EPFL

SCALING in THERMAL SYSTEMS

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micro-606

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Outline – Thermal Scaling Chapter

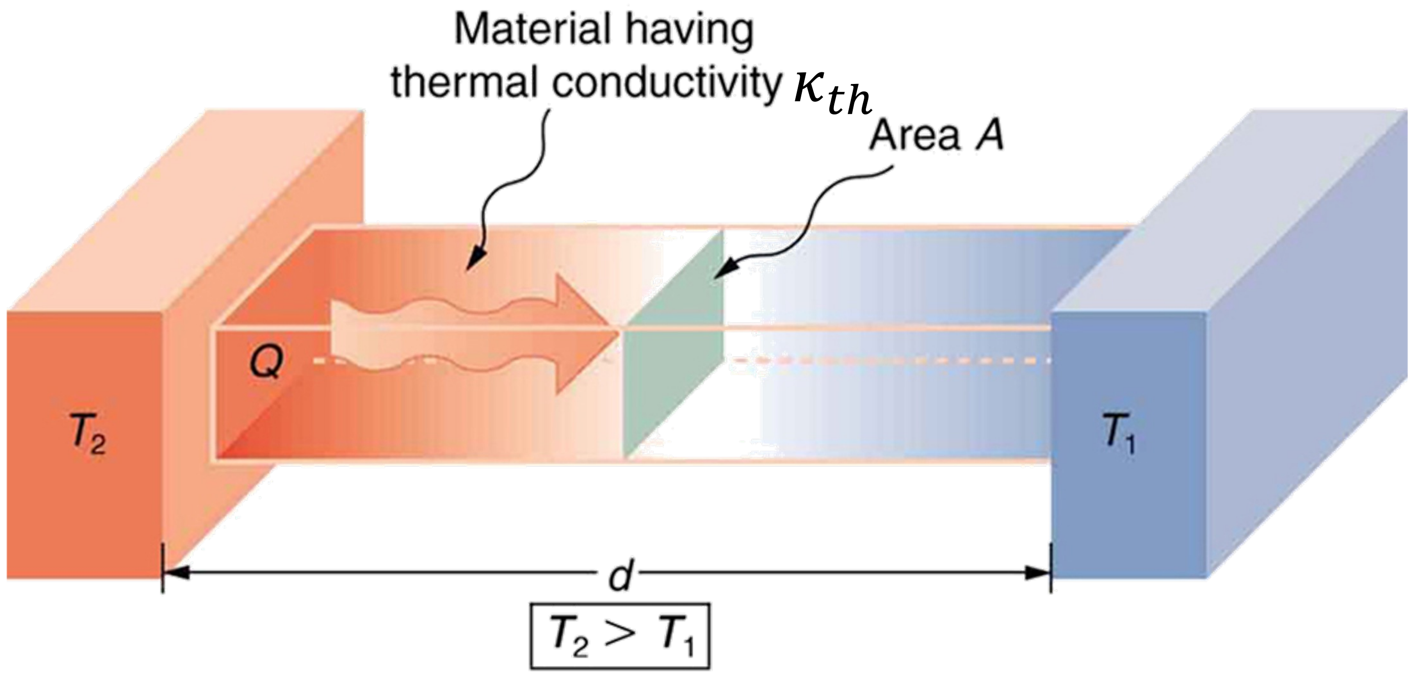
1. Heat transfer in solid
2. Heat conduction in a gas
3. Radiation
4. Thermal Time constants
5. ElectroThermal Actuators

Concepts to master in Thermal systems

- Thermal transport by conduction in solid
  - RC model of thermal time constant
  - scaling of R_{th} (or G_{th}) and c_{th}
- Thermal transport by conduction in a gas: length scale at which dominant
- Radiative heat transport, and scaling in small gaps
- Temperature profile along a uniformly heated wire for : a) only conduction, and b) conduction and convection
- Dynamics of thermal system in 1D
- How thermal accelerometer works
- How thermal inkjet works
- Electrothermal actuators
  - Main geometries (bimorph, hot/cold arm, chevron)
  - Energy density
  - efficiency

Heat conduction (ignoring radiation and convection for now)

- As soon as we put two elements with different temperatures in contact: we get a heat flux from the warmer side to the cooler one.
- Equilibrium state: temperature gradient (if both temperatures T_1 and T_2 are fixed)

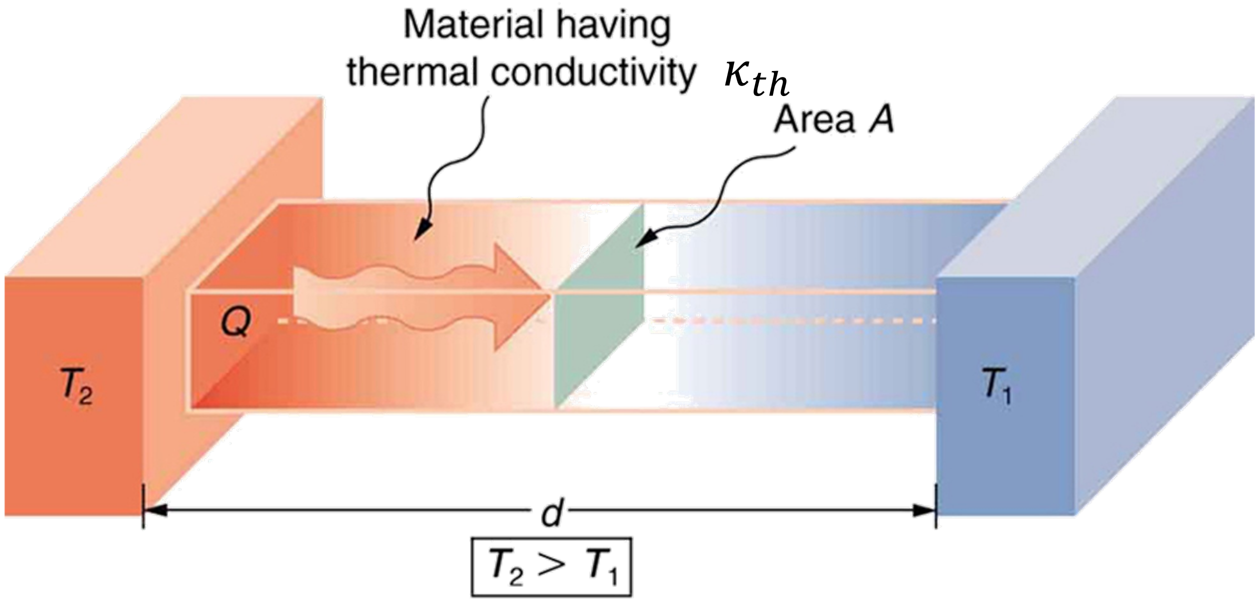


Heat conduction between 2 objects

- Heat flux is given by Fourier's law:

$$\vec{q} = -\kappa_{th}\,\vec{\nabla}T$$

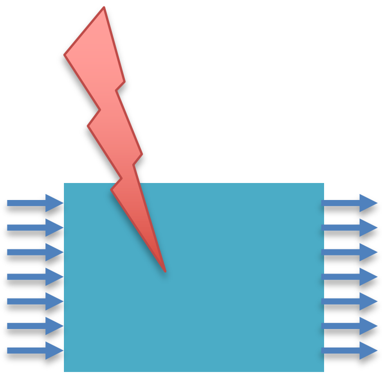
$$\dot{Q}_A = \vec{q}\vec{A} = -\kappa_{th}\,\vec{\nabla}T.\vec{A}$$



- q : rate of flow of thermal energy per unit area [W. m^{-2}]
- Q is the internal energy [J]
- dQ/dt is amount of heat transferred per unit time in [W]
- κ_{th} is thermal conductivity in [W/(m.K)]
- c_v is specific heat capacity [J/(kg.K)]
- C is heat capacity [J/K)]
- A is area [m^2]
- T is temperature [K]

1D heat flux with distributed heat source q(x)

- 1D Temperature $T(x,t)$ distribution given by:



$$\kappa_{th}\, A \frac{d^2T(x)}{dx^2} + q_{source}(x) = \rho\, A\, c_v \frac{\partial T}{\partial t}$$

