

# Materials Engineering I (MSE 214)

## Lecture 10: Metals – Mechanical Properties

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# Recap: Slip systems

Dislocations do not move with the same ease in all directions

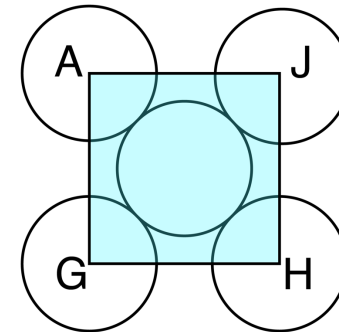
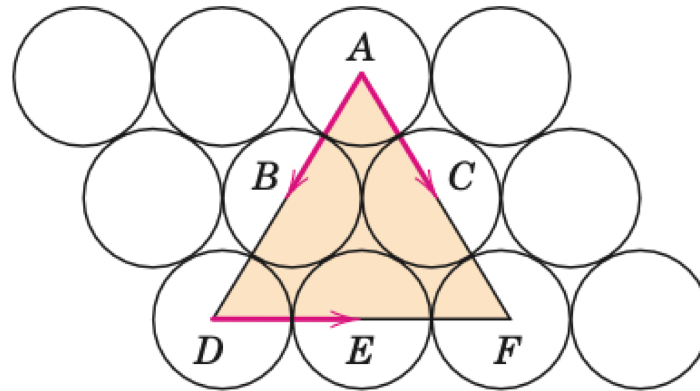
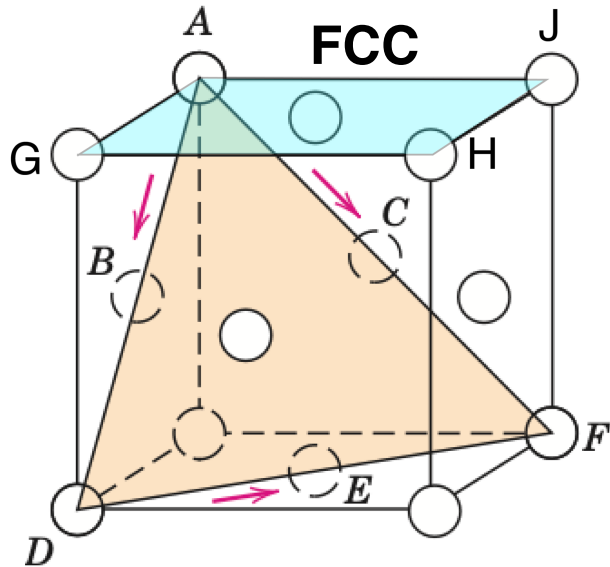
Preferred plane of dislocation movement → Slip plane

Preferred direction of movement → Slip direction



Slip system →

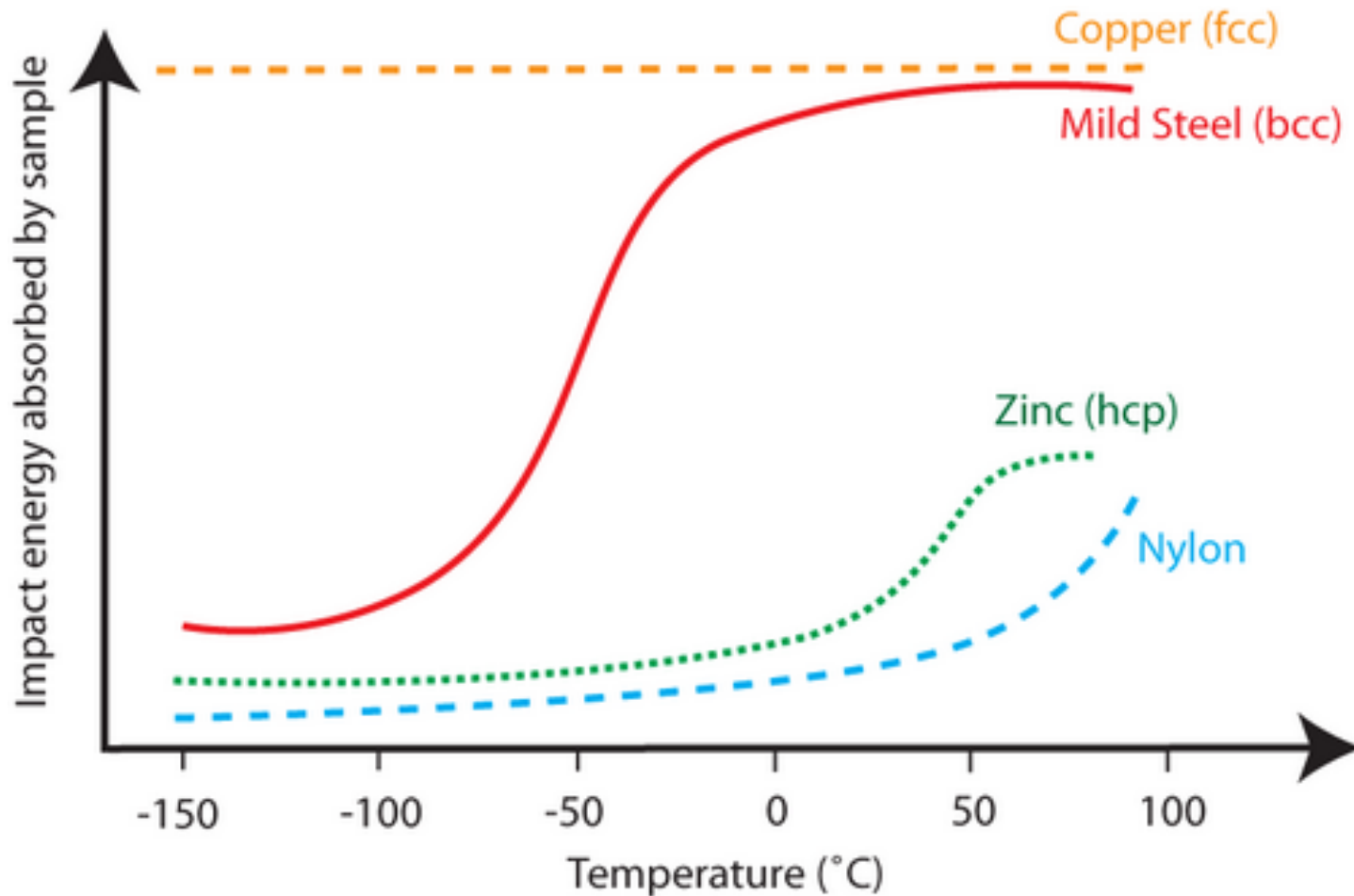
The system that allows dislocations to move with lowest applied energy



Which is easier?

Not all slip systems are created equal: Some need thermal energy to activate

# Recap: Ductile-to-Brittle Transition Temperature (DBTT)



Can think of energy absorbed as area under the stress-strain plot



More energy absorbed = more ductile

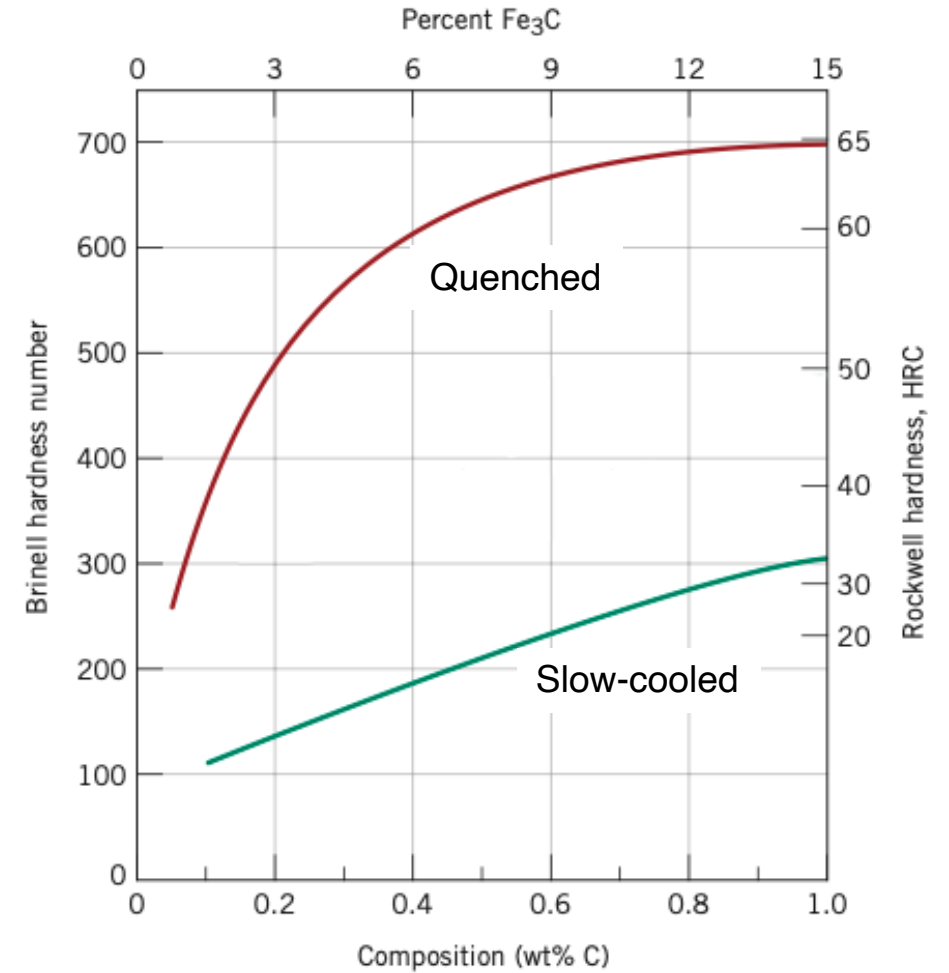
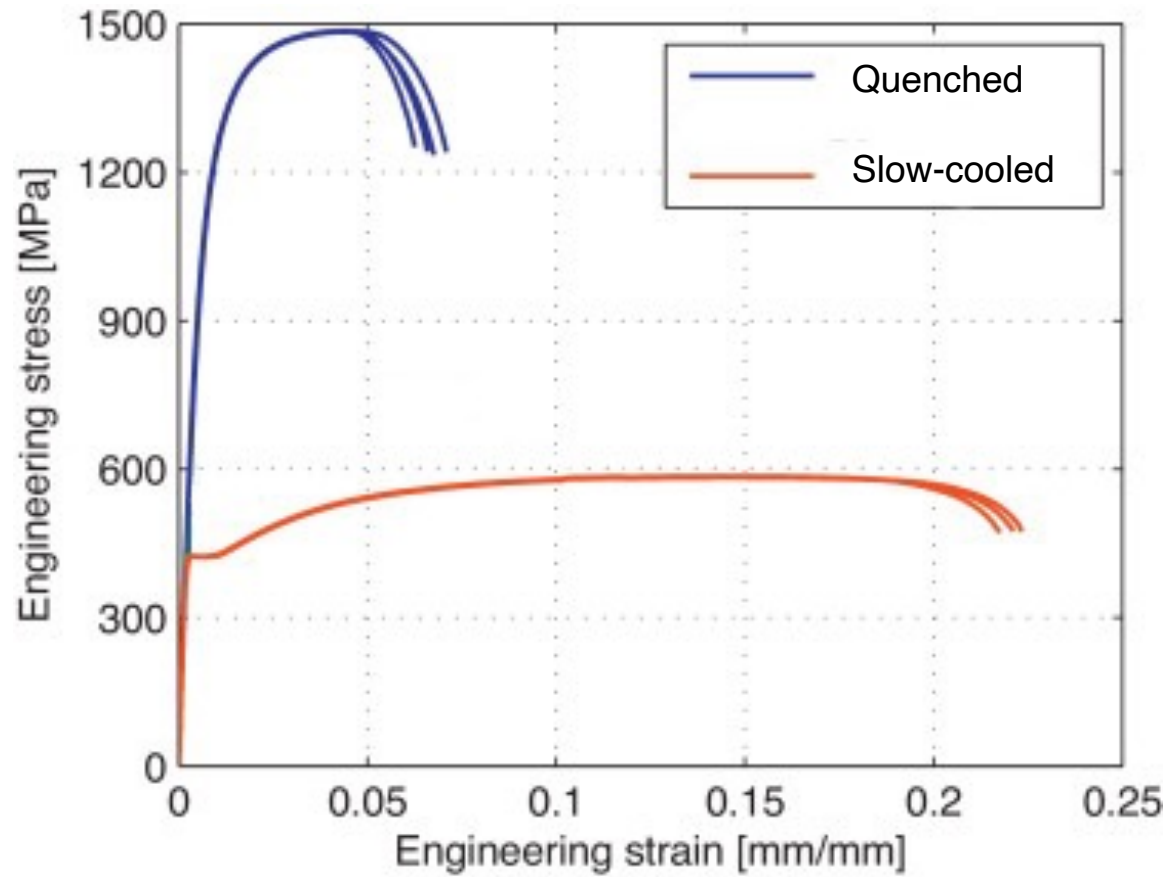
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Temperature does not affect FCC metals much

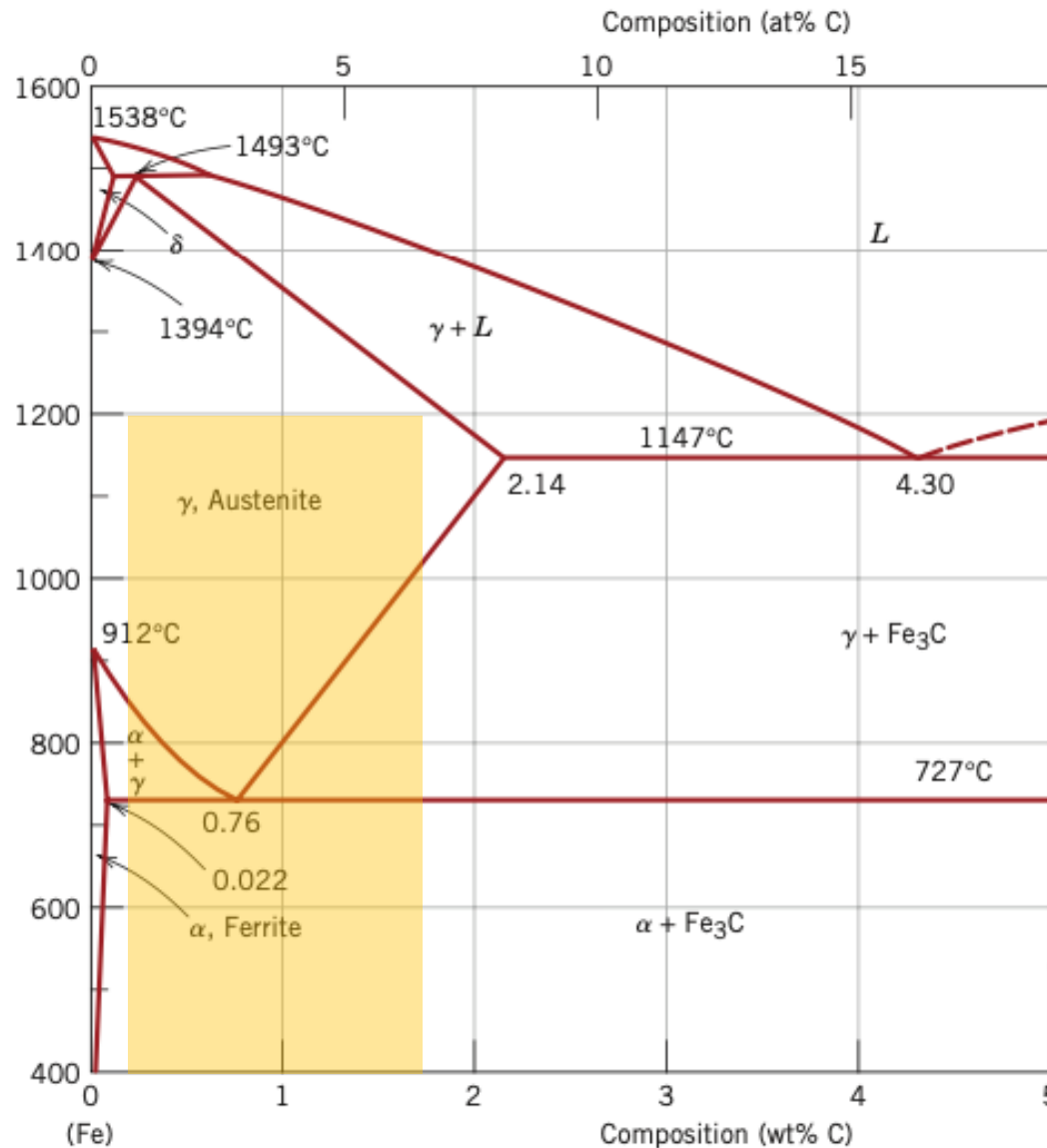
BCC and HCP have temperature-dependent behavior!

Nylon does as well. Why?  
(Recap polymers)

# Recap: Rapid cooling of steel makes it stronger

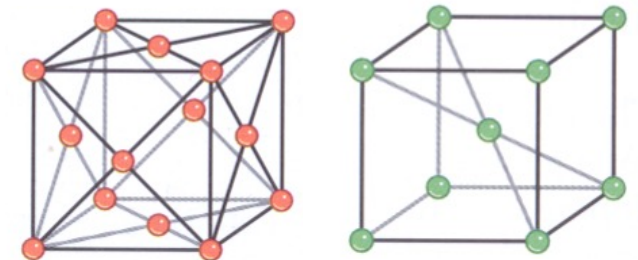


# Recap: Rapid cooling makes martensite



Rapid cooling → Quench

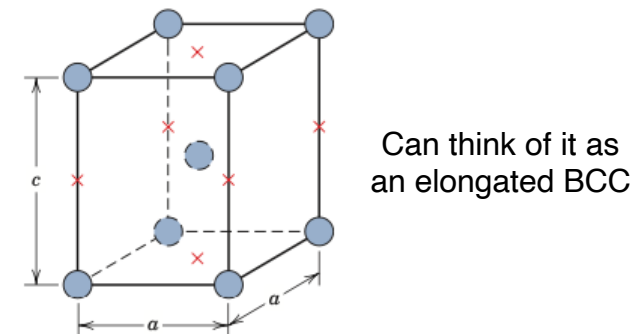
Austenite ( $\gamma$ ) is FCC  
 Ferrite ( $\alpha$ ) is BCC



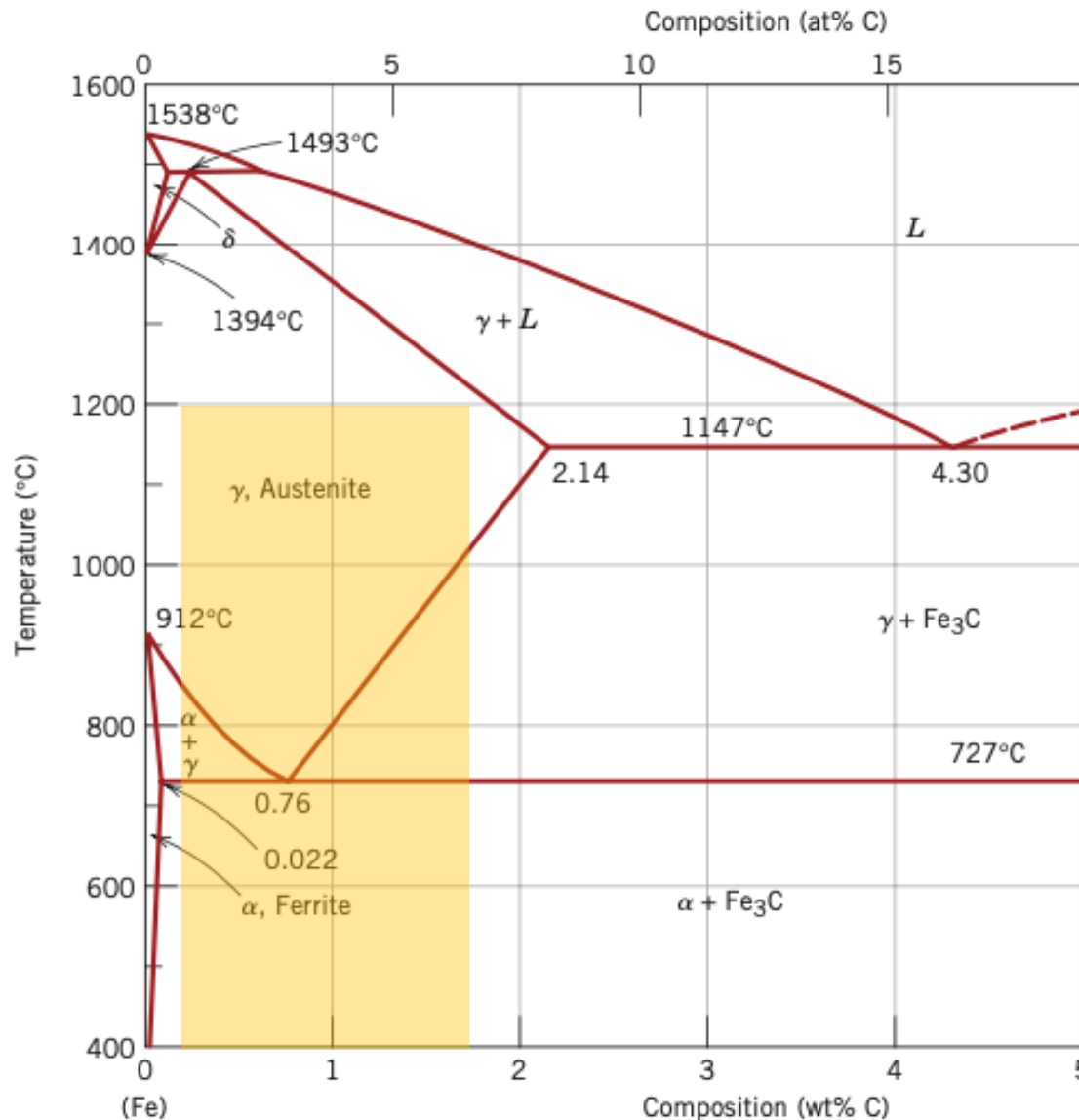
How do we rapidly transform from FCC to BCC?

You kinda can't...not enough time to rearrange the atoms!

Induce the formation of a new phase → Martensite  
 (Body Centered Tetragonal, BCT)



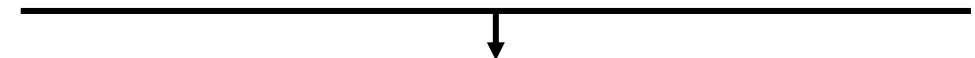
# Recap: Why is martensite so hard?



Supersaturated with carbon since carbon cannot diffuse away in time. Lattice is strained / distorted due to excess carbon

Martensite formation causes volume expansion (different densities) and shape change  $\rightarrow$  Internal stresses

BCT structure has fewer active slip systems than FCC and BCC

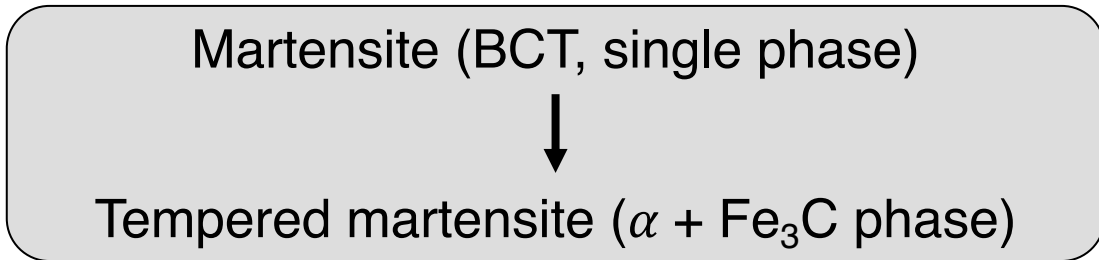


Impede dislocation movement!

