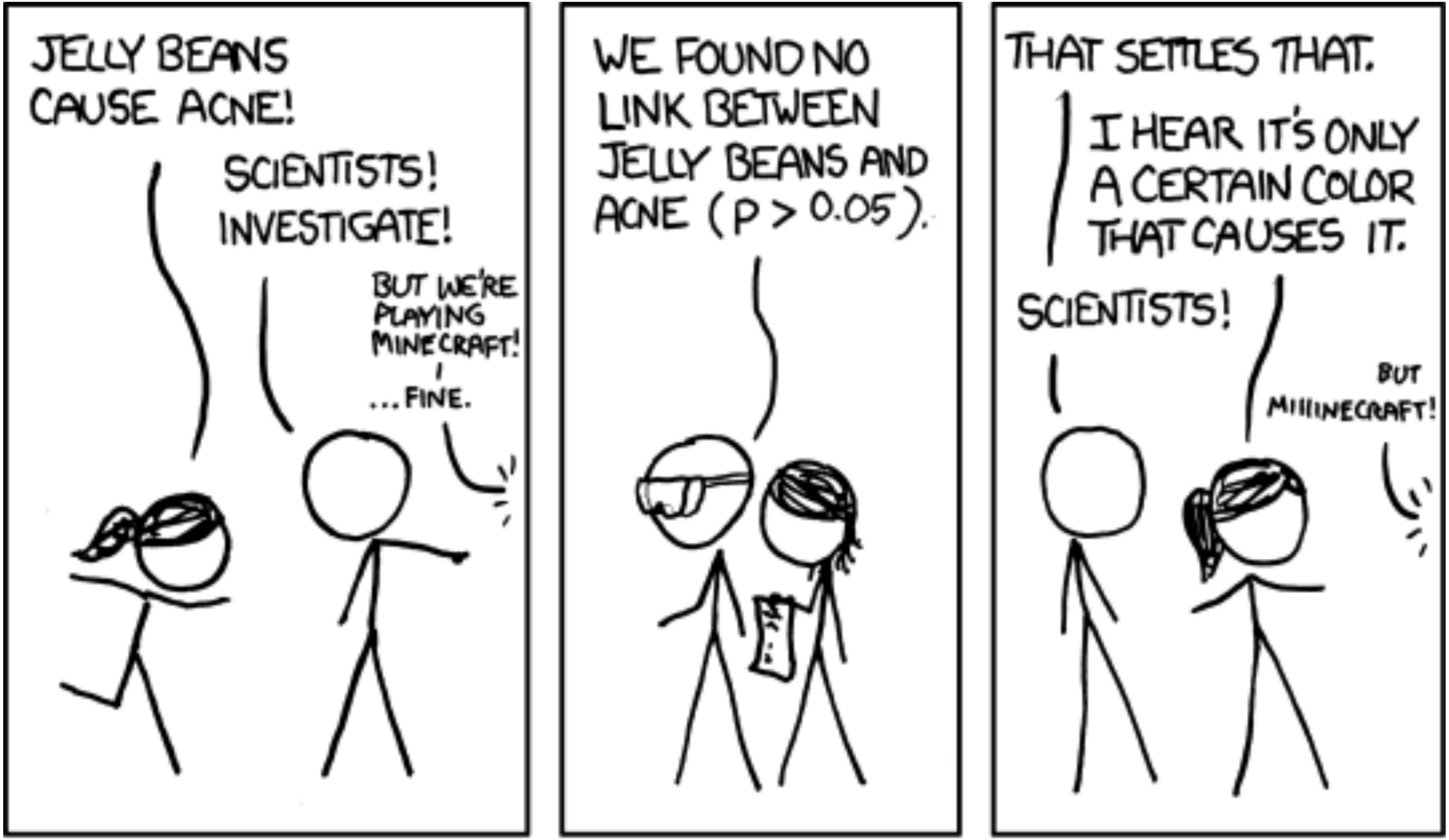
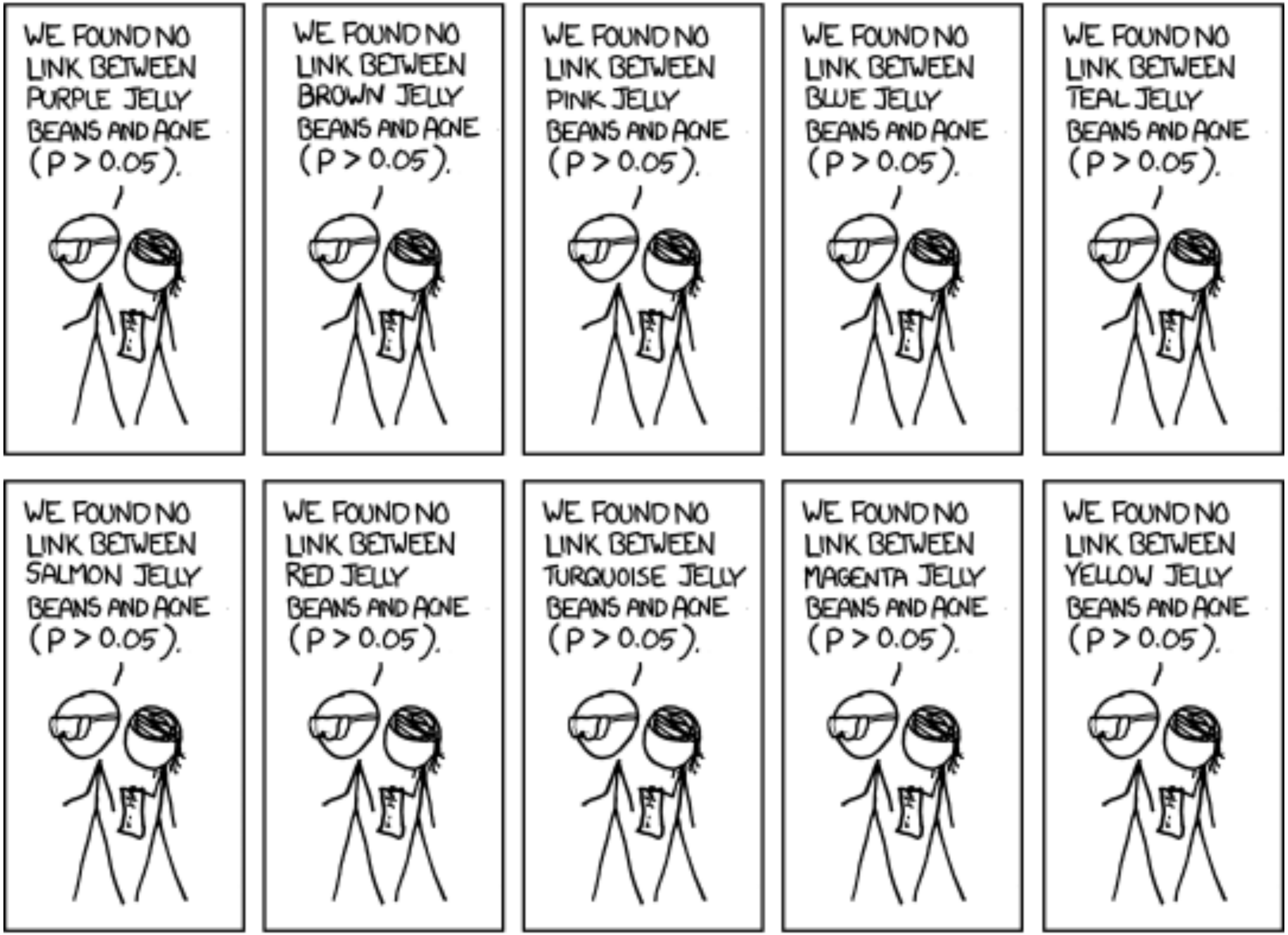
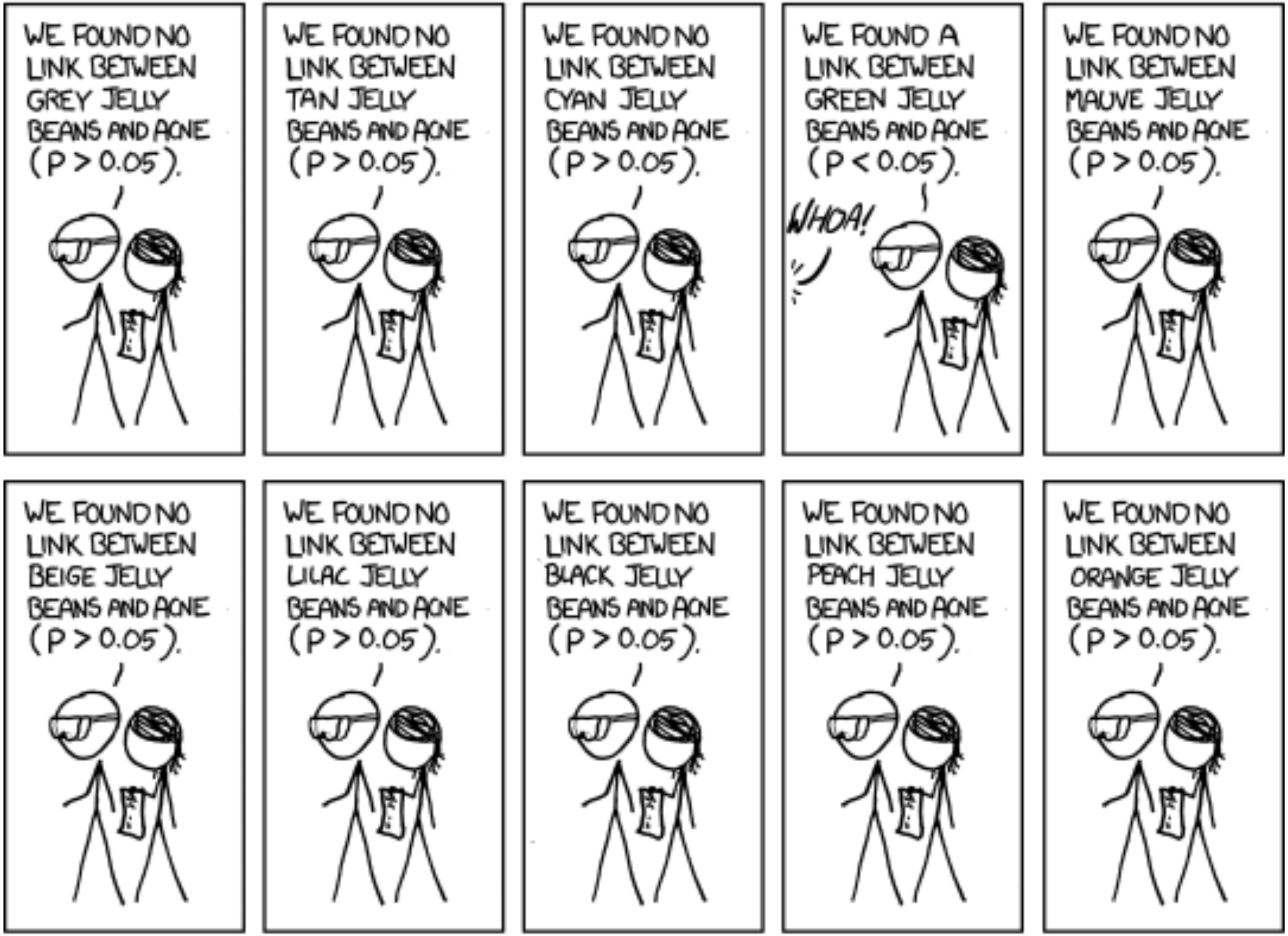