Steady state, 1D, no heat source

$$\kappa_{th} A \frac{d^2T(x)}{dx^2} + q_{source}(x) = \rho A c_v \frac{\partial T}{\partial t}$$

$$\kappa_{th} A \frac{d^2T(x)}{dx^2} = 0 \quad \rightarrow \; T(x) = ax + b$$

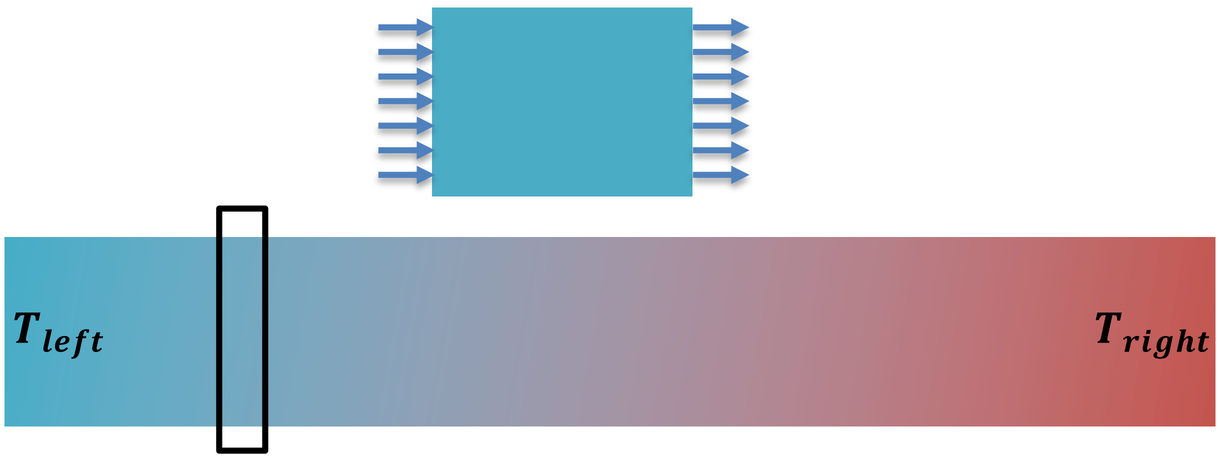
$$T(x) - T(0) = \frac{x}{L} \Delta T$$

constant heat flow:

$$\dot{Q}_A = -\frac{\kappa_{th}}{L} \Delta T$$

Thermal resistance $$R_{th} = \frac{L}{A\,\kappa_{th}}$$ $\propto L^{-1}$

Thermal conductance $$G_{th} = \frac{A\,\kappa_{th}}{L}$$ $\propto L$



Transient state (time-dependence), no source

- Temperature gradient:

$$\kappa_{th} A \frac{\partial^2 T(x,t)}{\partial x^2} + q_{source}(x) = \rho A c_v \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c_v}{\kappa_{th}} \frac{\partial T}{\partial t}$$

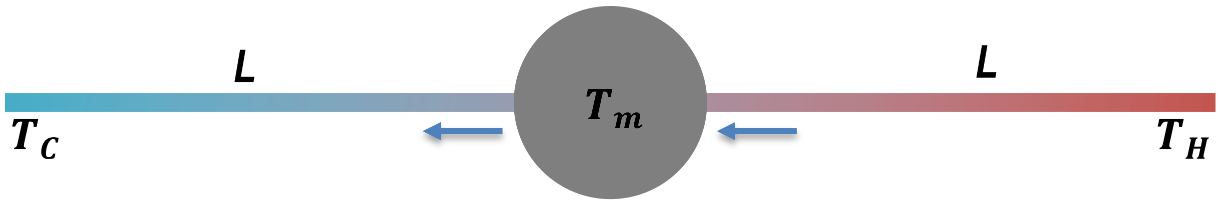
Partial derivatives make analytical solution a bit cumbersome

$$\frac{\kappa_{th}}{\rho c_v}$$

Thermal diffusivity: ability of an object to take heat out

Transient state for small mass with long anchors

- For a mass that is spatially small compared to the thermal links, we can calculate the transient easily if <u>we assume the mass has no temperature gradient</u> (=lumped):



$$\dot{Q}_{out} = \frac{\kappa_{th}}{L} A (T_m - T_c)$$

$$\dot{Q}_{in} = \frac{\kappa_{th}}{L} A (T_H - T_m)$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{\kappa_{th}}{L} A (T_H + T_c - 2T_m) = m.c_v \frac{\partial T_m}{\partial t}$$

$$T_m(t) = \frac{T_H + T_c}{2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{\rho c_v L.Vol}{2\kappa_{th} A} \quad \propto L^2$$

Time constant

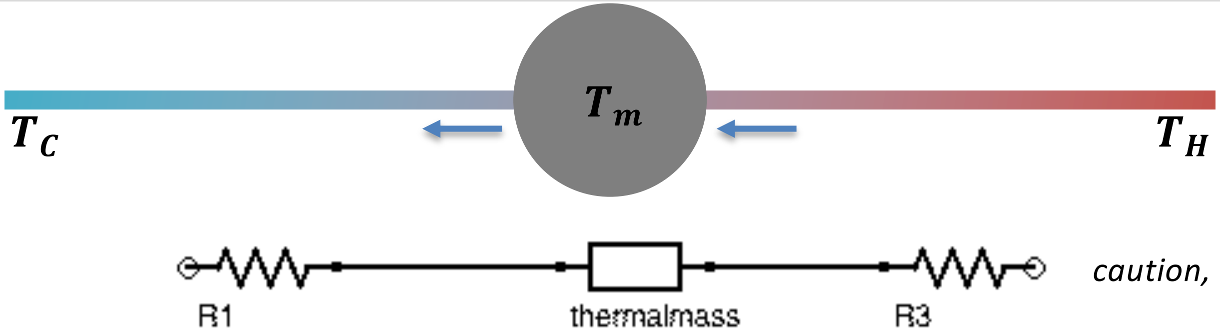
$$\tau = RC$$

$$C_{th} = \rho Vol.c_v = mc_v \quad \propto L^3$$

$$R_{th} = \frac{L}{\kappa_{th} A} \quad \propto L^{-1}$$

Initial conditions: $T_m(t=0) = 0$, and T_{hot} and T_{cold}

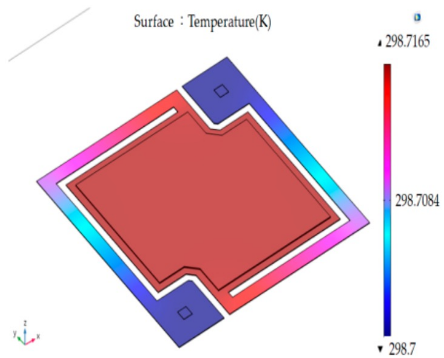
What allowed us to do this lumping?



caution, not electrical equivalent circuit !

