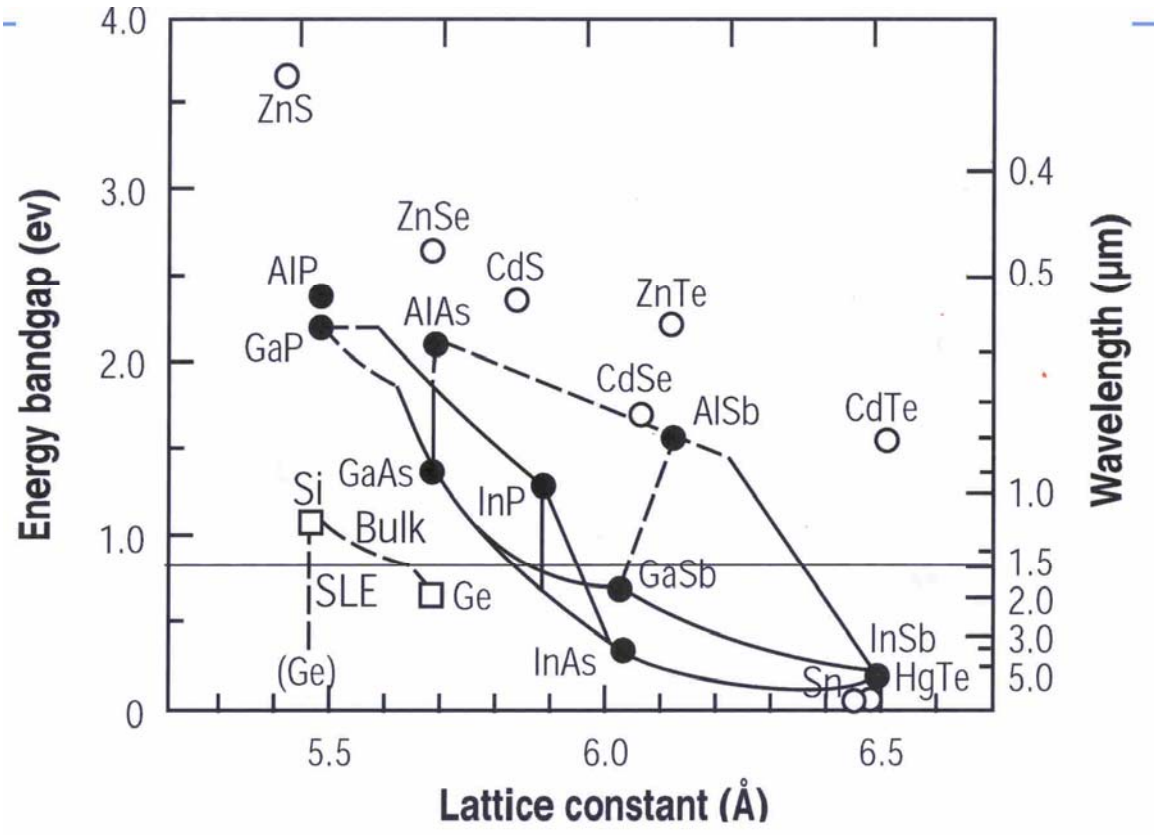
Prof Fink Notes:
Density of charge carriers in semiconductors

Identifying semi-conducting elements and compounds in the periodic table

II	III	IV	V	VI
	B	C	N	O
	Al	Si	P	S
Zn	Ga	Ge	As	Se
Cd	In	Sn	Sb	Te
Hg	Tl	Pb	Bi	Po



A number of ways to classify SC:

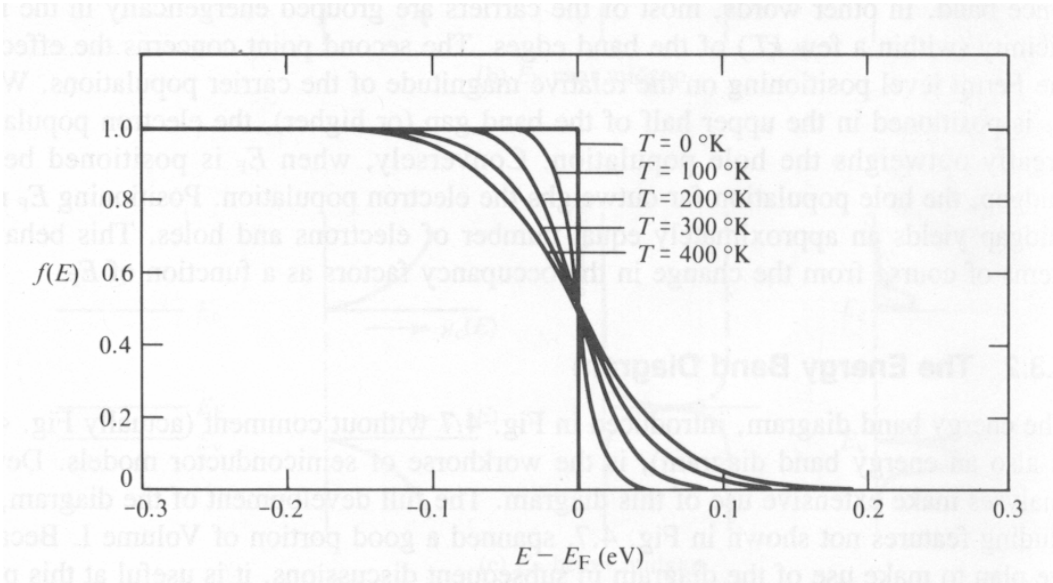
(a) Intrinsic vs. extrinsic
(b) Elemental vs. compound
(c) Direct vs indirect gaps

Plugging in the numbers into the Fermi function $\frac{\varepsilon_c - \mu}{k_B T} = \frac{0.55eV}{0.025eV} = 22$

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/k_B T} + 1} \approx 10^{-10}$$

thus fermi's function tells us that very few electrons make it to the conduction band in order to figure out how many states are actually populated we need to multiply the Fermi function $f(\varepsilon)$ with the density of states $g(\varepsilon)$ to give us the density of occupied states.

The Fermi function tells us that for $\frac{E_c - \mu}{kT} \gg 1$ the number of electrons promoted to the conduction band will be small and will occupy the lowest energy levels in that band.



Review: Band Structure – "parabolic shape"
The simplest band structure for a semiconductor is given by the following diagram:

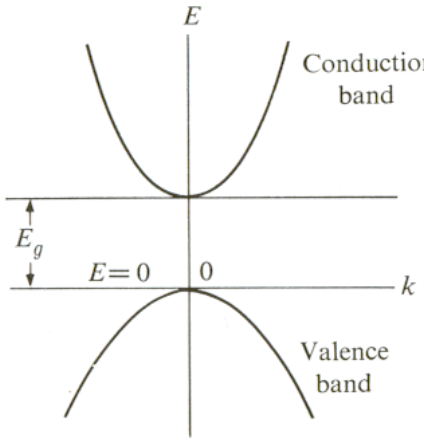


Fig. 6.3 Band structure in a semiconductor

(a) Allowed and forbidden bands
(b) Identification of the gap energy
(c) Curvature of bands – effective mass
(d) DOS
(e) Slope of the bands – group velocity

Near the band edges (extremum) the dispersion curve is given by

$$\varepsilon_c\left(\vec{k}\right) = E_g + \frac{\hbar^2 k^2}{2m_e^*}$$

$$\varepsilon_v\left(\vec{k}\right) = -\frac{\hbar^2 k^2}{2m_h^*}$$

Where E_g is the band gap and the effective masses are m* (given by the curvature of the conduction and valence band)

Holes as charge carriers
An empty state in a band filled with electrons is called a hole. This vacant state behaves in many ways as if it were a charge carrier of positive sign +e and equal mass to that of the missing electron.