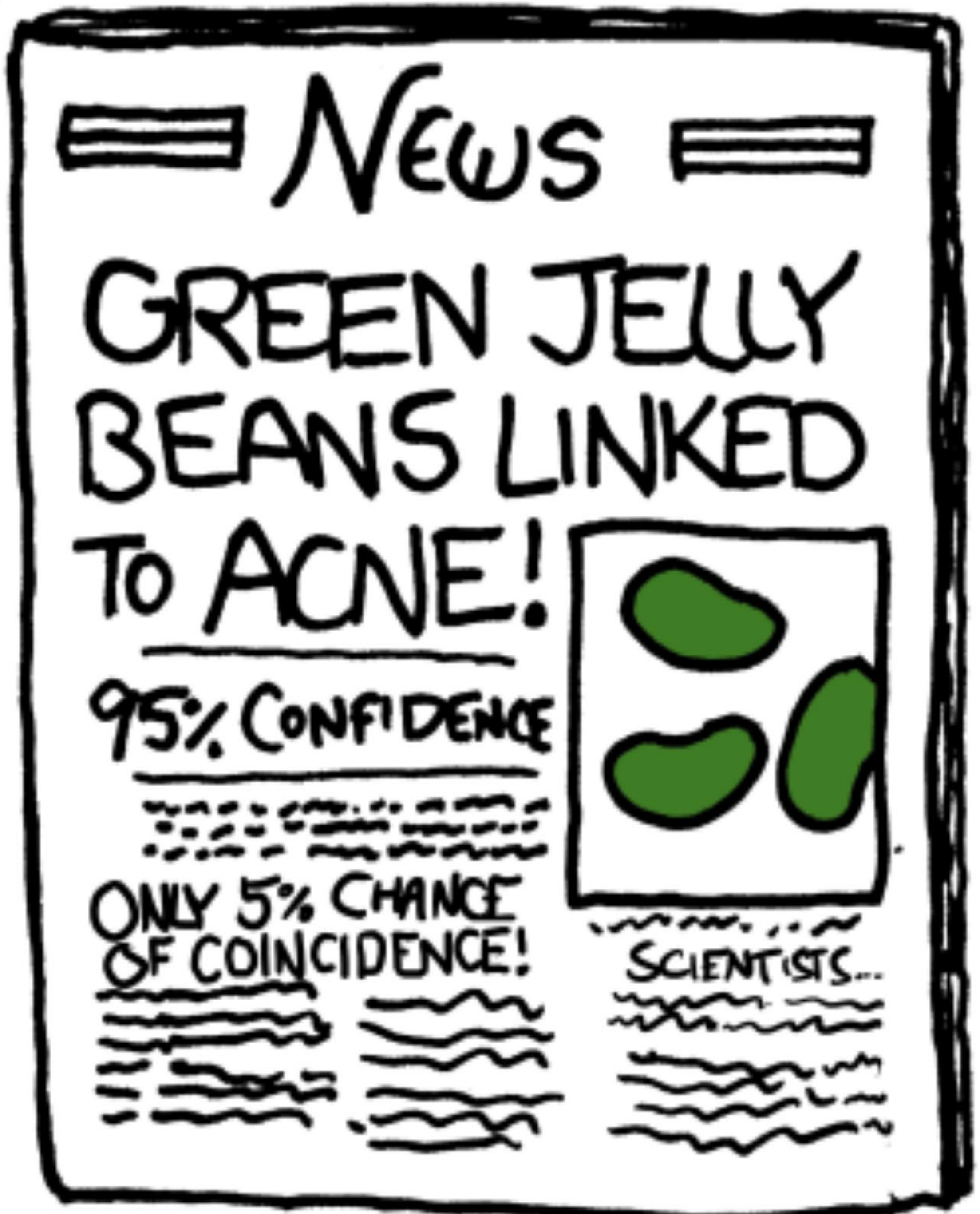
MSE-213
Probability and statistics for materials science
Lecture 7


JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
...FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
BUT MIIIINECRAFT!


WE FOUND NO LINK BETWEEN PURPLE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BROWN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PINK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLUE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TEAL JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN SALMON JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN RED JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TURQUOISE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE (P > 0.05).


WE FOUND NO LINK BETWEEN GREY JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN CYAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND A LINK BETWEEN GREEN JELLY BEANS AND ACNE (P < 0.05).
WHOA!
WE FOUND NO LINK BETWEEN MAUVE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE (P > 0.05).



NEWS
GREEN JELLY BEANS LINKED TO ACNE!
95% CONFIDENCE
ONLY 5% CHANCE OF COINCIDENCE!
SCIENTISTS...

The z-test (Gaussian test for the mean)

- I have measured N_S elements with mean $\bar{x}$.

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- I know (or assume) the standard deviation of the population (or random process) is σ.

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- I can assume a Gaussian/normal distribution for the means (CLT)

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When is this useful?

1. Fluctuations come from a well-characterized measurement method. The object (e.g. an atom) is considered fluctuation-free.
2. The full population is known, and I want to see if a subgroup is representative, or significantly different.
3. N is very large so I can assume the measured standard deviation is very close to the real one. (more on that later)

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- I can only to make some <u>probabilistic</u> statement about the real mean μ.

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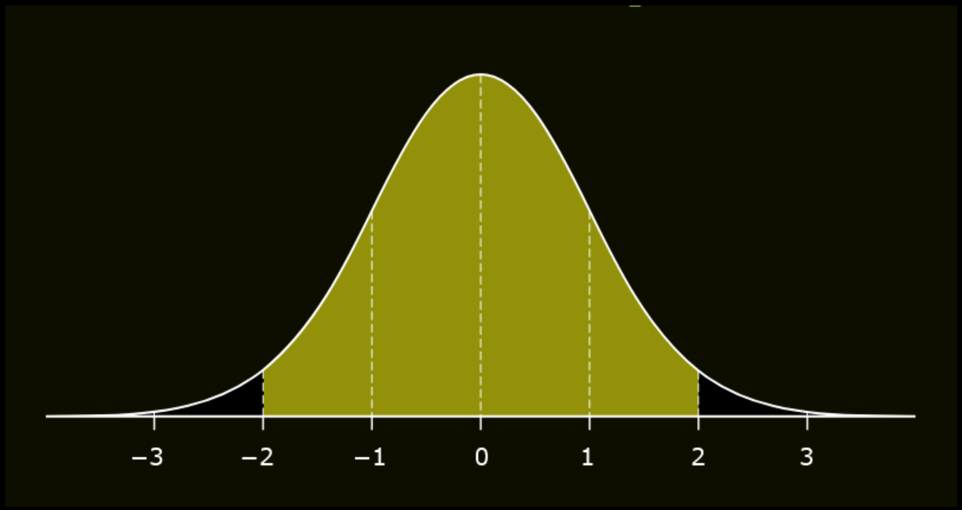
  2. The full population is known, and I want to see if a subgroup is representative, or significantly different.
  3. N is very large so I can assume the measured standard deviation is very close to the real one. (more on that later)
- I can only to make some probabilistic statement about the real mean μ.
- Note: I can only talk about ranges, not single values.

The z-test (Gaussian test for the mean)

Two-sided questions:

- I receive a molecule that is supposed to be C60. I measure the mass that deviates from the expectation. How probable is it, that this was just because of random deviations? [fixed deviation, search for P]



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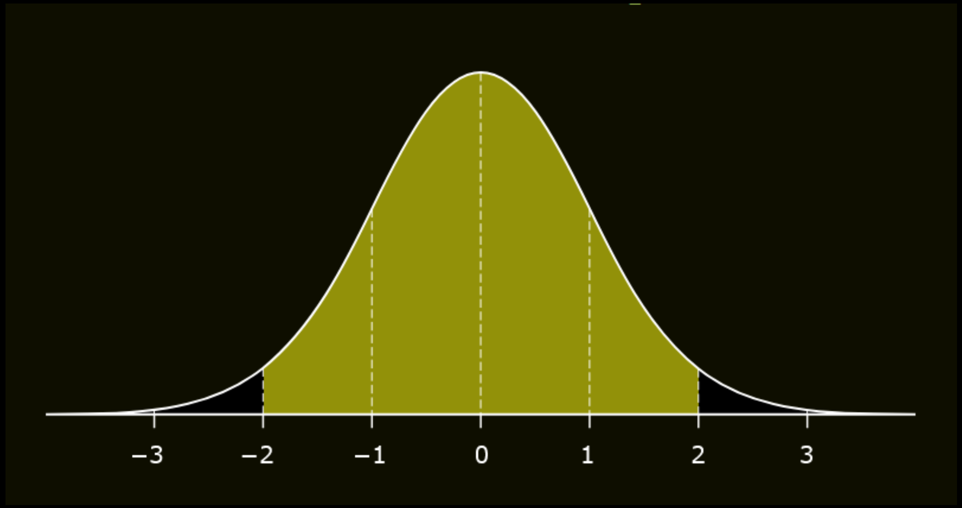
Two-sided questions:

- I receive a molecule that is supposed to be C60. I measure the mass that deviates from the expectation. How probable is it, that this was just because of random deviations? [fixed deviation, search for P]
- I measure the IQ of all people in this group and find a mean value that is not = 100. How probable is it that this is just a random fluctuation? [fixed deviation, search for P]



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Two-sided questions:

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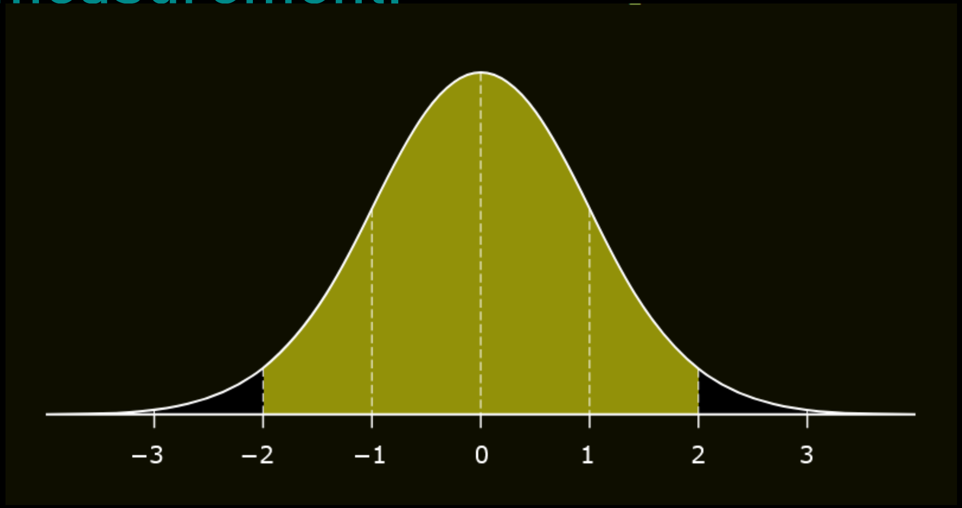
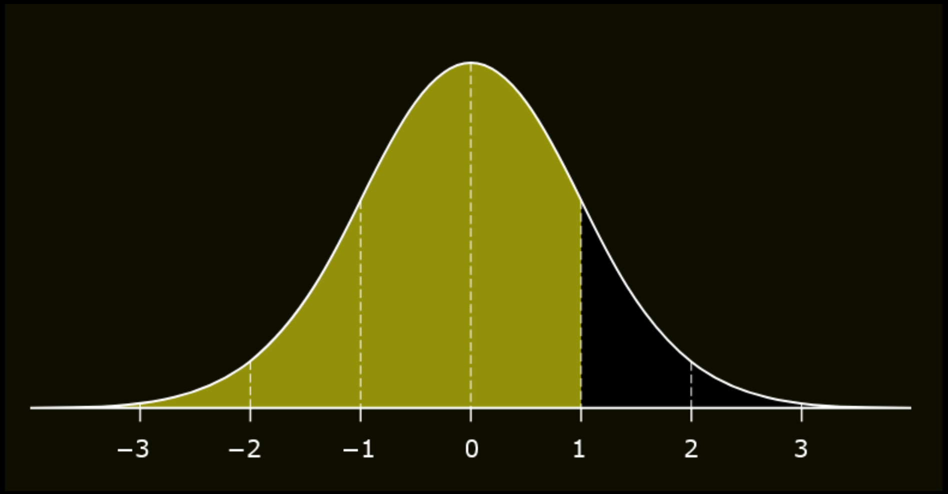
- I measure the IQ of all people in this group and find a mean value that is not = 100. How probable is it that this is just a random fluctuation? [fixed deviation, search for P]
- I measure the size of 10000 Corona viruses. In which (centred) range is the true mean size with 95% probability ("confidence")? [fixed probability, search for deviation] take σ from measurement.



The z-test (Gaussian test for the mean)

One-sided questions:

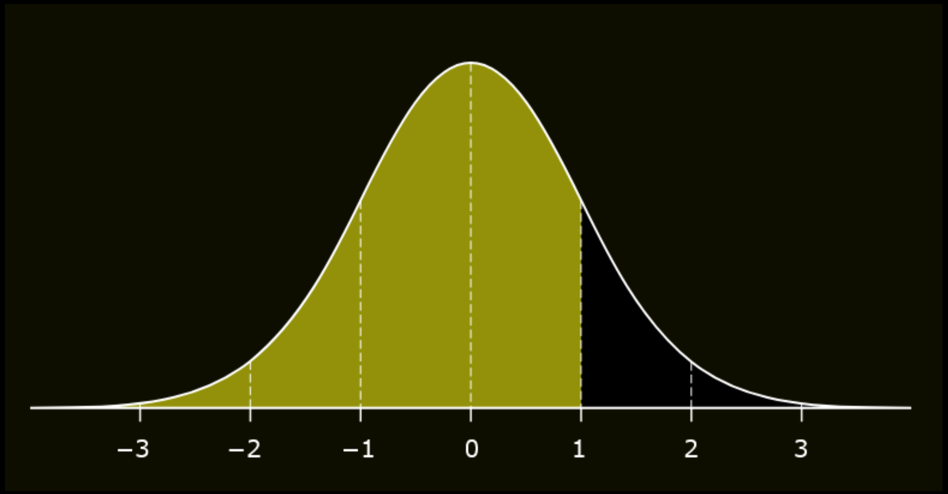
- I receive a molecule that is supposed to be C60. I measure the mass that deviates from the expectation. How probable is it, that I actually received a lighter molecule? [fixed deviation, search for P]



The z-test (Gaussian test for the mean)

One-sided questions:

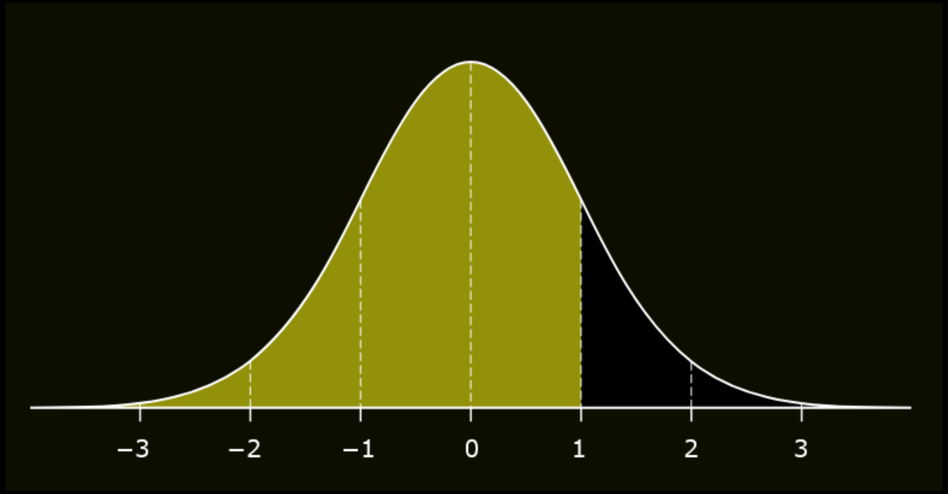
- I receive a molecule that is supposed to be C60. I measure the mass that deviates from the expectation. How probable is it, that I actually received a lighter molecule? [fixed deviation, search for P]
- I measure the IQ of all people in this group and find a mean value that is not = 100. How probable is it that the group is above-average intelligence? [fixed deviation, search for P]



The z-test (Gaussian test for the mean)

<u>One</u>-sided questions:

- I receive a molecule that is supposed to be C60. I measure the mass that deviates from the expectation. How probable is it, that I actually received a lighter molecule? [fixed deviation, search for P]
- I measure the IQ of all people in this group and find a mean value that is not = 100. How probable is it that the group is above-average intelligence? [fixed deviation, search for P]
- I measure the size of 10000 Corona viruses. Below which size is the true mean size with 95% probability ("confidence")? [fixed probability, search for deviation] take σ from measurement.



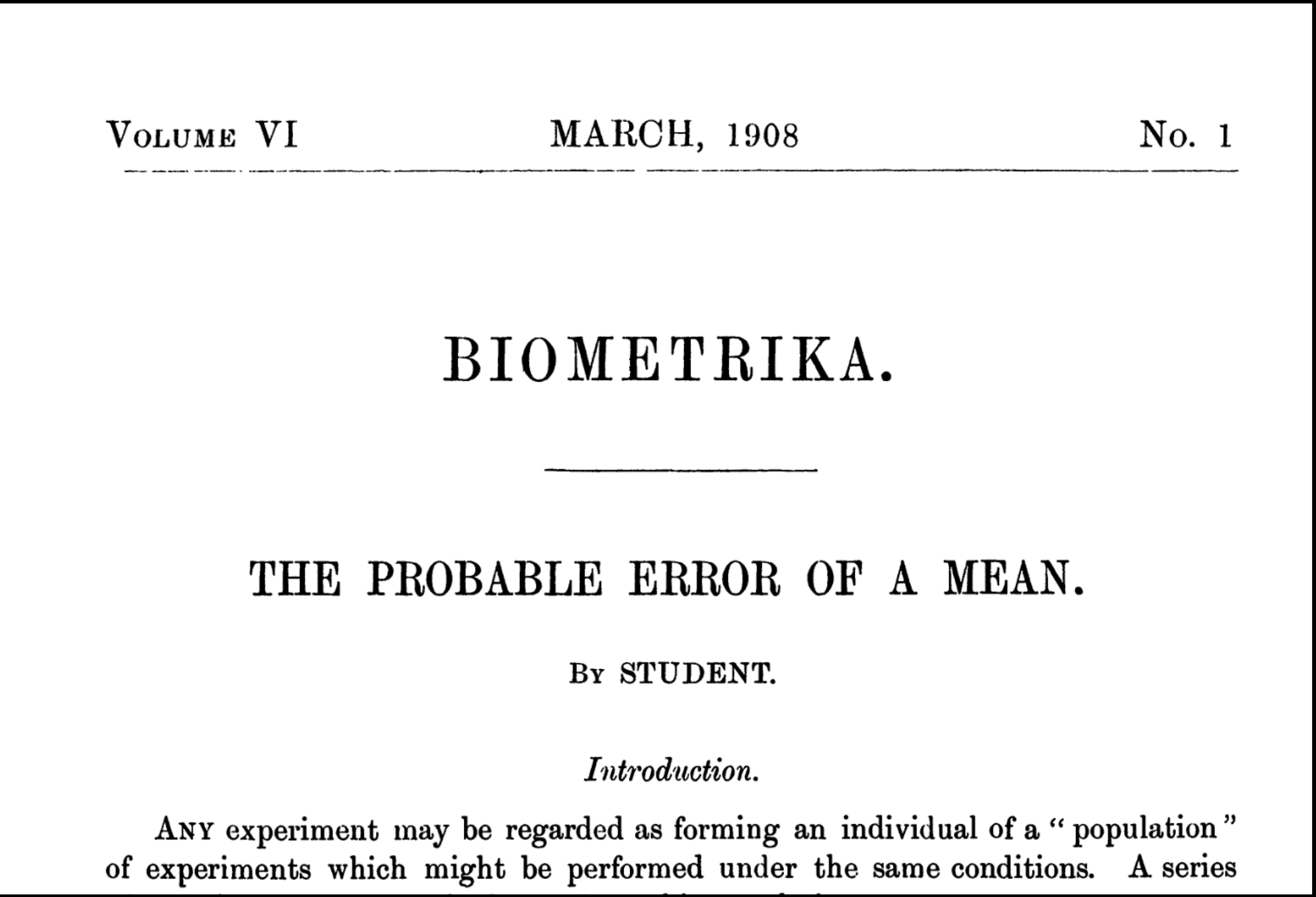
The z-test (Gaussian test for the mean)