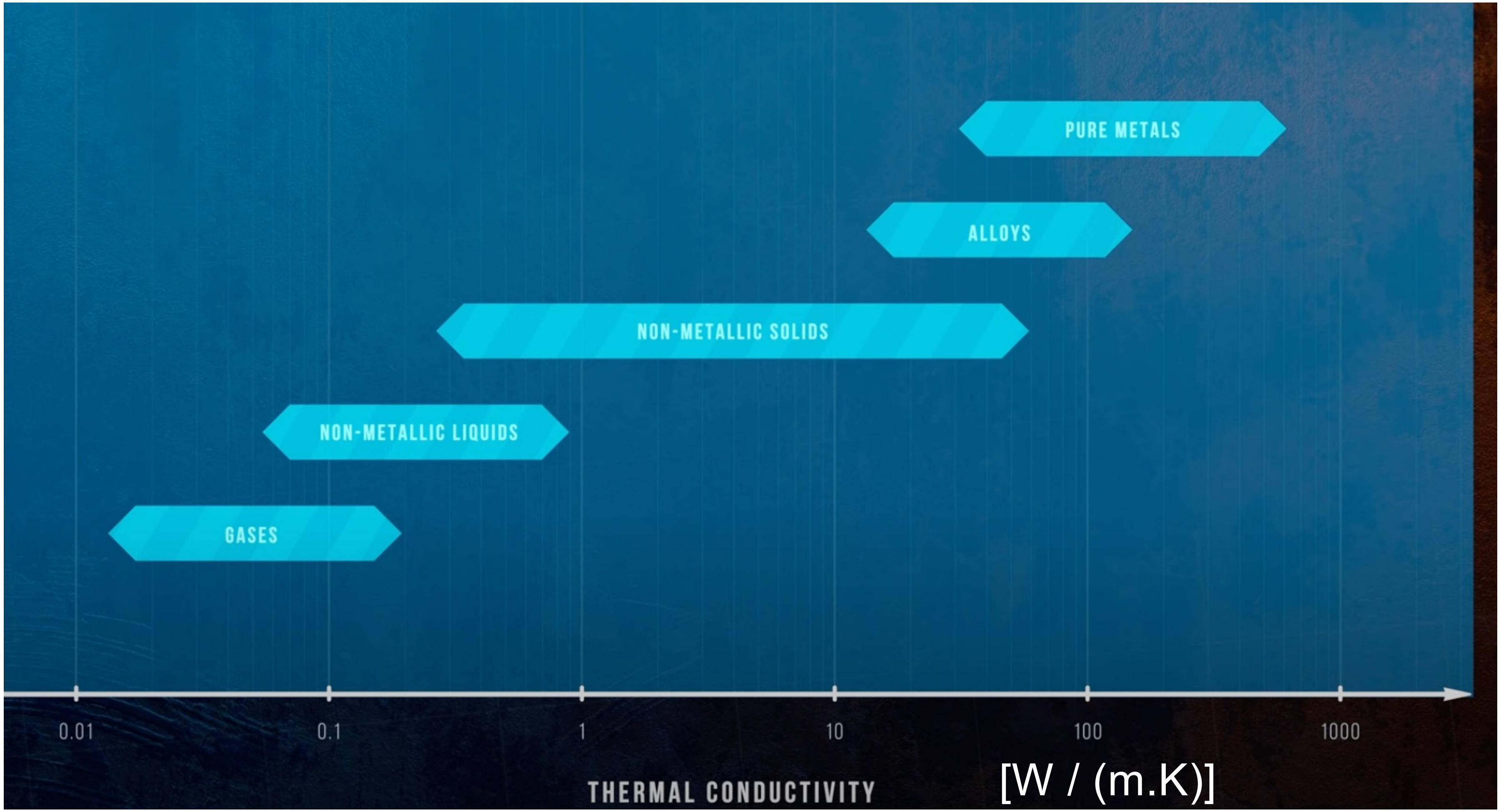
- We got rid of the partial derivatives $\frac{\partial^2 T(x,t)}{\partial x^2}$ by assuming the temperature in the middle region is uniform and by ignoring thermal mass of links
- This is only true if $\left(\frac{\kappa_{th}}{L}A\right)_{link} \ll \left(\frac{\kappa_{th}}{L}A\right)_{middle}$
- In other words: $R_{th,links} \gg R_{th,middle\ mass}$

Temperature of a bolometer plate



Uniform plate temperature due to cooling by conduction in arms

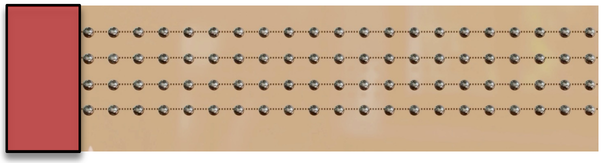
Thermal conductivity is a materials property



https://www.youtube.com/watch?v=6jQsLAqrZGQ

Thermal conductivity in solids

Dielectric



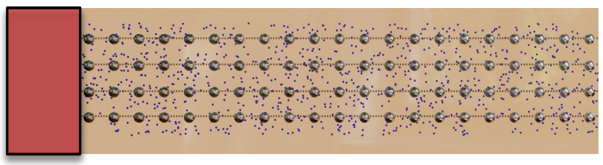
specific heat capacity

phonon group velocity

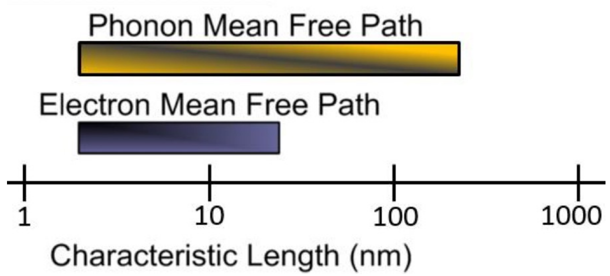
phonon mean free path

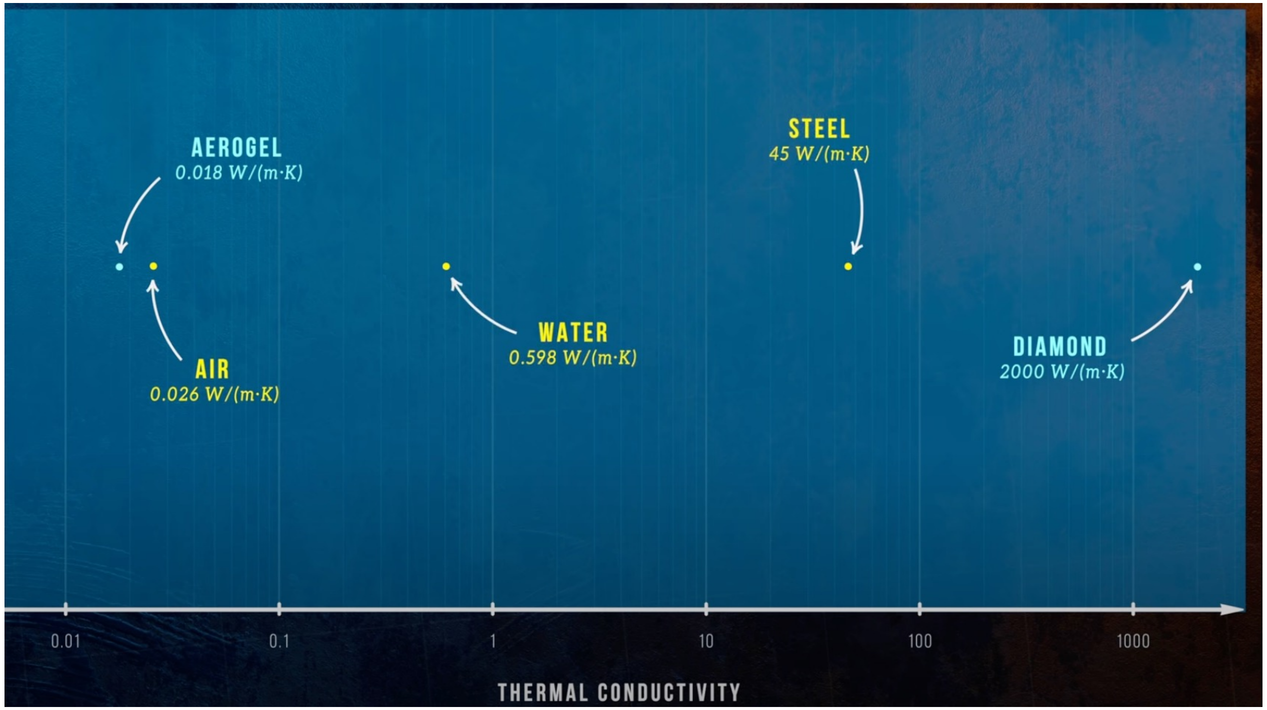
$$k_{th} = \frac{1}{3}cv\lambda$$

	metals:	dielectrics and semiconductors:
specific heat c:	c_e for electrons	c_s for phonons
molecular velocity v:	electron Fermi velocity v_e=1.4 10^6 m/s	sound velocity v_s=10^3 m/s
average mean free path:	λ_e= 1 to 10 nm	phonons λ_s=0.1 μm to 1 μm



Metal





Conductivity Numbers are for dimensions >> 1 µm

		Thermal conductivity k_{th} [W/m K]	Specific heat capacity c_v [J/kg K]	density ρ[kg/m3]
Gas	Air	**0.024**	1005	1.2
	He	0.14	5200	0.17
Liquids	Water	0.59	4200	1000
	Ethanol	0.18	2200	1100
Solids	Silicon	**170**	691	2300
	Al	**235**	879	3750
	Ni	91	444	8880
	SiO_2	**1.3**	840	2660

