Homework # 6

Exercise 1: Hydrogen atom in a magnetic field

Let us consider a hydrogen atom in the uniform magnetic field B (along the $\hat{z}$ direction). The spin combines with the angular momentum, and they couple via[1]

$$\hat{H}_{int} = \frac{\mu_B}{\hbar}(\hat{L}_z + 2\hat{S}_z)B. \tag{1}$$

1. Write the full Hamiltonian of the system described above, including the interaction term in Eq. (1). Does the angular momentum $\hat{L}_z$ commute with the rest of the Hamiltonian?
   (*Hint: For an electron in a central potential it is convenient to adopt spherical coordinates, where* ∇^2 *is made of a radial term and a term proportional to* $\hat{L}^2$)

In the following we will neglect the spin term proportional to $\hat{S}_z$ and consider only the angular momentum $\hat{L}_z$.

2. Compute the energy eigenvalues. Do they depend on quantum numbers other than the principal quantum number n?

3. Using the result of point 2 (see above), list the energy eigenvalues for the s, p, and d orbitals of the hydrogen atom in the uniform magnetic field B.

Exercise 2: Auf-bau principle

Consider the Li atom.

a) Using the Auf-bau filling scheme, determine the electronic configuration of its ground state.

b) Treating the atom as a hydrogen-like and neglecting the mutual repulsion between electrons, evaluate the total energy of the system.

[1] This formula is valid in the *strong magnetic field limit*, where we can neglect spin-orbit coupling or treat it afterwards as a perturbation.

Exercise 3: Noble gas in a weak magnetic field

The Ne atom has 10 electrons (and 10 protons in its nucleus).

1. Provide its electronic configuration and explain why it is a noble gas. Supposing that its electrons do not interact with each other (independent-electron approximation) write explicitly the energy levels (write the quantum numbers of the eigenstates) of all the electrons.

Suppose a weak magnetic field $\vec{B} = (0, 0, B_z)$ is then switched on.

2. Write the Hamiltonian that describes the electrons of the Ne atom in the magnetic field. (*Hint: look at the Excercise1*)
3. Using the expression for the Hamiltonian when an atom is in an external magnetic field (See: slides from Week 5 with Goudsmit and Uhlenbeck expression) compute the variation in energy of all electrons (You can try and represent it schematically).
   Note: *Pay attention to the values of quantum numbers n, l, m_l and m_s.*
   How much does the total energy change?
4. Is L_x conserved after the magnetic field is switched on? Was it before? What about S_y? Compute the expectation value of $\hat{L}_x^2 + \hat{L}_y^2$ for an electron of type s and one of type p (*Hint: start from specifying the quantum numbers that classify these electronic states and remember that $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$, so $\hat{L}_x^2 + \hat{L}_y^2$ is …?*).