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0,00	0,500	0,72	0,764	1,44	0,9251	2,16	0,9846	2,88	0,99801	3,80	0,9999277
0,02	0,508	0,74	0,770	1,46	0,9279	2,18	0,9854	2,90	0,99813	3,84	0,9999385
0,04	0,516	0,76	0,776	1,48	0,9306	2,20	0,9861	2,92	0,99825	3,88	0,9999478
0,06	0,524	0,78	0,782	1,50	0,9332	2,22	0,9868	2,94	0,99836	3,92	0,9999557
0,08	0,532	0,80	0,788	1,52	0,9357	2,24	0,9875	2,96	0,99846	3,96	0,9999625
0,10	0,540	0,82	0,794	1,54	0,9382	2,26	0,9881	2,98	0,99856	4,00	0,9999683
0,12	0,548	0,84	0,800	1,56	0,9406	2,28	0,9887	3,00	0,99865	4,04	0,9999733
0,14	0,556	0,86	0,805	1,58	0,9429	2,30	0,9893	3,02	0,99874	4,08	0,9999775
0,16	0,564	0,88	0,811	1,60	0,9452	2,32	0,9898	3,04	0,99882	4,12	0,9999811
0,18	0,571	0,90	0,816	1,62	0,9474	2,34	0,9904	3,06	0,99889	4,16	0,9999841
0,20	0,579	0,92	0,821	1,64	0,9495	2,36	0,9909	3,08	0,99996	4,20	0,9999867
0,22	0,587	0,94	0,826	1,66	0,9515	2,38	0,9913	3,10	0,99903	4,24	0,9999888
0,24	0,595	0,96	0,831	1,68	0,9535	2,40	0,9918	3,12	0,99910	4,28	0,9999907
0,26	0,603	0,98	0,836	1,70	0,9554	2,42	0,9922	3,14	0,99916	4,32	0,9999922
0,28	0,610	1,00	0,841	1,72	0,9573	2,44	0,9927	3,16	0,99921	4,36	0,9999935
0,30	0,618	1,02	0,846	1,74	0,9591	2,46	0,9931	3,18	0,99926	4,40	0,9999946
0,32	0,626	1,04	0,851	1,76	0,9608	2,48	0,9934	3,20	0,99931	4,44	0,9999955
0,34	0,633	1,06	0,855	1,78	0,9625	2,50	0,9938	3,22	0,99936	4,48	0,9999963
0,36	0,641	1,08	0,860	1,80	0,9641	2,52	0,9941	3,24	0,99940	4,52	0,9999969
0,38	0,648	1,10	0,864	1,82	0,9656	2,54	0,9945	3,26	0,99944	4,56	0,9999974
0,40	0,655	1,12	0,869	1,84	0,9671	2,56	0,9948	3,28	0,99948	4,60	0,9999979
0,42	0,663	1,14	0,873	1,86	0,9686	2,58	0,9951	3,30	0,99952	4,64	0,9999983
0,44	0,670	1,16	0,877	1,88	0,9799	2,60	0,9953	3,32	0,99955	4,68	0,9999986
0,46	0,677	1,18	0,881	1,90	0,9713	2,62	0,9956	3,34	0,99958	4,72	0,9999988
0,48	0,684	1,20	0,885	1,92	0,9726	2,64	0,9959	3,36	0,99961	4,76	0,9999990
0,50	0,691	1,22	0,889	1,94	0,9738	2,66	0,9961	3,38	0,99964	4,80	0,9999992
0,52	0,698	1,24	0,893	1,96	0,9750	2,68	0,9963	3,40	0,99966	4,84	0,9999994
0,54	0,705	1,26	0,896	1,98	0,9761	2,70	0,9965	3,42	0,99969	4,88	0,9999995
0,56	0,712	1,28	0,900	2,00	0,9772	2,72	0,9967	3,44	0,99971	4,92	0,9999996
0,58	0,719	1,30	0,903	2,02	0,9783	2,74	0,9969	3,46	0,99973	4,96	0,9999996
0,60	0,726	1,32	0,907	2,04	0,9793	2,76	0,9971	3,48	0,99975	5,00	0,9999997
0,62	0,732	1,34	0,910	2,06	0,9803	2,78	0,9973	3,50	0,99977	5,04	0,9999998
0,64	0,739	1,36	0,913	2,08	0,9812	2,80	0,9974	3,52	0,99978	5,08	0,9999998
0,66	0,745	1,38	0,916	2,10	0,9821	2,82	0,9976	3,54	0,99980	5,12	0,9999998
0,68	0,752	1,40	0,919	2,12	0,9830	2,84	0,9977	3,56	0,99981	5,16	0,9999999
0,70	0,758	1,42	0,922	2,14	0,9838	2,86	0,9979	3,58	0,99983	5,20	0,9999999

1,50	0,9332
1,52	0,9357
1,54	0,9382
1,56	0,9406
1,58	0,9429
1,60	0,9452
1,62	0,9474
1,64	0,9495
1,66	0,9515


ESTD 1759
GUINNESS

William Sealy Gosset


VOLUME VI MARCH, 1908 No. 1

BIOMETRIKA.

THE PROBABLE ERROR OF A MEAN.

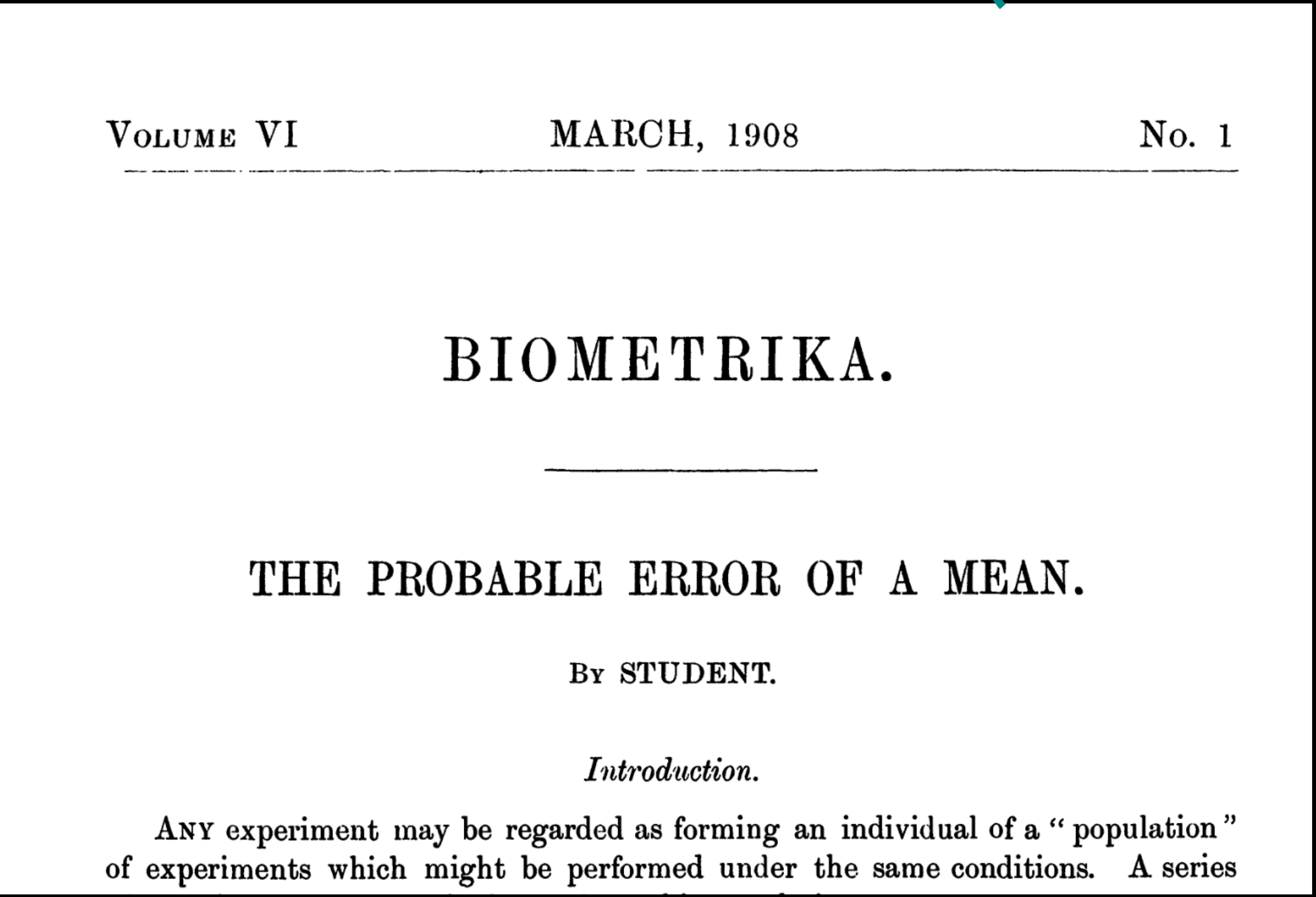
BY STUDENT.

