Solutions of homework # 1

Solar radiation and de Broglie equation

1. The Sun's spectrum is peaked in the green, so we can compute the corresponding black-body temperature simply by using Wien's law with λ_{max} in the green range(495 − 570 nm):

$$T = \frac{b}{\lambda_{max}} = \frac{2.898 \times 10^6 \text{nm} \cdot \text{K}}{500\text{nm}} \approx 5.8 \times 10^3 \text{K}. \tag{1}$$

2. We cannot use green (or any other visibile) light to probe the atomistic structure of materials. In order to do that, the radiation wavelength must be of the order of few angstroms. Instead, visible light is of the order of several hundreds of nanometers (i.e. thousands of times bigger).

3. We compute the energy of a UV-C photon by using the Planck relation:

$$E = h\frac{c}{\lambda} = 6.626 \times 10^{-34}\text{J} \cdot \text{s}\frac{3.00 \times 10^8 \text{m/s}}{100 \times 10^{-9}\text{m}} \approx 2.0 \times 10^{-18}\text{J} \approx 12e\text{V} \tag{2}$$

4. We use again the Planck relation, now for a photon with a wavelength of 1Å (see previous answers):

$$E = h\frac{c}{\lambda} = 6.626 \times 10^{-34}\text{J} \cdot \text{s}\frac{3.00 \times 10^8 \text{m/s}}{1 \times 10^{-10}\text{m}} \approx 2.0 \times 10^{-15}\text{J} \approx 1.2 \times 10^4 e\text{V} \tag{3}$$

   Such photon corresponds to the X-ray region of the electronmagnetic spectrum.

5. We use Wien's law:

$$T = \frac{2.898 \times 10^6 \text{nm} \cdot \text{K}}{0.1\text{nm}} \approx 3 \times 10^7 \text{K} \tag{4}$$

6. First we find the minimum velocity using de Broglie's equation ($\lambda \cdot p = h$):

$$v_n = \frac{h}{m_n \lambda} = \frac{6.626 \times 10^{-34}\text{J} \cdot \text{s}}{1.675 \times 10^{-27}\text{kg} \cdot 10^{-10}\text{m}} \approx 4 \times 10^3 \text{m/s}, \tag{5}$$

   where we used $p = mv$. Then we compute the corresponding kinetic energy:

$$E_n = \frac{1}{2} m_n v_n^2 = 1.3 \times 10^{-20}\text{J} \approx 0.08eV \tag{6}$$

7. We use the relation between kinetic energy and temperature:

$$T_n = \frac{2E_n}{3k_B} = \frac{2 \cdot 1.3 \times 10^{-20}\text{J}}{3 \cdot 1.38 \times 10^{-23}\text{J/K}} \approx 600\text{K} \tag{7}$$

8. We perform again the steps of the previous answer, now using the electron mass which is roughly 2000 times lighter than the neutron mass.

$$v_e = \frac{h}{m_e \lambda} = 2000 \cdot v_n \approx 8 \times 10^6 \text{m/s}, \tag{8}$$

$$E_e = \frac{1}{2} m_e v_e^2 = 2000 \cdot E_n \approx 160e\text{V}, \tag{9}$$

$$T_e = 2000 \cdot T_n = 10^6 K. \tag{10}$$

Hence we can use photons, neutrons and electrons to probe the crystal structure of materials. They are used in different techniques that give us complementary pieces of information; for instance electrons interact with the charge distribution while neutrons interact more with nuclei or with magnetic moments.