Exercise Set 5

Give your results with 2 significant digits precision e.g. 0.95 or 0.15%, or as a fraction e.g. 1/3

1 Steel cable resistance [basic-normal]

Steel cables for an elevator are tested for tensile resistance. Data (in MPa):

10.1	12.2	9.3	12.4	13.7	10.8	11.6	10.1	11.2	11.3
12.2	12.6	11.5	9.2	14.2	11.1	13.3	11.8	7.1	10.5

a) Compute mean $\bar{x}$ and median $\tilde{x}$ by hand and discuss their relationship. Also find the 25% and 75% quantiles, $\tilde{x}_{0.75}$ and $\tilde{x}_{0.25}$ - if they fall between two points you can take the mean of or just say "between x_i and x_j"

b) Draw a histogram with a bin width of 1MPa and a centres at integer MPa numbers (e.g. 10).

You are the process engineer. You revisit all cables which were measured and find that one of them had not been mounted correctly - it was the one which gave rise to the value of 7.1MPa. You decide to remove it from the dataset.

c) Recompute the quantities from (a) - you can re-do the whole calculation, but there is a faster way.

d) Naturally, the cable strength is very important for your industry partners. Given the empirical cumulative distribution function (with the outlier removed), what minimal strength do you find in 90% of your cables? How about in 95%? Based on the data you have, discuss if you can make a meaningful statement about this type of cable in general.

e) Compute the variance and the biased standard deviation $\tilde{s}$ of this dataset, by hand or in Python. How many datapoints fall into the window $[\bar{x} - \tilde{s}, \bar{x} + \tilde{s}]$?

2 Molecular Beam Epitaxy (MBE), Normal law and precision [normal]

A lab has developed an MBE technique to control the thickness of a film on a silicon wafer (see Dataset 1).

a) Compute the median $\tilde{x}$, mean $\bar{x}$ and standard deviation s of this sample dataset 1. Are there any hints for an asymmetric distribution?

b) Draw a histogram (choose an adequate bin size): is there only one population or more? outliers?

c) What fraction of the data lies within the intervals $[\bar{x} \pm s]$ and $[\bar{x} \pm 2s]$?

d) Model the data by a Gaussian distribution using the experimental values for mean and deviation, assuming $\sigma = s$ and $\mu = \bar{x}$. Compare the fractions computed in (c) with the ones of this Gaussian model, $[\mu \pm \sigma]$ and $[\mu \pm 2\sigma]$. What are the differences to (c)? Why? Can the distribution be represented by a Gaussian?

e) It turns out that the results were achieved by by two different technicians. One of the two repeats the experiment another time (Dataset 2).

f) Recompute the questions a) to d) with the second set and discuss whether the quality has increased or decreased?

g) Intel will only start a partnership with the lab if 95% of the chips lie within the interval [1.45, 1.55]. Assuming that you can infer the correct Normal/Gaussian distribution expected for future production based on this data: Can the lab certify this requirement for its chips? What is problematic about this inference?

h) Compute the 5% and the 95% quantiles. How does this compare against the experimental bounds that Intel required?

Dataset 1 (film thickness [nm]):

1.5	1.71	1.62	1.81	1.27	1.91	1.05	1.15	1.22	1.25	2.00	1.95
1.82	1.84	1.81	1.42	1.37	1.86	1.29	1.85	1.26	1.87	1.84	1.83

Dataset 2 (film thickness [nm]):

1.33	1.71	1.76	1.44	1.45	1.46	1.46	1.62	1.63	1.65	1.67	1.67	1.51
1.51	1.52	1. 58	1.52	1.57	1.57	1.56	1.55	1.56	1.54	1.53	1.53	

3 Maxwell-Boltzmann law and the particle speeds in an ideal gas [basic-avanced]

The speed of particles of ideal gas follow the Maxwell-Boltzmann distribution. The probability density function (PDF) to find a particle at a specific speed is given by:

$$f(v) = C \cdot v^2 \exp\left[\frac{-mv^2}{2k_B T}\right]$$

Here, v is the speed of the particle (i.e. the *absolute* velocity, meaning it is a scalar which is larger or equal to 0), m its mass, k_B the Boltzmann constant and T the absolute temperature in *Kelvin.*

a) Calculate the constant C to ensure $f(v)$ is a well-defined PDF. You can assume that the standard Gaussian integral (i.e. $\int_{-\infty}^{+\infty} \exp[-x^2]dx = \sqrt{\pi}$ is known, but you should derive the more general required Gaussian integrals).

b) Let us assume the gas is pure nitrogen. The weight of a nitrogen atom is $14u$, and each particle is a molecule of 2 atoms, N_2. We have the is "unified atomic mass" $u = 1.6710^{-27}$kg and the Boltzmann constant $k_B = 1.3810^{-23}$J/K. If the gas is at room temperature (25°C), compute the average speed of the gas.