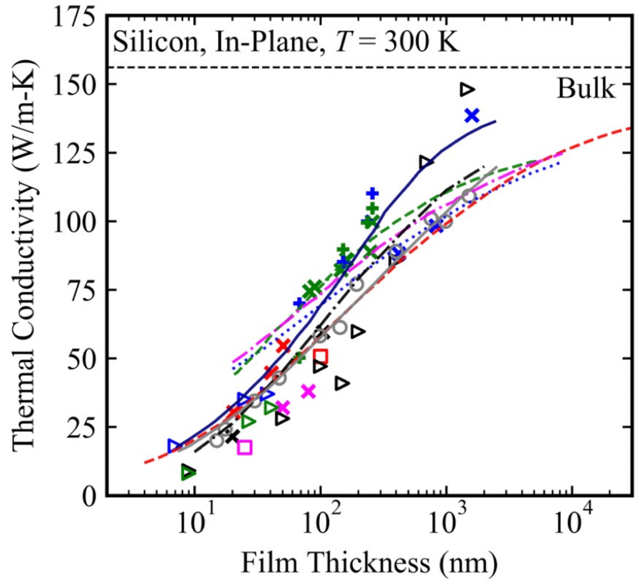
Thermal conductivity: lower in thin films and nanowires than in bulk

Phonon-boundary **scattering** reduces the bulk phonon mean free path (MFP) and thus reduces thermal conductivity.

Empirical correction factor in thin films (phonon confinement effect):

$$\frac{k_{th}^*}{k_{th}} = 1 - \frac{\lambda}{3h}$$

h: thickness

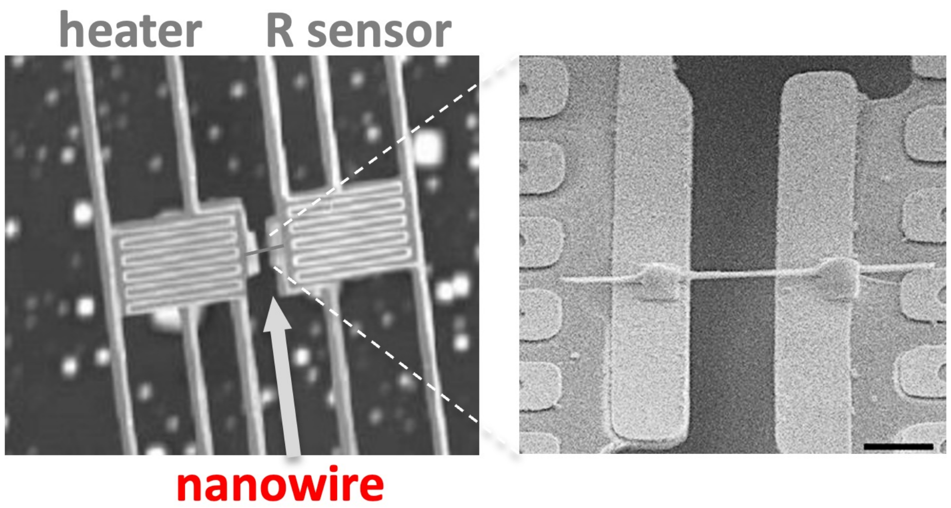


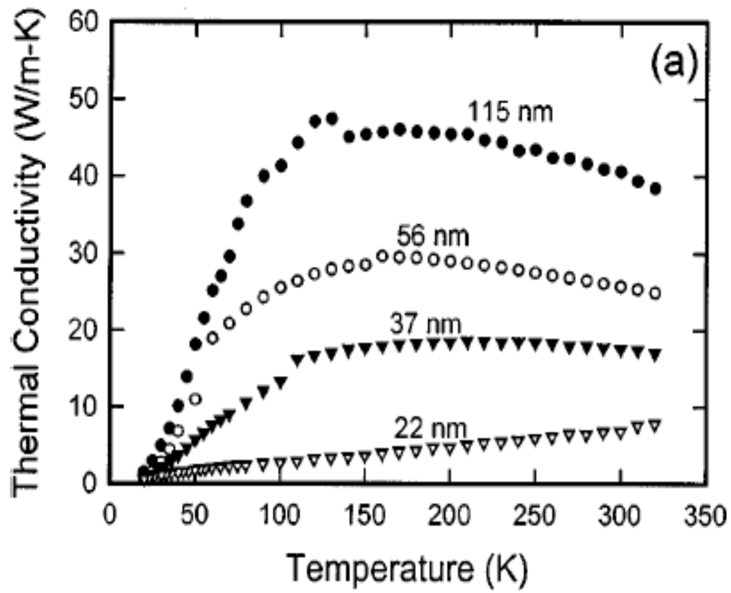
Fu et al. PHYSICAL REVIEW B 101, 045417 (2020)

Example:
a silicon film of 0.2 microns has a correction factor of 0.83
(MFP in Si ~ 100 nm)

The reduction of the phonon (lattice) thermal conductivity in thin films and nanowires is a result of the **phonon-boundary scattering** and **phonon spatial confinement** effects.

How can one measure the thermal conductivity of nanowires ?





At Room T, κ drops as size drops

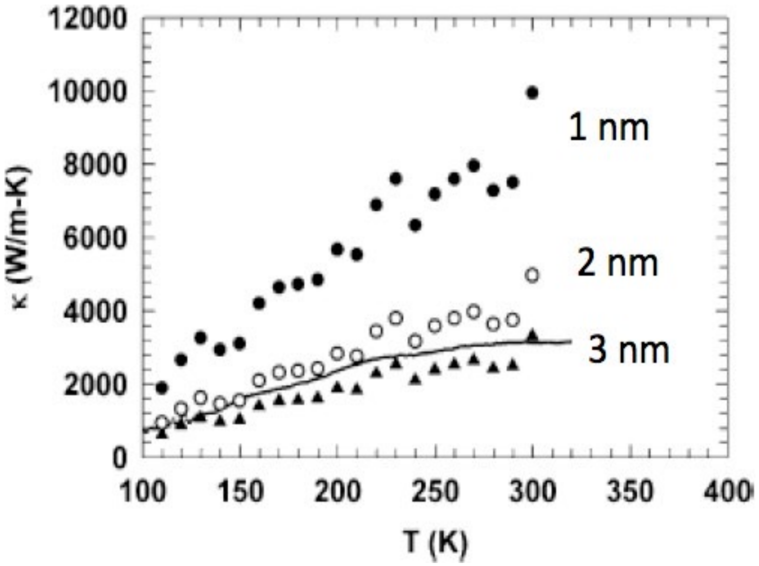
Effect of surface roughness:

- enhanced **boundary scattering** has a strong effect on phonon transport in Si nanowires

D. Li et al, Applied Physics Letters 83 (2003)

HOWEVER; Thermal conductivity is high in carbon nanotubes (CNT) and graphene

i) long range crystallinity,
ii) long phonon mean free path, and
iii) high speed of sound of the CNTs
lead to the high thermal conductivity of CNTs

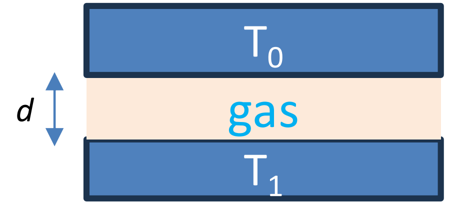


sample type	K (W/mK)
SLG	~4840–5300
MW-CNT	>3000
SW-CNT	~3500
SW-CNT	1750–5800

This is in sharp contrast with the reduced phonon thermal conductivity of other nanowires and thin films !!