# Recap: Tempering martensite makes it softer

**Temper = Reheat the quenched steel to change mechanical properties**

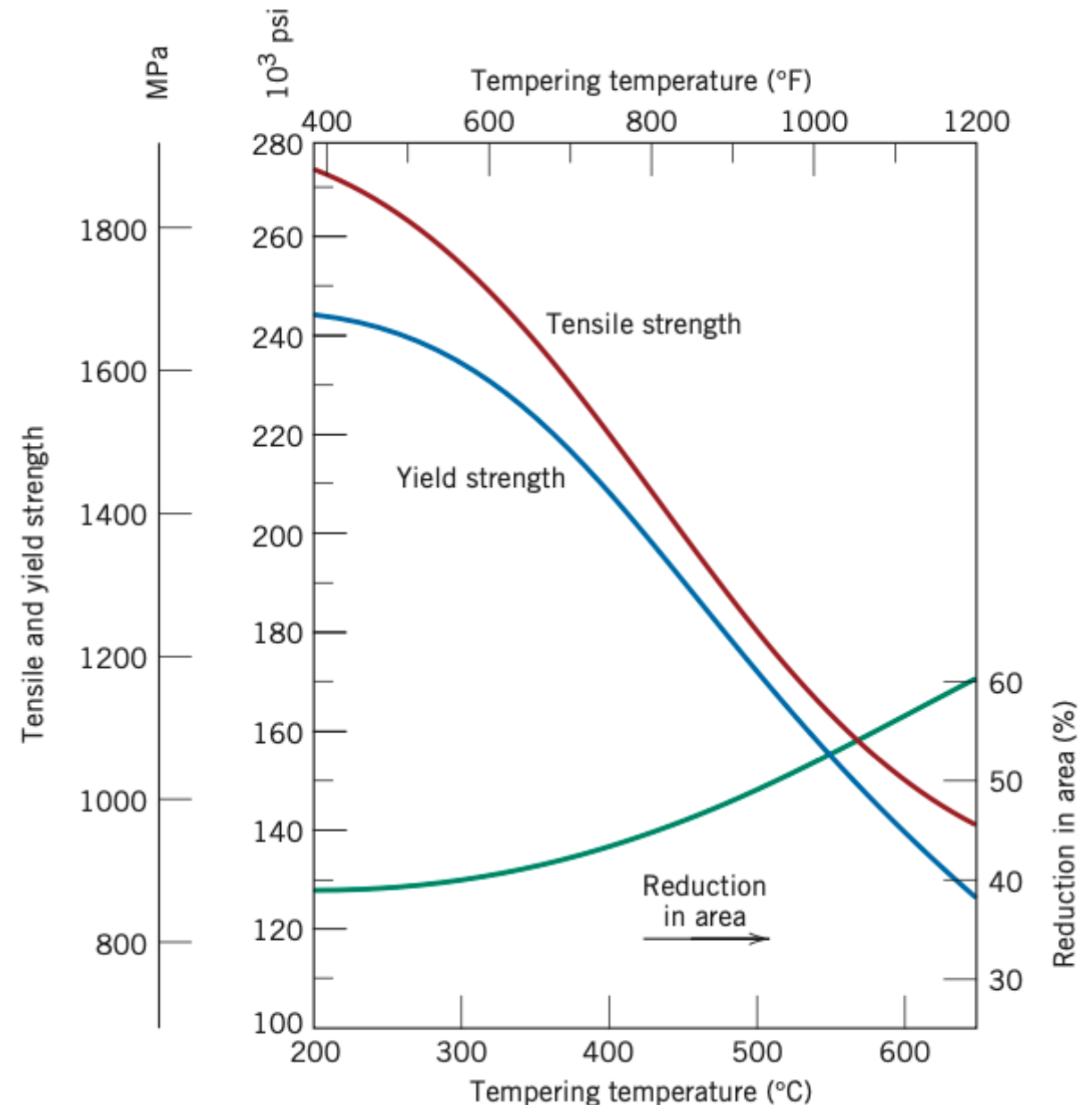
Effect 1: Allows carbon supersaturated martensite to transform to equilibrium phases



This generates a new microstructure that is tougher than pure martensite

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Effect 2: Reduce dislocation density due to increased atom mobility



# Recap: Solid-state transformations

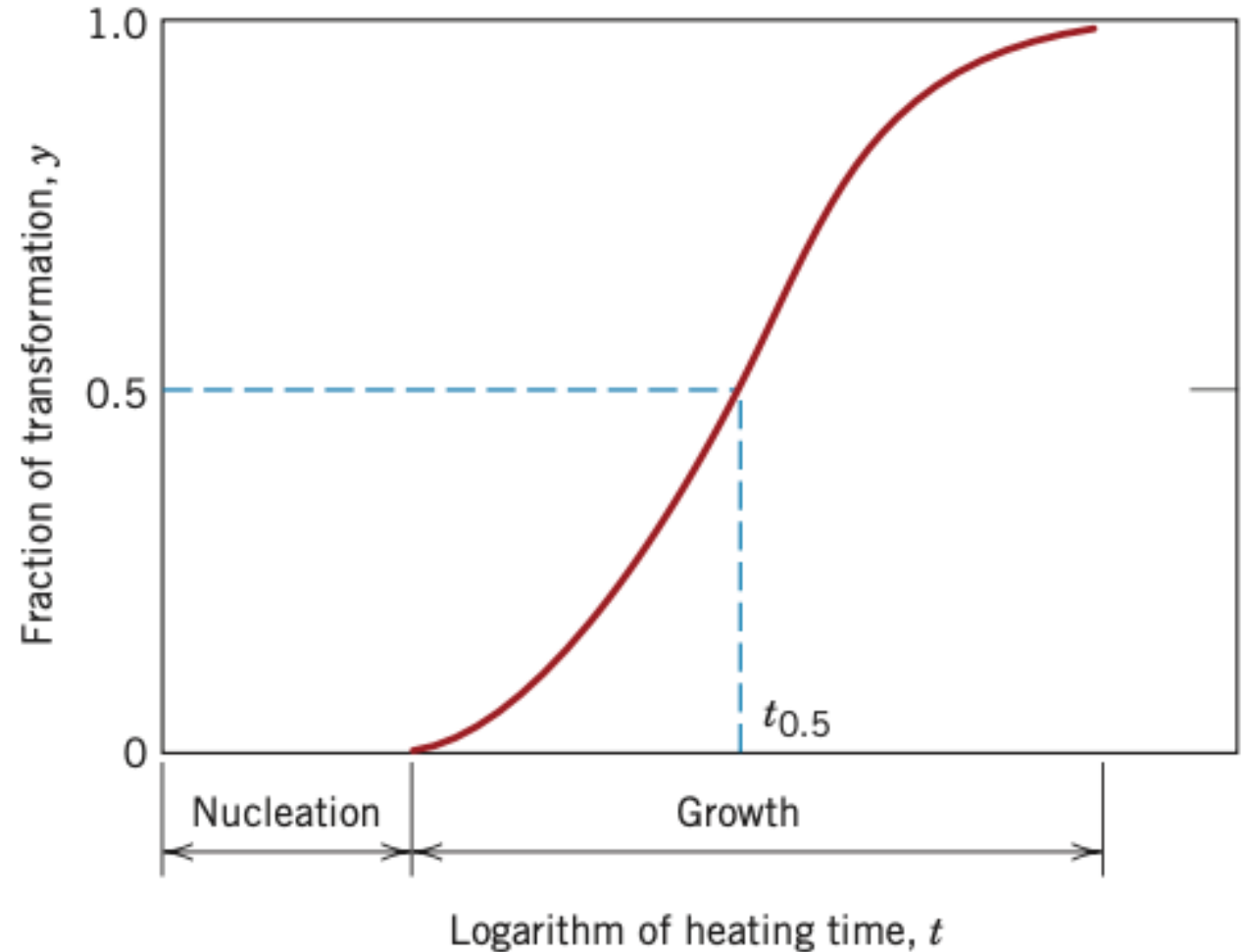
Solid-state transformation can be described by the Avrami equation:

$$y = 1 - e^{-kt^n}$$

$y$  = fraction of transformation  
 $k, n$  = time independent constants for the transformation

By convention, the rate of a transformation is taken as the reciprocal of time requires for 50% transformation

$$Rate = \frac{1}{t_{0.5}}$$



# Recap: Time-Temperature-Transformation (TTT) diagrams\*

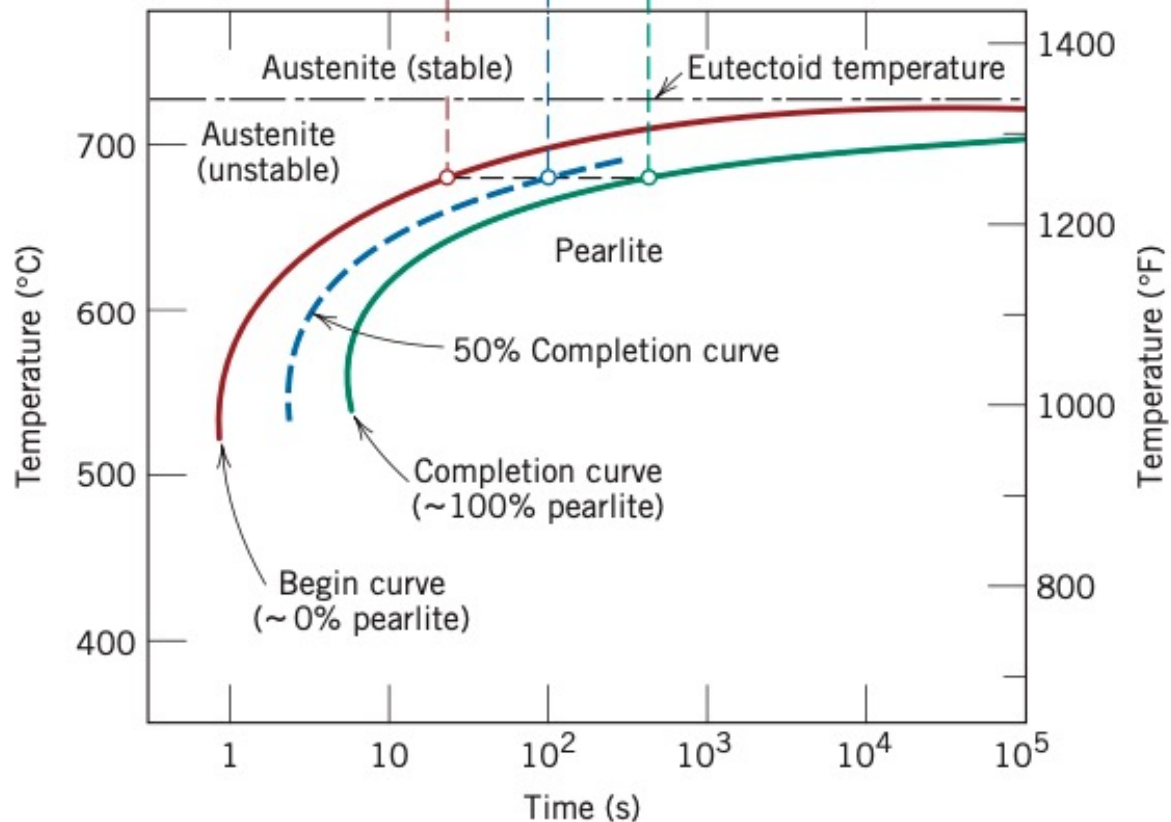
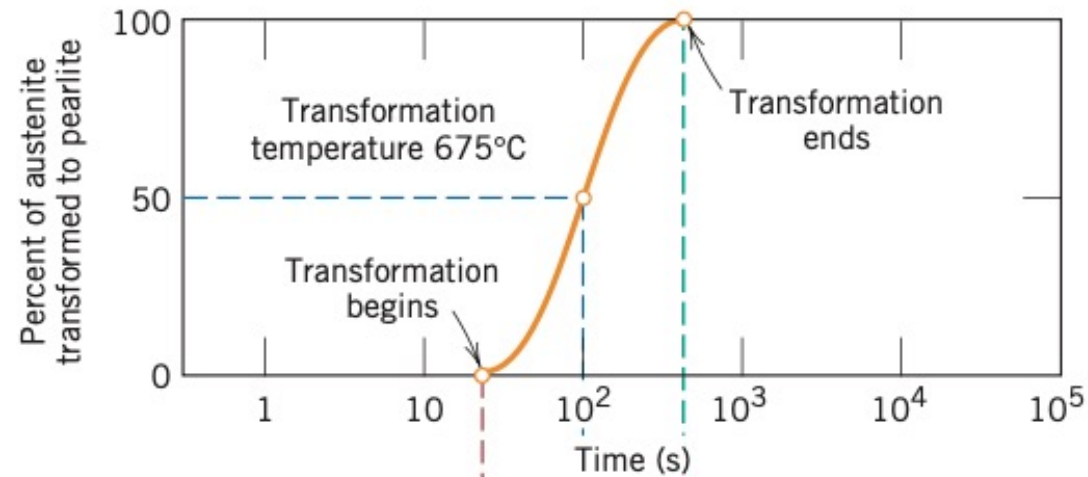
Let's walk through this diagram:

Above the eutectoid temperature ( $727^{\circ}\text{C}$ ), only stable austenite  $\rightarrow$  No change over any timeframe

The time needed for the austenite to pearlite transformation depends on the degree of supercooling

On the left is of the **red line** is 100% unstable austenite. It will transform to pearlite given enough time

On the right of the **green line** is 100% pearlite



## Recap: Let's use a TTT diagram to predict metastable microstructure

Starting from 760°C:

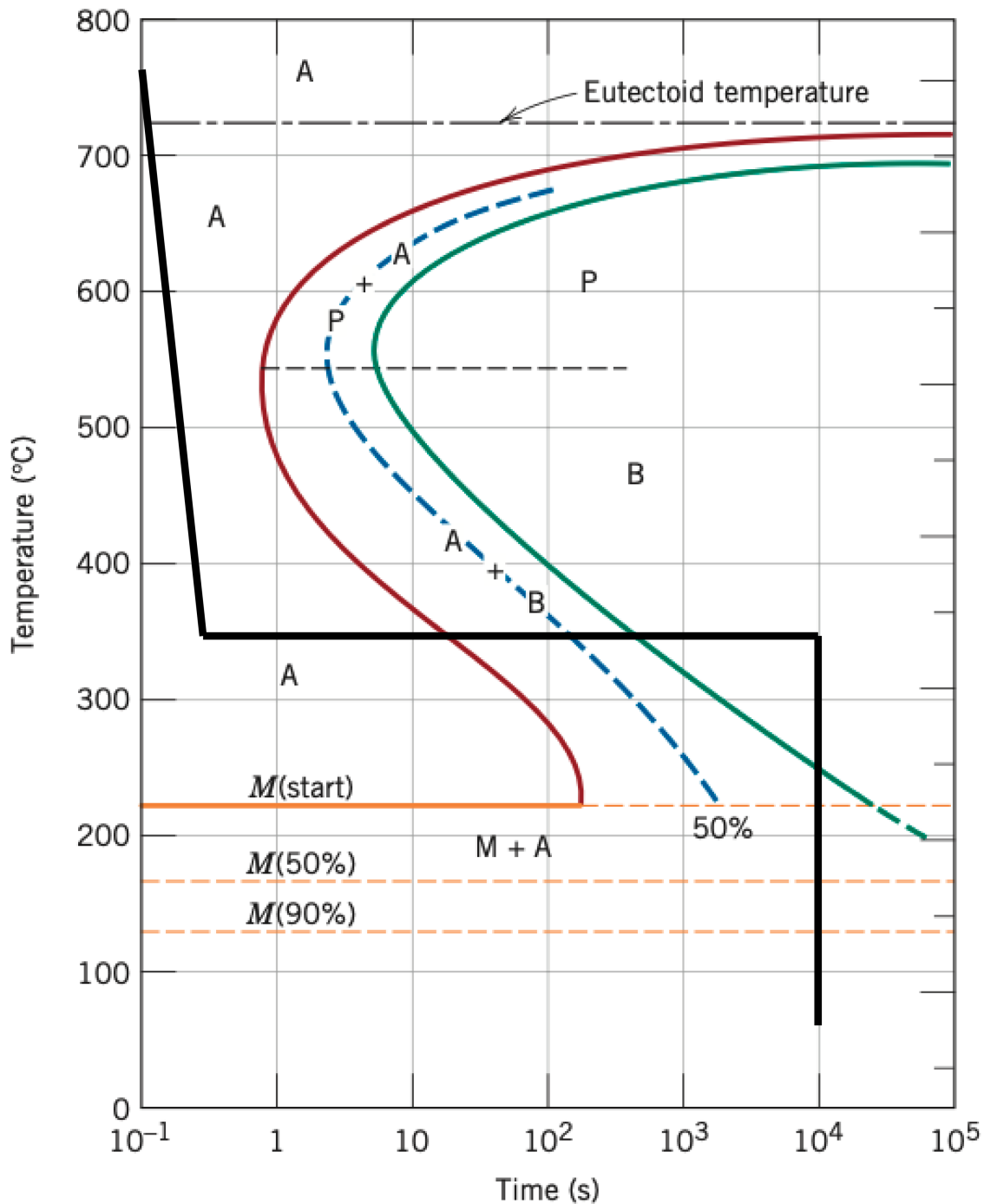
a) Rapidly cool to 350°C, hold for 10<sup>4</sup> s, quench to RT

Assume initial cooling is rapid enough to prevent any transformations from happening

Austenite (A) → Bainite (B) starts ~10s and is completed after ~600 s. After 10<sup>4</sup>s, it's 100% B

Quenching to RT does not produce any martensite even though it passes through the martensite region.

No austenite left to transform! Only austenite can transform to martensite



# Week 10 Learning Objectives

- Understand what a CCT diagram is and how it differs from a TTT diagram
- Understand how to qualitatively use a CCT diagram to predict the microconstituents of a cooling alloy
- Understand the difference between brittle and ductile fracture
- Understand what the fracture toughness of a material is
- Use the fracture toughness of a material to predict of a material will fracture
- Understand what fatigue is
- Be able to use Miner's rule to predict fatigue failure
- Be able to use Paris' Law to predict fatigue failure
- Understand the factors that affect fatigue fracture

## Let's try to use a TTT diagram to predict metastable microstructure

Starting from 760°C:

c) Rapidly cool to 600°C, hold for 4s, rapidly cool to 450°C, hold for 10s, then quench to RT

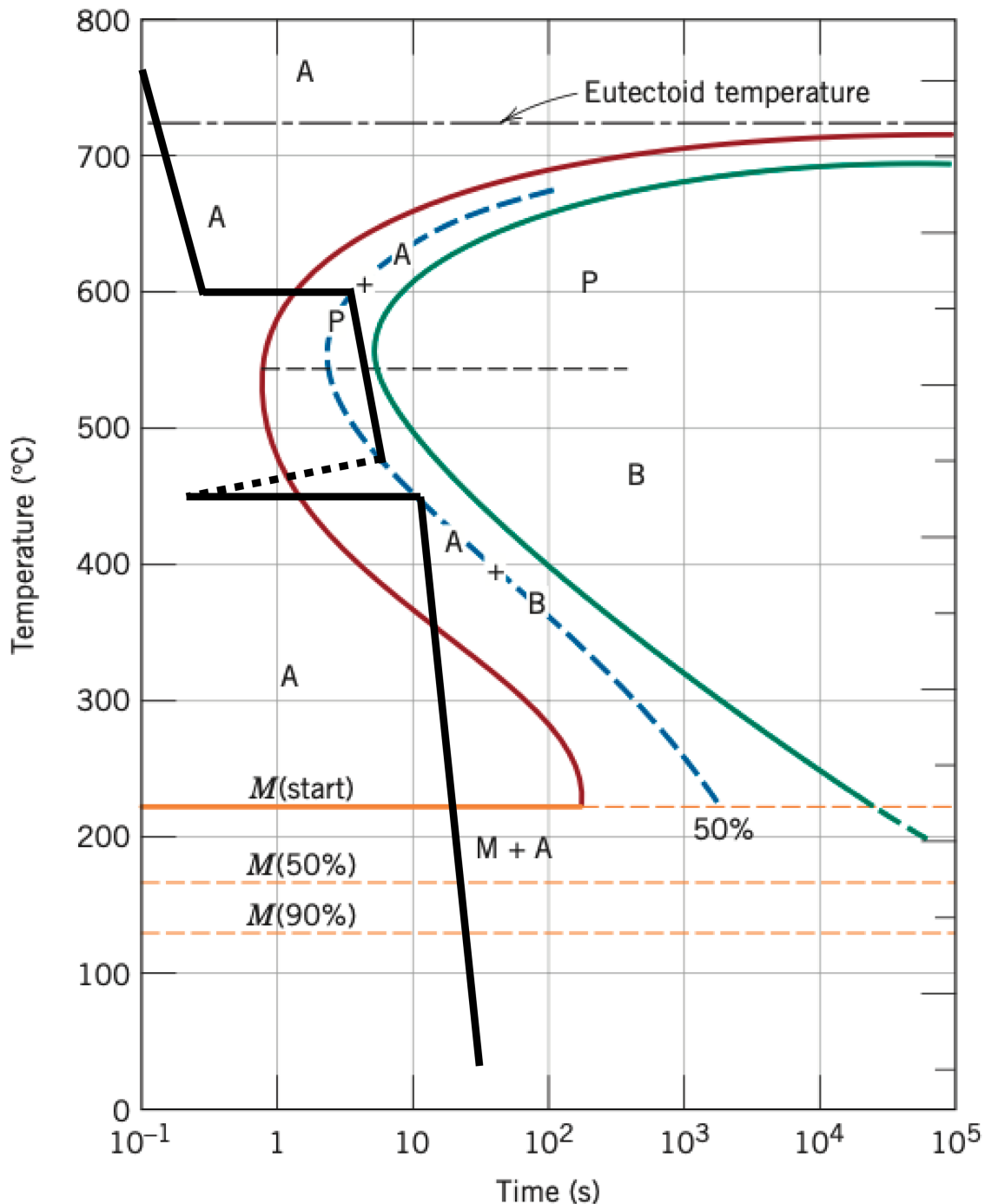
Holding at 600°C for 4s → ~50% A + P

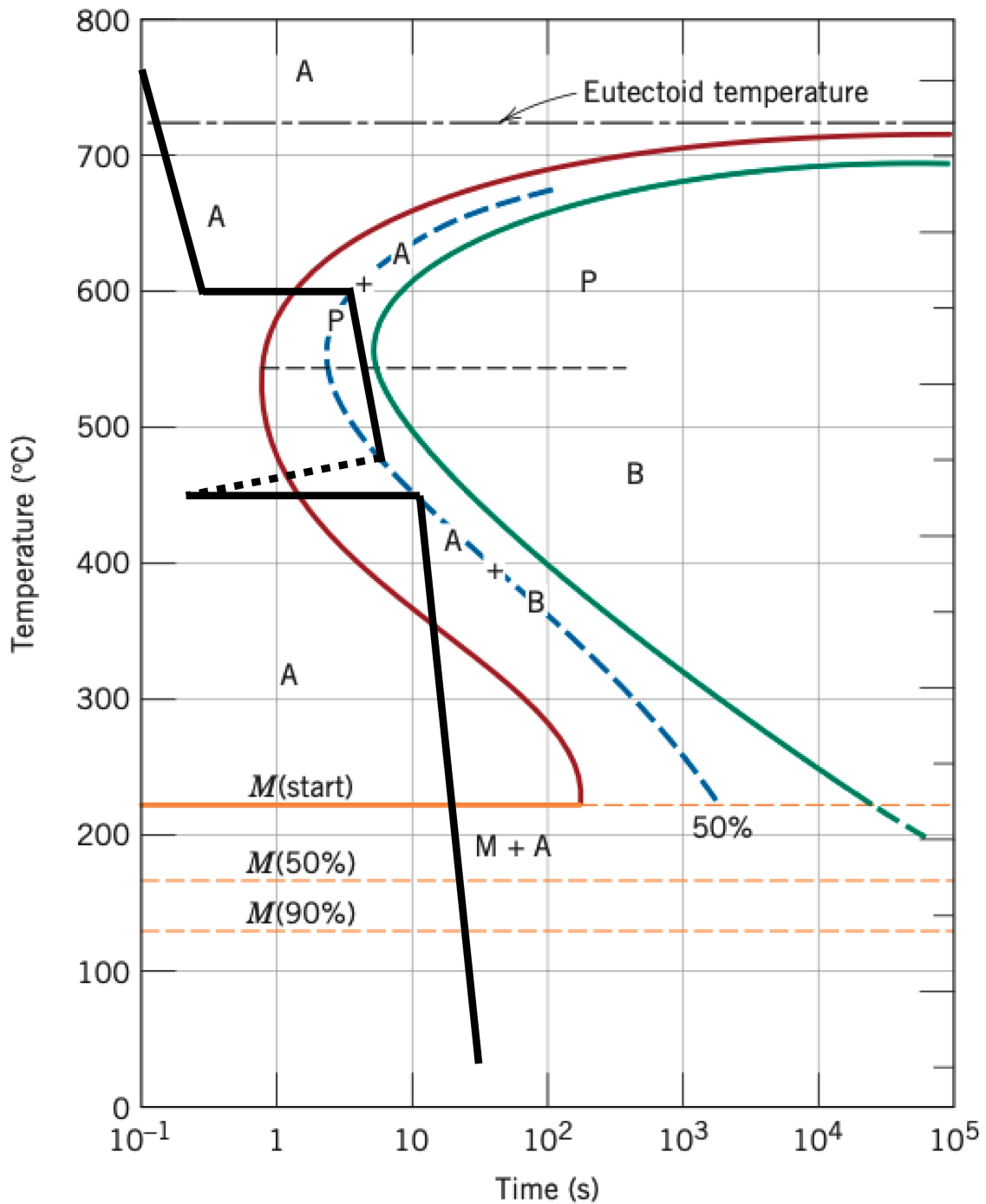
Assume during rapid cooling to 450°C, nothing happens

At 450°C, we start timing from 0 again to see what happens to the unstable austenite.

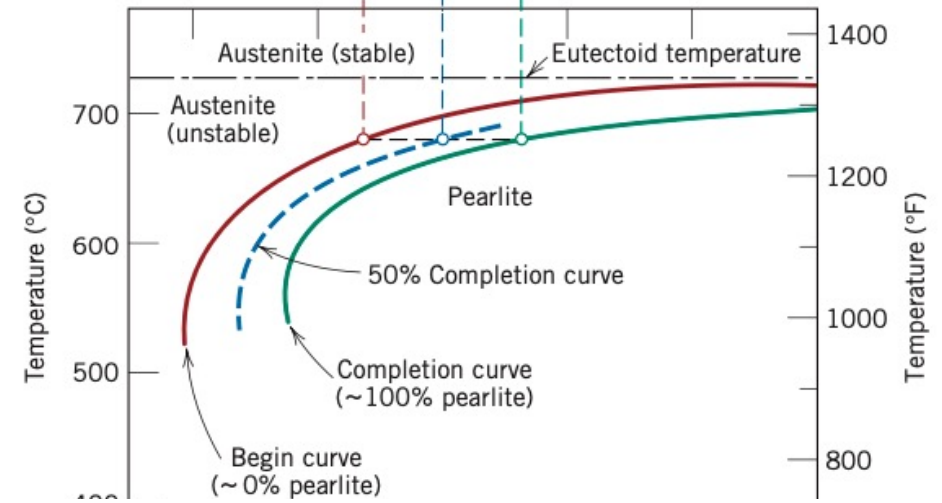
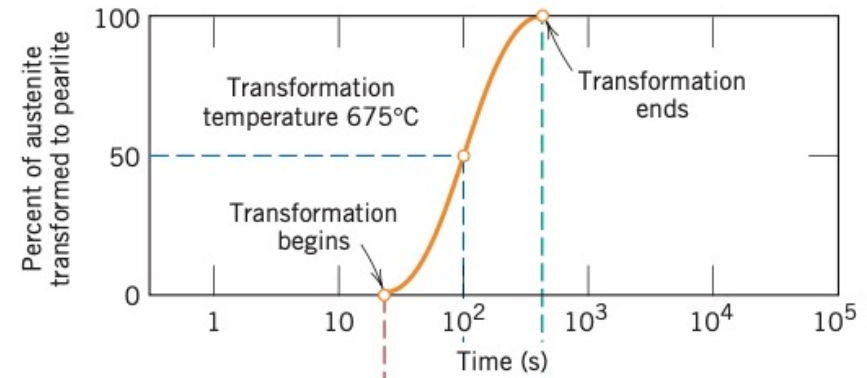
Holding at 450°C for 10s → 50% of remaining A till transform into B (25% of original sample is B)

Quenching to RT transforms the remaining A to M.





## Why do we go back in time?



Different transformations!

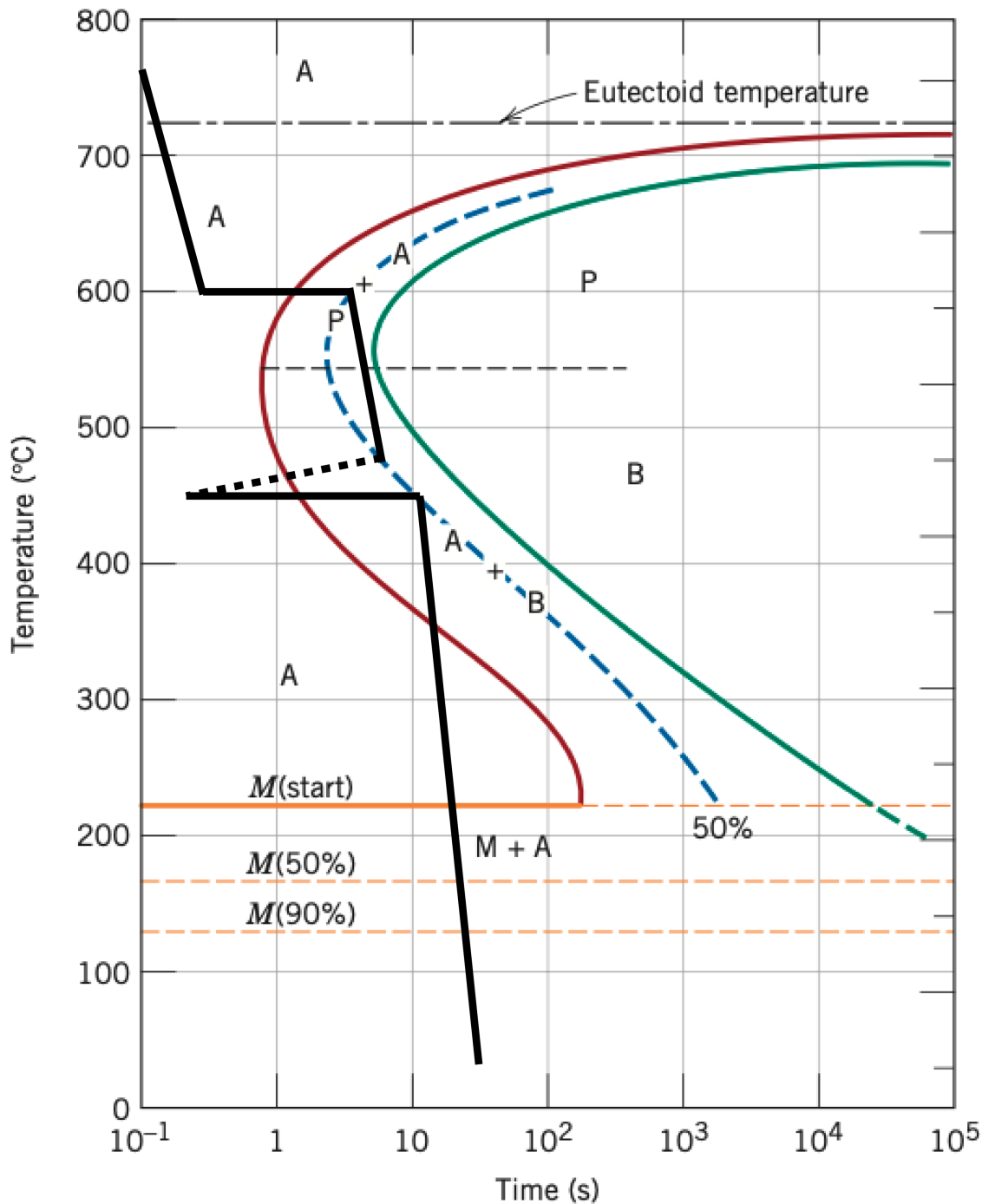
Reactions rate change with % transformation

# Limitations of TTT diagrams

TTT diagrams: Only valid for conditions of constant temperature

In real life, not practical to rapidly cool a metal to a set temperature and then hold it there for time.

