Solutions of homework # 13

Exercise 1: As-doped germanium

A) We use the formula given for $\varepsilon_c - \varepsilon_d$ and the approximations suggested in the text ($\epsilon \approx 16$ and $m_d^* \approx m_c = 0.22m$):

$$\varepsilon_c - \varepsilon_d = 0.22 \frac{1}{16^2} \text{ Ry} = 0.00086 \text{ Ry} = 0.012 \text{ eV} \tag{1}$$

The value obtained is much smaller than the band gap of germanium. Hence, the donor electrons are much easier to excite into the conduction bands than the valence electrons of pure germanium.
B) We use the formula for r_d given in the text:

$$r_d = \frac{1}{0.22} 16 \text{ Bohr} \approx 72 \text{ Bohr} \approx 38 \mathring{A}\,. \tag{2}$$

The radius is much larger than the the germanium lattice constant ($a \approx 5.7\mathring{A}$), hence the arsenic donor electrons are very delocalized in the germanium crystal.

Exercise 2: P-doped silicon

A) The P atom has one electron more than Si atom. Therefore, by doping silicon with phosphorous we obtain extra electrons, i.e. this is a n-doped semiconductor.

B) The electrical conductivity in semiconductors is given by the expression:

$$\sigma = \sigma_e + \sigma_p = n_c|e|\mu_e + p_v|e|\mu_p\,, \tag{3}$$

where σ_e and σ_p are the conductivities of electrons and holes, respectively, e is the electron charge, n_c and p_v are the densities of electrons in the conduction bands and of holes in the valence bands, respectively. μ_e and μ_p are the mobilities of electrons and holes, respectively, and we can take their values from the table of carrier mobilities shown during the lectures (for Si, $\mu_e = 1350 \,\frac{\text{cm}^2}{\text{Vs}}$ and $\mu_p = 480 \,\frac{\text{cm}^2}{\text{Vs}}$.). So, in order to determine the total conductivity, we need to know n_c and p_v.

Since the semiconductor is in the extrinsic regime [see point A)], mathematically this means that all donors are ionized, i.e. $n_c \approx N_d = 10^{16}$ cm^{-3}. Using the Law of Mass Action, by knowing n_i [see point B)] and n_c we can determine p_v. The intrinsic density of carriers can be determined using the following expression:

$$n_i = \sqrt{g_c g_v}\, 2.5 \left(\frac{m_c}{m}\right)^{3/4} \left(\frac{m_v}{m}\right)^{3/4} \left(\frac{T}{300\text{ K}}\right)^{3/2} e^{-\frac{E_g}{2k_B T}}\, 10^{19} \,\frac{1}{\text{cm}^3}. \tag{4}$$

where we also included the number g_v of equivalent maxima of the valence band and the number g_c of equivalent minima of the conduction band. Using the value of the effective masses and the value of the band gap for Si at 300 K and taking into account that the conduction band has 6 equivalent minima, we obtain:

$$n_i \simeq \sqrt{6} \cdot 2.5 \cdot (0.3)^{\frac{3}{4}} \cdot (0.55)^{\frac{3}{4}} \cdot e^{-\frac{1.12}{0.05}} \cdot 10^{19} \text{ cm}^{-3} \simeq 2.96 \cdot 10^9 \text{ cm}^{-3}. \tag{5}$$

Therefore, using the Law of Mass Action we obtain:

$$p_v = \frac{n_i^2}{n_c} \approx \frac{0.876 \cdot 10^{19}\,\text{cm}^{-6}}{10^{16}\,\text{cm}^{-3}} = 0.876 \cdot 10^3\,\text{cm}^{-3}\,. \tag{6}$$

Now we can proceed to the calculation of the conductivity. The charge of an electron is $e = -1.602 \cdot 10^{-19}$ C, therefore:

$$\sigma_e = 10^{16}\,\frac{1}{\mathrm{cm}^3} \cdot 1.602 \cdot 10^{-19}\,\mathrm{C} \cdot 1350\,\frac{\mathrm{cm}^2}{\mathrm{Vs}} \approx 216\,\frac{\mathrm{C}}{\mathrm{V\,s\,m}} = 216\,\frac{1}{\Omega \cdot \mathrm{m}}\,. \tag{7}$$

$$\sigma_p = 0.876 \cdot 10^3\,\frac{1}{\mathrm{cm}^3} \cdot 1.602 \cdot 10^{-19}\,\mathrm{C} \cdot 480\,\frac{\mathrm{cm}^2}{\mathrm{Vs}} \approx 674 \cdot 10^{-14}\,\frac{\mathrm{C}}{\mathrm{V\,s\,m}} \approx 0.674 \cdot 10^{-11}\,\frac{1}{\Omega \cdot \mathrm{m}}\,. \tag{8}$$

We can see that since the density of holes is much smaller than the density of electrons, the corresponding contributions to the total conductivity are such that $\sigma_p \ll \sigma_e$. Hence,

$$\sigma = \sigma_e + \sigma_p \approx \sigma_e = 216\,\frac{1}{\Omega \cdot \mathrm{m}}\,. \tag{9}$$

Now let us compare such a conductivity with the conductivity in the intrinsic case. From lectures you know that in the intrinsic case $n_c = p_v = n_i$. Therefore, the conductivity in the intrinsic case is:

$$\begin{aligned} \sigma_i &= n_i|e|\mu_e + n_i|e|\mu_p = n_i|e|(\mu_e + \mu_p) \\ &= 2.96 \cdot 10^9\,\frac{1}{\mathrm{cm}^3} \cdot 1.602 \cdot 10^{-19}\,\mathrm{C} \cdot 1830\,\frac{\mathrm{cm}^2}{\mathrm{Vs}} \approx 0.867 \cdot 10^{-4}\,\frac{\mathrm{C}}{\mathrm{V\,s\,m}}\,. \end{aligned} \tag{10}$$

Therefore, we have obtained that:

$$\frac{\sigma}{\sigma_i} \approx 10^6\,, \tag{11}$$

i.e. by doping silicon with $N_d = 10^{16}$ cm^{-3} of P impurities at room temperature we increase the electrical conductivity by six orders of magnitude.