Nano Lett., Vol. 8, No. 3, 2008

Thermal conduction in a Gas (no convection, no radiation)



Heat flow between two parallel plates: $Q_{th} = \Delta T \frac{\kappa_{th} A}{d}$ $Q = [W]$

A: area

Thermal conductance per surface unit: $g_{th} = \frac{\kappa_{th}}{d}$ $\left[\frac{W}{m^2 \cdot K}\right]$

This result **neglects convection** and **radiation**

Air: $\kappa_{th,air}$ = 0.024 W/mK at 1 bar, room temperature. (for He: 0.14 W/mK)

Scaling of conduction: $g_{th} \propto L^{-1}$

(this breaks down when $d = 0$, as we then have bulk conductivity of the walls)

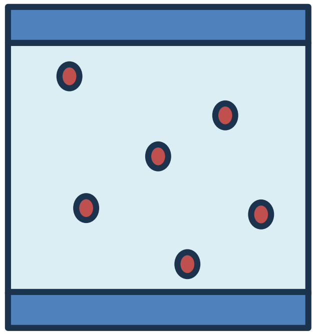
Small gap limit for gas thermal conduction

The thermal conductivity of the gas must be corrected when the gap is of order the molecular mean free path λ.

The mean free path of molecules in air (at room pressure and temperature) : $\lambda_{1atm} = 66\ nm$

An empirical rule for thermal conductance between parallel plates with d spacing:

$$g_{th} = \frac{Q_{th}}{\Delta T \cdot A} = \frac{\kappa_{th}}{d + 6\lambda} \qquad \left[W/m^2 K\right]$$



Ie, the effective thermal conductance in Knudsen (free molecule) regime becomes almost independent of distance.

Heat Convection

- Heat transfer due to movement of surrounding fluid

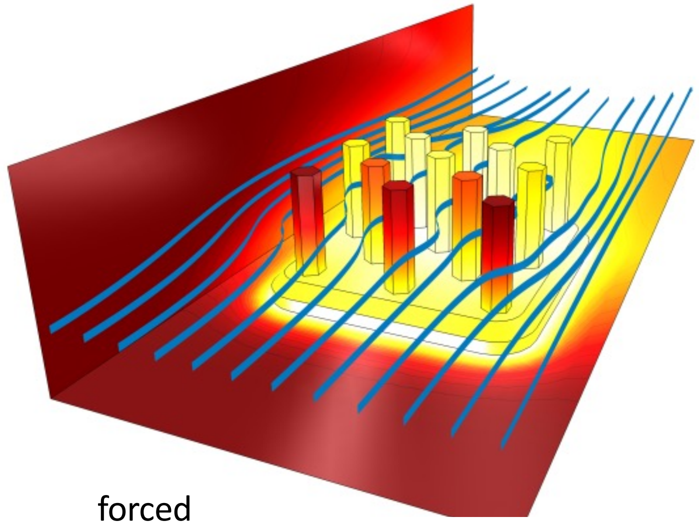
$$\frac{dQ}{dt} = h.A\left(T_{body} - T_{\infty}\right)$$

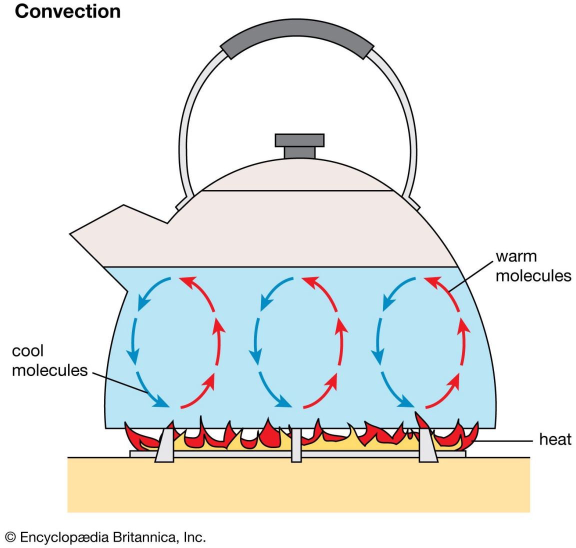
$h \rightarrow$ heat transfer coefficient $\left[\frac{W}{m^2K}\right]$

$$R_{th,convection} = \frac{1}{Ah}$$

$$G_{\ th,convection} = Ah$$

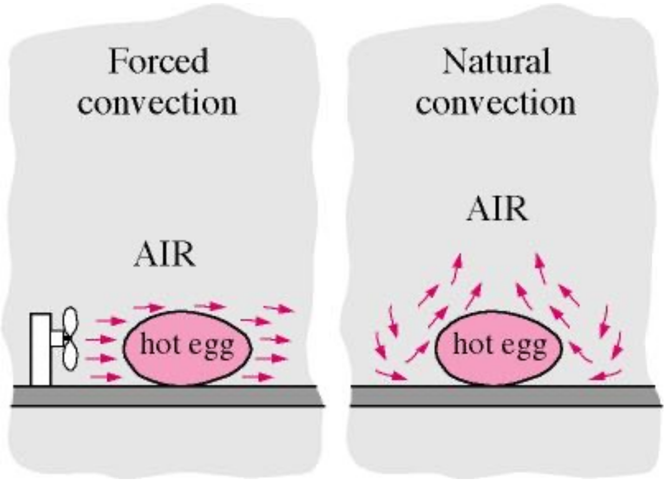
- Typical values for free convection in air of *h* are between 2 W / m^2 K and 25 W/ m^2 K
- We will often use 10 W/m^2K





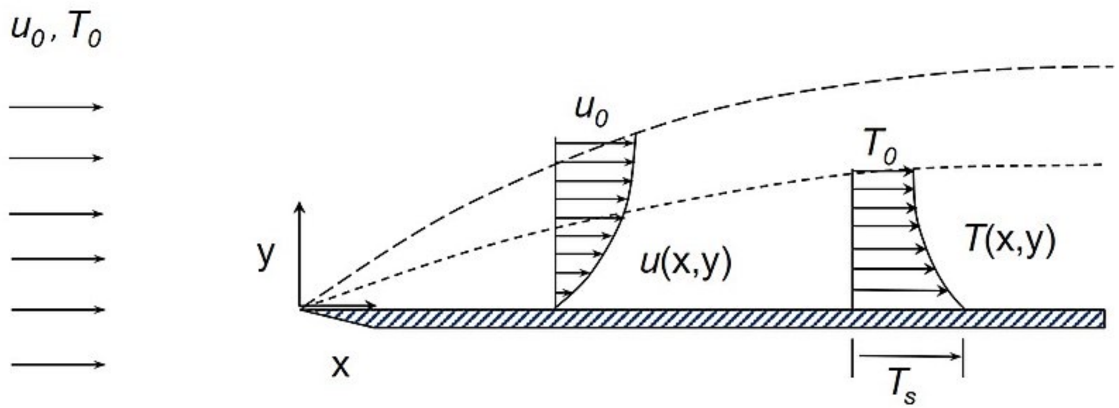
Heat transfer by convection in a gas (air)

- Natural convection depends on density change: needs gravity
  e.g. Vertical plate vs horizontal plate, not same convection
- Forced convection (fan)



For What happens to convection when systems are scaled down?

=> The thermal boundary layer depends on scale



$$\delta_T = 5\sqrt{x\frac{\kappa_{th}}{\rho\, u_0\, c_p}}$$

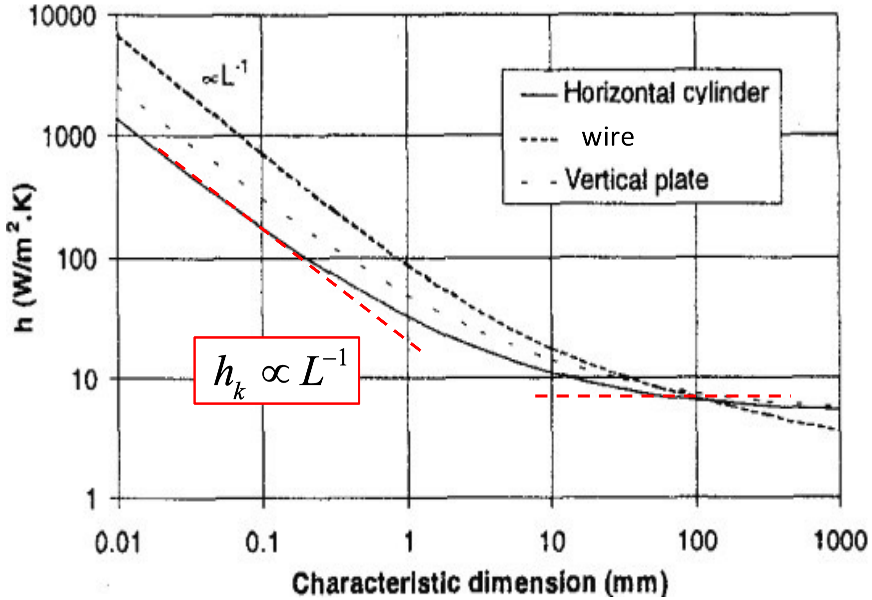
https://en.m.wikipedia.org/wiki/Thermal_boundary_layer_thickness_and_shape

