Exercise Set 13 - Solution

1 Finishing the two-factor ANOVA exercise

We look at the factor interaction. We have shown that both skiing and fondue increase happiness. But do these factors interact, so are people significantly happier if the have both on the same day, or do they just feel the same increase in happiness that fondue and skiing do separately?

To compute $SS_{Bskiing:fondue}$ we could either use the expression to get it directly, or we start from $SS_T = \sum_{i,j,k}(x_{i,j,k} - \bar{x}_{\bullet,\bullet,\bullet})^2 = 181.56$. For the second option, we know that the difference between the total sum of squares and the model with non-interacting factors is just given by the factor interaction. We find:

$$SS_{Bskiing:fondue} = SS_T - SS_{B,skiing} - SS_{B,fondue} - SS_E \approx 1.0$$

Because the interaction only has one degree of freedom $(I-1)(J-1) = 1$ we have $MS_{Bskiing:fondue} = SS_{Bskiing:fondue}$ This is a very small contribution compared to the other factors but also compared to the error. Constructing the updated ANOVA table:

Source	ν	SS	MS	F
Skiing	1	28.44	28.44	7.16
Fondue	1	25	25	6.29
Fondue:Skiing	1	1	1	0.25
Error	32	127.11	3.97	
Total	35	181.56		

Given this low F-value (compared to $\mathcal{F}_{1,32} = 4.149$), there is no statistically significant evidence for factor interaction. It seems people gain just the same happiness from fondue on a ski hill than at any other place. We have to tell the company that marketing on the ski hills will not be much more effective than any place else.

2 The tuna-nutella sandwich

As before, we construct an ANOVA table including factor interaction. First we compute the SS_B. Denoting nutella as factor I and tuna as factor J, we write $\bar{x}_{i,j}$ for the *mean* taste scores. Each group contains $n = 36/4 = 9$ Elements.

The partial means are:

$$\bar{x}_{0,\bullet,\bullet} = 5.39, \quad \bar{x}_{1,\bullet,\bullet} = 5.285, \quad \bar{x}_{\bullet,0,\bullet} = 5.285, \quad \bar{x}_{\bullet,1,\bullet} = 5.39$$

We see that all numbers are very similar, and close to the global mean. It is clear that there will be little effect of the main factors.

$$SS_{B,nutella} = Jn\sum_i(\bar{x}_{i,\bullet,\bullet} - \bar{x}_{\bullet,\bullet,\bullet})^2 = 0.099$$

$$SS_{B,tuna} = In \sum_j (\bar{x}_{\bullet,j,\bullet} - \bar{x}_{\bullet,\bullet,\bullet})^2 = 0.099$$

We are given the sum of squares for the errors within each group. The total SS_E is their sum, $SS_E = 14.67$. This leads to:

$$SS_{B,tuna:nutella} = SS_T - SS_{B,tuna} - SS_{B,nutella} - SS_E = 7.13$$

This allows us to construct the ANOVA table. Degrees of freedom for Tuna and Nutella are always 1 (2 levels only, constrained by one total mean), and hence the degrees of freedom for their interaction (the product of the individual degrees of freedom) is also 1. The total has 36 (as we have 36 elements and one total mean that is estimated from the data), so this leaves 32 for the error (which is just $(9-1) \times 2 \times 2$) :

Source	$df = \nu$	SS	MS	F
Tuna	1	0.099	0.099	0.22
Nutella	1	0.099	0.099	0.22
Tuna:Nutella	1	7.13	7.13	15.55
Error	32	14.67	0.46	
Total	35	22		

We see that just adding Nutella or Tuna to a sandwich has little effect, but adding both has a huge one. As in the exercise above, we compare this to the critical F-value of $\mathcal{F}_{1,32}(95\%) = 4.15$. We find that there is no effect of the individual factors factors. That means that on average just knowing one ingredient does not let you make a good guess about the average tastiness.

For example, if you only know that the sandwich contains tuna, you cannot say (at this level of confidence) if people will find it more tasty than a sandwich without tuna - because you don't know whether there is *also* nutella on it or not.

However, there clearly are statistically significant effects of the interaction. In fact, the the total variability is dominated by this interaction. Given that the mean for having tuna and nutella on the same sandwich is much smaller than the mean, we conclude that having both together tastes worse. Not surprisingly!