Exercise 3: non-degeneracy condition in doped silicon

Let us consider first the case of a donor-doped semiconductor (n-type). According to the definition of the chemical potential

$$\mu_{n-type}(T) = \varepsilon_c + k_B T \ln\left(\frac{N_D}{N_c(T)}\right)\,. \tag{12}$$

the second (temperature-dependent) term is proportional to the natural logarithm of the ratio $\frac{N_D}{N_c(T)}$. The value of $\ln\left(\frac{N_D}{N_c(T)}\right)$ is negative when $N_D < N_c(T)$, i.e. when the density of donors is smaller then the density of carriers in the conduction bands. The semiconductor is non-degenerate when $\mu_{n-type} \leq \varepsilon_c - 3k_BT$. The limiting case of the non-degeneracy $\mu_{n-type} = \varepsilon_c - 3k_BT$ corresponds to the maximum concentration of donors, which can be found from the condition

$$\mu_{n-type}(T) = \varepsilon_c + k_B T \ln\left(\frac{N_D}{N_c(T)}\right) = \varepsilon_c - 3k_BT\,, \tag{13}$$

which means

$$\ln\left(\frac{N_D}{N_c(T)}\right) = -3. \tag{14}$$

Using the expression for $N_c(T)$, with g_c the number of equivalent conduction band minima, **and considering that in silicon there are six equivalent conduction band minima**:

$$N_c(T) = g_c\frac{1}{4}\left(\frac{2m_c k_B T}{\pi\hbar^2}\right)^{3/2} = \mathbf{6} \times 2.5\left(\frac{m_c}{m}\right)^{3/2}\left(\frac{T}{300K}\right)^{3/2} 10^{19}\,\mathrm{cm}^{-3}\,, \tag{15}$$

we find that the maximum concentration of donors is

$$N_D^{\max} = N_c(T)e^{-3} = e^{-3}\,\mathbf{6}\times 2.5\left(\frac{m_c}{m}\right)^{3/2}\left(\frac{T}{300K}\right)^{3/2} 10^{19}\,\mathrm{cm}^{-3}$$
$$= e^{-3}\,\mathbf{6}\times 2.5\,(0.3)^{3/2}10^{19}\,\mathrm{cm}^{-3} \approx 12.3\times 10^{17}\,\mathrm{cm}^{-3}. \quad (16)$$

In the case of p-type doping the semiconductor is non-degenerate when $\mu_{p-type} \geq \varepsilon_v + 3k_BT$. The limiting case of the non-degeneracy $\mu_{p-type} = \varepsilon_v + 3k_BT$ corresponds to the maximum concentration of acceptors which can be found from the condition

$$\mu_{p-type}(T) = \varepsilon_v - k_BT\ln\left(\frac{N_A}{P_v(T)}\right) = \varepsilon_v + 3k_BT\,, \quad (17)$$

which means

$$\ln\left(\frac{N_A}{P_v(T)}\right) = -3. \quad (18)$$

Using the expression for $P_v(T)$ from lectures

$$P_v(T) = \frac{1}{4}\left(\frac{2m_vk_BT}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_v}{m}\right)^{3/2}\left(\frac{T}{300K}\right)^{3/2} 10^{19}\,\mathrm{cm}^{-3}\,, \quad (19)$$

we find that the maximum concentration of donors is

$$N_A^{\max} = P_v(T)e^{-3} = e^{-3}\,2.5\left(\frac{m_v}{m}\right)^{3/2}\left(\frac{T}{300K}\right)^{3/2} 10^{19}\,\mathrm{cm}^{-3}$$
$$= e^{-3}\,2.5\,(0.55)^{3/2}10^{19}\,\mathrm{cm}^{-3} \approx 5.1\times 10^{17}\,\mathrm{cm}^{-3}. \quad (20)$$

Exercise 4: p-n junction

The built-in voltage is defined as a difference between the chemical potentials in the n-type material and p-type material:

$$eV_{bi} = \mu_n - \mu_p\,. \quad (21)$$

During the lectures it was explained that using the expressions for the chemical potentials for μ_n and μ_p it can be shown that the built-in voltage can be written as:

$$eV_{bi} = k_BT\ln\left(\frac{N_dN_a}{n_i^2}\right)\,. \quad (22)$$

In order to compute the built-in voltage V_{bi} for GaAs p-n junction using the equation above, we need to know the density of carriers in the intrinsic case n_i. We can compute it using Eq. (**??**). At $T = 300$ K we obtain:

$$n_i = 2.5\cdot(0.063)^{3/4}\cdot(0.51)^{3/4}\cdot 1\cdot e^{-\frac{1.43}{0.0517}}\cdot 10^{19}\,\frac{1}{\mathrm{cm}^3} \approx 1.84\cdot 10^6\,\frac{1}{\mathrm{cm}^3}\,. \quad (23)$$

Thus, the built-in voltage at $T = 300$ K is:

$$V_{bi} = \frac{1}{e}\cdot 0.025\,\mathrm{eV}\,\ln\left(\frac{10^{15}\cdot 10^{16}}{(1.84\cdot 10^6)^2}\right) \approx 1.1\,\frac{\mathrm{eV}}{e} = 1.1\,\mathrm{V}\,. \quad (24)$$