Instead, we cool the metal continuously over time



**TTT**

(Time-Temperature-Transformation)



**CCT**

(Continuous cooling transformation)

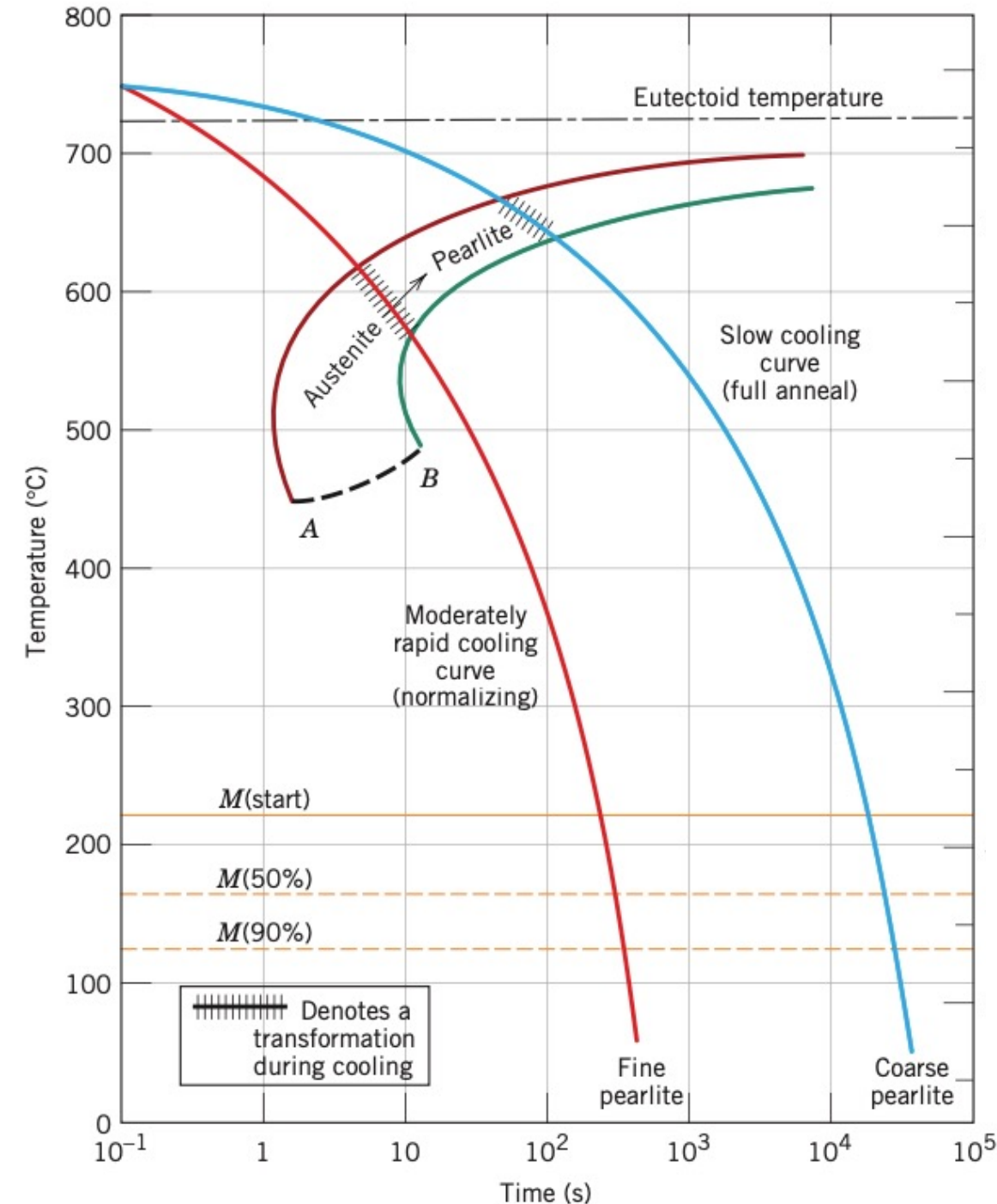
## ← CCT diagram

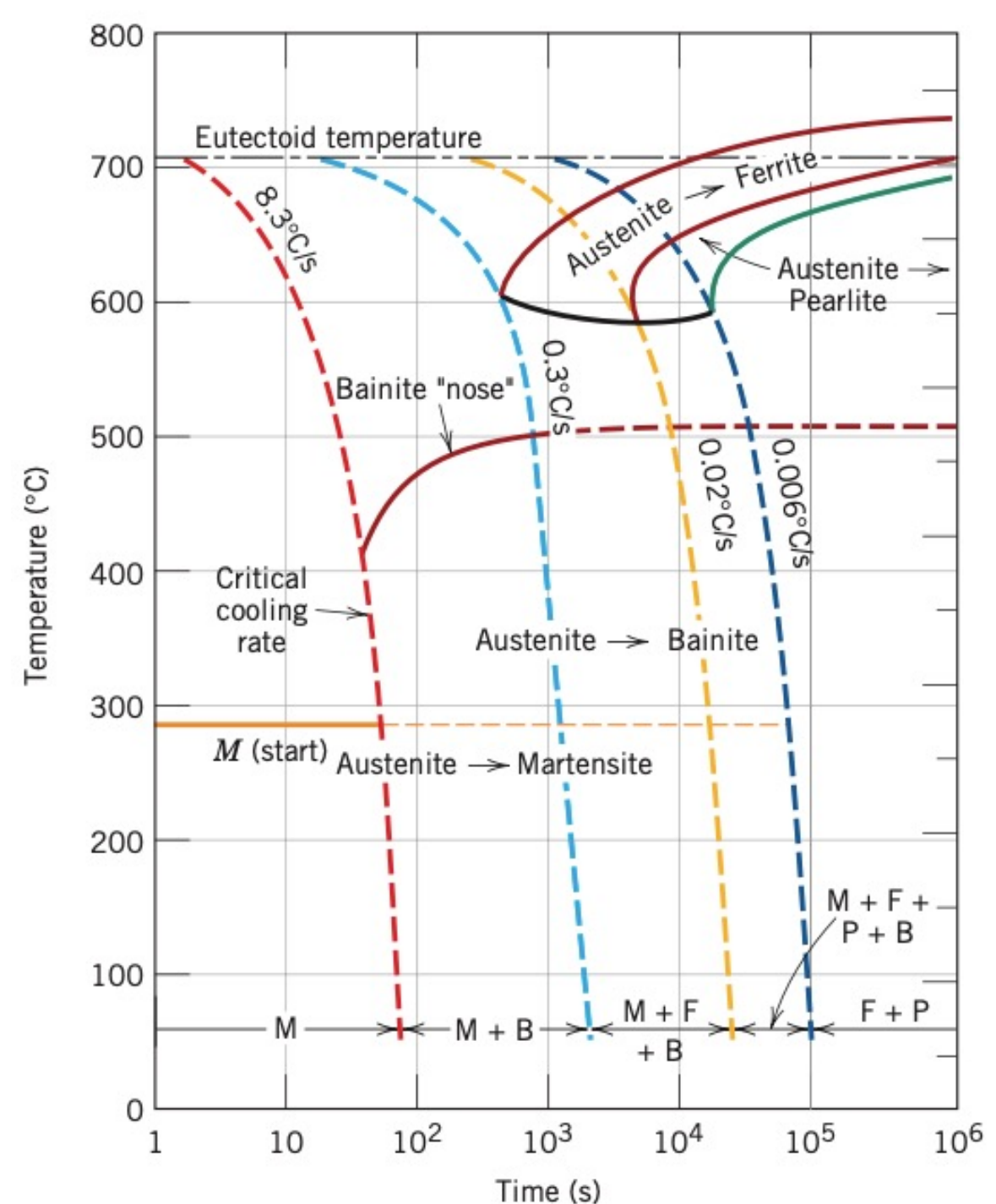
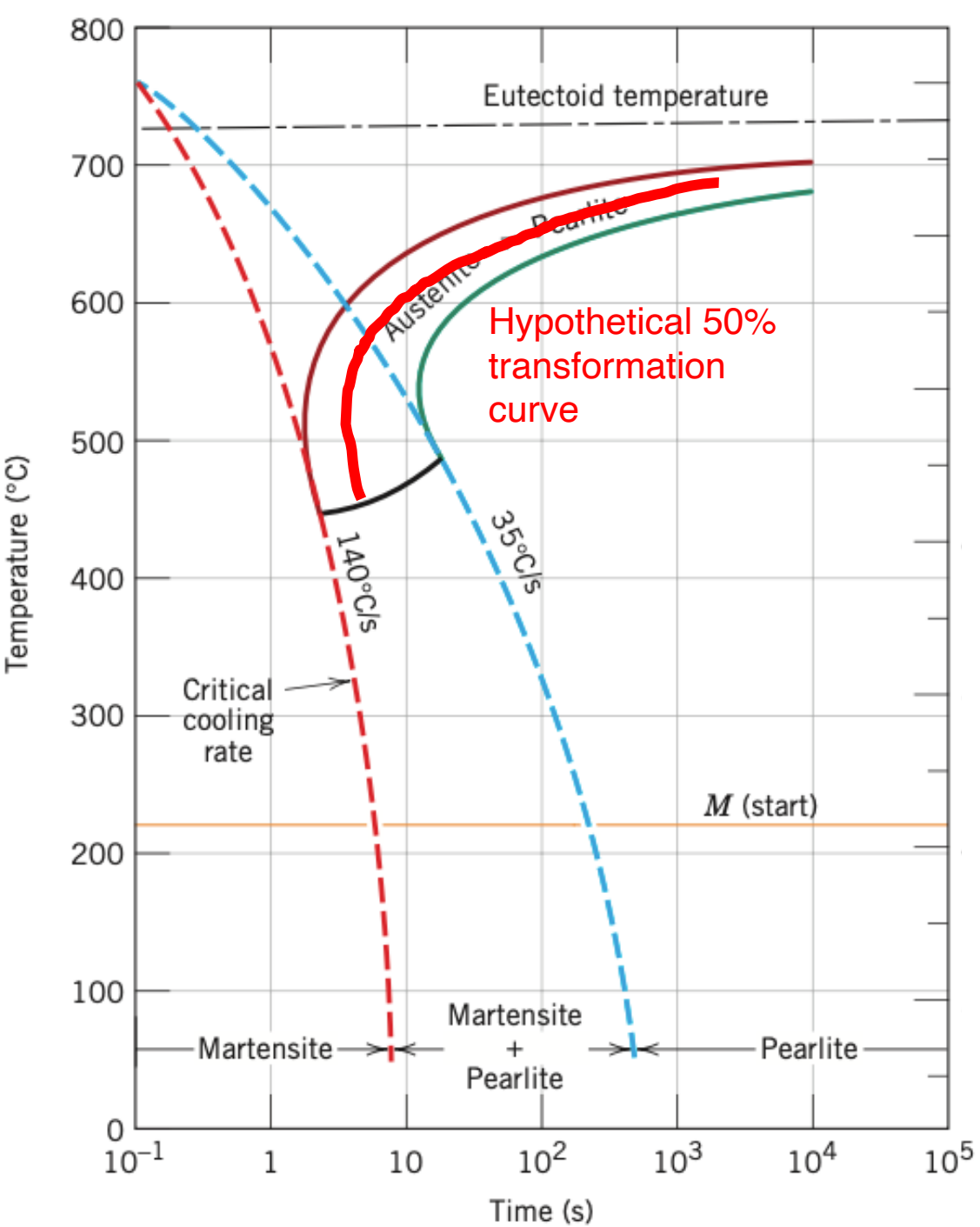
CCT diagrams can look similar to TTT diagrams!

Cooling curves (cooling at different rates) superimposed onto CCT diagrams

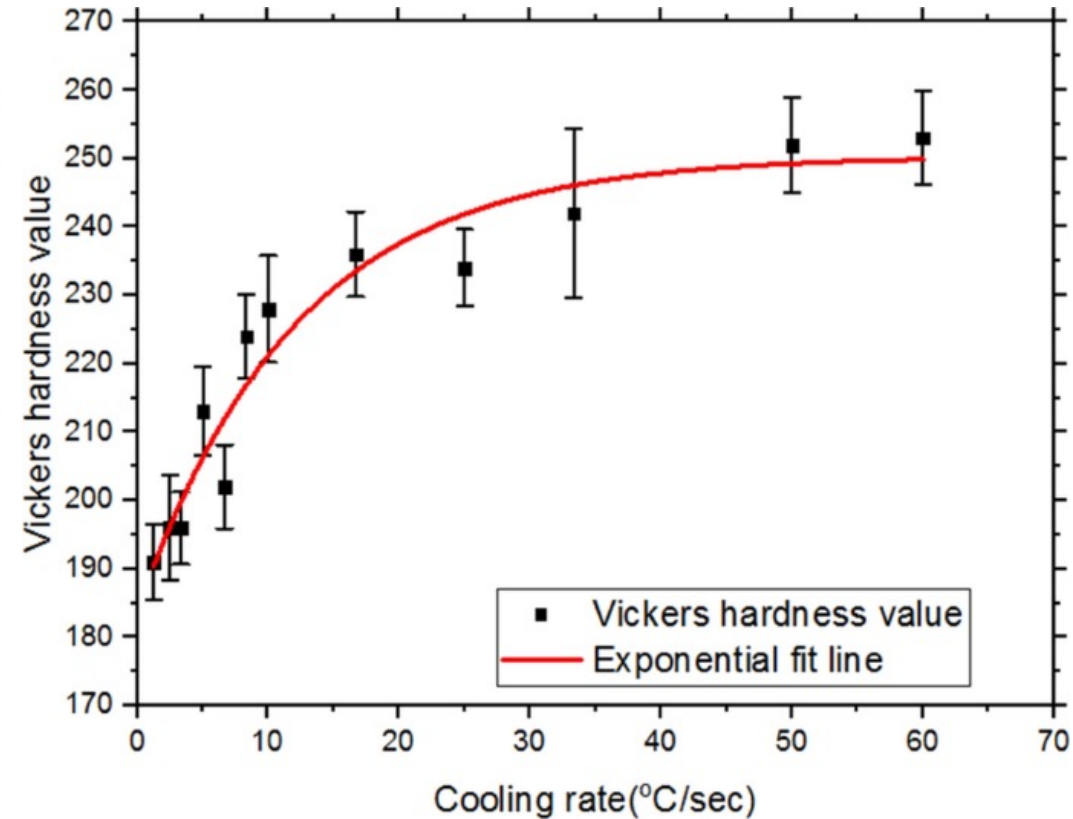
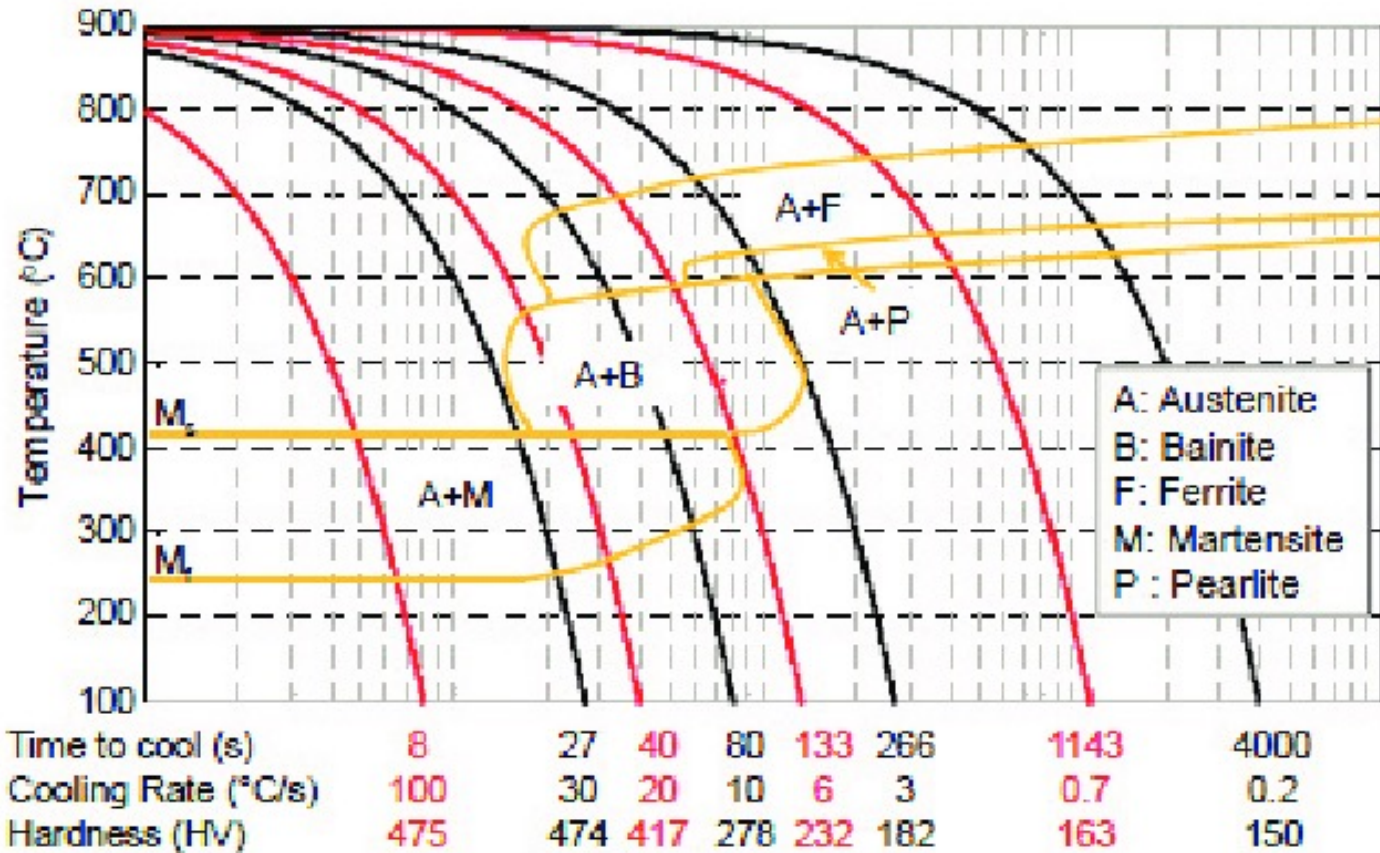
Transformation starts when the cooling curve intersects the beginning reaction curve and is completed when it crosses the completion curve.

If the cooling curve exits the region before crossing the completion curve, only some fraction of the metal transformed



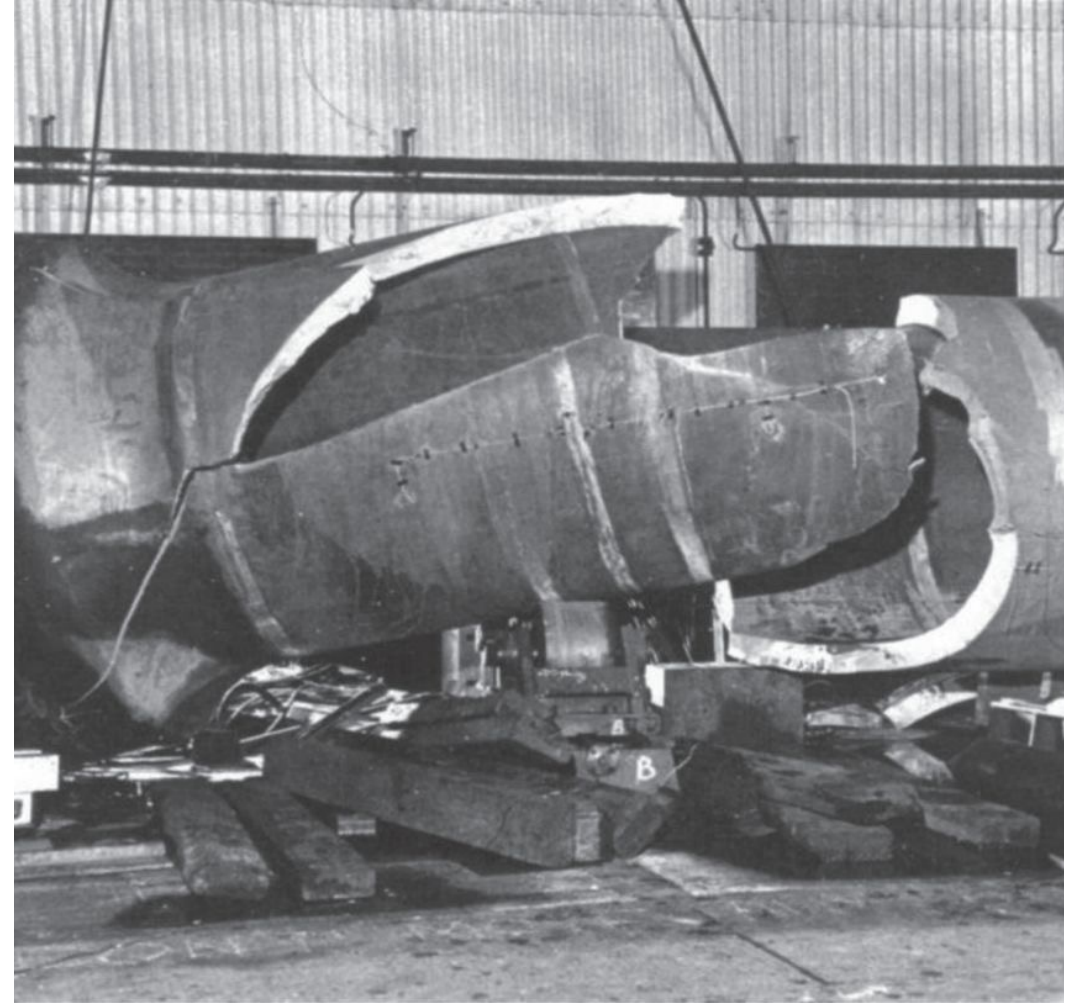
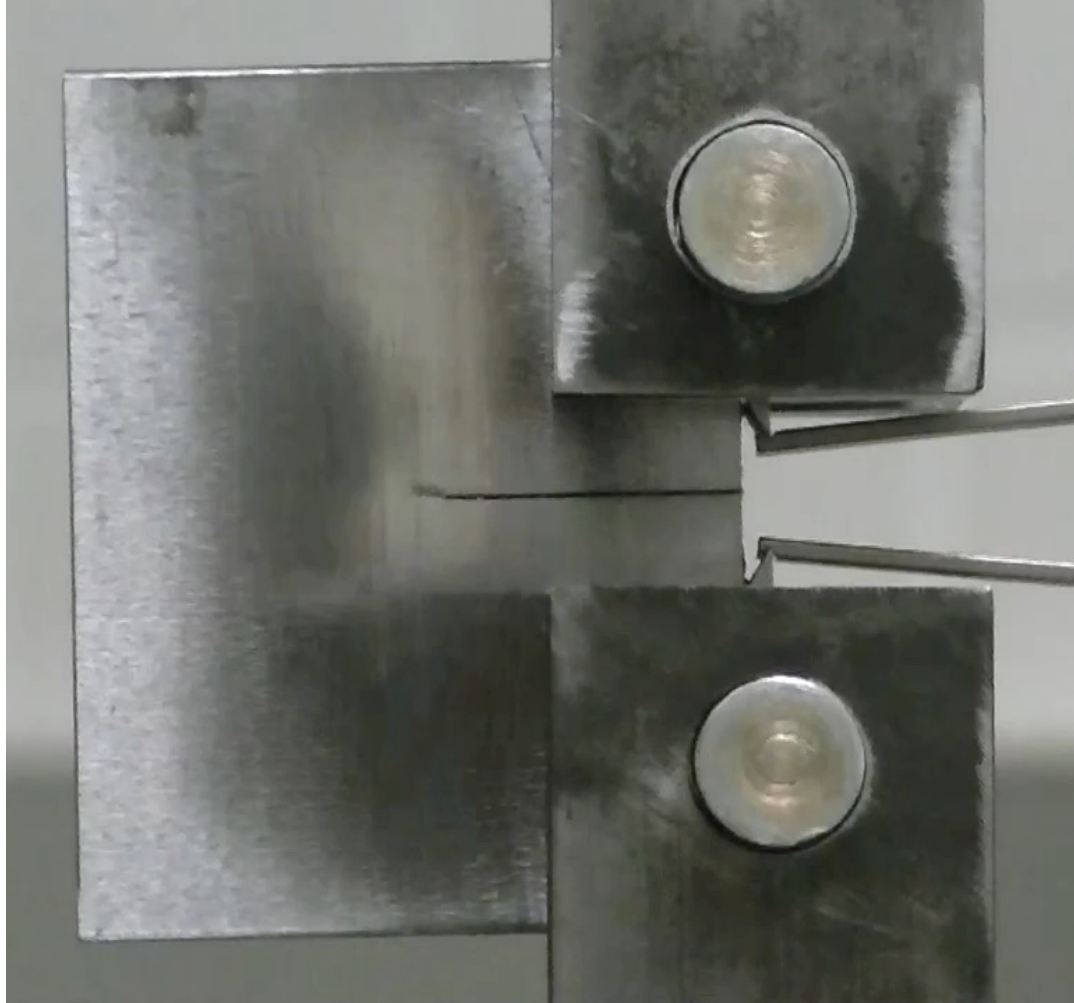


# Cooling Rate → Microstructure → Mechanical Properties



**Key takeaway:** When dealing with metals/materials, it is important to know that cooling rate plays a large role in material properties

# How do metals / materials fail? — Fracture

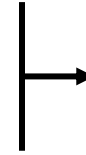


# How do metals / materials fail? — Fatigue



# Fracture

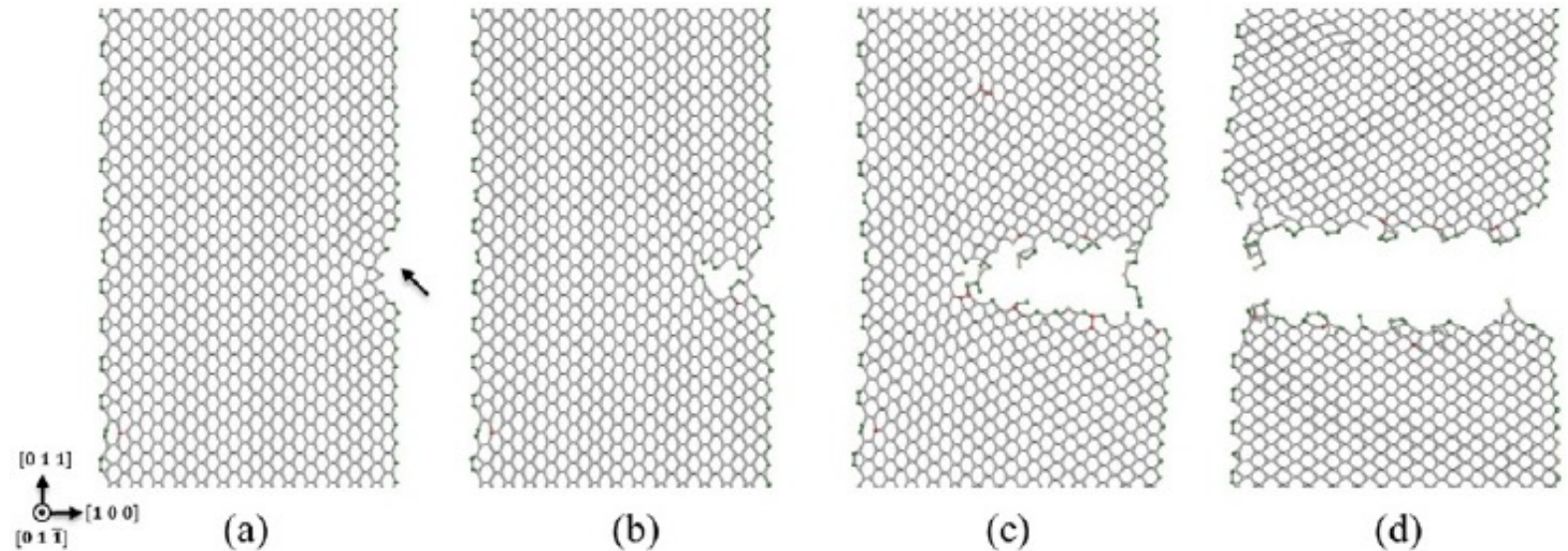
Simple definition: Separation of a body into two or more pieces in response to an imposed stress



