Solutions to Soft Matter Exercise - Chapter 2: Liquid Crystals

1. Liquid Crystals

A thermotropic liquid crystal is a pure material and does not contain any solvent. Its phase changes if the temperature is changed.

A lyotropic liquid crystal is composed of molecules dispersed in a solvent. Its phase is a function of both mesogen concentration and temperature.

2. Structure

Short range order means that there is some order in the orientation or spacing of the molecules (or atoms) over a short distance (there is a correlation between the distances of nearest neighbors), but there is no order over longer distances. The order of liquid crystals is higher than that in liquids, which have no long or short range order at all, but is lower than that in crystals, which have short and long range order.

3. Thermodynamics

The free energy decreases because the intermolecular forces increase, which results in a decrease in the enthalpy ($\Delta H = H_{final} - H_{intitial}$). In this case, the intermolecular forces in the liquid state are smaller than those in the nematic state, such that $H_{initial} > H_{final}$, therefore $\Delta H < 0$. This decrease in the enthalpy exceeds the reduction in entropy such that the Gibbs free energy, ΔG, decreases.

4. Quarter-Wave Plate

The definition of a quarter-wave plate is that $\delta = \pi/2 = 90°$.
Thus, we obtain:

$$\delta = \frac{2\pi}{\lambda} \Delta n d = \frac{\pi}{2}$$

Such that:

$$d = \frac{\lambda}{4\Delta n} = \frac{488\ nm}{4 \times 0.22} = 0.55\ \mu\text{m}$$

5. Birefringence

Birefringence is an optical property of a material where the refractive index depends on the polarization of the light. Only materials with refractice indices that are anisotropic can be birefringent. They are birefringent if the refractive indices in the two directions that are perpendicular to the propagation direction of the light are different. For example, if the light is propagating along the z-direction, a material is birefringent if the refractive index in the x-direction is different from that in the y-direction.

a. homeotropically aligned nematic phase: This phase is birefringent if the light propagation direction is not parallel to the director.

b. planar-aligned smectic A phase: This phase is birefringent if the light propagation direction is not parallel to the director.
c. cholesteric phase: This is birefringent because it is made of chiral mesogens which, if aligned, are birefringent.
d. isotropic phase of a discotic material: This is not birefringent because its properties are isotropic.

6. Twisted Nematic Liquid Crystal

a. To determine the switching voltage, U, we use $U = E \times d$ where E is the electric field and d is the distance between the plate. We know:

$$E_{crit} = \frac{2\pi}{d}\sqrt{\frac{\pi}{\Delta\epsilon}}[K_1 + \tfrac{1}{4}(K_3 - 2K_2)]^{\frac{1}{2}}$$

$$= \frac{2\pi}{10\times10^{-6}\ m}\sqrt{\frac{\pi}{0.7\times8.85\times10^{-12}\frac{F}{m}}}[5.3\times10^{-12}\ N + \tfrac{1}{4}(7.45\times10^{-12}\ N - 2\times 2.2\times10^{-12})]^{\frac{1}{2}} = 1.1\times10^{6}\ V$$

and

$$U = E_{crit}d = 11\ V$$

Note that the switching voltage is independent of the sample thickness!

b. To reduce the switching voltage, the elastic constants could be reduced. This could be achieved by reducing the intermolecular forces by weakening the Van-der-Waals interaction forces or by increasing the intermolecular distances.