Exercise Set 1

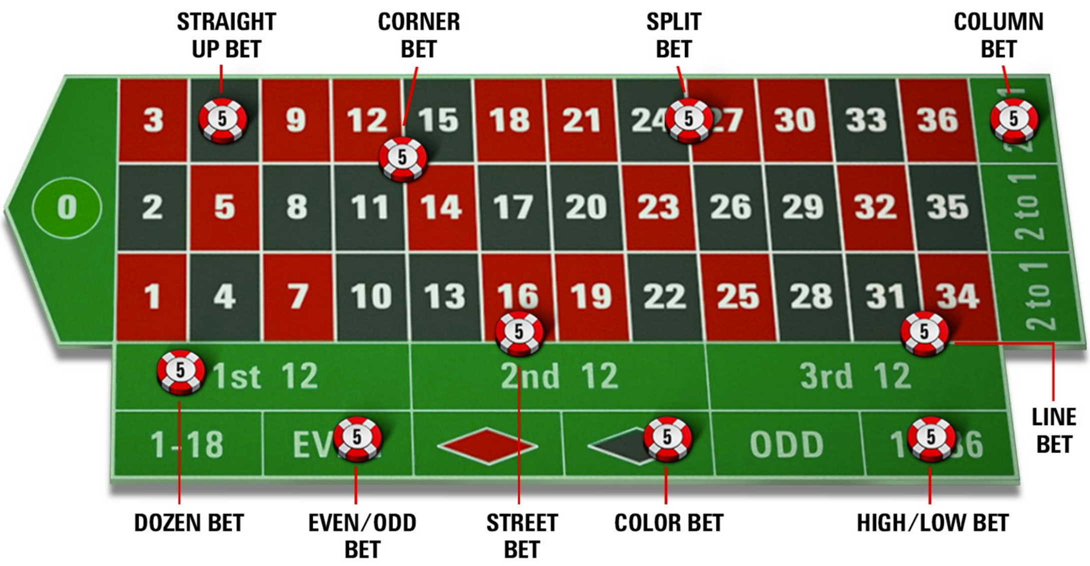
1 Elementary events operation [normal]

a) Let A and B be independent events. Prove that the complementary events $\bar{A}$ and $\bar{B}$ are also independent.
b) Let E, F and G be three events and assume their individual probabilities P(E), P(F), P(G) are known. Express the probabilities of the following composite events in terms of the probabilities of E,F, and G as well as their intersections:

1) "none of them is realized"?
2) "at maximum one is realized"
3) "exactly two are realized"
4) "E and F are realized but not G"

Drawing Venn diagrams can help you with this task.

2 Your odds at roulette [normal]



Roulette lives from composite events. Above, a roulette board is shown. Assume that all numbers [0,36] can be realized with equal probability (elementary events).

a) Compute the following probabilities:

1) Single number.

2) Red [32,19,21,25,34,27,36,30,23,5,16,1,14,9,18,7,12,3].

3) First dozen.

4) Even.

b) Are "Red" and "Even" independent events?

3 Modeling a random coin game experiment [normal]

In this game, we throw a coin four times and record the ordered results: heads or tails?

a) Describe the sample space Ω. How many elementary events does Ω contain?

b) A useful way to record this game is a vector (coin1,coin2,coin3,coin4), assigning the number 1 for heads and 0 for tails. For example, $(1, 0, 0, 1)$ denotes the elementary event "first coin heads, second and third coin tails, fourth coin heads". Write down the following composite events in terms of the elementary ones:
- "We get heads at the second and fourth throw"
- "We get heads only at the second and fourth throw"
- "We get at least one heads out of the four throws"
- "We get at least two heads out of the four throws"

c) Let assume the coin is fair and both sides have equal probability. Compute the probabilities of all the events the you just expressed.

d) Lets think about a completely different situation. We now consider a family with four children. Using the oversimplified assumption of a 50% probability for giving birth to a boy and a 50% probability for giving birth to a girl, and no correlation (i.e. the probability at each birth does not depend on previous births), what are the probabilities to have:
- only boys?
- only girls?
- two boys and two girls?

e) Comment on whether we could use the same model for the hair colour of four children from the same parents.