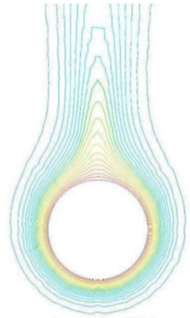
Convection in thin wires and plates

Heat exchange by air convection becomes very efficient in small structures (if they are far from walls)

$$h_k = \left(2.68 + \frac{0.11}{\sqrt{d}}\right)^2$$

$$h_k \propto L^{-1}$$





for large d: $h_k \to 2.68^2 \approx 7$

h_k is independent of size if d >10 mm

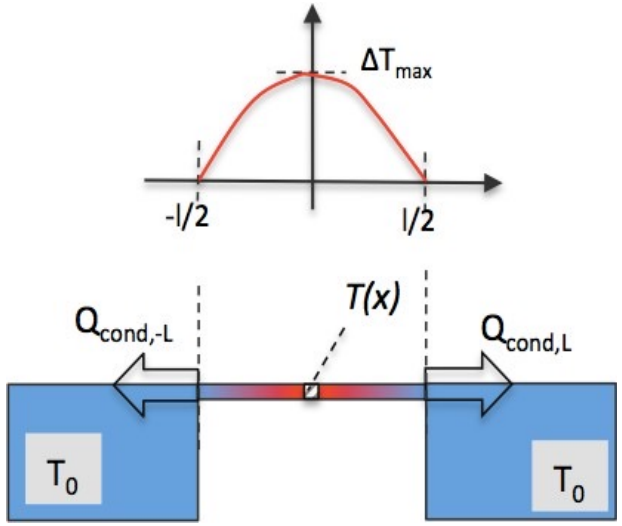
However: the behavior of free convection also depends on size of cavities around the element.
In a small cavity, the Re number is small => less convective flow
So we do not see this enhancement in MEMS

Peirs, J.; Reynaerts, D.; Van Brussel, H., "Scale effects and thermal considerations for micro-actuators," Robotics and Automation, 1998. Proceedings. 1998

Temperature profile of uniformly heated beam (only conduction to anchors)

- <u>only</u> conduction along beam
- <u>no</u> convection and <u>no</u> radiation losses.
- Uniform heating along the beam (eg Joule heating along the wire)
- Thermally clamped at both ends $T_{end}=T_0$

$$\frac{\partial^2 T}{\partial x^2} = -\frac{q_{in}}{\kappa_{th}}$$



T profile (at equilibrium): $\Delta T(x) = \frac{q_{in}}{2\kappa_{th}}\left(\left(\frac{l}{2}\right)^2 - x^2\right)$

$$\Delta T_{max} = \frac{q_{in}\, l^2}{8\, \kappa_{th}} \propto L^2$$

@ cst power density

q_{in}= internal heating power $\left[W \cdot m^{-3}\right]$

<u>Numerical examples</u>:

beam: l =500 µm, w=20 µm, e=1 µm $q_{in}=10^{12}$ W/m³   this is P =10 mW total power

Silicon beam	k_{th}=170 W/m.K	**ΔT_{max} = 184 K (ie T=200°C)**
Same total power (10 mW), longer beam (l=1mm)		ΔT_{max} = 368 K
Platinum beam, 500 µm,	k_{th}=76 W/Km	**ΔT_{max} = 411 K**

Easy to heat small structures to high T with little power !

important

Temperature profile of uniformly heated beam with convection and with conduction

- **Convection and conduction along beam (L x b x h)**
- **No radiative losses.**
- **Uniform heating along the beam (eg Joule heating along the wire)**
- Thermally clamped at both ends $T = T_0$

q_{in}= internal heating power $\left[W \cdot m^{-3}\right]$ $q_{in} = \rho_{el} j^2$

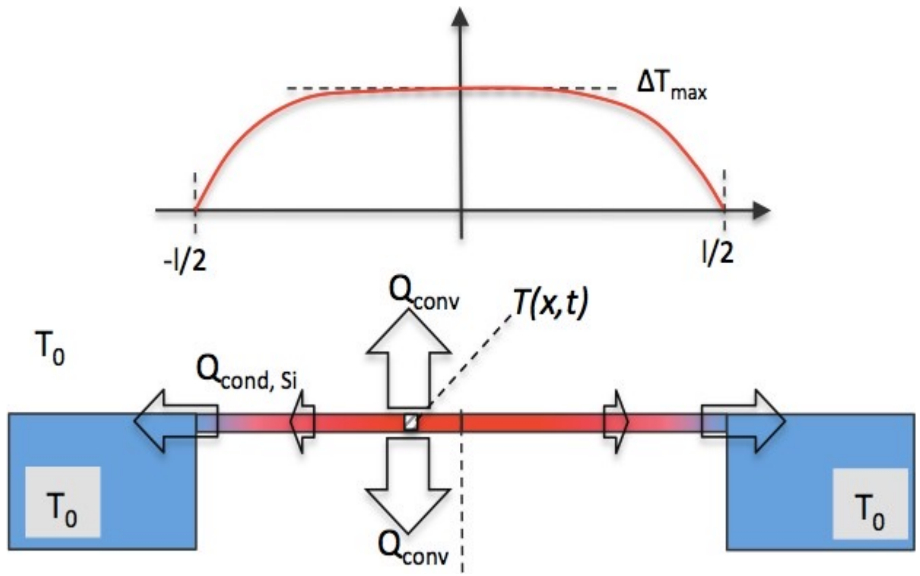
Y(x)= losses $\left[W \cdot m^{-3}\right]$ convection and radiation losses

$$\rho c \frac{\partial T}{\partial t} = \kappa_{th} \frac{\partial^2 T}{\partial x^2} + q_{in} - Y(x)$$

Convection:

$$Y(x) = 2\frac{h_k LbT}{V} = 2\frac{h_k LbT}{Lbh} = \frac{2}{h} h_k T(x)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_{in}}{\kappa_{th}} - \frac{2h_k}{\kappa_{th} h} T(x) = 0$$



General solution:

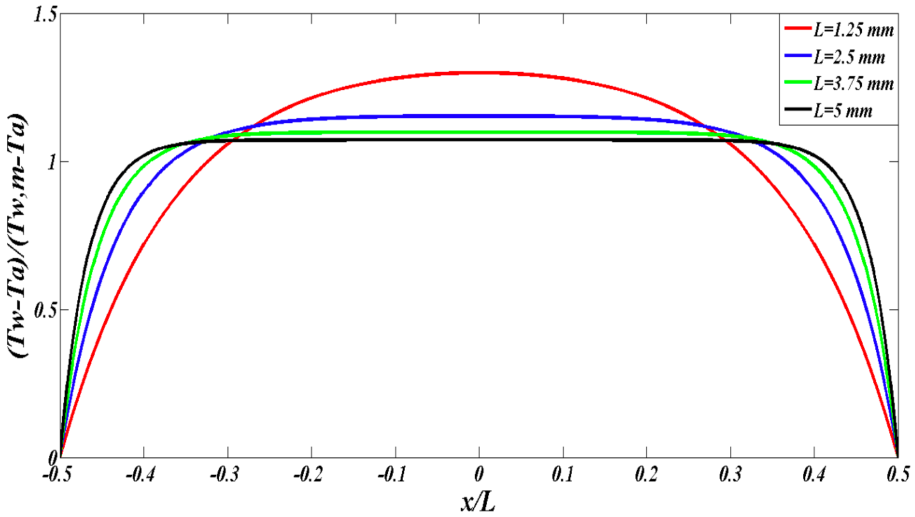
$$T(x) = C_1 e^{\Gamma x} + C_2 e^{-\Gamma x} + C_3$$