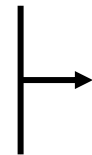
Stress can be tensile, compressive, shear, torsional, or combinations of them

Fracture involves two steps:

1. Crack formation
2. Crack propagation

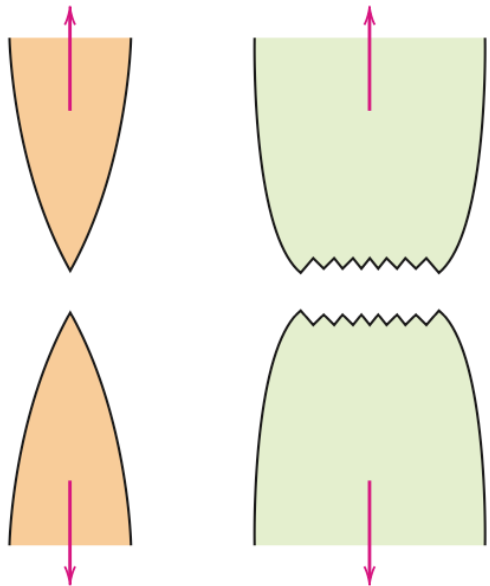
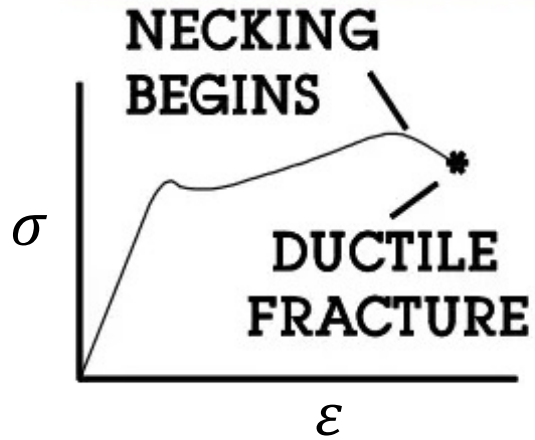


Fracture can be either **ductile** or **brittle**



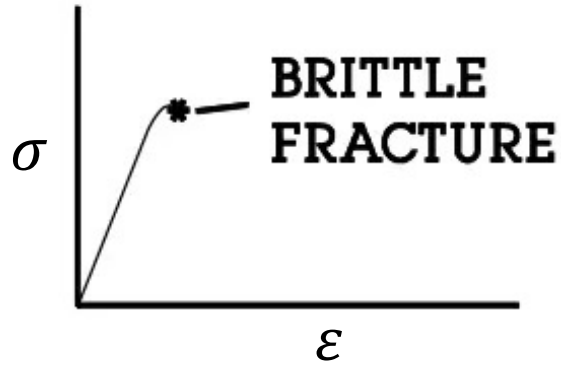
Depends on temperature, strain rate, and how the stress is being applied

# Ductile Fracture



- High energy absorption before failure
- Substantial plastic deformation in the vicinity of the growing crack
- Crack grows relatively slowly. Often called stable crack growth
- Crack growth requires additional stress to be applied
- Can be identified by deformation at fracture surface (twisting/tearing/etc.)

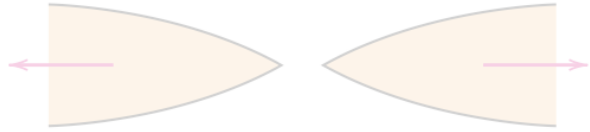
# Brittle Fracture



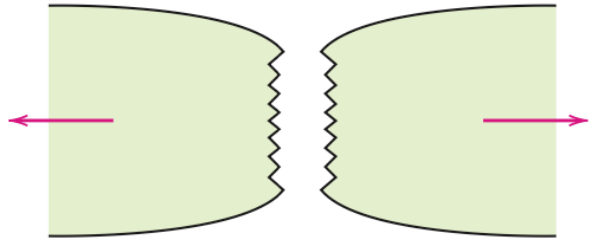
- Little energy absorption before failure
- Little to no plastic deformation in the vicinity of the growing crack
- Crack can grow rapidly. Often called unstable crack growth
- Crack growth, once started, can continue without additional stress
- Can be identified by relatively flat fracture surface

# Fracture Surfaces

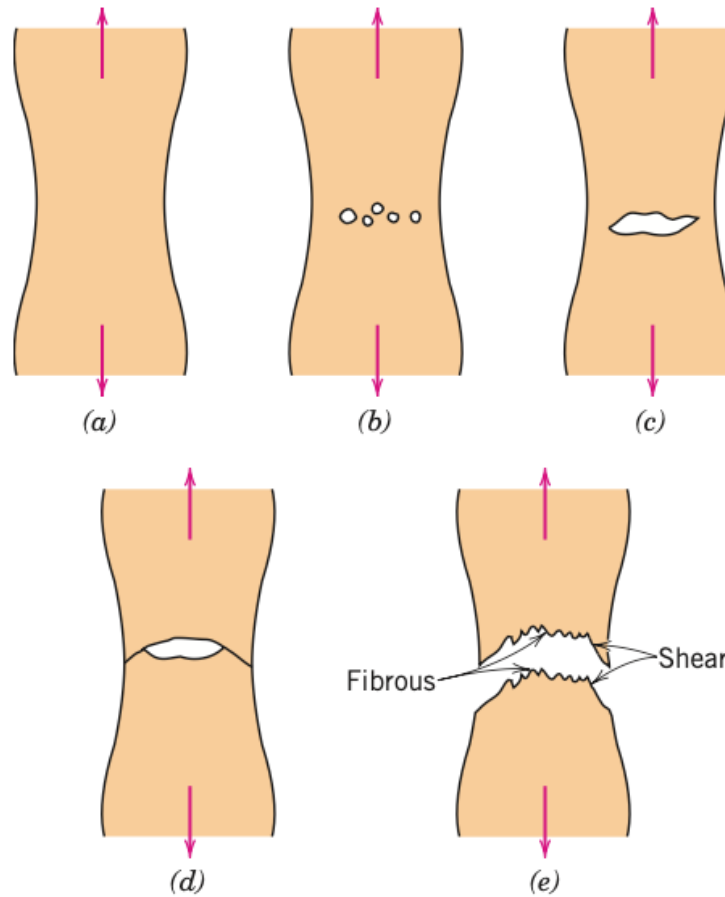
## Ductile fracture



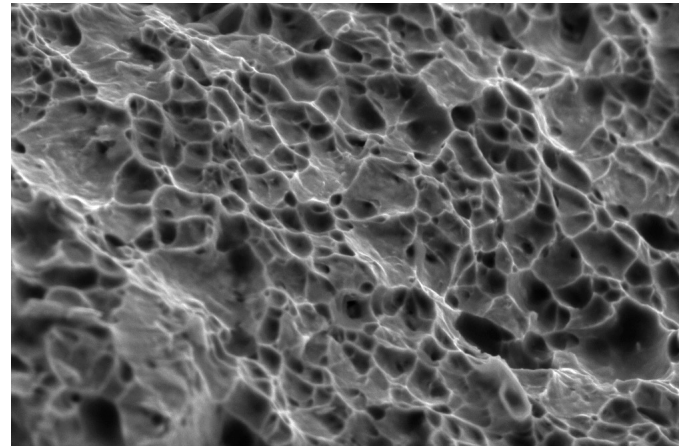
Highly ductile metals  
(Neck down to a point fracture)



Moderately ductile metals  
(Most common type of profile  
for ductile metals)



- a) Initial necking
- b) Microvoid formation
- c) Microvoids enlarge and combine to form elliptical crack
- d) Crack propagation via microvoid formation and combination
- e) Final shear fracture



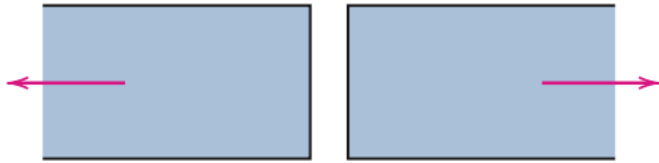
Ductile fracture surfaces often look fibrous.

Dimples come from the microvoids

# Fracture Surfaces

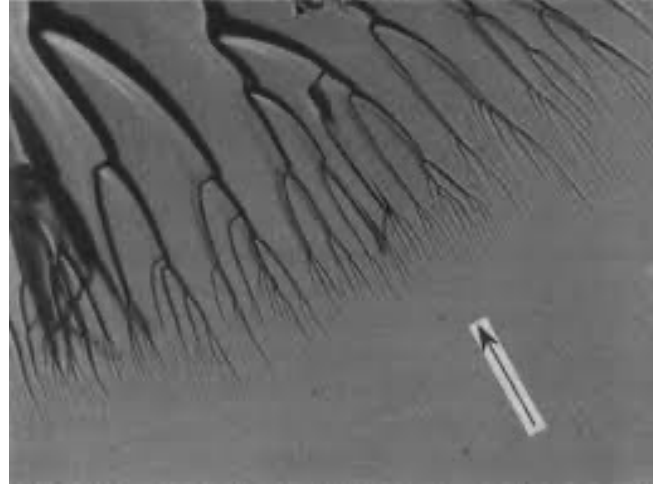
## Some features of brittle fracture surfaces

### Brittle fracture

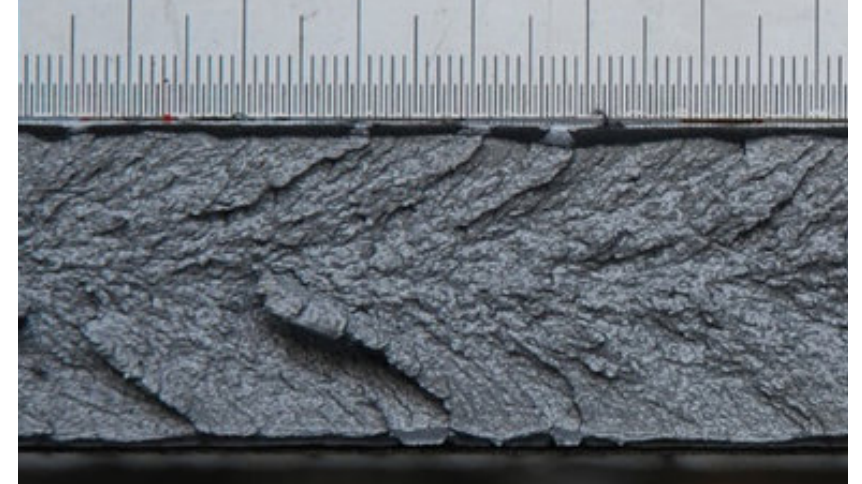


Brittle metals  
(Fracture with minimal  
plastic deformation)

### River Lines



### V-shaped lines

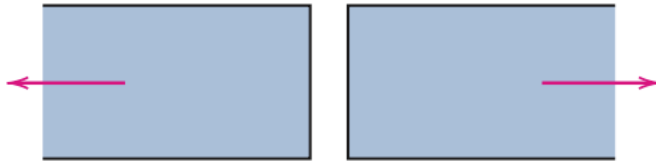


### Fan-shaped ridges



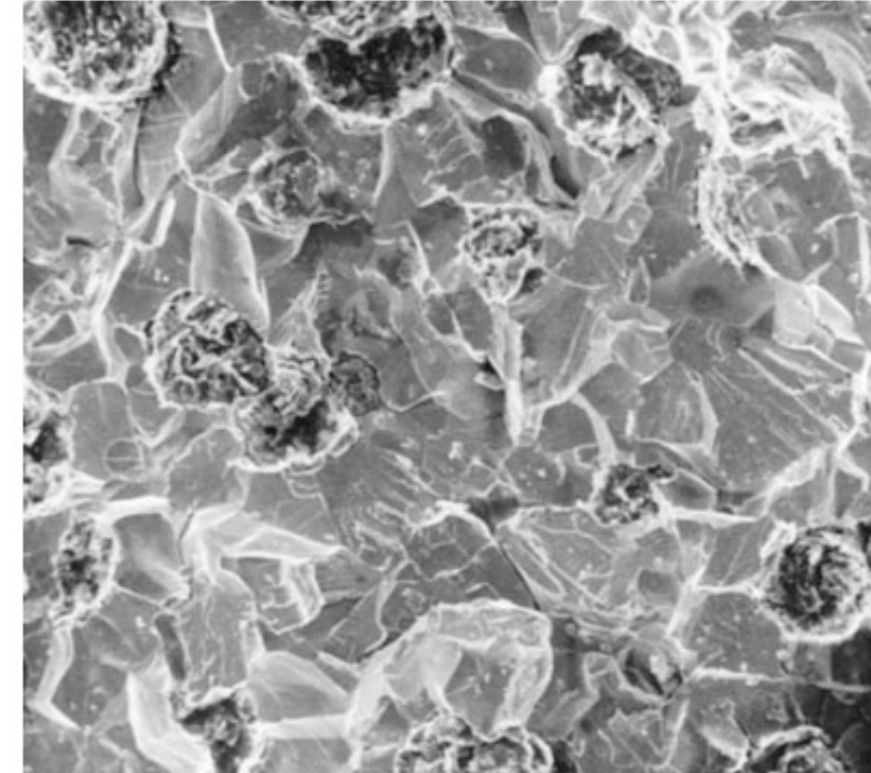
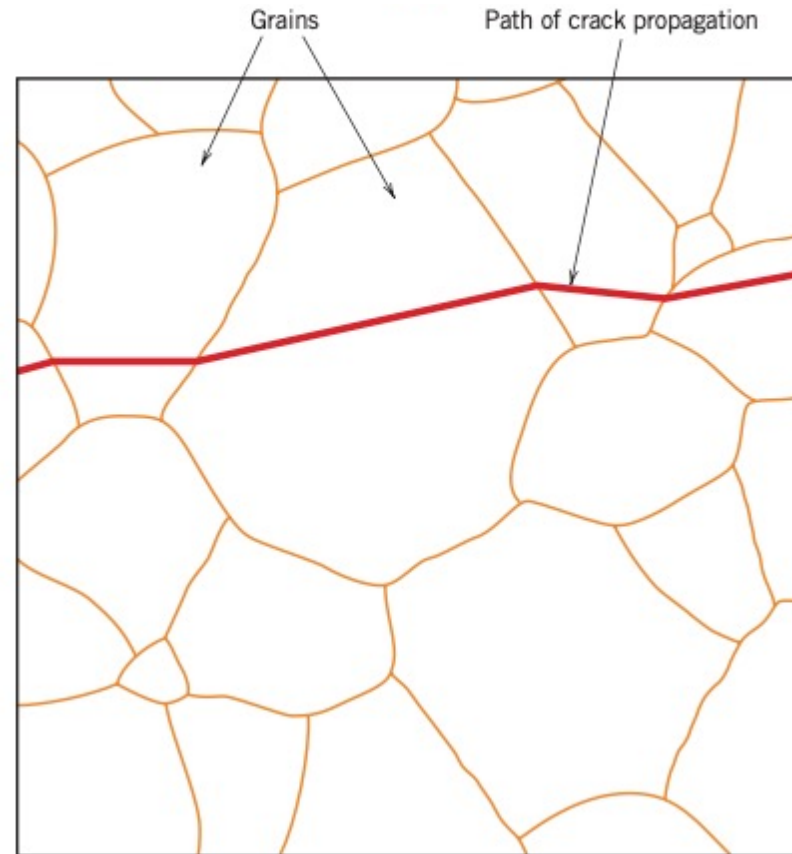
# Fracture Surfaces

## Brittle fracture



Brittle metals  
(Fracture with minimal  
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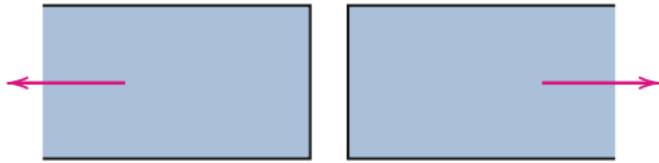
## **Transgranular** fracture: Through the grains



Crack propagates through specific planes in the crystal structure

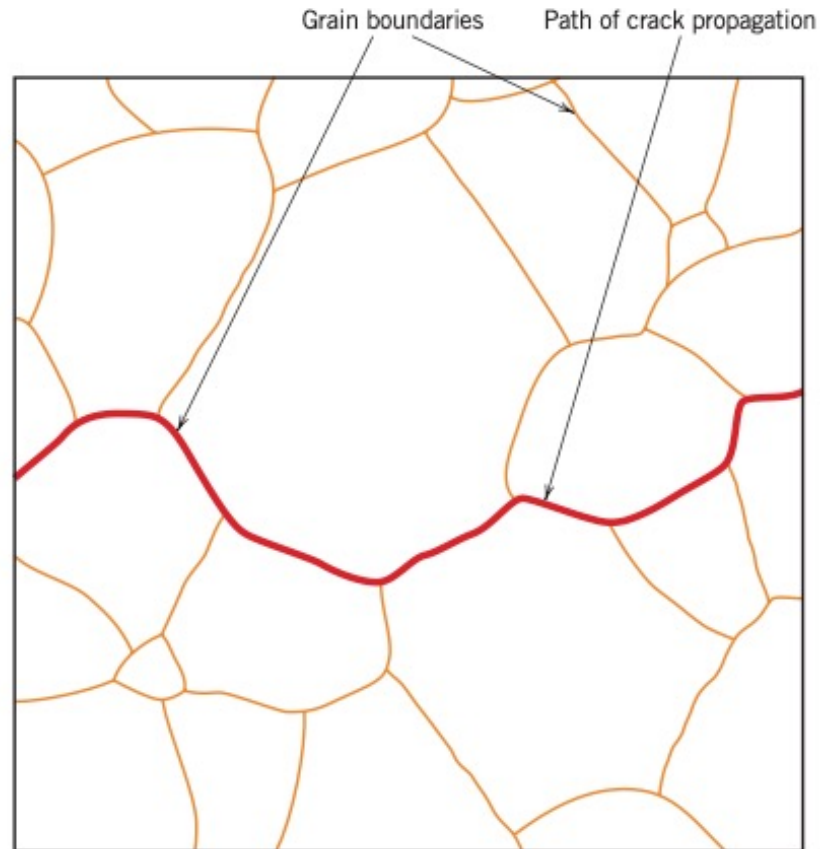
# Fracture Surfaces

## Brittle fracture



Brittle metals  
(Fracture with minimal  
plastic deformation)

## Intergranular fracture: Along the grain boundaries

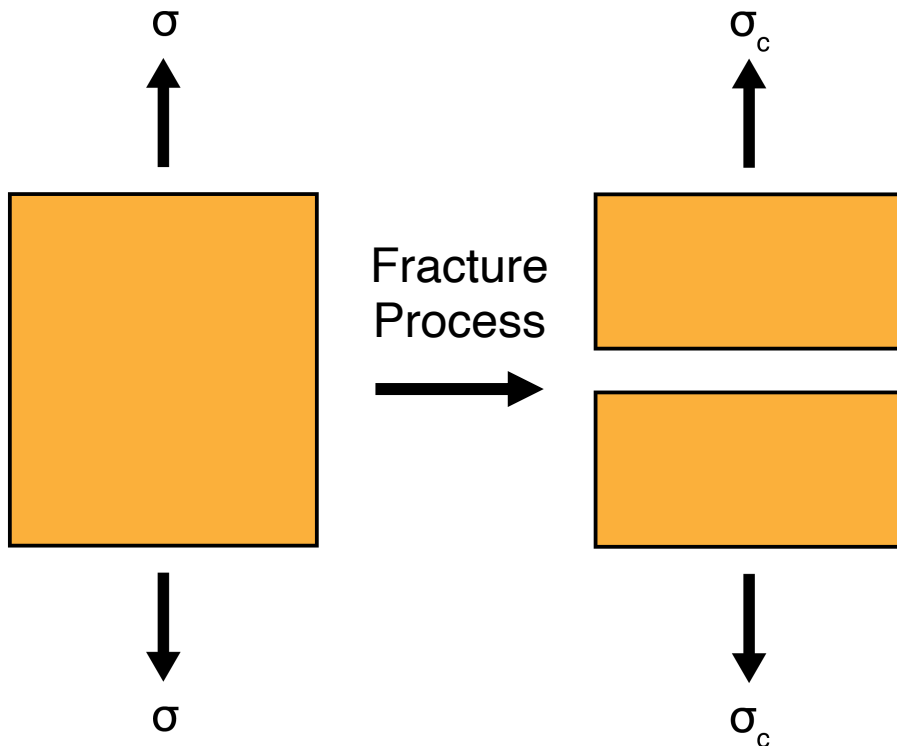


Crack propagates through weaker/brittle grain boundaries

# Introduction to Fracture Mechanics

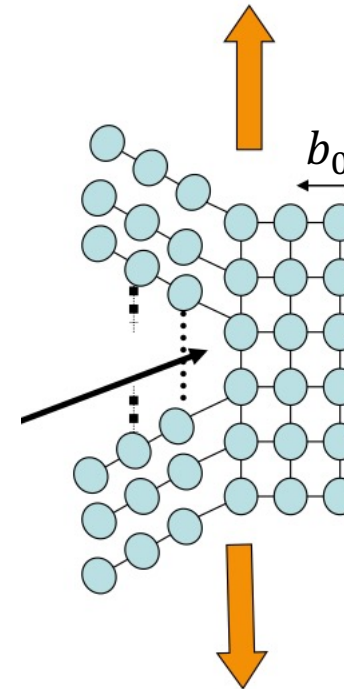
Fracture mechanics → Quantitative way to understand fracture in materials

What is the theoretical fracture strength\* of a material?



Created 2 new surfaces:

A: Surface area  
 $\gamma$ : Surface energy



Energy balance

Energy released when the crack propagated

vs

Energy created by new surface

A lot of math later...  $\sigma_c = \sqrt{\frac{E\gamma}{b_0}}$

# Introduction to Fracture Mechanics

Fracture mechanics → Quantitative way to understand fracture in materials

What is the theoretical fracture strength\* of a material?

Let's consider glass:

$$\begin{array}{l} E \sim 100 \text{ GPa} \\ b_0 \sim 5 \times 10^{-10} \text{ m} \\ \gamma \sim 1 \text{ J/m}^2 \end{array} \left| \rightarrow \right. \sigma_c = \sqrt{\frac{E\gamma}{b_0}} = E \sqrt{\frac{\gamma}{Eb_0}} \approx \frac{E}{7}$$

The theoretical strength of glass is ~ 14 GPa

In general, the theoretical strength of most materials is ~  $E/10$

Experimental strength of glass is ~ 0.01 GPa

The experimental strength of most materials is ~  $E/10000$  to  $E/1000$

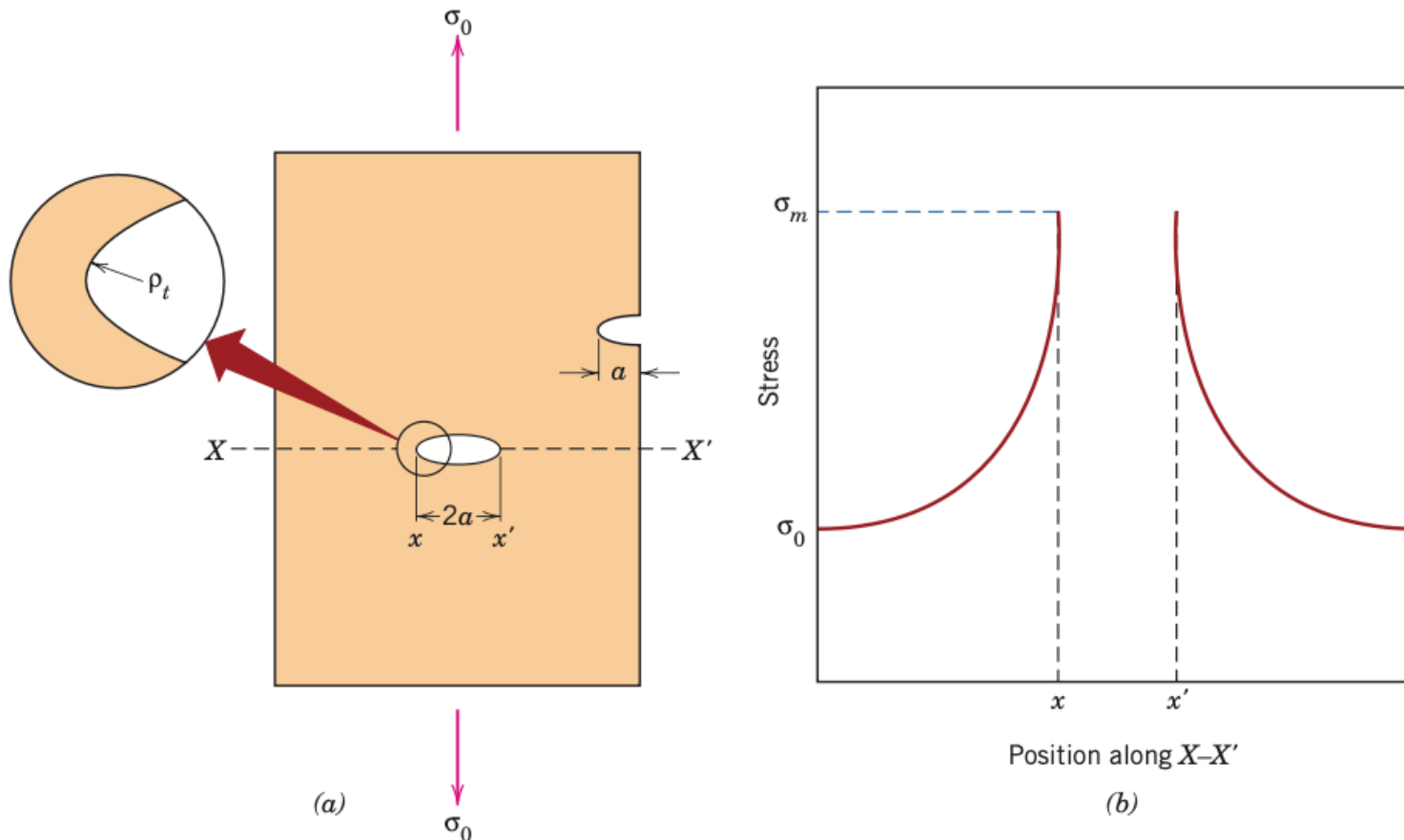


**Stress concentration** at the crack tip is not accounted for!

\*Different from when we looked at dislocations, that was in shear. This is in tension

# Stress concentration

Consider an elliptical crack of width  $2a$  in the center of a body subjected to an applied stress  $\sigma_0$