Introduction.

ANY experiment may be regarded as forming an individual of a "population" of experiments which might be performed under the same conditions. A series

William Sealy Gosset

The t-test (Student's t-test)



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William Sealy Gosset

The t-test (Student's t-test)

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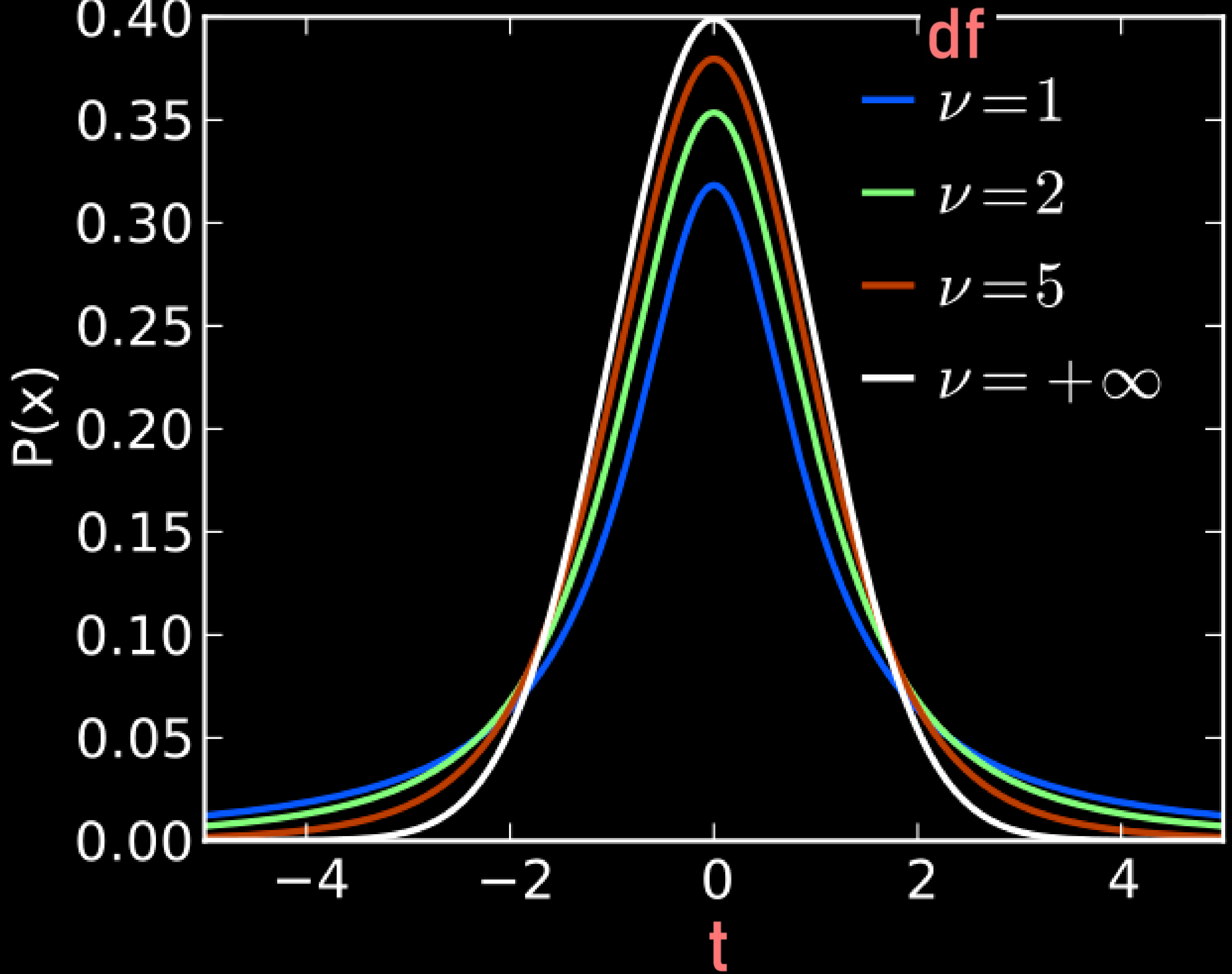
- I have measured n elements with mean $\bar{x}$.
- I can assume a Gaussian/normal distribution for the means (CLT)
- I DO NOT know the standard deviation of the population (or random process) and will estimate it from the sample standard situation, and n is not large

The t-test (Student's t-test)

