Exercise Set 6

Give your results with 2 significant digits precision e.g. 0.95 or 0.15%, or as a fraction e.g. 1/3

1 The Binomial distribution and the grocer [refresher, normal]

A grocer buy apples from a farmer. 25% of these apples are not suitable for clients. The grocer asks a worker to make packages with 5 apples, but the worker does not care whether the apple is suitable or not. Whenever a client buys a package, if two or more apples are not suitable the client comes back to the grocer to complain.

a) Compute the probability distribution of X = number of good apples in the package.

b) What is the probability for a client to come back and complain?

c) If 100 clients buy a package this day, how many complaints will there be on average? (You can assume that an individual client will only come back to complain at most once).

d) The grocers sells the packages for 2CHF and buys them from the farmer for 1CHF. If the package was bad though, the customer returns to the store, complains and receives a new package for free. Does the grocer make profit on average (ignoring any other costs)?

e) A more friendly grocer, in case of a complaint, also returns back the 2CHF that the customer paid. Does this grocer make a profit on average?

2 Photodetector efficiency [normal]

You are working on producing a new type of photodetector. Your aim is to beat your company's current best detector for 2500nm light, made of InGaAs, which as a responsivity of 1.2A/W.

Your company policy is that improved prototypes are only considered for further development if they are shown to provide an improvement with 99% probability.

You have measured your device and found 1.3A/W, using a testing method that has a standard deviation of 10% of the measured value (from that, you can compute the standard deviation in A/W). You repeat the measurement 4 times. Now you want to convince your company to consider further development.

a) Choose the correct way to test this.

b) Perform the test. Does your photodiode fulfil the requirements?

c) The conclusion of such a test depends on the choice of the confidence level. Here we took the choice of 0.99. Compute the confidence level at which the reference value of 1.2A/W would be exactly at the edge. In that case, what would be the probability, that your phododiode does actually perform worse than the reference value, and your measurement was just a statistical fluctuation?

d) How many measurements would you have to perform to get just over the 99% level?

3 Two different estimators of a macromolecule length measurement [optional, advanced]

A researcher has developed an experimental method to randomly cut a macromolecule and simultaneously measure the length of the molecule she cut off.

Before cutting, all molecules are of equal but unknown length we denote by θ. As the molecules are cut at any point with equal probability, the length distribution can be assumed to follow a uniform law $\mathcal{U}(0,\theta)$, i.e. a continuous distribution whose probability density $f(x)$ is constant and equal to c form 0 to θ and null everywhere else:

$$f(x) = \begin{cases} c & [0,\theta] \\ 0 &]-\infty,0[\cup]\theta,\infty[\end{cases}$$

a) Compute the cumulative distribution function of this density and express the constant c as a function of θ.

b) The observations are noted X_1, X_2, ..., X_n and follow the law $\mathcal{U}(0,\theta)$. Two estimators for θ are proposed:

$$\begin{aligned} \theta_1 &= 2 \cdot \frac{X_1 + X_2 + ... + X_n}{n} \\ \theta_2 &= \max(X_1, X_2, ..., X_n) \end{aligned}$$

   i. Are the estimators biased? Prove your result.

   ii. In the case that one of them is biased, compute its expected mean and suggest a solution to remove the bias.

c) Based on the previous results, compute the mean square error of the two unbiased estimators.

d) Which estimator performs better?