Magnitude of localized stress decreases with distance away from the crack tip

Far from the crack tip, the stress is just the applied stress  $\sigma_0$

**Maximum stress** at the crack tip

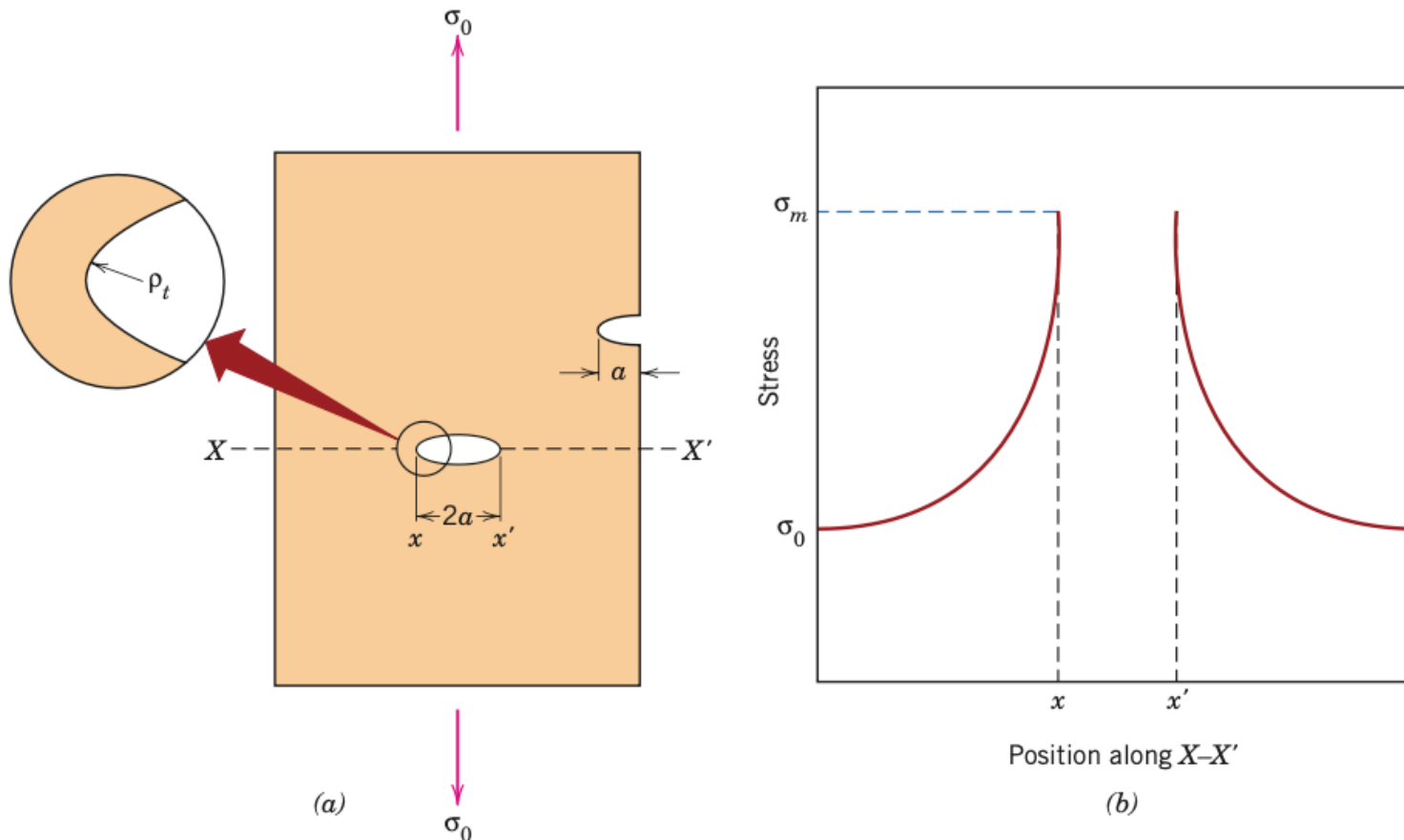
$$\sigma_m = 2\sigma_0 \sqrt{\frac{a}{\rho_t}}$$

$a$ : length of surface crack or half the length of an internal crack

$\rho_t$ : the radius of curvature of the crack tip

# Stress concentration

Consider an elliptical crack of width  $2a$  in the center of a body subjected to an applied stress  $\sigma_0$



**Maximum stress** at the crack tip

$$\sigma_m = 2\sigma_0 \sqrt{\frac{a}{\rho_t}}$$

$a$ : length of surface crack or half the length of an internal crack

$\rho_t$ : the radius of curvature of the crack tip

**Stress concentration factor  $K_t$ :**

$$K_t = \frac{\sigma_m}{\sigma_0} = 2 \sqrt{\frac{a}{\rho_t}}$$

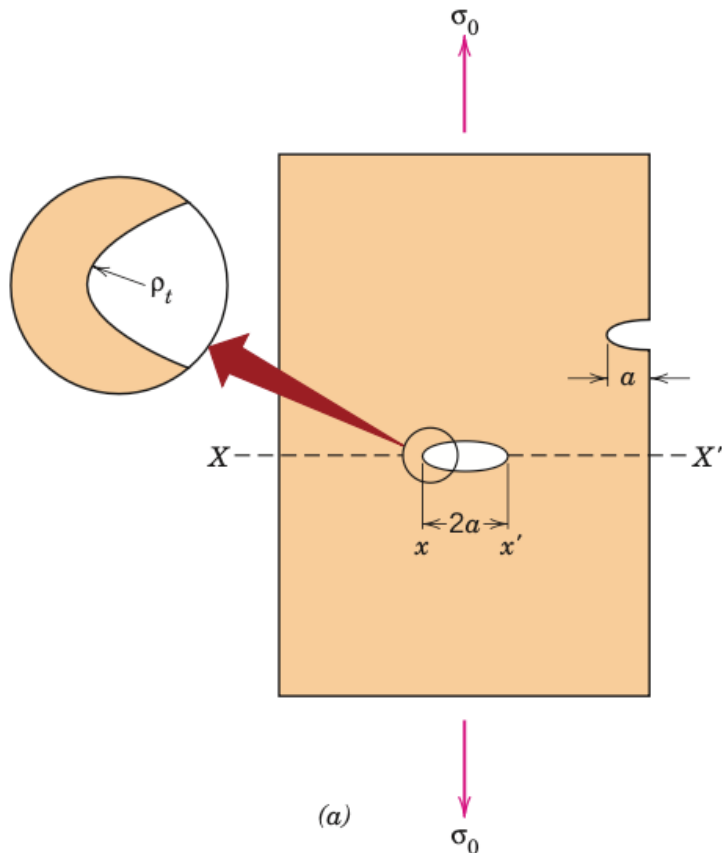
For a long crack with small crack tip radius,  $K_t$  can be a large value!

# Griffith Theory of Brittle Fracture

When a crack propagates:

Release in elastic strain energy (decrease energy)

Create new surfaces (increase energy)



Griffith in 1921

Crack grows under a constant applied stress  $\sigma_c$  when the decrease in elastic strain energy = Energy required to form new surfaces



No change in total energy of the system with crack growth

**Surface energy ( $U_s$ )**

$$U_s = 4a\gamma$$

$a$ : Crack length

$\gamma$ : Surface energy

2 surfaces created!

**Elastic strain energy\*  $U_{SE}$**

$$U_{SE} = -\frac{\pi\sigma^2 a^2}{E}$$

$\sigma$ : Applied stress

**Change in potential energy from crack growth ( $\Delta U$ )**

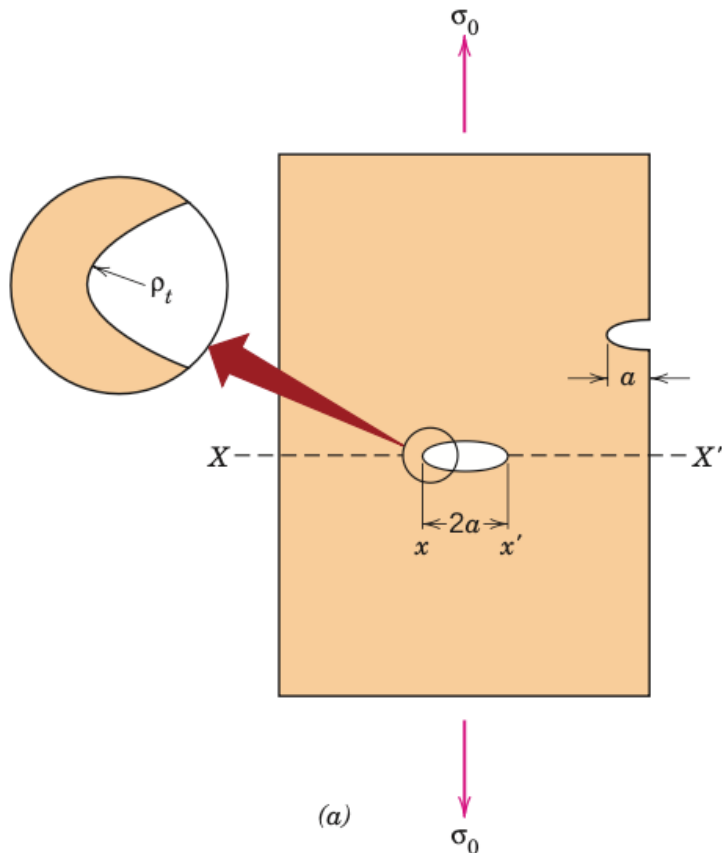
$$\begin{aligned} \Delta U &= U_s + U_{SE} \\ &= 4a\gamma - \frac{\pi\sigma^2 a^2}{E} \end{aligned}$$

# Griffith Theory of Brittle Fracture

When a crack propagates:

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Griffith in 1921

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No change in total energy of the system with crack growth

**Griffith's criterion for crack growth**

$$\frac{d\Delta U}{da} = \frac{d}{da} \left( 4a\gamma - \frac{\pi\sigma_c^2 a^2}{E} \right) = 0$$

$$4\gamma - \frac{2\pi\sigma_c^2 a}{E} = 0$$

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

For a crack of length  $a$ ,  $\sigma_c$  is the stress required for crack propagation

# Griffith Theory of Brittle Fracture

When a crack propagates:

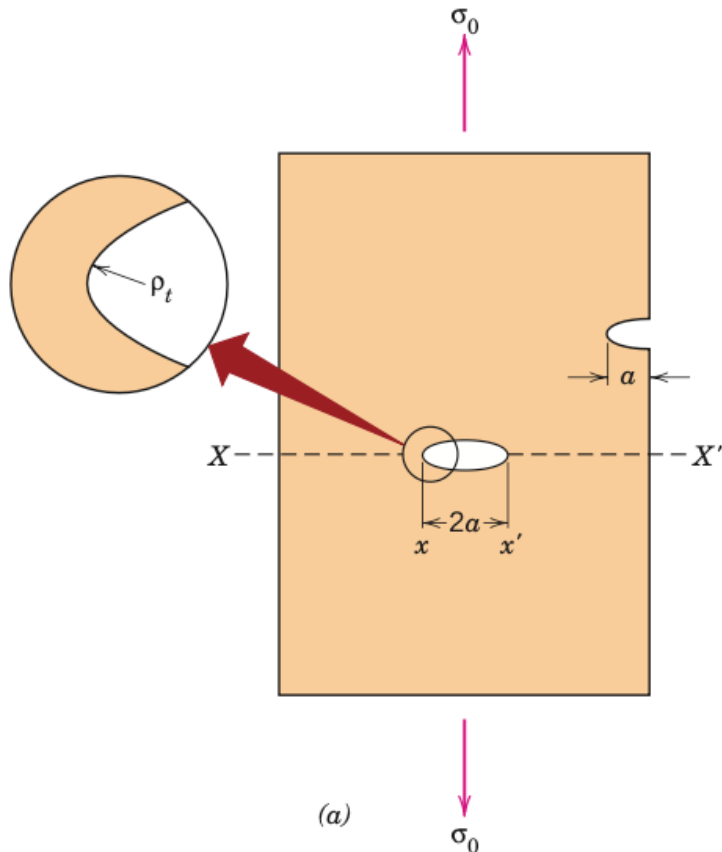
Release in elastic strain energy (decrease energy)

Create new surfaces (increase energy)

Griffith in 1921

Crack grows under a constant applied stress  $\sigma_c$  when the decrease in elastic strain energy = Energy required to form new surfaces

↓  
No change in total energy of the system with crack growth



Let's look at glass again

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

$E \sim 100 \text{ GPa}$  ( $100E9 \text{ J/m}^3$ )

$\gamma \sim 1 \text{ J/m}^2$

$\sigma_c \sim 0.01 \text{ GPa}$

If you do the math,  $a \sim 600 \mu\text{m}$

**For a crack of length  $\sim 600 \mu\text{m}$   
only 0.01 GPa of stress is  
needed for brittle fracture!**

# Fracture Toughness

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

$$\sigma_c \sqrt{\pi a} = \sqrt{2E\gamma} \rightarrow \text{Material properties}$$

$$\sigma_c \sqrt{\pi a} = K_c \rightarrow \text{Critical stress intensity factor or Fracture Toughness}$$

More generally:

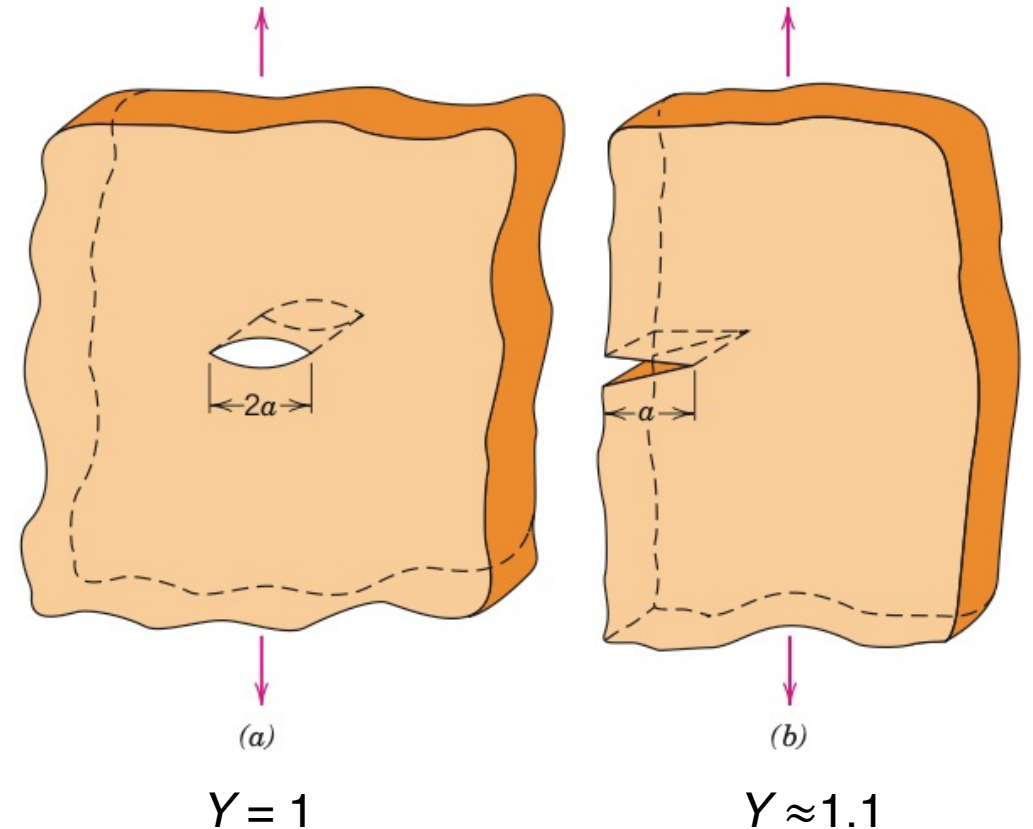
$$K_c = Y \sigma_c \sqrt{\pi a}$$

$Y$ : Dimensionless geometric constant

$K_c$  has units of  $\text{MPa} \sqrt{m}$

$Y$  depends on the crack, specimen, and loading conditions

For cracks that are much shorter than the sample width,  $Y$  is  $\sim 1$



$Y$  can be very complex!

# Fracture Toughness

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

$$\sigma_c \sqrt{\pi a} = \sqrt{2E\gamma} \rightarrow \text{Material properties}$$

$$\sigma_c \sqrt{\pi a} = K_c \rightarrow \text{Critical stress intensity factor or Fracture Toughness}$$

More generally:

$$K_c = Y \sigma_c \sqrt{\pi a}$$

Y: Dimensionless geometric constant

$K_c$  has units of MPa  $\sqrt{m}$

## Why is $K_c$ important?

If you know  $K_c$  and  $a$ , you can solve for stress needed to cause fracture ( $\sigma_c$ )

If you know  $K_c$  and the stress in the system ( $\sigma_c$ ), you can solve for the maximum crack size that can be tolerated before fracture ( $a$ )

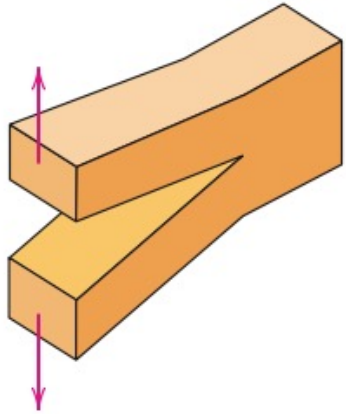
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Note 1:  $K_c$  is independent of sample thickness only when sample thickness is much greater than crack dimensions

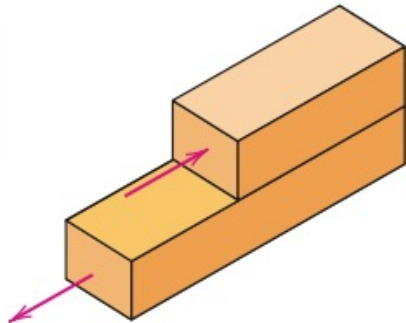
Note 2: The constant value  $K_c$  is the one that is most often reported. It is called the *plane strain fracture toughness*

# There are different kinds of Fracture Toughness

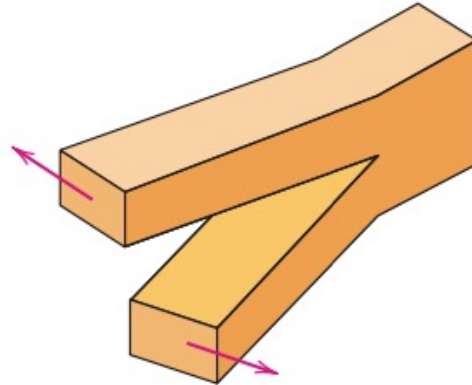
**Mode I**  
(Opening)



**Mode II**  
(In-plane shear)

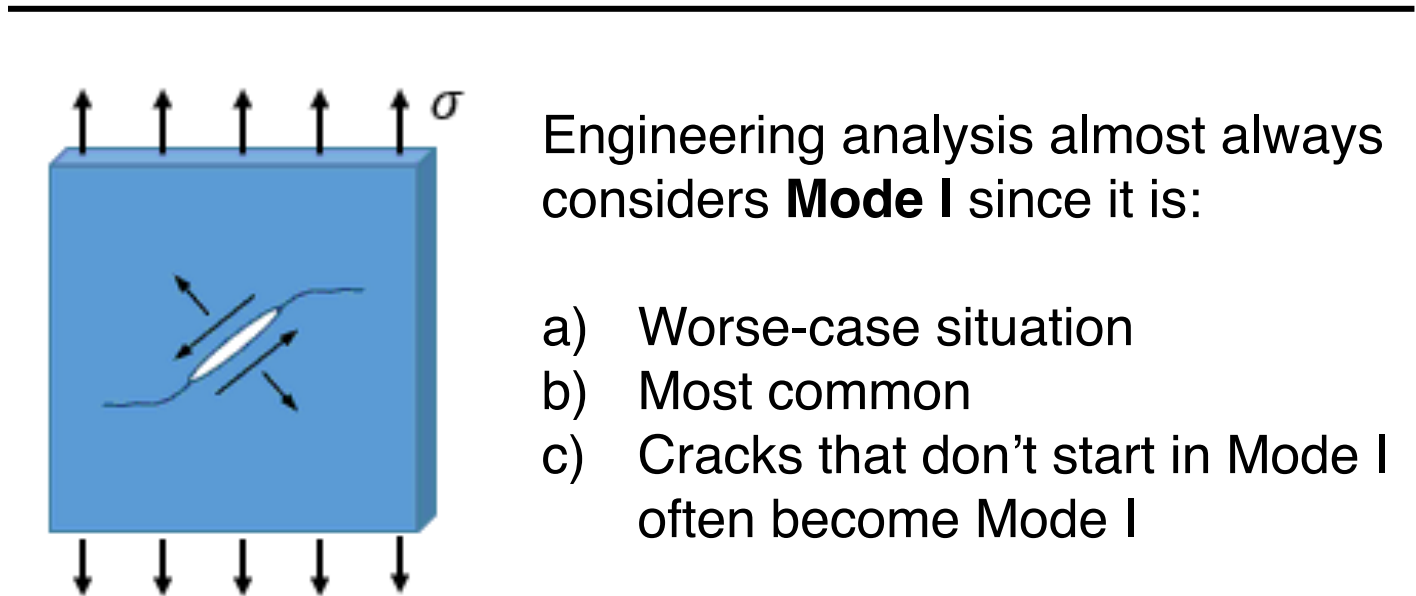


**Mode III**  
(Out-of-plane shear)



Each loading mode has its own associated fracture toughness value!

$$K_{Ic}, K_{IIc}, K_{IIIc}$$



Engineering analysis almost always considers **Mode I** since it is:

- a) Worse-case situation
- b) Most common
- c) Cracks that don't start in Mode I often become Mode I

*Material*

*$K_{Ic}$*   
*MPa√m*

Aluminum alloy <sup>a</sup> (7075-T651)	24
Aluminum alloy <sup>a</sup> (2024-T3)	44
Titanium alloy <sup>a</sup> (Ti-6Al-4V)	55
Alloy steel <sup>a</sup> (4340 tempered @ 260°C)	50.0
Alloy steel <sup>a</sup> (4340 tempered @ 425°C)	87.4
Polystyrene (PS)	0.7–1.1
Poly(methyl methacrylate) (PMMA)	0.7–1.6
Polycarbonate (PC)	2.2

# Let's look at an example of how to use $K_{IC}$

An aircraft component is made from an aluminum alloy with a  $K_{IC}$  of  $40 \text{ MPa}\sqrt{m}$

Fracture results at a stress of 300 MPa when the critical internal crack length is 4.0 mm

For this component and alloy, will fracture occur at a stress of 260 MPa with an internal crack length of 6.0 mm?

Why or why not?

Initial information:

$$K_{IC} = 40 \text{ MPa}\sqrt{m}$$

$$\sigma = 300 \text{ MPa}$$

$$2a = 4.0 \text{ mm}$$

$$K_{IC} = Y\sigma_c\sqrt{\pi a}$$

Normally assume  $Y = 1$ , but in this case, we have to solve for  $Y$

$$Y = \frac{40 \text{ MPa}\sqrt{m}}{300 \text{ MPa} \sqrt{\pi \cdot 0.002}}$$
$$= 1.682$$

Scenario:

$$K_{IC} = 40 \text{ MPa}\sqrt{m}$$

$$\sigma = 260 \text{ MPa}$$

$$Y = 1.682$$

$$K_{IC} = Y\sigma\sqrt{\pi a_c}$$

$$a_c = 2.3 \text{ mm} < 3.0 \text{ mm}$$

At these conditions, the component will fail at a crack length of 2.3 mm. Since the crack length is 3.0 mm, **fracture will occur.**

# Let's look at an example of how to use $K_{IC}$

An aircraft component is made from an aluminum alloy with a  $K_{IC}$  of  $40 \text{ MPa}\sqrt{\text{m}}$

Fracture results at a stress of  $300 \text{ MPa}$  when the critical internal crack length is  $4.0 \text{ mm}$

For this component and alloy, will fracture occur at a stress of  $260 \text{ MPa}$  with an internal crack length of  $6.0 \text{ mm}$ ?

Why or why not?

Initial information:

$$K_{IC} = 40 \text{ MPa}\sqrt{\text{m}}$$

$$\sigma = 300 \text{ MPa}$$

$$2a = 4.0 \text{ mm}$$

$$K_{IC} = Y\sigma_c\sqrt{\pi a}$$

Normally assume  $Y = 1$ , but in this case, we have to solve for  $Y$

$$Y = \frac{40 \text{ MPa}\sqrt{\text{m}}}{300 \text{ MPa} \sqrt{\pi \cdot 0.002}} = 1.682$$

Scenario:

$$2a = 6.0 \text{ mm}$$

$$\sigma = 260 \text{ MPa}$$

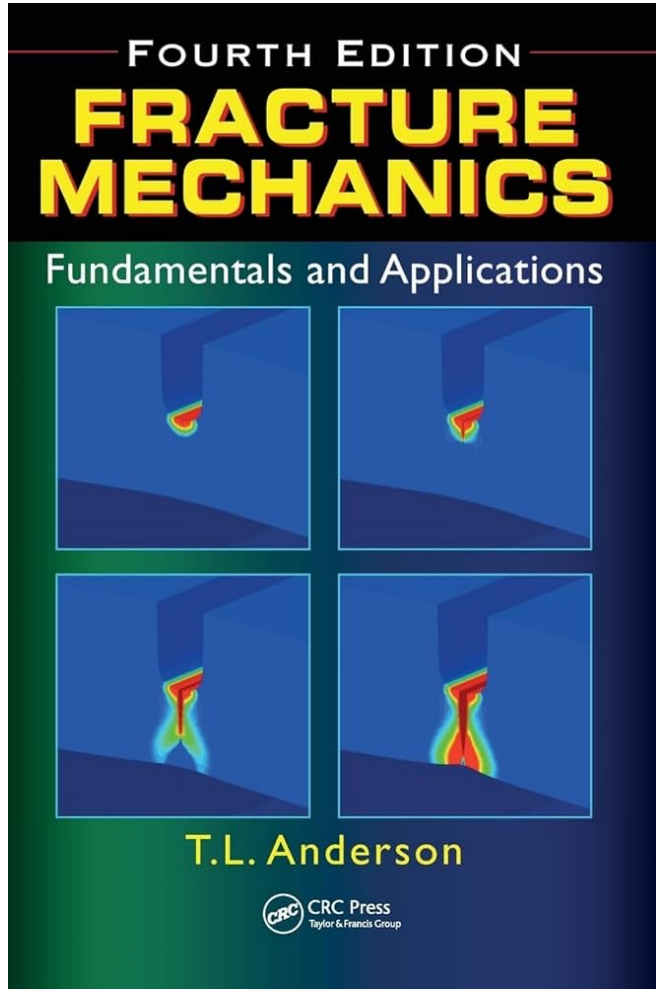
$$Y = 1.682$$

$$K = Y\sigma\sqrt{\pi a}$$

$$K = 42.44 \text{ MPa}\sqrt{\text{m}}$$

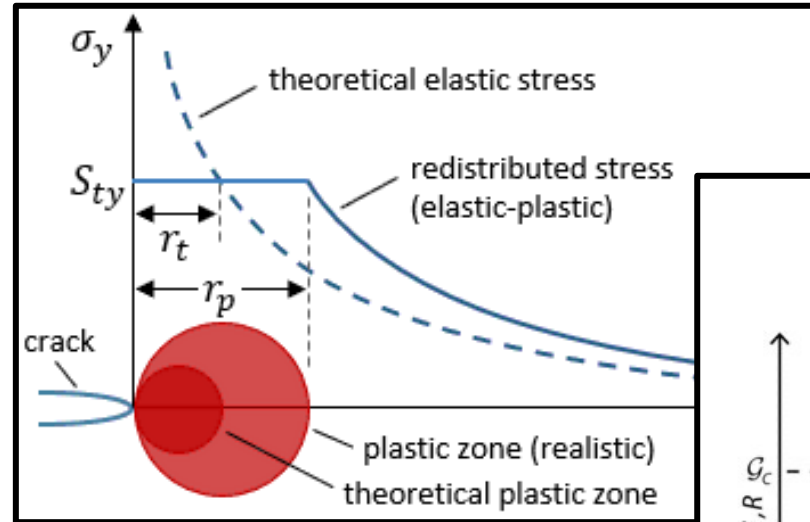
As the stress intensity factor ( $K$ ) is larger than the fracture toughness of the material ( $K_{IC}$ ), **fracture will occur.**

# We have only just scratched the surface of fracture mechanics

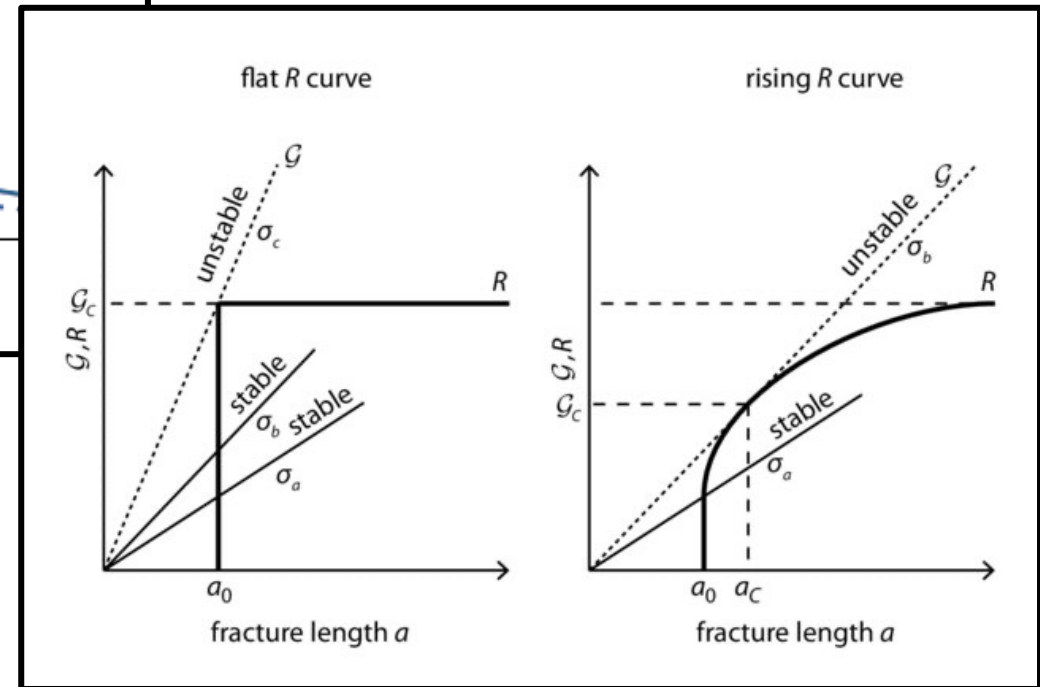


Whole field by itself!

