$I=2 \quad J=2$

3a) (2)

1pt

$$\underline{\bar{X}_{1,\cdot,\cdot}} = \frac{1}{J}\sum_j \bar{X}_{1,j,\cdot} = \frac{67+83}{2} = \underline{75}$$

$$\underline{\bar{X}_{2,\cdot,\cdot}} = \frac{139+211}{2} = \underline{175}$$

$$\underline{\bar{X}_{\cdot,1,\cdot}} = \frac{1}{I}\sum_i \bar{X}_{i,1,\cdot} = \frac{67+139}{2} = \underline{103}$$

$$\underline{\bar{X}_{\cdot,2,\cdot}} = \frac{83+211}{2} = \underline{147}$$

$$\left[\bar{X}_T = \bar{X}_{\cdot,\cdot,\cdot} = \frac{67+83+139+211}{4} = 125 \left(\text{or via } \frac{75+175}{2} \text{ or } \frac{103+147}{2}\right)\right]$$

optional

1pt.

$J \backslash I$	Pt	Mo	all
RT	67	139	103
LN	83	211	147
all	75	175	125

optional

Transpose is ok.

3b) $SS_{E,i,j} = S_{i,j}^2 \cdot (N_s - 1) = S_{ij}^2 \cdot 10$

$\xleftarrow{} 11$

2

0.5 pt

$SS_{E\,1,1} = 11^2 \cdot 10 = \underline{1210}$

$SS_{E\,1,2} = 8^2 \cdot 10 = \underline{640}$

$SS_{E\,2,1} = 14^2 \cdot 10 = \underline{1960}$

$SS_{E\,2,2} = 17^2 \cdot 10 = \underline{2890}$

(1p if correct without explicit formula)

1pt $\underline{\underline{SS_{E(TOT)}}} = \sum_i \sum_j SS_{E\,i,j} = 1210 + 640 + 1960 + 2890$

$= \underline{\underline{6700}}$

3c) $SS_{B,I} = J \cdot N_s \cdot \sum_i (\bar{X}_{i,\cdot,\cdot} - \bar{X}_T)^2$ $N_s = 11$

1

$= 2 \cdot 11 \cdot [(75-125)^2 + (175-125)^2] = 2 \cdot 11 \cdot 5000$

$= \underline{\underline{110'000}}$

$SS_{B,J} = 2 \cdot 11 [(103-125)^2 + (147-125)^2] =$

$= 2 \cdot 11 \cdot 968$

$= \underline{\underline{21'296}}$

~~3~~ 3 d) for this we have to compare the

(1) MEAN SQUARE ERROR, $\frac{SS_{B,\cancel{A}}}{\nu_B} = MS_B$

$\nu_{B,I} = \nu_{B,J} = 2-1 = 1$ so $SS_B = \cancel{SS} MS_B$ here.

$MS_{B,I} > MS_{B,J}$

So factor I ("type of metal") is dominant.

0.5pt if only compared $SS_{B,I}$ to $SS_{B,J}$

$\nu_{B,I:J} = (I-1)(J-1) = 1\cdot 1 = \underline{1}$ 0.5pt

3e) $\nu_{B,I} = \nu_{B,J} = 2-1 = \underline{1}$ 0.5pt

5

$\nu_E = I\cdot J\cdot (N_s-1) = 2\cdot 2\cdot (11-1) = \underline{40}$ 0.5pt

$$\underline{MS_{B,I}} = \frac{SS_{B,I}}{\nu_{B,I}} = \underline{110'000}$$

$$\underline{MS_{B,J}} = \frac{SS_{B,J}}{\nu_{B,J}} = \underline{21'296}$$

$$\underline{MS_E} = \frac{SS_E}{\nu_E} = \frac{6700}{40} = \underline{167.5}$$

0.5pt

$$\underline{SS_{B,I:J}} = N_s \sum_i \sum_j \left(\bar{x}_{i,j,\cdot} - \bar{x}_{i,\cdot,\cdot} - \bar{x}_{\cdot,j,\cdot} + \bar{x}_T\right)^2$$

$$= 11\cdot\Big[(67 - 75 - 103 + 125)^2 + (83 - 75 - 147 + 125)^2 + (139 - 175 - 103 + 125)^2 + (211 - 175 - 147 + 125)^2$$

$$= 11\left[14^2 + (-14)^2 + (-14)^2 + 14^2\right]$$

all same as interaction has only one df

$$= 11\cdot 4\cdot 196$$

$$= \underline{8624}$$

$$\underline{MS_{B,I:J}} = \frac{SS_{B,I:J}}{\nu_{B,I:J}} = \frac{8624}{1} = \underline{8624}$$

