MSE213 - Lecture 7

Hypothesis Testing: Your friend throws 10 times "6" with a die ...

H_A: "The die is fair" $p_1 = p_2 \dots = p_6 = \frac{1}{6}$

$P(10 \text{ times } 6 \mid H_A) = \left(\frac{1}{6}\right)^{10} = 0.00000017\%$ → seems unreasonable...

H_B: The die has $p_6 = 0.9$, $p_2 = 0.1$ others $= 0$

$P(10 \text{ times } 6 \mid H_B) = (0.9)^{10} = 35\%$ → plausible

$p_6 = 0.99$, $p_5 = 0.01$

H_C: $P(10 \times 6 \mid H_C) = 0.99^{10} = 90\%$ → also plausible...

H_D: $p_6 = 1$ others $= 0$

$P(10 \times 6 \mid H_D) = 1^{10} = 1$ ... → also plausible...?

↯ $P(\text{OUTCOME} \mid \text{HYPOTHESIS}) \neq P(\text{HYPOTHESIS TRUE})$

Hypothesis examples:
- The mean weight of cars in the USA > in Italy → YES/NO
- Smoking makes cancer more likely than not smoking → Y/N
- There is a particle at 125 GeV

$\boxed{5}$ Hypothesis testing procedure

1) Get a YES/NO Hypothesis, as a statement about a probability distribution. → "H_1"

"My friend cheats" → H_1
↳ "The die does not follow a Laplace distribution." → cannot calculate P

2) Get the logical opposite of H_1, i.e. something that covers all other scenarios except H_1. → "H_0", the "null hypothesis"

→ "The die does follow a Laplace distribution" → H_0
→ can calculate P

3) Choose a level of "statistical significance" α. This is the chance you will reject H_0 (and believe H_1) even if it is true.

$\alpha = 5\%$ is common... should it be?

4) Compute $P(\text{outcome} \mid H_0)$ for some measurement/observation.
Is $P < \alpha$ → reject H_0, believe H_1

Die example: $P(10 \times 6 \mid H_0) = 0.0000017\% < 5\%$ → reject H_0...
→ "Friend cheats"

2 throws only: $P(2 \times 6 \mid H_0) = \frac{1}{36} = 2.8\% < 5\%$ ⇝ would you reject H_0?

1 RECIPE FOR Z-TEST (GAUSSIAN TEST FOR THE MEAN)

1) Define $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{N_s}}$

2) Fixed probability?

set P (e.g. 95%)

3) Ask:

(a) Above which value μ_0...

(b) Below wich value μ_0...

(c) In which range $\bar{x} \pm |\mu_0 - \bar{x}|$

... is the true mean that represents my distribution, with probability P?

4) Answer:

(a) $\Phi(+z) = P$ (b) $\Phi(-z) = P$

(c) $2\Phi(|z|) - 1 = \Phi(|z|) - \Phi(-|z|) = P$

→ Find z from table.

→ Compute μ_0.

Fixed value μ_0 / deviation z?

determine z (e.g. 1.5)

3) Ask:

With which probability is the true mean that represents my population...

(a) above μ_0?

(b) below μ_0?

(c) in the range $\bar{x} \pm |\mu_0 - \bar{x}|$?

4) Answer:

(a) $P = \Phi(z)$ (b) $P = \Phi(-z) = 1 - \Phi(z)$

(c) $P = 2\Phi(|z|) - 1 = \Phi(|z|) - \Phi(-|z|)$

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$\boxed{2}$ Two-sample Gauss / z-test.

- We are interested in the difference of two independent distributions X and Y. · We assume σ_X and σ_Y are known.

Distribution of the difference: Mean: $\bar{X} - \bar{Y}$

Variance: $V(X) + V(Y)$

(as $V(-Y) = (-1)^2 V(Y)$)

and is still Gaussian / Normal.

So we now use
$$z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{N_{SX}} + \frac{\sigma_Y^2}{N_{SY}}}}$$
and proceed as before.

$\boxed{3}$ One-sample Student t-test

We do not know σ, but estimate it from the unbiased estimator for σ, $s = \sqrt{\frac{1}{N_S - 1} \sum_{i=1}^{N_S} (X_i - \bar{X})^2}$.

We then get the "t-value" $t = \frac{\bar{X} - \mu_0}{s / \sqrt{N_S}}$ (like z, but $\sigma \to s$).

We now have an estimator (s) in our equation, so the "degrees of freedom" (df) for our system are reduced by 1. $\boxed{df = N_S - 1}$

t now follows a "T-distribution" which is equal to a Gaussian ~~but~~ for $df \to \infty$ but broader (implying more uncertainty) for small df.

We need to look up its CDF in a "t table" now, which depends on df, but otherwise the recipe is the same as in $\boxed{1}$.