## Plasticity at crack tip



## Crack growth resistance

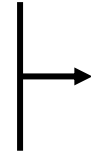


Highly complex topic, but necessary for any material application

Typically a graduate level course

# Fatigue

Form of fracture in structures subjected to dynamic and fluctuating stresses

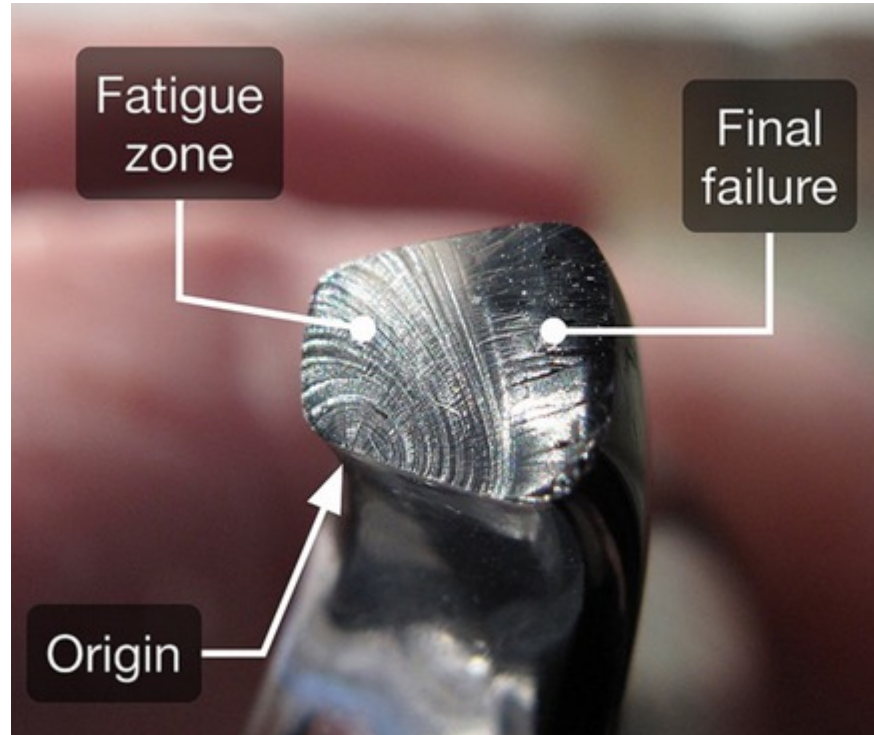


Here, fracture can occur at a stress level **much lower** than the tensile or yield strength

Fatigue fracture is often sudden and without warning

Fatigue fracture is brittle-like even in normally ductile metals

Little to no plastic deformation associated with fracture

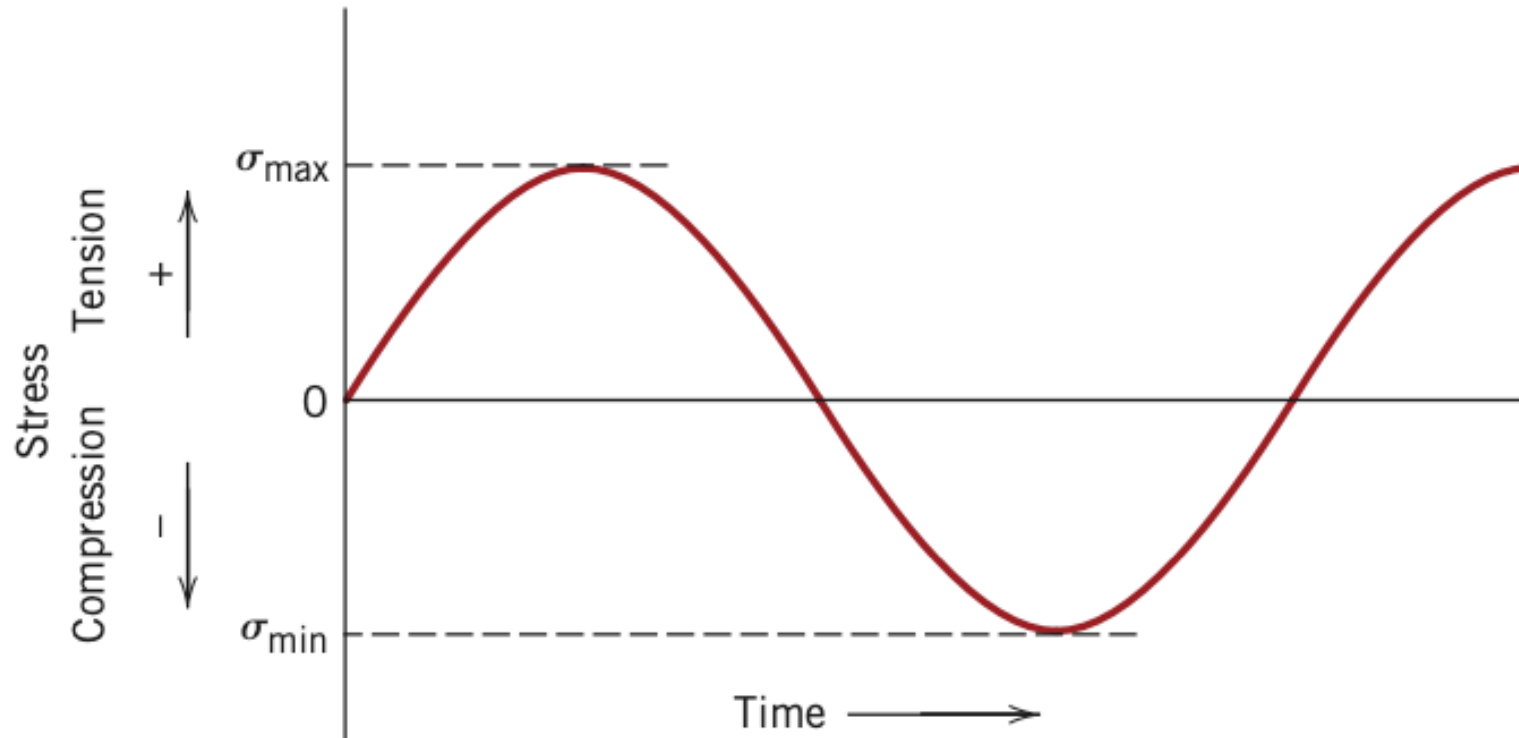


Fatigue fracture looks like brittle failure!

# Fatigue involves cyclic stresses

The applied stress may be axial, flexural, torsional, or combinations of them

In general, three different varying stress-time modes are possible:



## Reversed stress cycle

Regular and sinusoidal

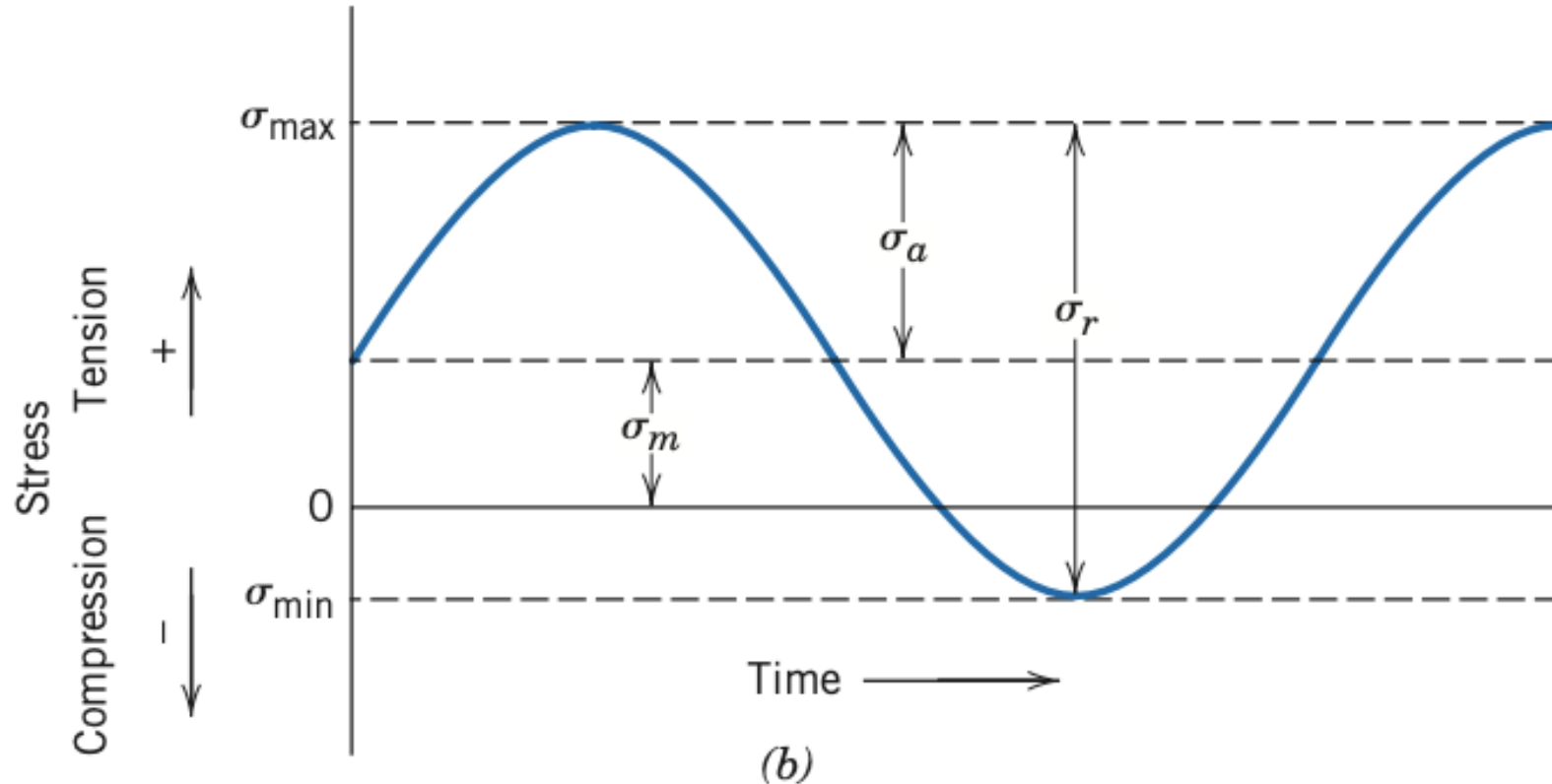
Amplitude is symmetrical about a mean zero stress level

Alternative from maximum tensile to minimum compression stress of equal magnitude

# Fatigue involves cyclic stresses

The applied stress may be axial, flexural, torsional, or combinations of them

In general, three different varying stress-time modes are possible:



Repeated stress cycle

Regular and sinusoidal

Maxima and minima are asymmetrical relative to the zero stress level

Symmetrical about the mean stress  $\sigma_m$

$\sigma_m$  = mean stress

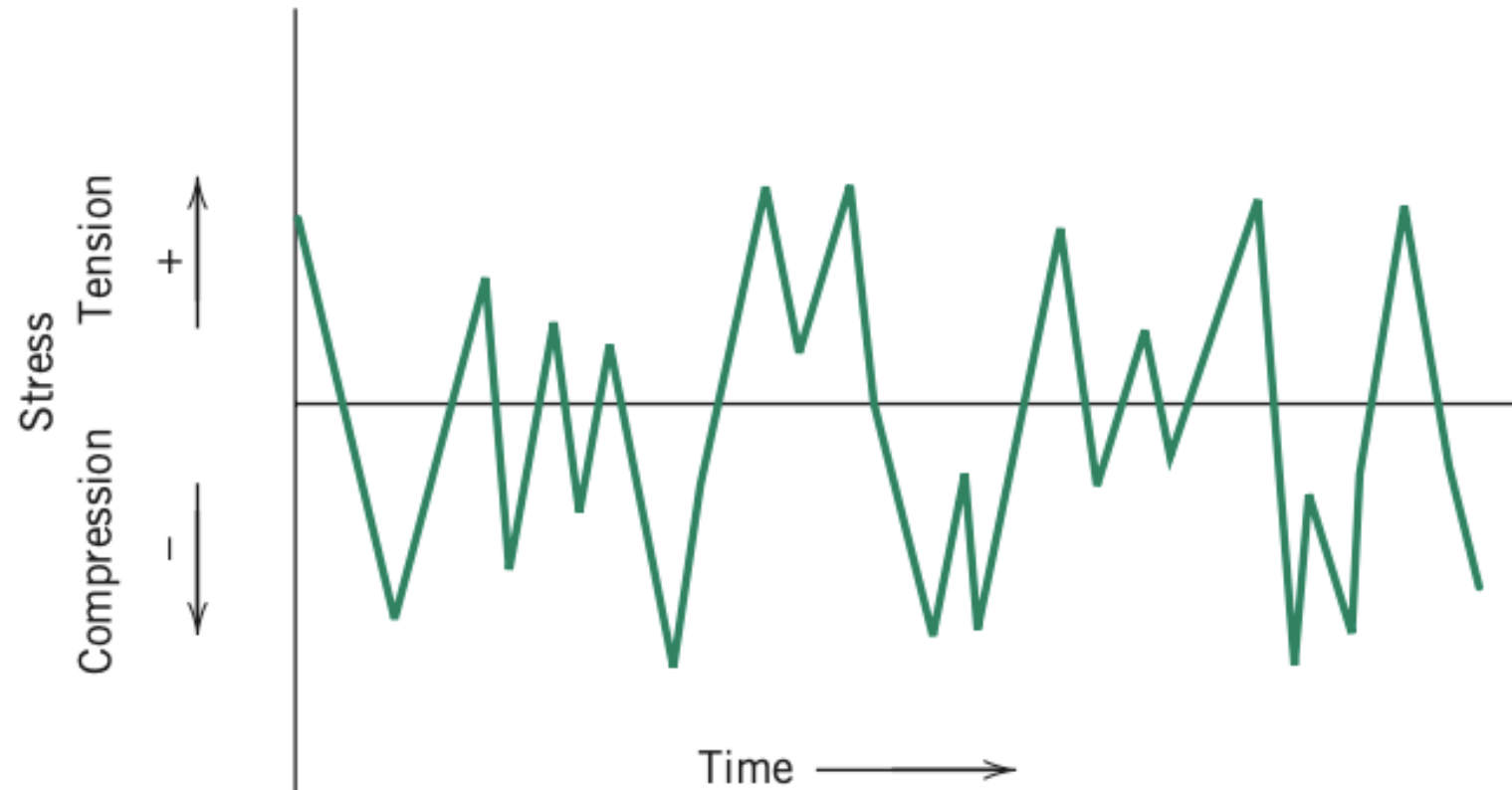
$\sigma_r$  = range of stress

$\sigma_a$  = stress amplitude

# Fatigue involves cyclic stresses

The applied stress may be axial, flexural, torsional, or combinations of them

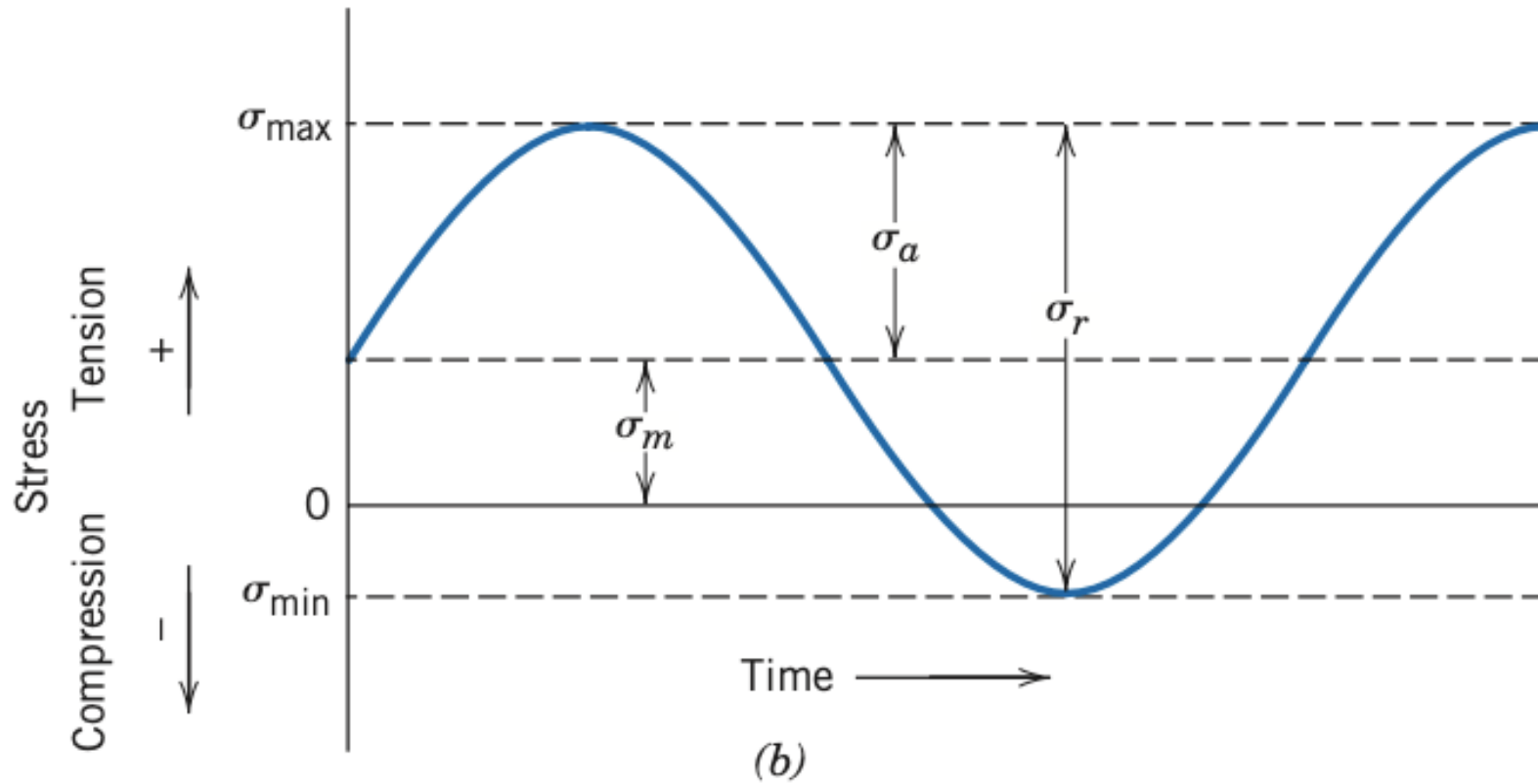
In general, three different varying stress-time modes are possible:



Random stress cycle

Stress level varies in amplitude and frequency

# Parameters to characterize a stress cycle



$\sigma_m$  = mean stress  
 $\sigma_r$  = range of stress  
 $\sigma_a$  = stress amplitude

Mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Range of stress

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Stress amplitude

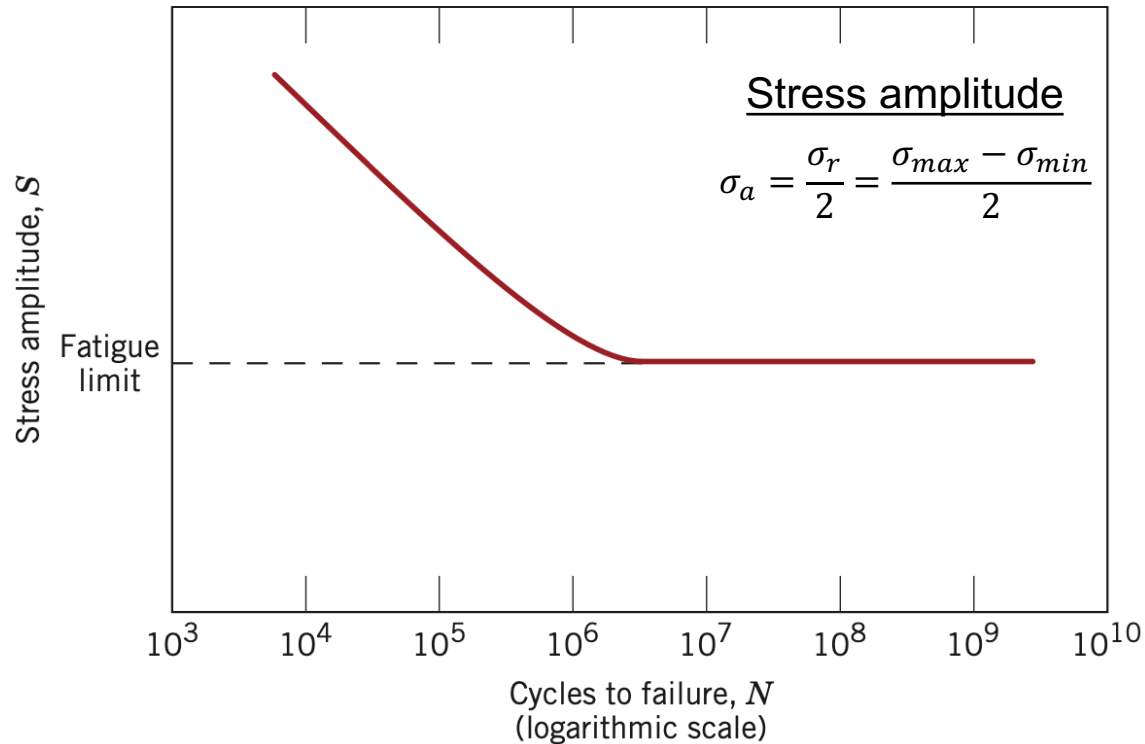
$$\sigma_a = \frac{\sigma_r}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Stress ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

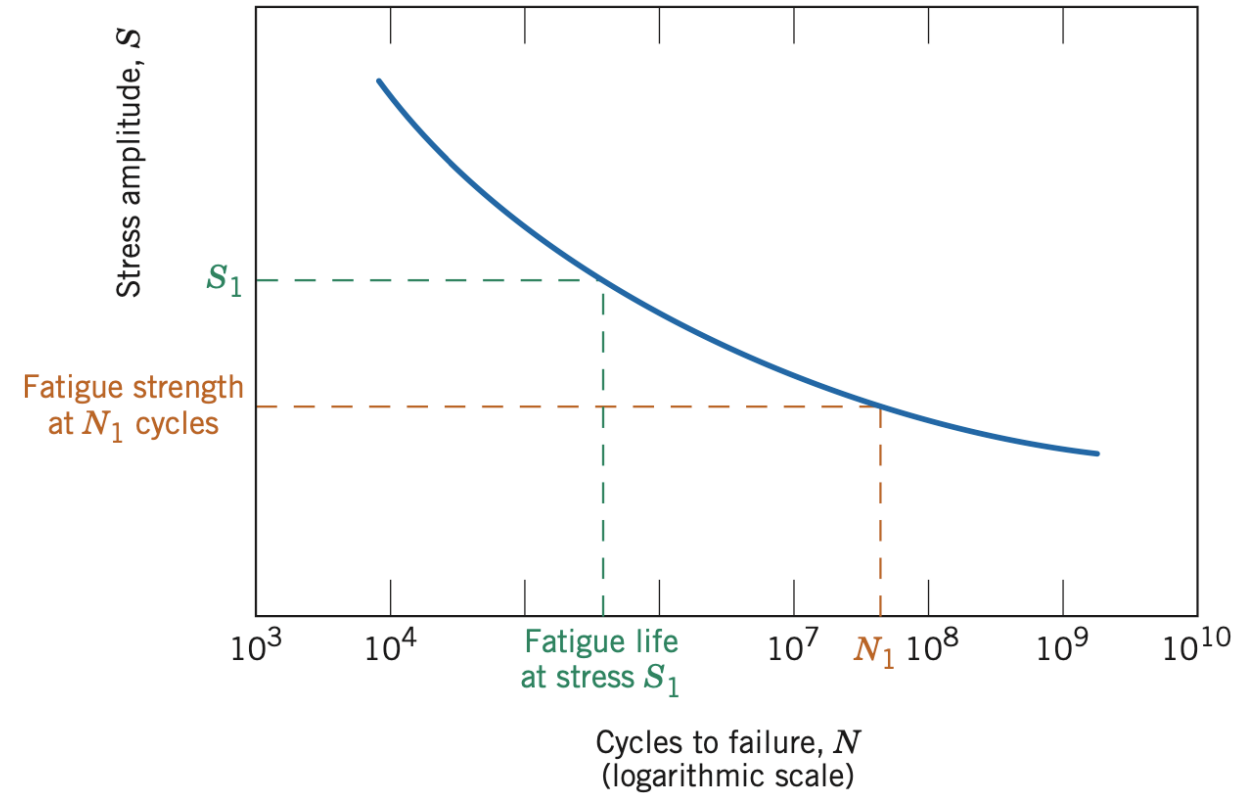
# S-N Curve (Applied Stress vs. Number of cycles to failure)

Possesses a fatigue limit



No fatigue fracture below the fatigue limit  
(Seen in some ferrous and titanium alloys)