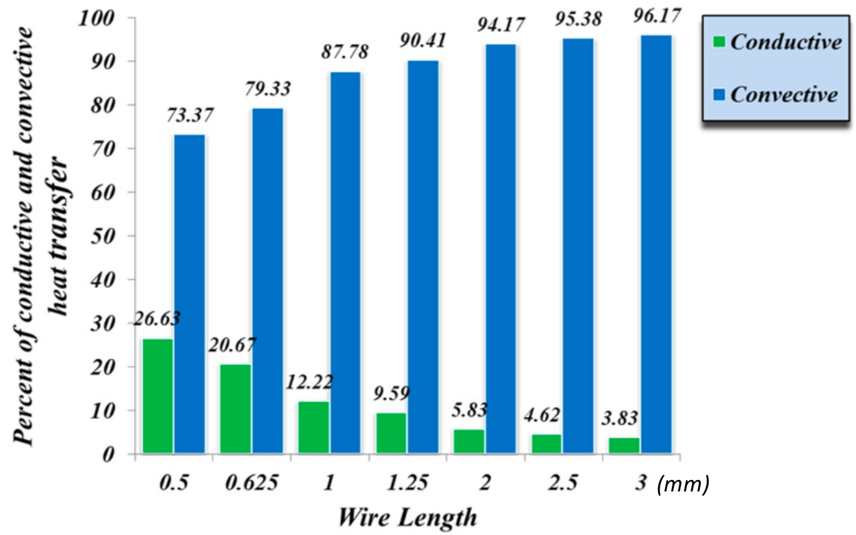
with $C_3 = \frac{q_{in} e}{2h_k}$ *and*

$$C_1 = C_2 = -\frac{q_{in} e}{2\lambda_k} \cdot \frac{1}{\left(e^{\Gamma l/2} + e^{-\Gamma l/2}\right)}$$

more info in : Ki Bang Lee, Principles of Microelectromechanical Systems, Wiley, 2011

Temperature profile of a uniformly heated wire (doubly clamped, T_0 at each end)



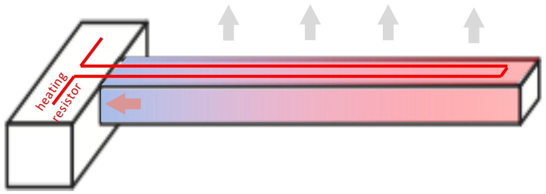


ΔT_{max} lower when convention than when only conduction

- Diameter of wire is 5 µm
- Air velocity is 20 m/s
- Wire far from any wall

Dehghan M and Kazemi M (2012) "Analytical and Experimental Investigation About Heat Transfer of Hot-Wire Anemometry. An Overview of Heat Transfer Phenomena." InTech. http://dx.doi.org/10.5772/51989.

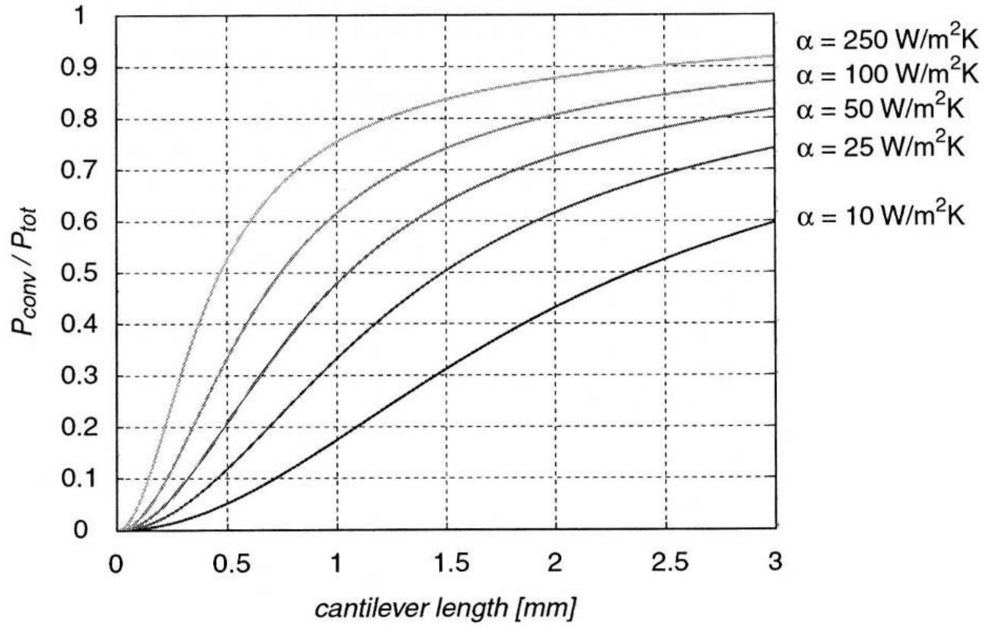
Temperature profile of a cantilever, with conduction along a beam and FORCED convection in air



Numerical example:

Thickness h=1 μm , Nickel
Convection coeff: h_k=10 W/m²K

l =1 mm => P_{conv}/P_{tot}=18%
l =0.3 mm => P_{conv}/P_{tot} =3%

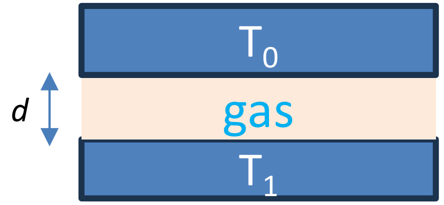


Simply supported beam, uniformly (Joule) heating: computed relative convection losses for different convection coefficients. (radiation is neglected)
From G. Lammel thesis, EPFL

Conclusion: In short beams (typ. below 500 μm), **convection** in air can be **neglected**.

In MEMS, conduction generally dominates

Conduction vs convection in MEMS



What is the "Equivalent gap" for which losses from conduction are equal to losses from convection (h_k=10 W/m^{2}K) ?

$$q_{th} = \kappa_{th}\frac{\Delta T}{d} = h_k\,\Delta T$$

→ **for gaps thinner than 1 mm, the main thermal transfer mechanism is typically conduction, rather than convection in the air gap.**

$$d_0 = \frac{\kappa_{th}}{h_k} = \frac{0.024}{10} = 2.4\ \text{mm}$$

This is Nusselt Number =1

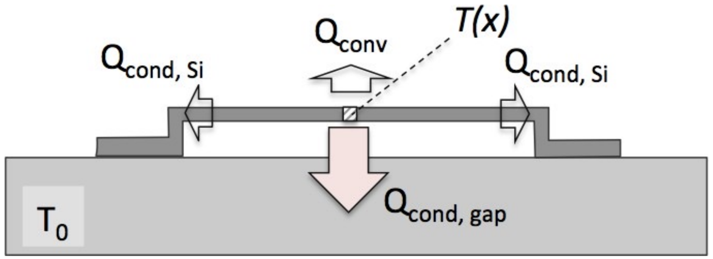
$$Nu = \frac{convection}{conduction} = \frac{h_k}{k_{th}/L} = \frac{h_k\,L}{k_{th}}$$

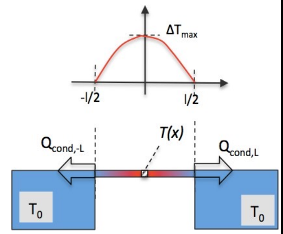
Heated beam/plate near a surface

$$Q_{gap,cond} = \Delta T \frac{\kappa_{th,air}}{d} A$$

$$= \Delta T \frac{\kappa_{th,air}}{d} l\, b$$

$$\Delta T = \frac{Q_{gap} \cdot d}{k_{th} \cdot l \cdot b}$$





<u>Numerical example</u>:

power 10 mW, l=500 μm, b=20 μm d=2 μm

=> ΔT= 83 °C

The conduction along the beam model gave ΔT_{max}=184 °C for a beam far away from the surface (i.e., when losses are dominated by conduction along the beam)

An air gap of 2 μm has a thermal conductance (per area) of $\mathbf{g_{th,\ conduction,area} = 10^4\ W/m^2K}$