2pt

$F_I = MS_{B,I}/MS_E = \frac{110'000}{167.5} = 656.7\ldots \approx \underline{657}$

$F_J = 21'296/167.5 \approx \underline{127}$

$F_{I:J} = 8624/167.5 \approx 51.50.. \approx \underline{52}$

0.5pt

	ν	SS	MS	F
FACT I	1	110'000	110'000	657
FACT J	1	21296	21'296	127
INTER I:J	1	8'624	8'624	52
ERROR	40	6700	167.5	/

0.5pt [for table]

3 f) $F_{OBS, I:J} = 51$

2

$qF_{\nu_1=1, \nu_2=40}(P=95\%) = 4.085$

1 pt.

$F_{OBS\ I:J} > 4.085 \Rightarrow$

We reject the no-interaction hypothesis at the 5% significance level.

1pt

3 g)

3

2pt

$$\frac{\partial SS_E}{\partial \hat{a}} = \frac{\partial}{\partial \hat{a}} \sum_{i=1}^{N_T} \left[K_i - (\hat{a} + \hat{b}_1 T_i) \right]^2$$

$$= (-1) \cdot 2 \sum_{i}^{N} \left[K_i - (\hat{a} + \hat{b}_1 T_i) \right]$$

$$= \cancel{-2} \left(\cancel{N_T} \bar{K} - \cancel{N_T} \cdot \hat{a} - \cancel{N_T} \cdot \hat{b}_1 \cdot \bar{T} \right) \stackrel{!}{=} 0$$

$$\Rightarrow \underline{\underline{\hat{a} = \bar{K} - \hat{b}_1 \bar{T}}}$$

1pt

This shows it is an extremum. To show it is a <u>minimum</u> we also need to show $\frac{\partial^2 SS_E}{\partial \hat{a}^2} > 0$ at $\hat{a} = \bar{K} - \hat{b}_1 \bar{T}$

$$\frac{\partial^2 SS_E}{\partial \hat{a}^2} = \frac{\partial}{\partial \hat{a}} \left[-2 N_T \left(\bar{K} - \hat{a} - \hat{b}_1 \bar{T} \right) \right]$$

$$\underline{\underline{= 2 N_T > 0}} \quad \checkmark$$

$= N_{TOT} - 1 - \nu_m$

3 h) $\nu_{E,m1} = 7 - 1 - 1 = \underline{\underline{5}} = \nu_{E,\text{restricted}}$

1 slope parameter

(1)

$\nu_{E,m2} = 7 - 1 - 2 = \underline{\underline{4}} = \nu_{E,\text{full}}$

3 i) $\underline{\underline{F_{m1vsm2}}} = \dfrac{\Delta SS_E / \Delta \nu_m}{SS_{E,\text{full}} / \nu_{E,\text{full}}}$

(2)

$= \dfrac{(1255-405)/(2-1)}{405/4}$

$= \dfrac{850}{101.25} = 8.39506\ldots \approx \underline{\underline{8.4}}$

1

$qF_{\Delta\nu,\nu_{E,\text{full}}}(p=95\%) = qF_{1,4}(p=95\%) = \underline{7.709}$ 0.5

$F_{OBS} > qF \Rightarrow$ The second modell is statistically significantly (at 5% significance) better.

0.5

3 j) calibratedData = measuredData * 1.2

1 (numpy arrays multiply element-wise)

3 k) [68.4 72.1]

1 (for 0:2 you get elements 0 and 1).

0.5 pts for [68.4 72.1 1200.2]

3 l) I) print(np.max (calibratedData))

2 II) print(calibratedData [calibratedData > 1000])

III) for i in calibratedData:

```
    if i > 1000:
        print("spike")
    else:
        pass
```

etc...

3 m) I) use

2

print(np.vectorize(myFilk)(calibraten Data))

(ok. in words)

II)

run a loop:

for i in range(calibraten Data):

calibraten Data[i] = myFilk(calibraten Data[i])

etc...