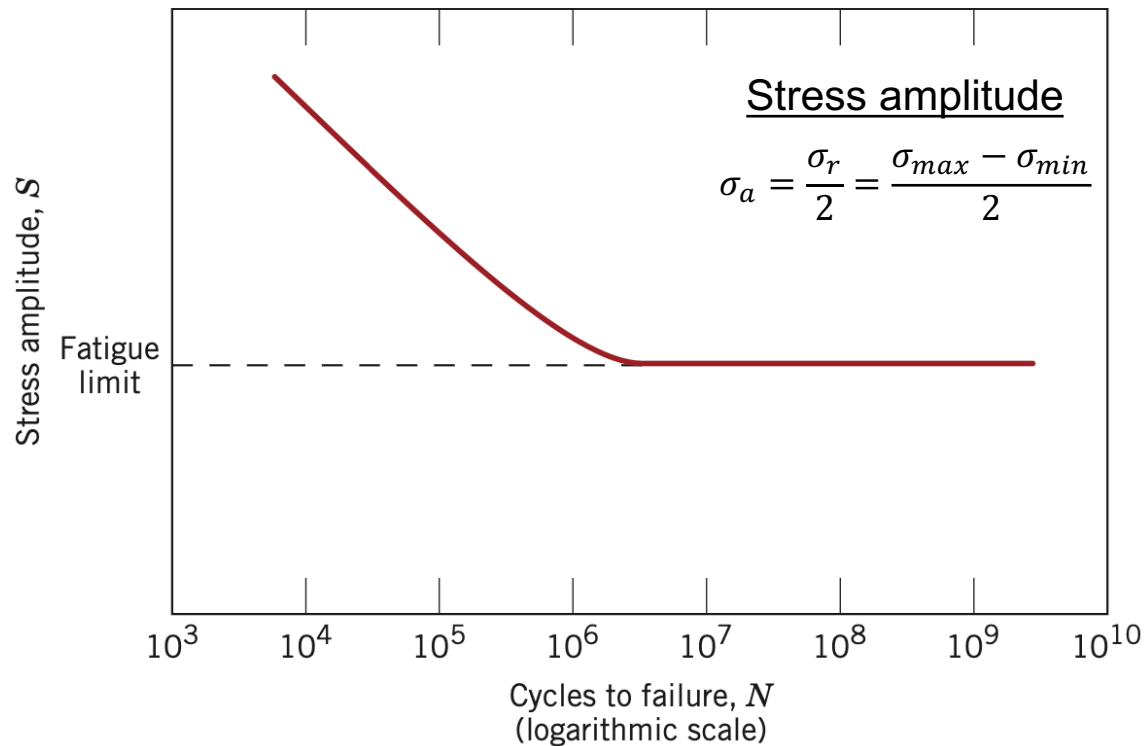
Does not possess a fatigue limit



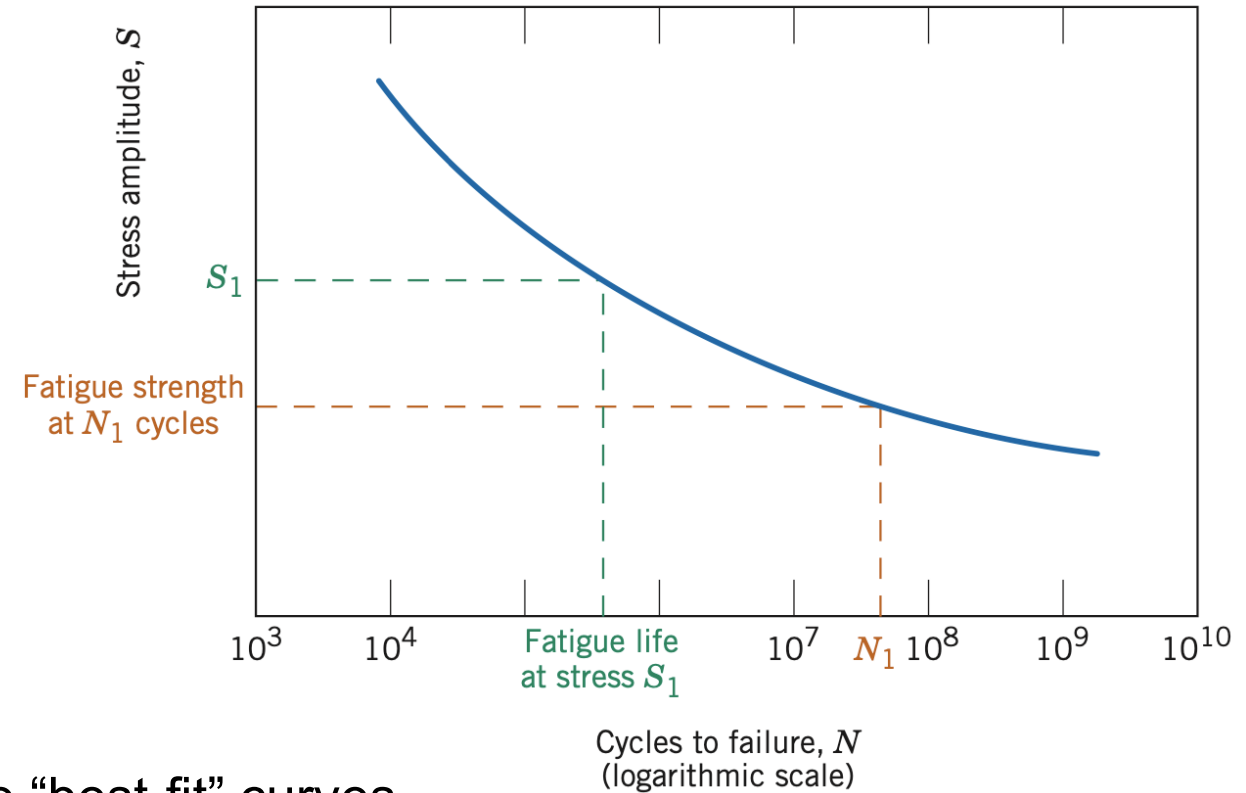
Fatigue fracture will occur eventually  
Fatigue strength is defined with respect to a  
specified number to failure  
(Often seen in non-ferrous alloys)

# S-N Curve (Applied Stress vs. Number of cycles to failure)

Possesses a fatigue limit



Does not possess a fatigue limit



These curves are “best-fit” curves

Represents the average stress amplitude values to fatigue fracture

High stresses → Elastic and plastic strain → Low cycle fatigue (<10<sup>5</sup> cycles)

Low stresses → Only elastic strain → High cycle fatigue (>10<sup>5</sup> cycles)

# Constant probability S-N curves

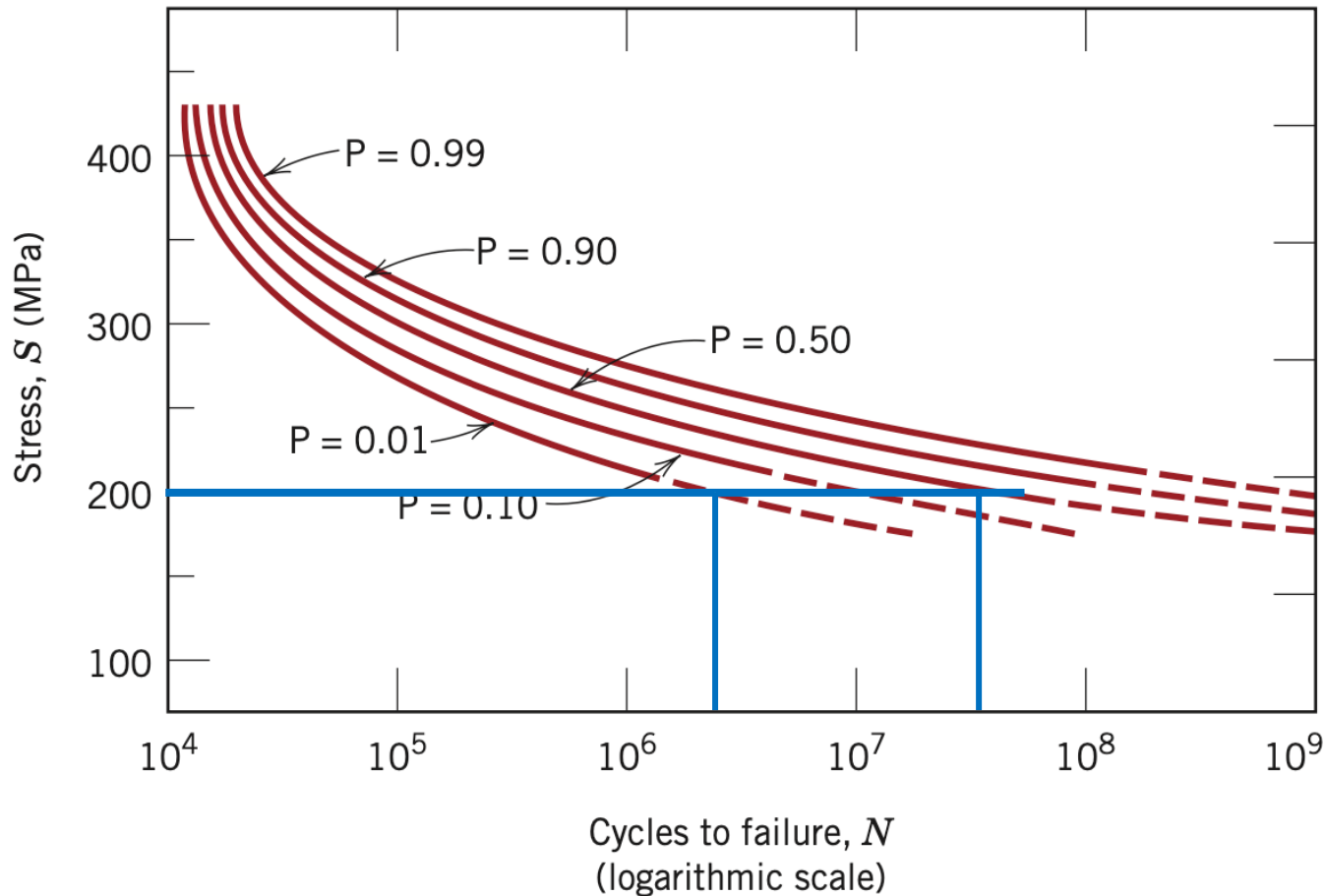
Fatigue is sensitive to many variables that are hard to control precisely → Significant scatter in fatigue data

## How to read a constant probability S-N curve

P values represent the probability of failure

For a certain stress level, draw a horizontal line and observe where it intersects the probability curves

Constant probability S-N curves



---

At 200 MPa, we can see that ~ 1% of samples will fail at ~ 4 x 10<sup>6</sup> cycles

At 200 MPa, we can see that ~ 50% of samples will fail at ~ 6 x 10<sup>7</sup> cycles

# Miner's Rule

$$\sum_{i=1}^k \frac{n_i}{N_i} = C$$

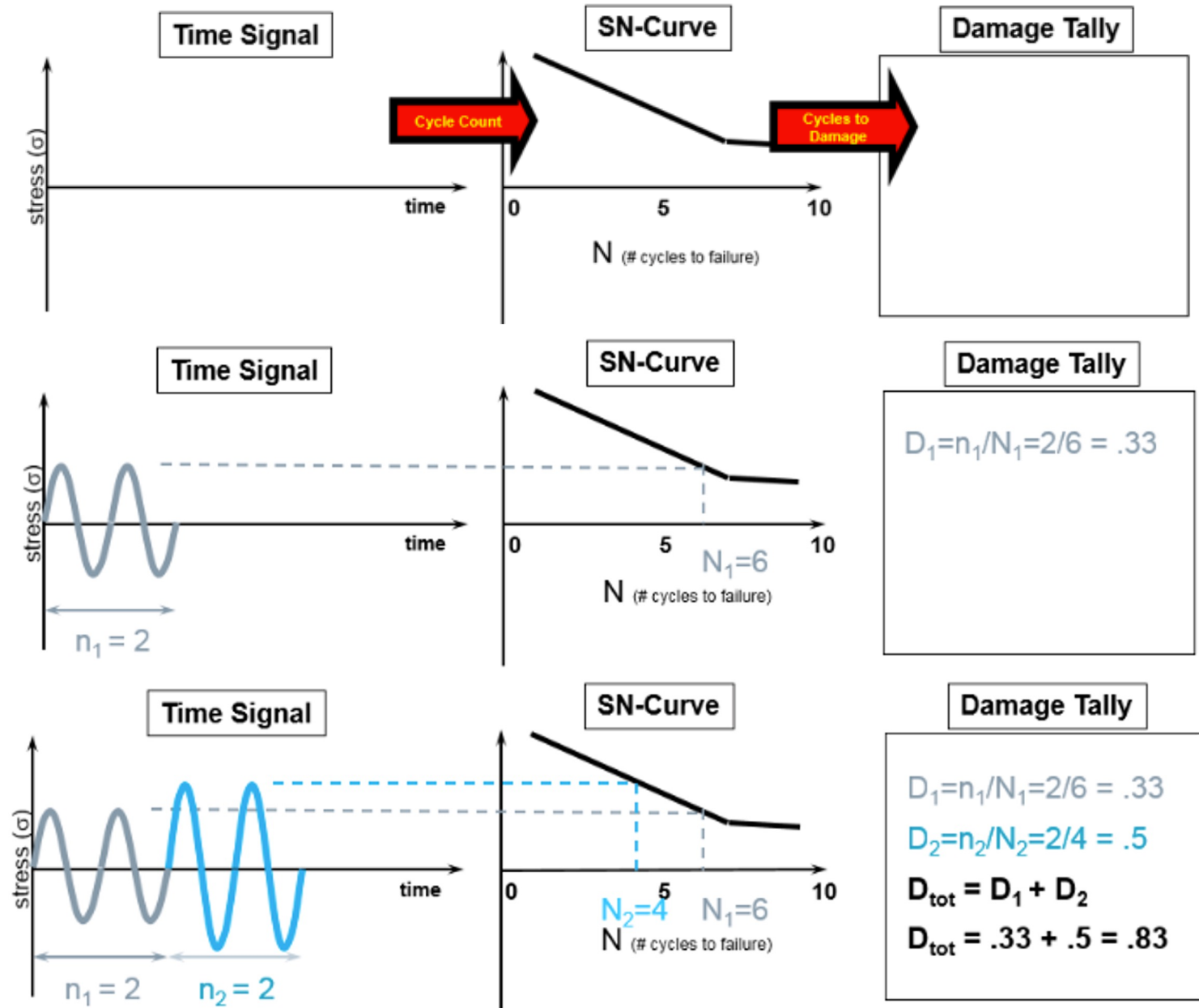
$n_i$  is the number of cycles accumulated at stress  $S_i$

$N_i$  is the average number of cycles to failure at stress  $S_i$

$C$  is the fraction of life consumed  
Sometimes denoted as  $D$

Can think of Miner's rule as determining the fraction of life consumed at each stress level and then summing them to predict how much "life" is left

## Miner's rule: Simple predictions of fatigue failure



# Miner's Rule

$$\sum_{i=1}^k \frac{n_i}{N_i} = C$$

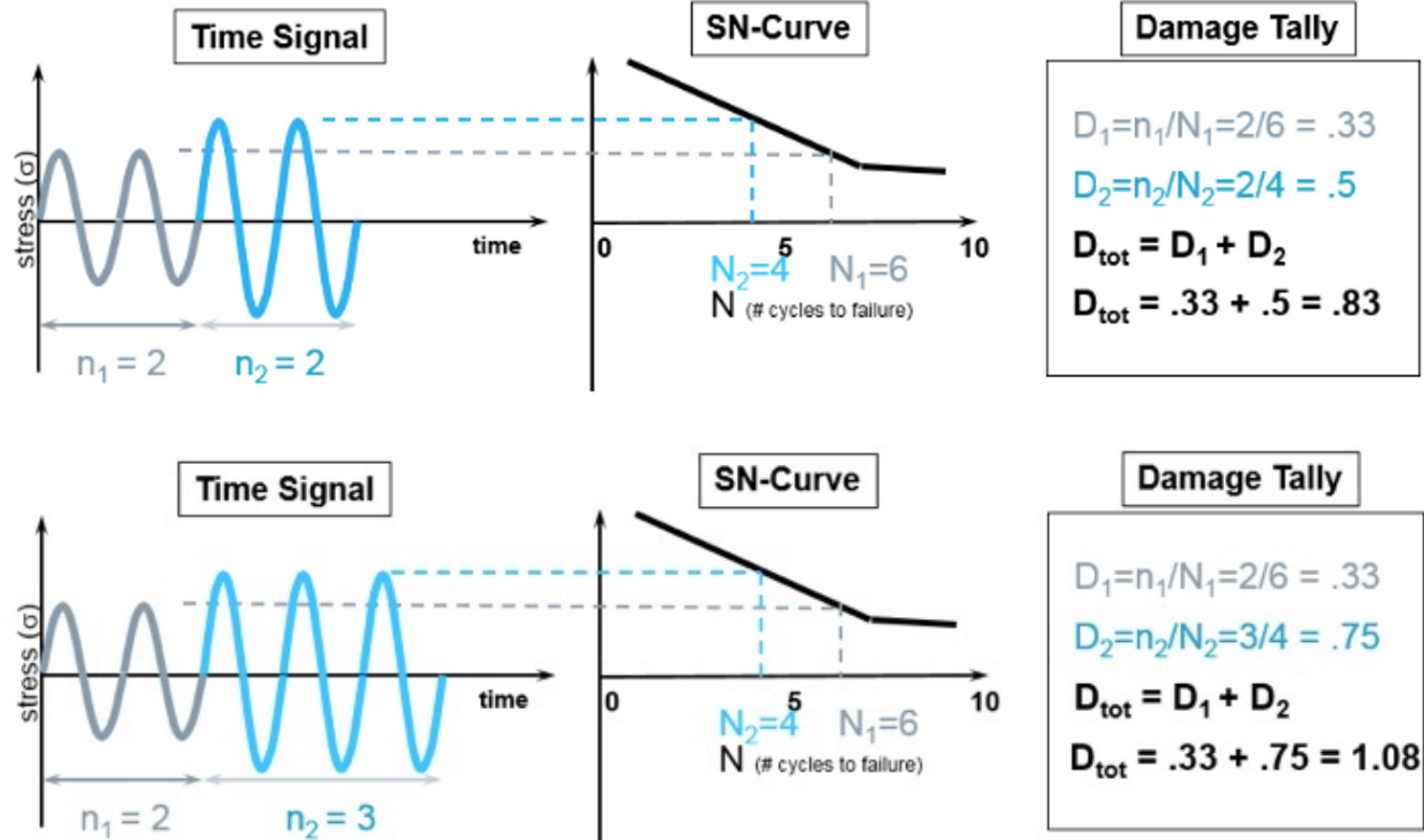
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Can think of Miner's rule as determining the fraction of life consumed at each stress level and then summing them to predict how much "life" is left

## Miner's rule: Simple predictions of fatigue failure



Since  $D_{tot} > 1$ , Miner's rule predicts that the application of one more stress cycle will result in failure

# Miner's Rule

$$\sum_{i=1}^k \frac{n_i}{N_i} = C$$

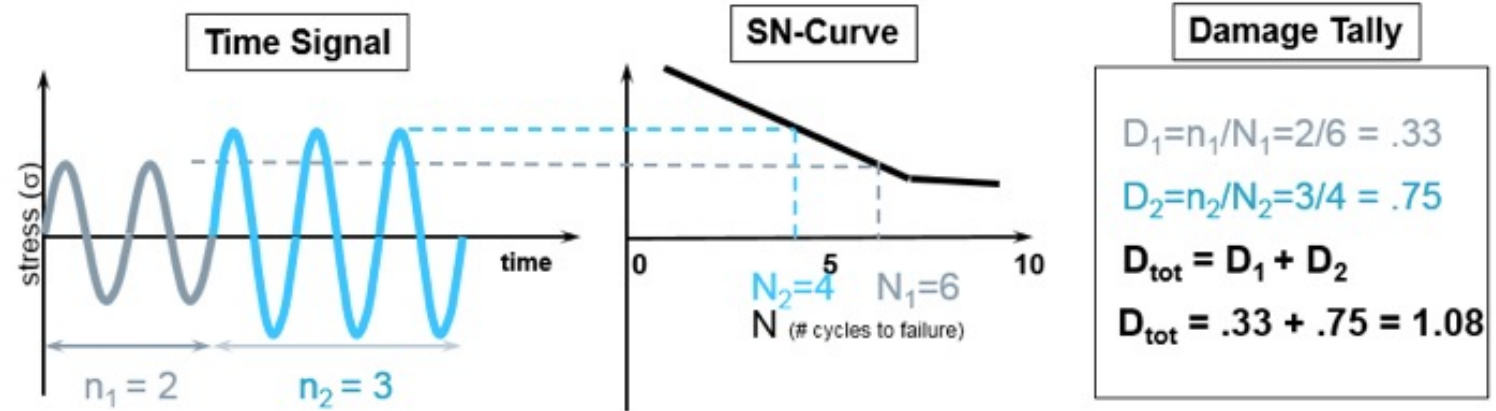
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$C$  is the fraction of life consumed  
Sometimes denoted as  $D$

Can think of Miner's rule as determining the fraction of life consumed at each stress level and then summing them to predict how much "life" is left

## Limitations of Miner's Rule



**Independent damage:** Damage from each stress cycle is independent of the previous stress cycles

**No effect of load sequence:** The order of applied stress does not matter. High stress  $\rightarrow$  Low stress = Low stress  $\rightarrow$  High stress

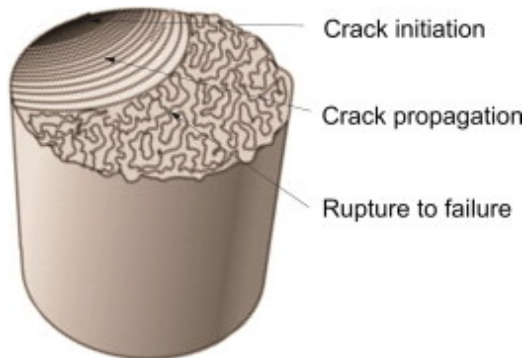
**Linear accumulation of damage:** Damage from each stress level is additive. Ignores possibility of higher-order effects

**Does not include probability:** Only uses average values, ignores probabilistic component of fatigue fracture

# Fatigue crack initiation and propagation

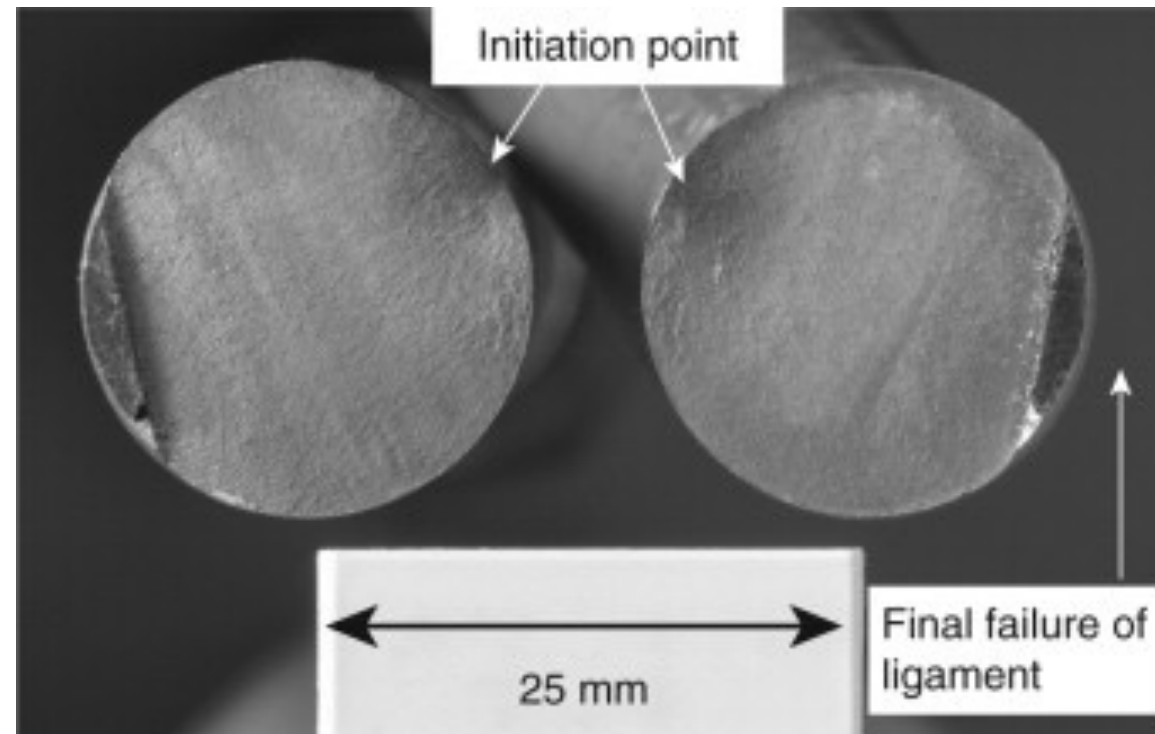
Fatigue failure is characterized by:

1. **Crack initiation** where a small crack forms at some point of high stress concentration
2. **Crack propagation** where the crack advances incrementally with each stress cycle
3. **Failure** once the crack reaches a critical size



Initiation almost always starts from the surface

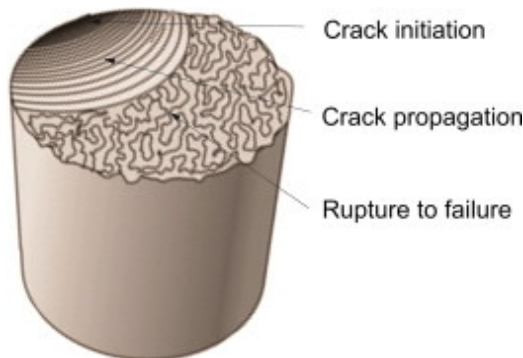
Could be from a scratch, threads, dents, etc.  
Could even be from dislocation slip!