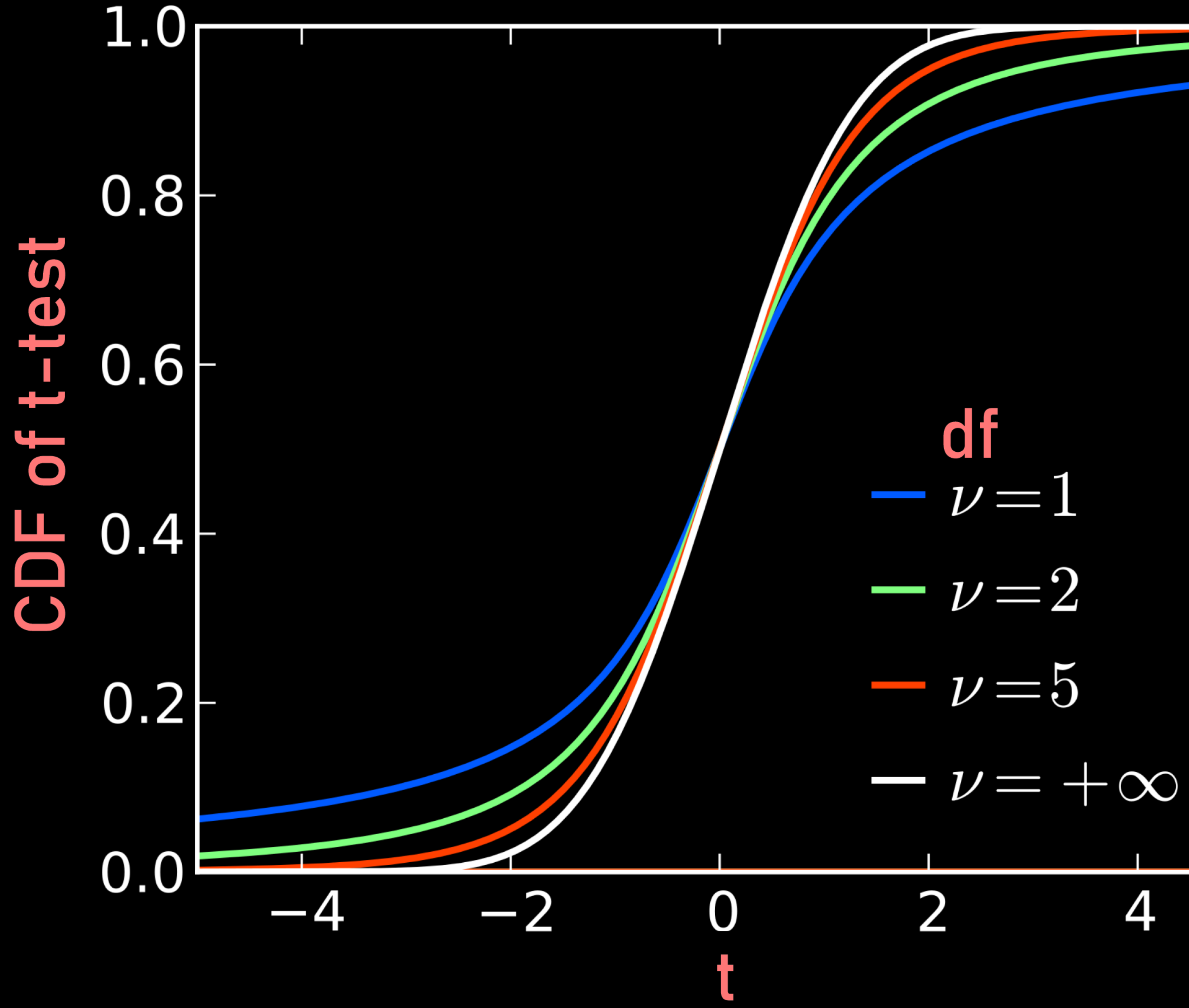
Compared to the z-test, would expect the resulting interval at a given P=99% to be...

- Larger
- Same
- Smaller

The t-distribution



The t-distribution



One-sample t-test

3

df	$1-\alpha$			
	0.95	0.975	0.99	0.995
1	6.3138	12.706	31.821	63.657
2	2.9200	4.3027	6.9646	9.9248
3	2.3534	3.1824	4.5407	5.8409
4	2.1318	2.7764	3.7469	4.6041
5	2.0150	2.5706	3.3649	4.0321
6	1.9432	2.4469	3.1427	3.7074
7	1.8946	2.3646	2.9980	3.4995
8	1.8595	2.3060	2.8965	3.3554
9	1.8331	2.2622	2.8214	3.2498
10	1.8125	2.2281	2.7638	3.1693
11	1.7959	2.2010	2.7181	3.1058
12	1.7823	2.1788	2.6810	3.0545
13	1.7709	2.1604	2.6503	3.0123
14	1.7613	2.1448	2.6245	2.9768
15	1.7531	2.1314	2.6025	2.9467
16	1.7459	2.1199	2.5835	2.9208
17	1.7396	2.1098	2.5669	2.8982
18	1.7341	2.1009	2.5524	2.8784
19	1.7291	2.0930	2.5395	2.8609
20	1.7247	2.0860	2.5280	2.8453
30	1.6973	2.0423	2.4573	2.7500
40	1.6839	2.0211	2.4233	2.7045
50	1.6759	2.0086	2.4033	2.6778
60	1.6706	2.0003	2.3901	2.6603
70	1.6669	1.9944	2.3808	2.6479
80	1.6641	1.9901	2.3739	2.6387
90	1.6620	1.9867	2.3685	2.6316
100	1.6602	1.9840	2.3642	2.6259
200	1.6525	1.9719	2.3451	2.6006
300	1.6499	1.9679	2.3388	2.5923
400	1.6487	1.9659	2.3357	2.5882
500	1.6479	1.9647	2.3338	2.5857

df	$1-\alpha$
	0.95
1	6.3138
2	2.9200
3	2.3534
4	2.1318
5	2.0150
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300	1.6499
400	1.6487
500	1.6479

One-sample t-test

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9	1.8331	2.2622	2.8214	3.2498
10	1.8125	2.2281	2.7638	3.1693
11	1.7959	2.2010	2.7181	3.1058
12	1.7823	2.1788	2.6810	3.0545
13	1.7709	2.1604	2.6503	3.0123
14	1.7613	2.1448	2.6245	2.9768
15	1.7531	2.1314	2.6025	2.9467
16	1.7459	2.1199	2.5835	2.9208
17	1.7396	2.1098	2.5669	2.8982
18	1.7341	2.1009	2.5524	2.8784
19	1.7291	2.0930	2.5395	2.8609
20	1.7247	2.0860	2.5280	2.8453
30	1.6973	2.0423	2.4573	2.7500
40	1.6839	2.0211	2.4233	2.7045
50	1.6759	2.0086	2.4033	2.6778
60	1.6706	2.0003	2.3901	2.6603
70	1.6669	1.9944	2.3808	2.6479
80	1.6641	1.9901	2.3739	2.6387
90	1.6620	1.9867	2.3685	2.6316
100	1.6602	1.9840	2.3642	2.6259
200	1.6525	1.9719	2.3451	2.6006
300	1.6499	1.9679	2.3388	2.5923
400	1.6487	1.9659	2.3357	2.5882
500	1.6479	1.9647	2.3338	2.5857

df	$1-\alpha$
	0.95
1	6.3138
2	2.9200
3	2.3534
4	2.1318
5	2.0150
6	1.9432
7	1.8946
8	1.8595
9	1.8331
100	1.6602
200	1.6525
300	1.6499
400	1.6487
500	1.6479

Z table

1,48	0,9306
1,50	0,9332
1,52	0,9357
1,54	0,9382
1,56	0,9406
1,58	0,9429
1,60	0,9452
1,62	0,9474
1,64	0,9495
1,66	0,9515