eta_0} = \exp\Big(\frac{B}{T-T_0}\Big)$$

The viscosity is related to the shear modulus, G_0, and the shear rate, τ, which is related to the experimental time scale as:

$$\eta = G_0\tau$$

With G_0 being a constant, we find:

$$\frac{\tau}{\tau_0} = \exp\Big(\frac{B}{T - T_0}\Big)$$

To determine the temperature-dependent viscosity, we must calculate τ_0:

$$\tau_0 = \tau \exp\Big(-\frac{B}{T - T_0}\Big) = 1000 \text{ s} \times \exp\Big(-\frac{710}{(101.4+273) - (50+273)}\Big) = 0.001 \text{ s}$$

At T$_g$: $\tau_{exp} = \tau_{conf}$, using

$$T = \frac{B}{\ln(\frac{\tau}{\tau_0})} + T_0$$

We find

for $\tau_{conf} = 10$ s, $T_g = 127°$C
for $\tau_{conf} = 100$ s, $T_g = 112°$C
for $\tau_{conf} = 10^6$ s, $T_g = 84°$C

Hence, T_g decreases with increasing τ_{conf}.

b. For $T_g = T_0 + 50°\text{C} = 100°\text{C}$:

$$\tau = \tau_0 \exp\Big(\frac{B}{T - T_0}\Big) = 0.001 \text{ s} \times \exp\Big(\frac{710\ K}{50\ K}\Big) = 1472 \text{ s} = 24.5 \text{ min}$$

For $T_g = T_0 + 10°\text{C} = 60°\text{C}$:

$$\tau = \tau_0 \exp\Big(\frac{B}{T - T_0}\Big) = 0.001 \text{ s} \times \exp\Big(\frac{710\ K}{10\ K}\Big) = 6.8 \times 10^{27} \text{ s}$$

This is not experimentally feasible.

5. Elasticity

The Young's modulus is determiend as $E = \frac{\sigma}{\epsilon}$. To determine the stress, σ, we use $\sigma = \frac{F}{A}$, where

$$A_{final} = \frac{A_{initial}}{1.5} = \frac{0.4\ mm^2}{1.5} = 0.267\ mm^2$$

hence

$$\sigma = \frac{F}{A} = \frac{10\ N}{0.267 \times 10^{-6}\ m^2} = 37.5 \text{ MPa}$$

and

$$E = \frac{\sigma}{\epsilon} = \frac{3.75 \times 10^7\ Pa}{0.5} = 75 \text{ MPa}$$