# Exercise Set 2

*Give your results with 2 significant digits precision e.g. 0.95 or 0.15%, or as a fraction e.g.* 1/3

## 1 Refresher on probabilities [normal]

When Mr Schmitt has a cold, he always wears his red jumper. When he does not have a cold, there is a 6% chance he will wear his red jumper. On average, Mr Schmitt has a cold on 15 days in one year.

a) Draw a probability tree diagram and fill in the probabilities that are given.

b) What is the overall probability for Mr Schmitt to wear his red jumper?

c) You see Mr. Schmitt *not* wearing his red jumper. What is the probability that he has a cold?

d) Mr. Schmitt is wearing his red jumper. What is the probability that he has a cold?

e) Are the probabilities for "Mr Schmitt has a cold" and "Mr Schmitt wears his red jumper" independent? Prove your statement.

## 2 Bonus round [basic]

In a quiz show, you win the chance to play a bonus game. One dice is rolled, and you receive the number of eyes *squared* in 100 CHF bills. For example, if the dice shows the number 2, you win $100 \times 2^2 = 400$ CHF.

a) What are the minimal and maximal possible amounts you can win?

b) How much can you expect to win on average?

c) Why is the result in (b) not $100 \times 3.5^2$ CHF

## 3 The probability distribution of dice [normal]

a) Find the probability distribution of rolling a fair, single dice with 6 faces (normal dice). Compute the expectation value, variance and standard deviation.

b) Now you throw 2 fair dice in parallel, and you play a game that depends on the sum of the faces. Say, $X_1$ denotes the result of die 1, and $X_2$ that of die 2. You are now interested in their sum, $X = X_1 + X_2$. Find the probability of $X$, and compute its expectation value $\mu$, variance and standard deviation $\sigma$.

c) Draw the probability mass distribution and cumulative distribution.

d) Someone offers you to gamble in the following game: You throw 2 dice. If the sum $X$ is below 10, you have to pay 1 CHF, but if it is 10 or higher, you win 2 CHF. Is it a good idea for you to play this game (is it fair)?

## 4 Binomial distribution and coin game [normal-advanced]

A fair coin has two possible outcomes, "heads" or "tails", that occur with equal probability ($p = 1/2$). The coin toss is the paradigmatic realization of a binomial experiment. We denote the number of coin tosses by $n$, and we are interested only in the total number of "heads" and "tails" in the $n$ tosses, not their order. This is described by the random variable $X = \sum_{i=j}^{n} X_j$, where $X_j$ denotes the value assigned to the outcome of the j-th coin throw. You define $X_j = 1$ if the j-th throw shows "heads" and $X_j = 0$ if it shows "tails".

a) You throw a coin ten times. Compute the probabilities of the following events:

   1) only "heads" in every throw
   2) either only "heads" or only "tails"
   3) exactly 5 "heads" and 5 "tails"

b) 1) Express the probability $P(X = k)$ and $P(X \leq k)$ for any integer k with $0 \leq k \leq n$.
   2) Compute the expectation value $\mathbb{E}[X]$ and its standard deviation $\sigma = \sqrt{\mathbb{V}[X]}$ of the Binomial distribution for general n and state the numbers for the example above, with n=10.
   3) The variance apparently is zero for two special cases, $p = 0$ and $p = 1$. Why is that?
   4) Consider now an unfair coin. It has been made to show "heads" with a very low probability, $p = \frac{1}{6}$. On average, how many times do you expect to throw the coin before you find the first "heads"?