Number of carriers at thermal equilibrium

The number of charge carriers per unit volume at a given temperature is the most important property of any semiconductor. The values of these are highly dependent on the number of impurities. We will first consider the relations which hold regardless of whether the material is doped or not.

$$n_c(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon g_c(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/k_B T}+1}$$

$$p_v(T) = \int_{-\infty}^{\varepsilon_v} d\varepsilon g_v(\varepsilon)\left(1-\frac{1}{e^{(\varepsilon-\mu)/k_B T}+1}\right)$$

$$= \int_{-\infty}^{\varepsilon_v} d\varepsilon g_v(\varepsilon)\left(\frac{1}{e^{(\mu-\varepsilon)/k_B T}+1}\right)$$

Where n_c is the number of electrons in the conduction band and p_v is the number of holes in the valence band.The impurities effect the values of n_c and p_v through their effect on the chemical potential.

We will use an approximation to the DOS function which is based on our 3D free electron gas

$$g_c(\varepsilon) = \sqrt{2(\varepsilon-\varepsilon_c)}\frac{m_c^{3/2}}{\pi^2\hbar^3}$$

$$g_v(\varepsilon) = \sqrt{2(\varepsilon_v-\varepsilon)}\frac{m_v^{3/2}}{\pi^2\hbar^3}$$

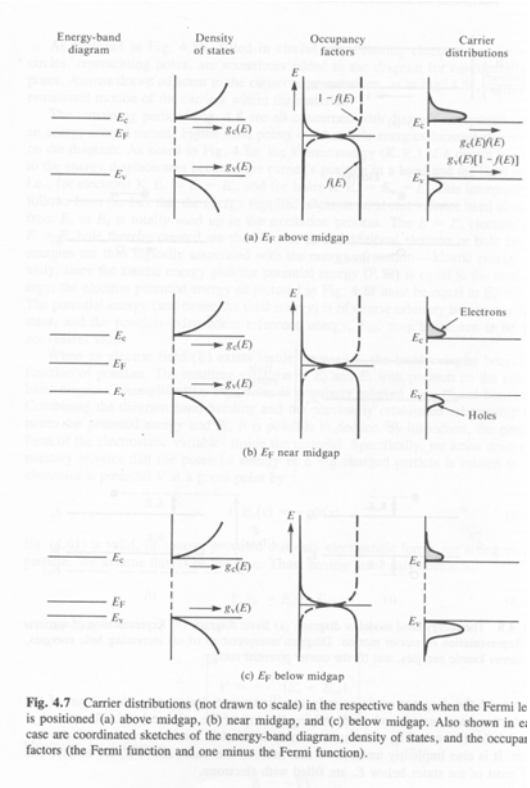


Fig. 4.7 Carrier distributions (not drawn to scale) in the respective bands when the Fermi le is positioned (a) above midgap, (b) near midgap, and (c) below midgap. Also shown in ea case are coordinated sketches of the energy-band diagram, density of states, and the occupa factors (the Fermi function and one minus the Fermi function).

Degenerate and non-degenerate semiconductors

In non-degenerate SC the chemical potential satisfies:

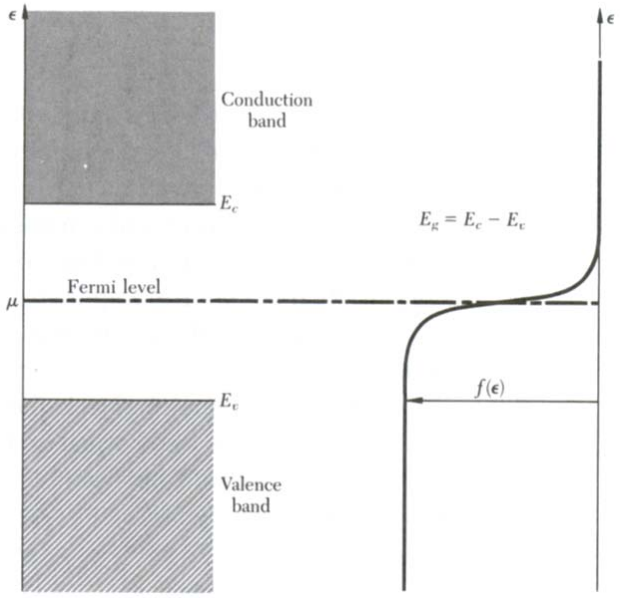
$$\varepsilon_c - \mu >> k_B T$$

$$\mu - \varepsilon_v >> k_B T$$

which leads to the following approximation, note: that if the electron energy is far from the Fermi energy the FD distribution approximates a MB distribution – the electrons behave like a classical gas.

$$\frac{1}{e^{(\varepsilon-\mu)/k_B T}+1} \approx e^{-(\varepsilon-\mu)/k_B T},\ \varepsilon>\varepsilon_c$$

$$\frac{1}{e^{(\mu-\varepsilon)/k_B T}+1} \approx e^{-(\mu-\varepsilon)/k_B T},\ \varepsilon<\varepsilon_v$$



the expressions we derived for the carrier densities are given approximately by:

$$n_c(T) = \underbrace{\int_{\varepsilon_c}^{\infty} d\varepsilon g_c(\varepsilon) e^{-(\varepsilon-\varepsilon_c)/k_B T}}_{N_c(T)} e^{-(\varepsilon_c-\mu)/k_B T}$$

$$p_v(T) = \underbrace{\int_{-\infty}^{\varepsilon_v} d\varepsilon g_v(\varepsilon) e^{-(\varepsilon_v-\varepsilon)/k_B T}}_{P_v(T)} e^{-(\mu-\varepsilon_v)/k_B T}$$

Because of the rapidly decaying function in the integrand only energies that are within $k_B T$ of the band edge will contribute significantly, therefore we can assume a quadratic density of states of the following form:

$$N_c(T) = \frac{1}{4}\left(\frac{2m_c k_B T}{\pi\hbar^2}\right)^{3/2}$$

$$P_v(T) = \frac{1}{4}\left(\frac{2m_v k_B T}{\pi\hbar^2}\right)^{3/2}$$

we cannot calculate the carrier density without specific knowledge of the chemical potential but the product of the hole and electron carrier densities does not depend on the chemical potential.

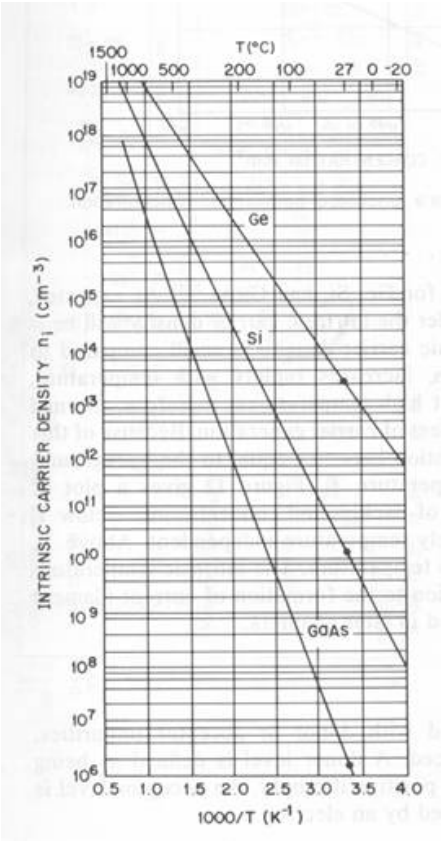
$$n_c p_v = N_c P_v e^{(\varepsilon_v - \varepsilon_c)/k_B T} = N_c P_v e^{-E_g/k_B T}$$

This is called the *law of mass action* what it says that at a given temperature it is enough to know the density of one carrier type to calculate the other. Up till now the results were valid regardless of whether the SC is intrinsic or extrinsic, the only assumption made was that the material was non-degenerate. One of the results of the law of mass action is that the product np is constant, by adding a large number of carriers of one type we cause the carrier concentration of the other type to decline. Leading to an overall decline in the total number of charge carriers.

