
Exercise Set 3

Give your results with 2 significant digits precision e.g. 0.95 or 0.15%, or as a fraction e.g. 1/3

1 Expectation values and variances [basic-normal]

Let random variables X_1, X_2, \dots, X_n follow all have the same mean μ , and variance σ^2 and all be *independent* with respect to each other. \mathbb{E} denotes the expectation value. Calculate:

- a) $\mathbb{E}(10X_1 + 2X_2)$
- b) $\mathbb{E}((X_1 + X_2 + \dots + X_n)/n)$
- c) $\text{Var}(10X_1 + 2X_2)$
- d) $\text{Var}((X_1 + X_2 + \dots + X_n)/n)$
- e) Using the expressions for $\sum_{i=1}^n i$ and $\sum_{i=1}^n i^2$ from the lecture, prove that

$$\mathbb{V}(X) = \frac{(n+1)(n-1)}{12}$$

for throwing a die with n sides (i.e. you have n elements which all can appear with equal probability).

2 Bernoulli law, Normal law and car insurance [normal]

Table 1 shows a list of different car accidents in Switzerland and their average costs for the insurance. The total operating budget of all insurance firms (paying all employees, renting the building, etc. but not the payout of insurance cases) is 1.7 billion CHF, and the total number of insured cars in Switzerland is 4.4 million.

219 dead	300'000 CHF
3654 seriously injured	100'000 CHF
17'759 lightly injured	5'000 CHF

Table 1: Yearly accidents on the swiss roads and respective average costs.

- a) Assume that the results in the table are *representative*. We will discuss more what that means, but essentially it means that relative frequency is directly giving the probability. How much does the average client cost a company? For this, first compute the probability p for a client to have an accident of any type (assume one client does not have multiple accidents). Then get the total cost and from that get the average cost per client. After that you can ignore the types of accident and just say "with probability p the client has an accident with a certain cost."

- b) We now model the distribution by a Gaussian with a centre and standard deviation following the Binomial law. How much should a client pay such that an insurance with 10'000 clients is profitable with a probability of 95%?
- c) How would b) change with 1'000'000 clients?
- d) The average yearly insurance cost in Switzerland is 1'100 CHF. What profit do car insurance companies make?

What is the profit per employees, if there are 10'000 employees for car insurance?

3 The distribution function of a mean [advanced]

- a) Let there be two independent normal/Gaussian distributions, both with a mean of 0. One has a standard deviation of 4 while the other has a standard deviation of 3. Derive the probability distribution for the mean of these two distributions (assuming that each was sampled once and they are given equal weight).

We assume that we know the usual Gaussian integral:

$$\int_{-\infty}^{+\infty} \exp[-t^2] dt = \sqrt{\pi} \tag{1}$$

(for which a number of elegant proofs exist).

- b) What is the mean and standard deviation of the resulting distribution? How does this compare to the result you expect from the rules for adding variances for independent random variables?