EPFL
Autumn Semester 2025

Probability and statistics for materials science
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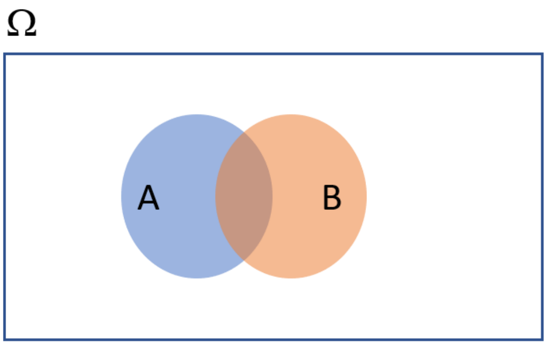
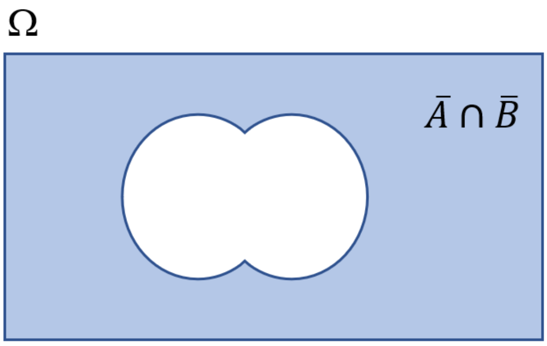
Exercise Set 1 - Solutions

1 Elementary events operation

a) if A and B are independent events, show that $\bar{A}$ and $\bar{B}$ (the complementary events) are also independent.





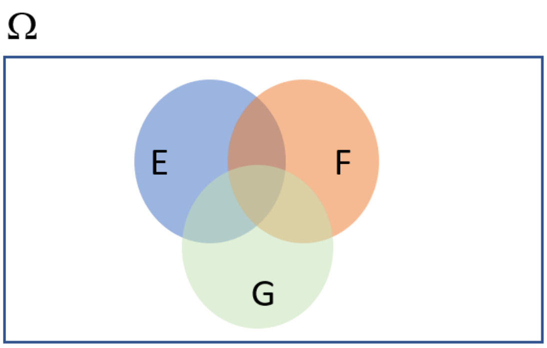
Two events, A and B, are stochastically independent if and only if $P(A \cap B) = P(A) * P(B)$. We need to show that $P(\bar{A} \cap \bar{B}) = P(\bar{A}) * P(\bar{B})$ to prove independence.

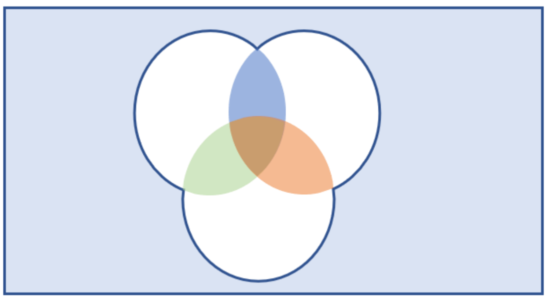
Drawing the Venn diagram illustrates that $P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B) + P(A \cap B) = 1 - P(A) - P(B) + P(A) * P(B)$, as A and B are independent.

$1 - P(A) - P(B) + P(A) * P(B) = (1 - P(A)) * (1 - P(B)) = P(\bar{A}) * P(\bar{B})$.

Therefore, also the complementary events of independent events are themselves independent.

b) Let E, F and G be three events. Find the expressions in terms of the probabilities P(E),P(F),P(G) for:





- "none of them is realized"

$$P(\Omega \setminus E \setminus F \setminus G) = 1 - P(E) - P(F) - P(G) + P(E \cap F) + P(F \cap G) + P(E \cap G) - P(E \cap F \cap G)$$

- "at maximum one is realized"

$$1 - P(E \cap F) - P(F \cap G) - P(E \cap G) + 2P(E \cap F \cap G)$$

- "exactly two are realized"

$$P(E \cap F) + P(F \cap G) + P(E \cap G) - 3P(E \cap F \cap G)$$

- "E and F are realized but not G"

$$P(E \cap F) - P(E \cap F \cap G)$$

2 Your odds at Roulette

a)

1) There are a total of 37 numbers possible. The odds to find a single number are thus $P = \frac{1}{37} \sim 2.7\%$.
2) The zero counts as colorless, the other 36 are half black, half red. Hence the Laplacian probability is given by $P = \frac{18}{37} \sim 48.6\%$.
3) $P = \frac{12}{37} \sim 32.4\%$.
4) $P = \frac{18}{37} \sim 48.6\%$. (zero counts as neither even nor odd)

b) We have to show that $P(Even \cap Red) = P(Even) * P(Red)$. We know from 2) and 4) that $P(Even) * P(Red) = \frac{324}{1369} \sim 23.7\%$. We also need to find the intersection, meaning we need to count all numbers that are both even and red at the same time numbers. A look at the list gives 8 even red numbers. Thus, $P(Even \cap Red) = \frac{8}{37} \sim 21.6\%$.

Hence, the events "red" and "even" are *not* independent. This means that if a roulette ball has been thrown, and your friend knows the number but only tells you the color that has been realized, you have a slightly higher chance than 50% to guess right because the events are dependent.

3 Modeling a random coin game experiment

In this game, we throw a coin four times and record the results.

a) Each coin throw has 2 possible outcomes, "heads" or "tails". We assign numbers to these cases, "heads"=0 and "tails"=1. Four consecutive coin throws form an elementary event in this game, denoted by a vector (coin1,coin2,coin3,coin4). For example, $(1, 0, 0, 1)$ denotes the elementary case "first coin tails, second and third coin heads, fourth coin tails".

There are 2 outcomes per coin, and 4 coin throws. Therefore Ω contains $2^4 = 16$ elementary events.

b)

- "we get heads at the second and fourth throw"

$\{(0,0,0,0),(1,0,0,0),(0,0,1,0),(1,0,1,0)\}$

- "we get heads only at the second and fourth throw"

$\{(1,0,1,0)\}$

- "we get at least one heads out of the four throws"

$\Omega \setminus \{(1,1,1,1)\}$

- "we get at least two heads out of the four throws"

$\{(1,1,0,0),(1,0,1,0),(1,0,0,1),(0,1,1,0),(0,1,0,1),$
$(0,0,1,1),(0,0,0,1),(0,0,1,0),(0,1,0,0),(1,0,0,0),(0,0,0,0)\}$

c) This can be computed by counting the number n of cases in each event in c). The total number of events is $2^4 = 16$. Therefore, the probability of each event is $\frac{n}{16}$.

d) Even though the family composition and coin throws are very different processes, within the over-simplified assumption used here they share the same mathematical description: independent random processes with a probability of $p = 0.5$. Hence the results from c),d) are just the same, mapping "heads/tails" to "girl/boy".

The reality in the latter case is much more complex of course, for biological and social reasons.

And even coin flips are not exactly 50:50 - there is a slightly higher chance for the coin to fall onto the same orientations that it started, see https://doi.org/10.48550/arXiv.2310.04153 , and one has to deal with the (small) probability that the coin can land on the edge.

Remember, mathematical models are a description of reality which is often useful, but ultimately a human creation.

e) For the hair colour of two children there are a number of things that would be badly captured:

- It is more difficult to decide how to reduce the outcomes to only two options
- For any choice of two options, a $50:50$ probability is less likely to be a good approximation.
- The approximation that the probabilities of consecutive results are uncorrelated is of course a very bad assumption: If one child has blonde hair, the probability of another child having blonde hair is much higher than it would be in general.