Intrinsic Case

If the crystal is pure such that there is a negligible contribution of the impurities to the charge density – it is called an intrinsic SC.

$$n_c(T) = p_v(T) \equiv n_i(T)$$



$$n_i = \sqrt{n_c p_v} = \sqrt{N_c P_v} e^{-E_g/2k_B T}$$
$$n_c(T) = N_c(T) e^{-(\varepsilon_c - \mu)/k_B T}$$
$$p_v(T) = P_v(T) e^{-(\mu - \varepsilon_v)/k_B T}$$

As we saw above, while the density of electrons in the conduction band and holes in the valence band depends on the chemical potential the product of vacancies and electrons depends only on the magnitude of the bandgap. Such that:

$$np = N_e P_v e^{-\frac{E_g}{k_B T}}$$

It is important to remember that this expression is valid for both intrinsic and extrinsic SC, satisfying the non-degeneracy condition.

	Energy gaps at 300 K $\mathcal{E}_g$, eV	Quantum concentrations of electrons and holes at 300 K n_{ci}, cm^{-3}	n_{vi}, cm^{-3}	Density-of-states effective masses, in units of the free electron mass m_e^*/m	m_h^*/m	Dielectric constants, relative to vacuum ϵ/ϵ_0
Si	1.14	2.7×10^{19}	1.1×10^{19}	1.06	0.58	11.7
Ge	0.67	1.0×10^{19}	5.2×10^{18}	0.56	0.35	15.8
GaAs	1.43	4.6×10^{17}	1.5×10^{19}	0.07	0.71	13.13
InP	1.35	4.9×10^{17}	6.9×10^{18}	0.073	0.42	12.37
InSb	0.18	4.6×10^{16}	6.2×10^{18}	0.015	0.39	17.88

The position of the chemical potential is obtained from the expressions for the charge carrier density. In intrinsic SC the number of electrons in the conduction band equals the number of holes in the valence band, thus:

$$n_c(T) = N_c(T) e^{-(\varepsilon_c - \mu)/k_B T} = e^{-(\mu - \varepsilon_v)/k_B T} = p_v(T) = n_i(T) = \sqrt{P_v(T) N_c(T)} e^{-\frac{Eg}{2k_B T}}$$

which in turn enables us to solve for the chemical potential:

$$\mu = \varepsilon_v + \frac{1}{2} E_g + \frac{3}{4} k_B T \ln\left(\frac{m_v}{m_c}\right)$$

since

$$\left(\frac{m_v}{m_c}\right) \approx 1$$

the position of the chemical potential is close to the gap center in an intrinsic SC. Note that the only assumption we used was that of non-degeneracy.

Conductivity in SC

$$\sigma = n_e e \frac{e\tau_e}{m_e} + n_h e \frac{e\tau_h}{m_h}$$

$$\mu_e = \frac{e\tau_e}{m_e}$$

$$\mu_h = \frac{e\tau_h}{m_e}$$

where μ_e and μ_h is the mobility of the electrons and holes respectively. As we can see the mobilities are related to the band curvature through the effective mass parameter, such that larger effective masses will lead to smaller mobilities.

Table 3 Carrier mobilities at room temperature, in $cm^2/V\text{-}s$

Crystal	Electrons	Holes	Crystal	Electrons	Holes
Diamond	1800	1200	GaAs	8000	300
Si	1350	480	GaSb	5000	1000
Ge	3600	1800	PbS	550	600
InSb	800	450	PbSe	1020	930
InAs	30000	450	PbTe	2500	1000
InP	4500	100	AgCl	50	—
AlAs	280	—	KBr (100 K)	100	—
AlSb	900	400	SiC	100	10–20