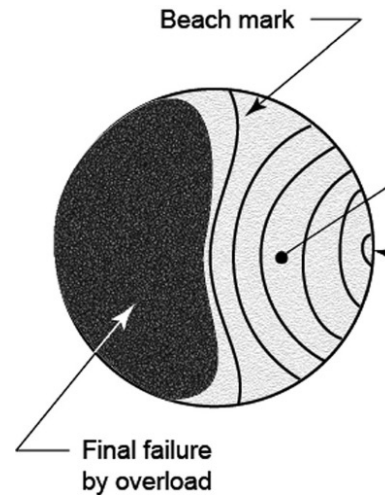
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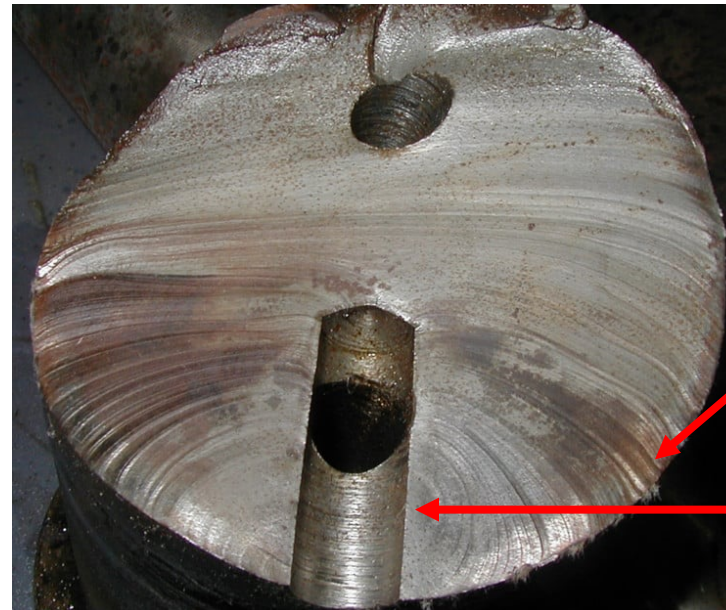


Fatigue crack propagation has distinct features



**Beachmarks** are macroscopic features and can be seen by eye

Each beachmark band represents a period of time where crack growth occurred



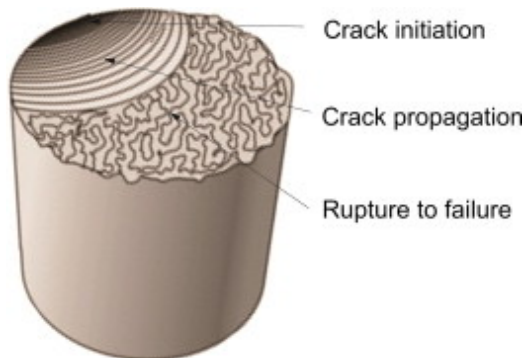
Beachmark bands

Crack initiation  
(concentric rings coming out of it)

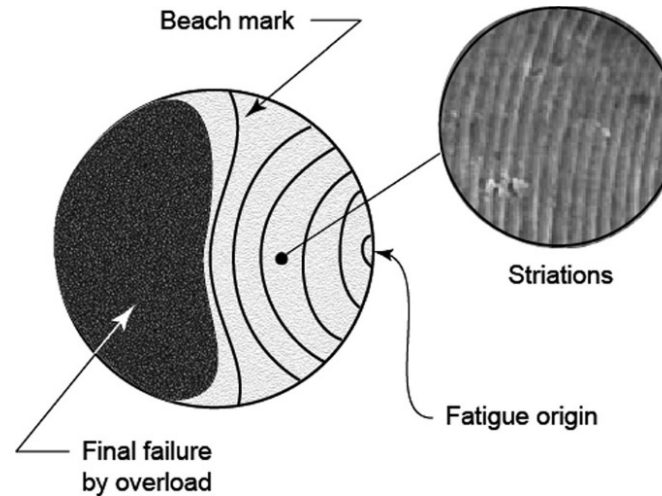
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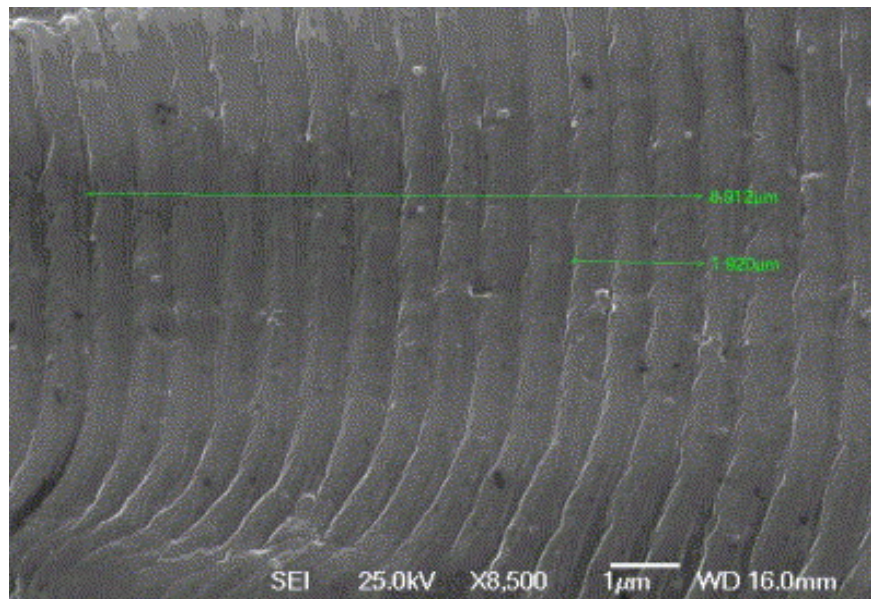


Fatigue crack propagation has distinct features



Within beachmarks are **striations**

These are microscopic and have to be seen with an electron microscope

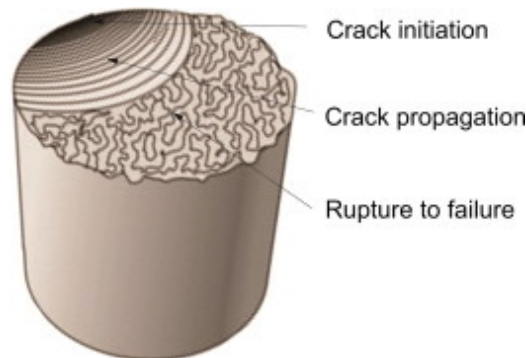


Can think of each striation as the amount of crack growth per stress cycle

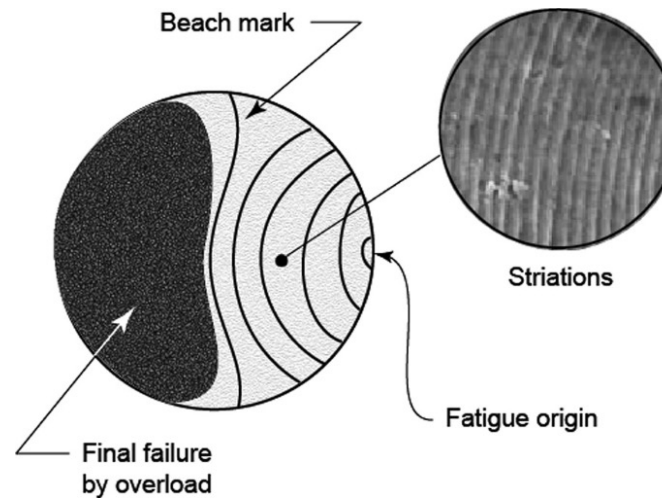
# Fatigue crack initiation and propagation

Fatigue failure is characterized by:

1. **Crack initiation** where a small crack forms at some point of high stress concentration
2. **Crack propagation** where the crack advances incrementally with each stress cycle
3. **Failure** once the crack reaches a critical size



Fatigue crack propagation has distinct features



**Beachmarks:** Macroscopic  
**Striations:** Microscopic

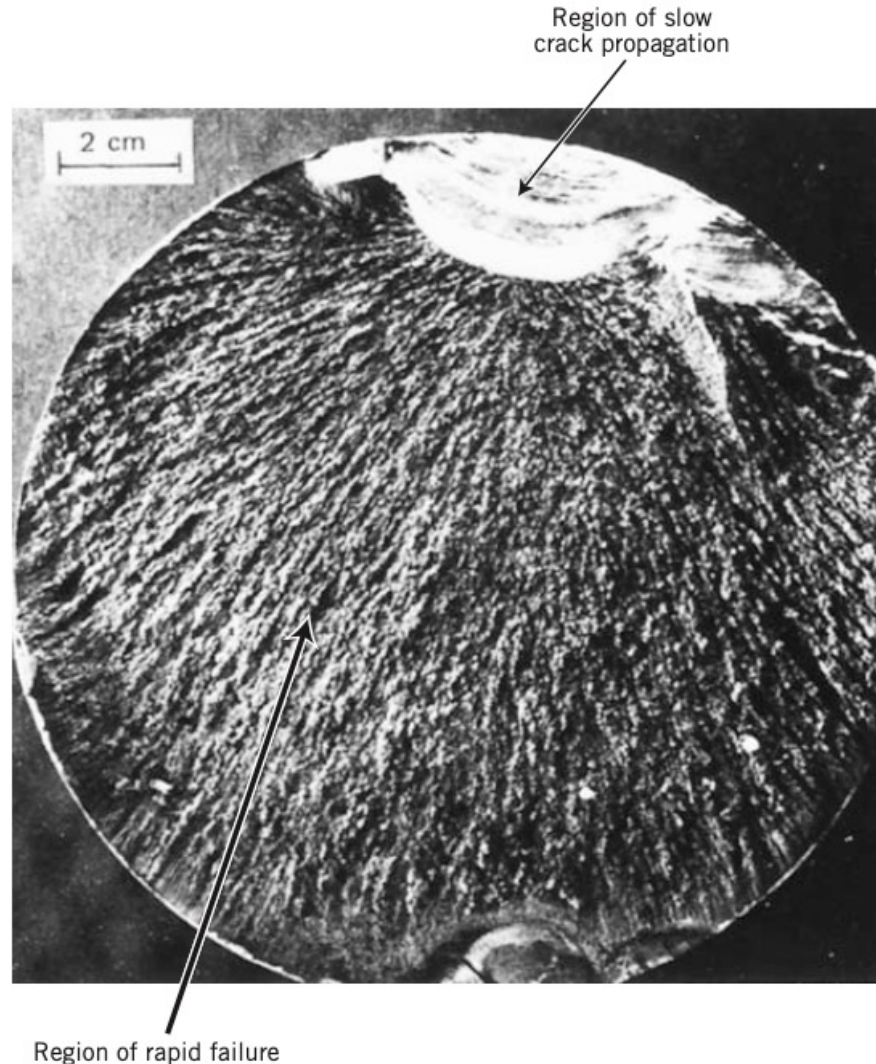
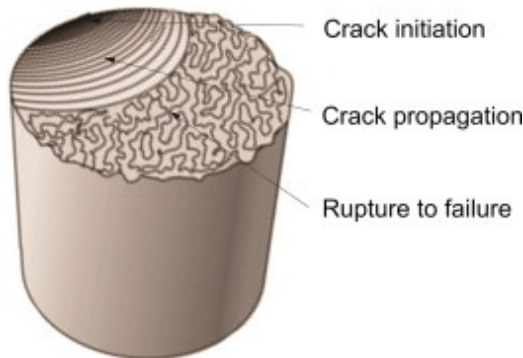
If beachmarks/striations are observed on fracture surface  
→ Fatigue played a role in fracture

If beachmarks/striations are NOT observed on fracture surface  
→ Fatigue could have played a role in fracture also

# Fatigue crack initiation and propagation

Fatigue failure is characterized by:

1. **Crack initiation** where a small crack forms at some point of high stress concentration
2. **Crack propagation** where the crack advances incrementally with each stress cycle
3. **Failure** once the crack reaches a critical size

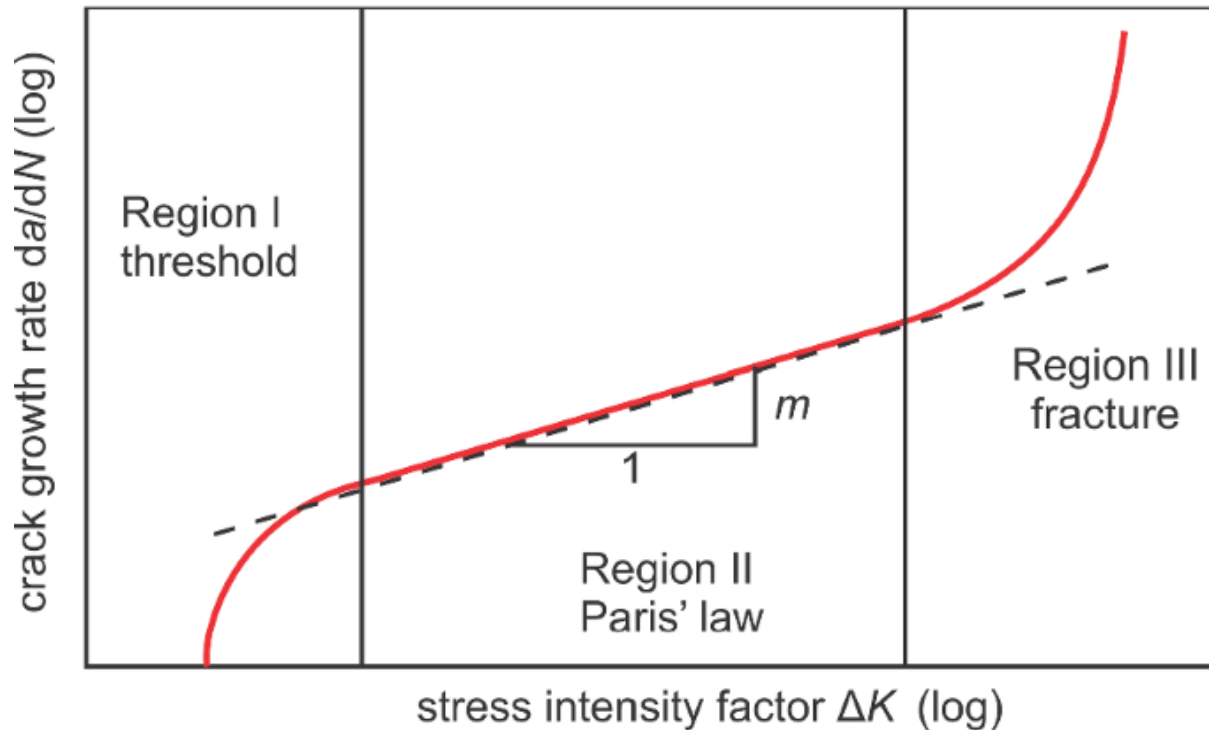


The rapid failure region can be ductile or brittle.

Plastic deformation will be observed if it was a ductile fracture

# Paris' Law (Crack growth equation)

Crack growth equations are used to calculate the size of a fatigue crack growing from cyclic loads



Typical plot of crack growth vs stress intensity range

$$\Delta K = K_{max} - K_{min}$$

Maximum and minimum stress intensity factors in a stress cycle

## Paris' Law

$$\frac{da}{dN} = C(\Delta K)^m$$

$C$  and  $m$  are determined experimentally

Paris' law only holds for mid-range of crack growth.

Can be used to estimate cycles to failure

# Paris' Law in action

A large plate contains a crack of length  $2a_0 = 10$  mm in the center

The plate is subjected to a constant amplitude cyclic tensile stress ranging from 100 MPa to 200 MPa.

Assume the fatigue crack growth rate is given by Paris' Law with:

$$\frac{da}{dN} = 0.42 \times 10^{-11} (\Delta K)^3$$

1) What is the crack growth rate?

$$2a = 10 \text{ mm}$$

$$\Delta K = K_{max} - K_{min}$$

$$K = Y\sigma\sqrt{\pi a}$$

$$\Delta K = Y\Delta\sigma\sqrt{\pi a}$$

$$\Delta K = 1(100)\sqrt{\pi \cdot 0.005}$$

$$\Delta K = 12.53 \text{ MPa}\sqrt{\text{m}}$$

$$\frac{da}{dN} = 0.42 \times 10^{-11} (12.53)^3 = 8.26 \times 10^{-9} \text{ m/cycle}$$

# Paris' Law in action

A large plate contains a crack of length  $2a_0 = 10$  mm in the center

The plate is subjected to a constant amplitude cyclic tensile stress ranging from 100 MPa to 200 MPa.

Assume the fatigue crack growth rate is given by Paris' Law with:

$$\frac{da}{dN} = 0.42 \times 10^{-11} (\Delta K)^3$$

2) Assuming that the fracture toughness of this material is  $60 \text{ MPa}\sqrt{\text{m}}$ , estimate the number of cycles to failure

$$2a = 10 \text{ mm}$$

$$K_{IC} = 60 \text{ MPa}\sqrt{\text{m}}$$

Let us first determine the critical crack size under these conditions

$$K_{IC} = Y \sigma_{max} \sqrt{\pi a_c}$$

$$a_c = 28.7 \times 10^{-3} \text{ m}$$

At these conditions, complete and rapid fracture occurs at a crack length of  $28.7 \times 10^{-3}$  m.

# Paris' Law in action

A large plate contains a crack of length  $2a_0 = 10$  mm in the center

The plate is subjected to a constant amplitude cyclic tensile stress ranging from 100 MPa to 200 MPa.

Assume the fatigue crack growth rate is given by Paris' Law with:

$$\frac{da}{dN} = 0.42 \times 10^{-11} (\Delta K)^3$$

Knowing  $a_c$ , how do we find the cycles to failure?

$$a_c = 28.7 \times 10^{-3} \text{ m}$$

$$\int_0^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{0.42 \times 10^{-11} (\Delta K)^3}$$

$$N_f = \int_{5 \times 10^{-3}}^{28.7 \times 10^{-3}} \frac{da}{0.42 \times 10^{-11} (\Delta K)^3}$$

$$\Delta K = Y \Delta \sigma \sqrt{\pi a}$$

$$\Delta K = 1(100) \sqrt{(3.14)a}$$

$$\Delta K = 177.2 \sqrt{a}$$

$$N_f = \int_{5 \times 10^{-3}}^{28.7 \times 10^{-3}} \frac{da}{2.34 \times 10^{-5} \cdot a^{\frac{3}{2}}}$$

# Paris' Law in action

A large plate contains a crack of length  $2a_0 = 10$  mm in the center

The plate is subjected to a constant amplitude cyclic tensile stress ranging from 100 MPa to 200 MPa.

Assume the fatigue crack growth rate is given by Paris' Law with:

$$\frac{da}{dN} = 0.42 \times 10^{-11} (\Delta K)^3$$

$$N_f = \int_{5 \times 10^{-3}}^{28.7 \times 10^{-3}} \frac{da}{2.34 \times 10^{-5} \cdot a^{\frac{3}{2}}}$$

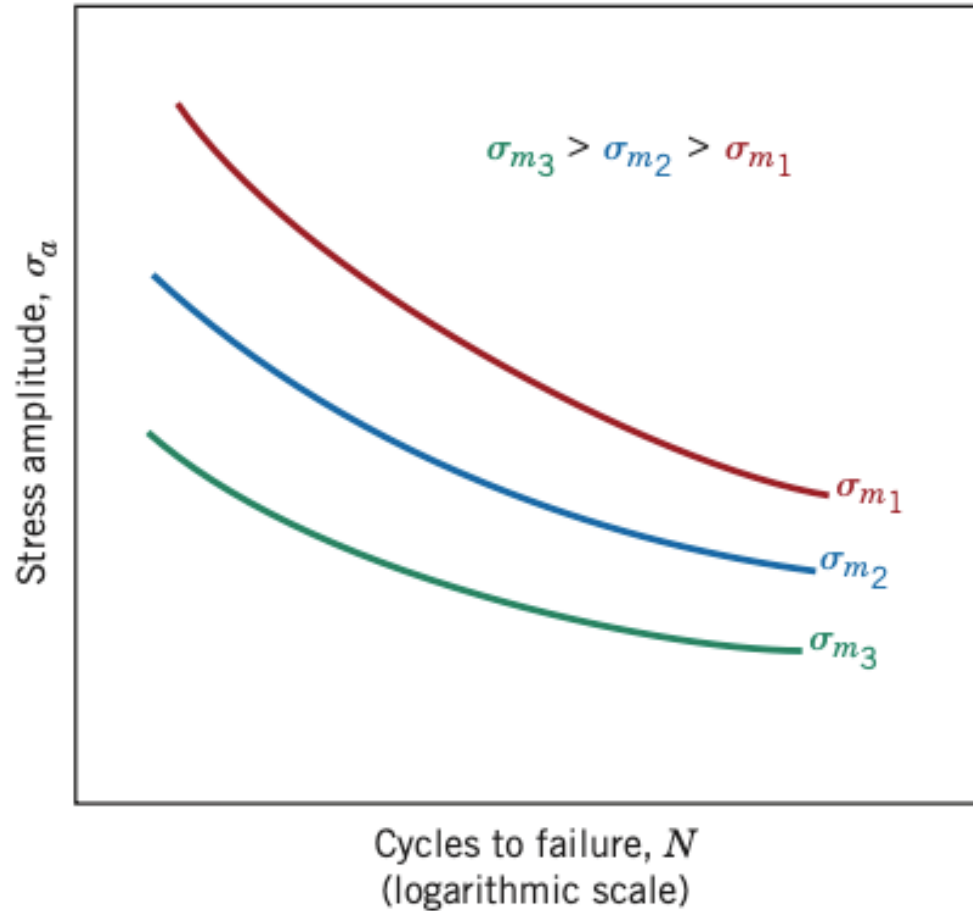
$$N_f = 42791 \int_{5 \times 10^{-3}}^{28.7 \times 10^{-3}} a^{-\frac{3}{2}} da$$

$$N_f = 42791 \left[ (-2) a^{-\frac{1}{2}} \right]_{5 \times 10^{-3}}^{28.7 \times 10^{-3}}$$

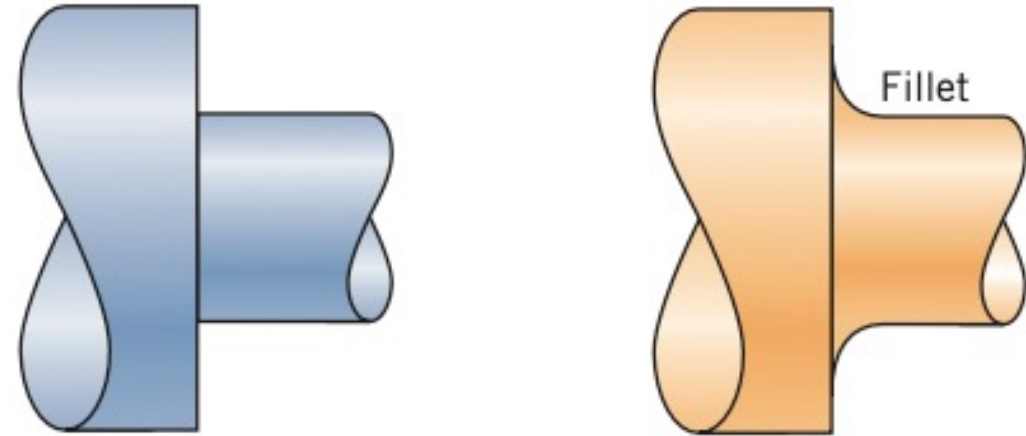
$$N_f = 7.05 \times 10^5$$

# What affects fatigue?

## Mean stress level



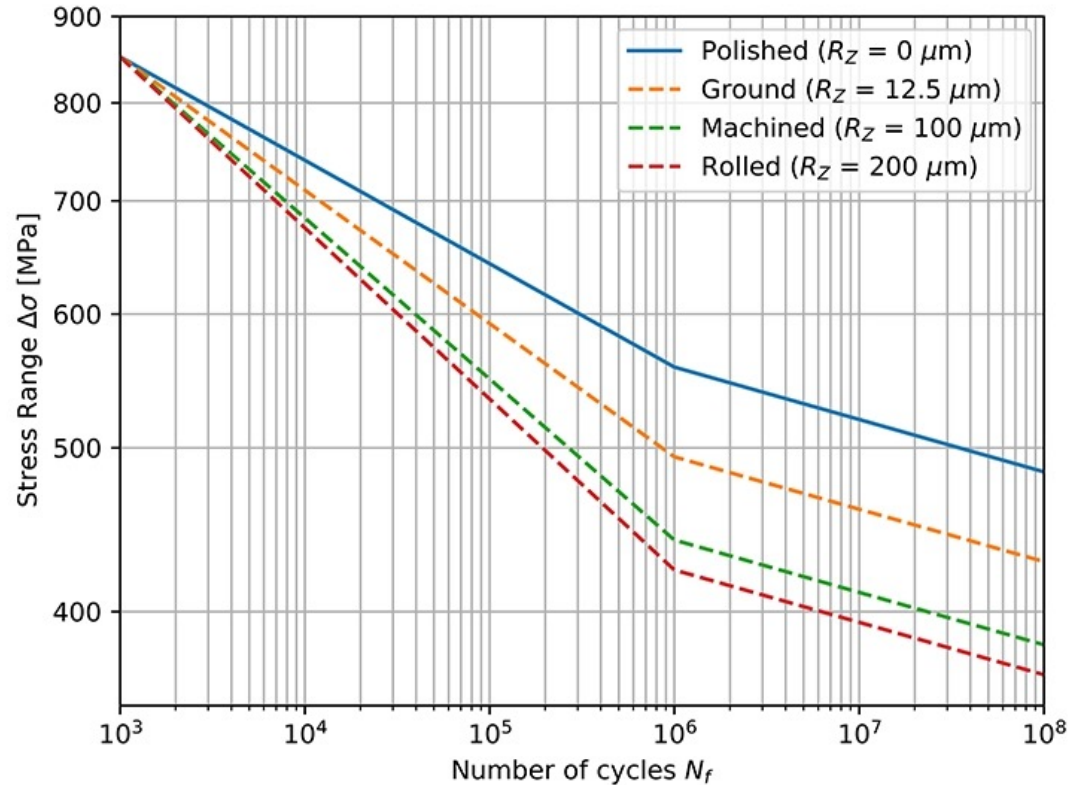
## Design of part



Sharp corners can act as stress concentrators and initiate a surface crack. This surface crack can initiate fatigue crack propagation

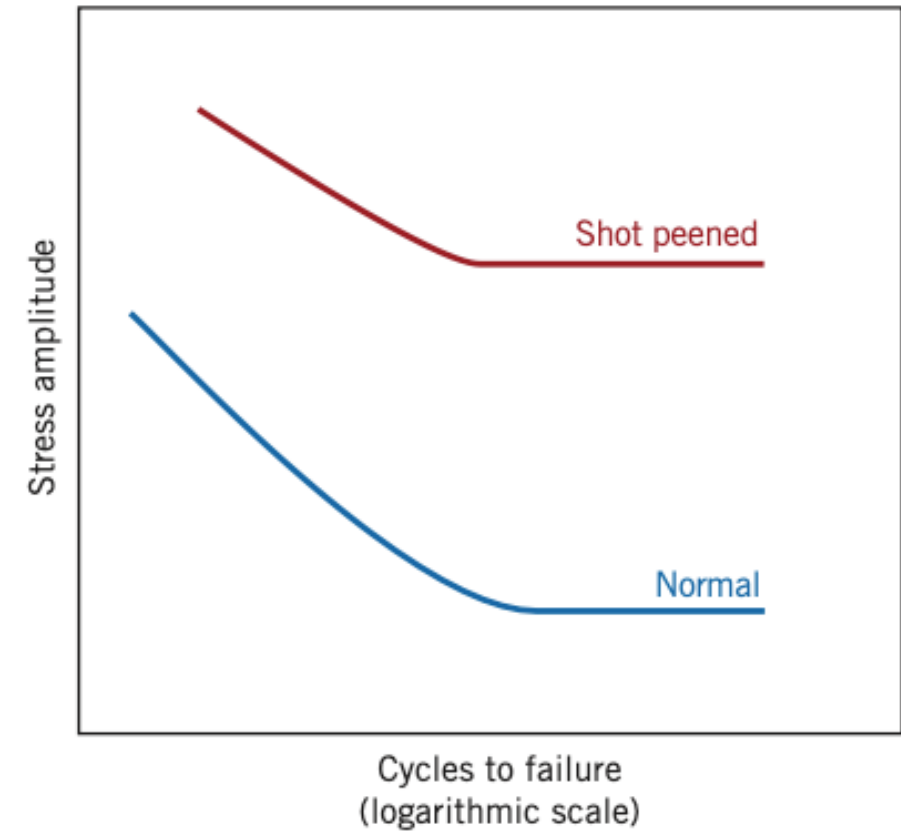
# What affects fatigue?

## Surface Treatment (Polishing)



Polishing removes surface flaws that could initiate fatigue cracking

## Surface Treatment (Shot peening)



Cracks can only grow in tension

Shot peening induces a compressive stress onto the surface. Need to overcome this stress before crack can grow

# Week 10 Learning Objectives

- **Understand what a CCT diagram is and how it differs from a TTT diagram**
- **Understand how to qualitatively use a CCT diagram to predict the microconstituents of a cooling alloy**
- **Understand the difference between brittle and ductile fracture**
- **Understand what the fracture toughness of a material is**
- **Use the fracture toughness of a material to predict of a material will fracture**
- **Understand what fatigue is**
- **Be able to use Miner's rule to predict fatigue failure**
- **Be able to use Paris' Law to predict fatigue failure**
- **Understand the factors that affect fatigue fracture**