Exercise Set 10

1 A new post-it chemical [basic]

Post-its are magical things. The glue used in them sticks quite well to most surfaces, yet they can be easily removed without residue. Even after a long time, the glue does not chemically react with the surfaces. In fact, this glue is a prime example of a serendipitous discovery: While the researchers initially looked for an extremely strong glue, they found a particularly removable one.

Over the years, the company continues to optimize the properties of the glue. Post-it has started an open competition to researchers, to challenge the quality of their glue. Two teams of scientists have made it into the finals. Each team prepares 40 post-its with their glues, and the sticking coefficients are measured by an independent laboratory. The group means and standard deviations are listed below (higher number means it sticks better). Is there statistical evidence on the $\alpha = 0.01$ level that the glues are not all the same?

The provided table does not include this level of significance, but you can use Python or https://fvaluecalculator.com/ to find the critical F-value.

	mean	sd
EPFL team	21.4	4.5
ETH team	16.9	5.5
post-it original	19.1	5.8

2 Is size a good predictor of weight? (a linear regression) [normal]

The weight and height of six female students are presented in the table below:

Height (cm)	170	167	171	181	158	166
Weight (kg)	71	58	56	85	45	53

a) Determine and plot the regression line for this table.

b) Establish the ANOVA table for a linear regression with one slope and run the F-test at a level of significance $\alpha = 0.05$.

c) Compute the "goodness of fit" coefficient R^2. What can you conclude from it?

3 Exam question from 2024

On the following page you will find an exam question from 2024. The exam lasted three hours.

The questions on the next page accounted for 18 out of a total of 68 points - with proportional time allocation, this would correspond to about 48 minutes.

In the exam tables for the z-test, t-test, χ^2 test and F-test were provided (see the StatTables pdf on the moodle) .

Of course now that you do not have your condensed notes yet, you should use your course notes or favourite textbook, and can expect to take longer because of that.

Superconducting crystals

You create a new method for producing superconducting crystals. You find that 20% of the crystals have superconducting critical temperatures above 10K (we call this probability $P(A) = 0.2$). Of those crystals, 60% are paramagnetic at room temperature (we call this probability $P(B|A) = 0.6$). Furthermore[1], 72% of the crystals are non-paramagnetic and do not have a superconducting critical temperature above 10k, $P(\bar{A} \cap \bar{B}) = 0.72$

{1p} a) What is the probability for a randomly chosen crystal to have a superconducting critical temperature above 10K and not be paramagnetic at room temperature, $P(A \cap \bar{B})$?

{2p} b) Create a probability tree diagram, including the values for $P(A)$, $P(\bar{A})$, $P(B|A)$,$P(\bar{B}|A)$,$P(B|\bar{A})$ and $P(\bar{B}|\bar{A})$

{2p} c) Given that a randomly chosen crystal is paramagnetic, what is the probability that it has a superconducting critical temperature above 10K?

Having produced a very large number of these crystals, you find that their resistivity (always measured in units of nΩm) is well-described by a normal/Gaussian distribution with a mean of 150 and a standard deviation[2] of 10.

{1p} d) What is the probability for a randomly chosen crystal to have a resistivity below 160?

{2p} e) What is the probability for a randomly chosen crystal to have a resistivity between 130 and 140?

{2p} f) What is the probability for the *mean* resistivity of a sample of 25 randomly chosen crystal to be below 155?

In a new research project, you develop another method to grow a new type of crystals. Your aim is to show that this new method produces crystals with a mean superconducting critical temperature that is larger than 20K. You produce 4 of those crystals and find critical temperatures (measured in K) of $[21, 19, 22, 26]$.

{2p} g) Compute the mean and the unbiased[3] estimator for the variance from this data.

{2p} h) Which type of statistical test would be suitable to show if you were successful?

{4p} i) Perform this test, using a confidence level[4] of 99% (significance level[5] of 1%)

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