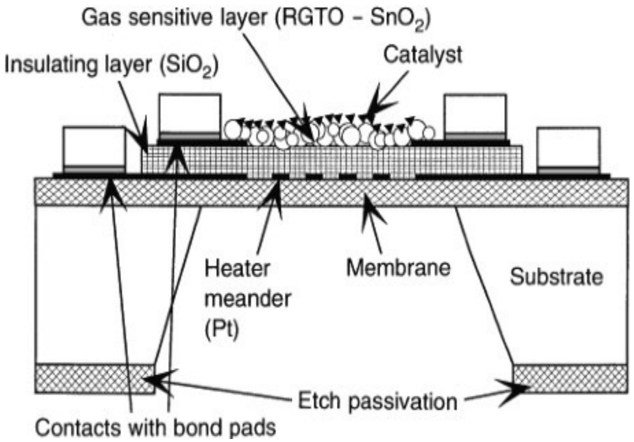
Compare this to unforced convection of about $\mathbf{g_{th,\ convection,area} = 10\ W/m^2K}$

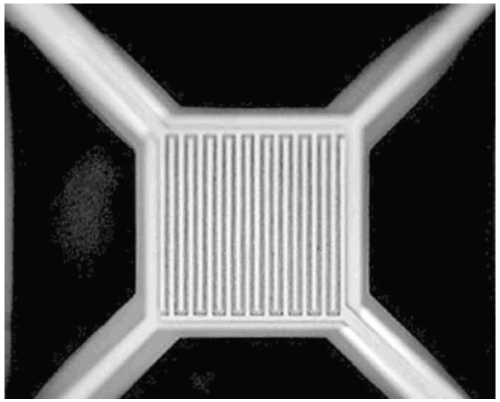
=> for gaps thinner than 1 mm, <u>conduction</u> through the air gap is typically the main thermal transfer mechanism

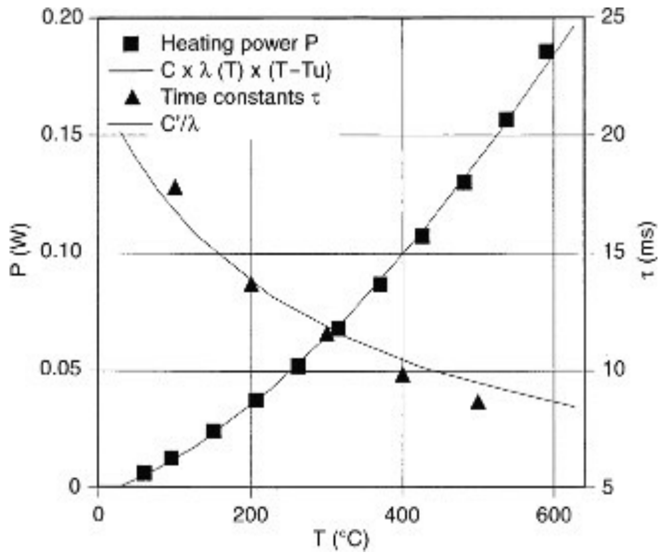
Microhotplates: suspended by narrow hinges to minimize heat loss

Gas sensors (semiconductor SnO_2 and catalytic) are based on the use of surfaces heated to 200-400 °C

To decrease the heat losses, the plate consists of a **thin dielectric membrane** with thin-film metal heaters.







The **time constant** is directly **related** to the **heat losses** ($\tau = RC$, see elsewhere in this chapter).

If we want to rapidly modulate the temperature (which is useful in some detection modalities), there will be a **trade-off** between heat loss and time constant.

Low heat loss (by conduction): thin long arms. $R_{th} = \frac{L}{A\,\kappa_{th}}$

Thermal accelerometer – based on <u>convection</u>

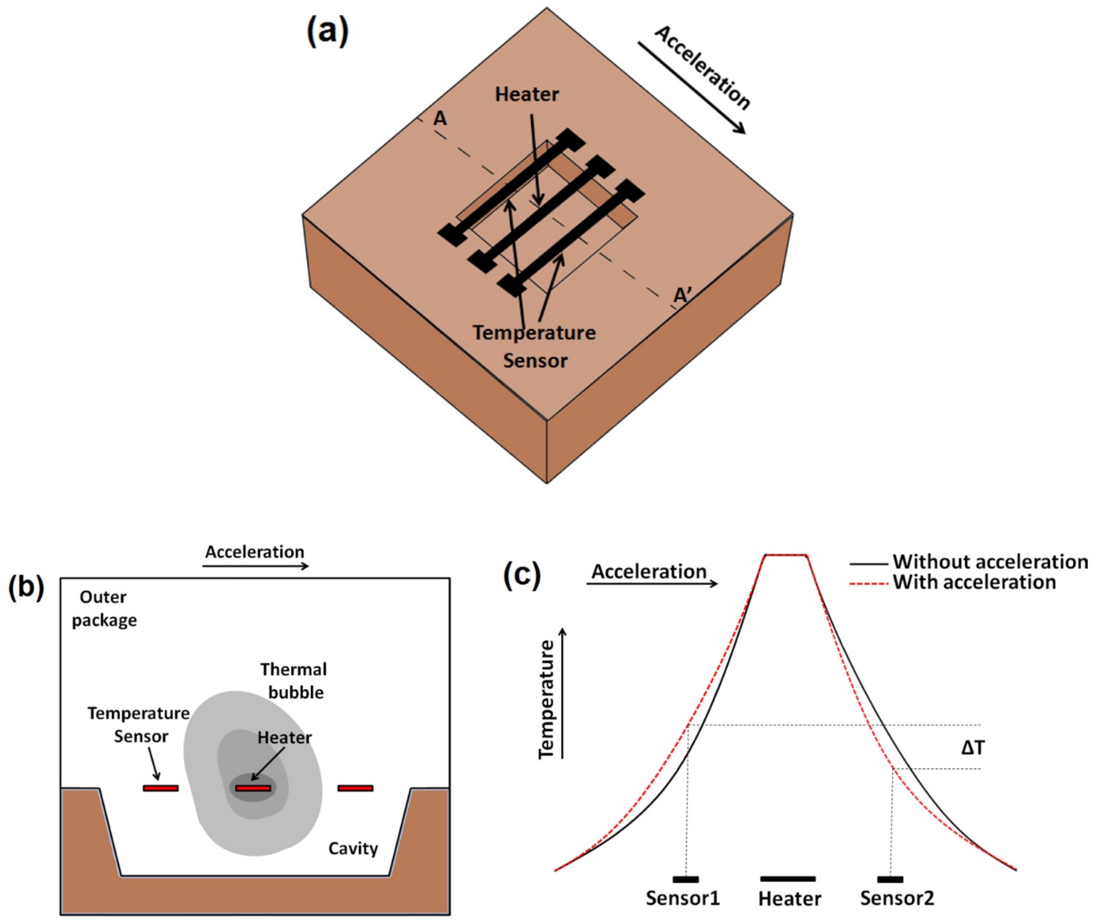
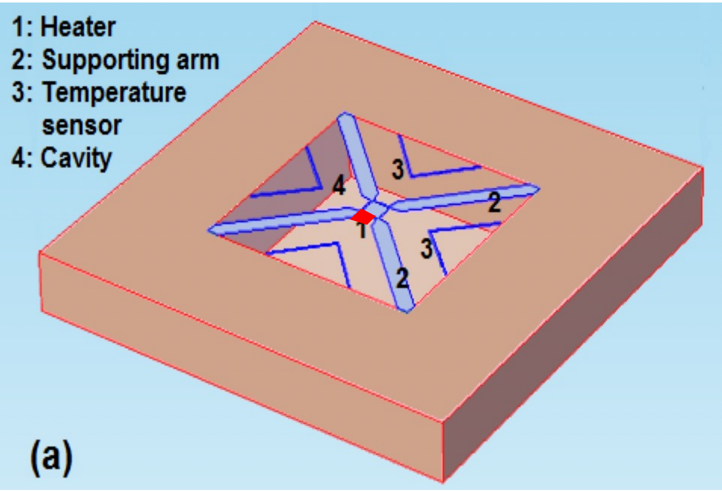


Figure 1. (a) Schematic view of thermal accelerometer, (b) cross-sectional view along AA′ line, and (c) temperature profile along AA′.



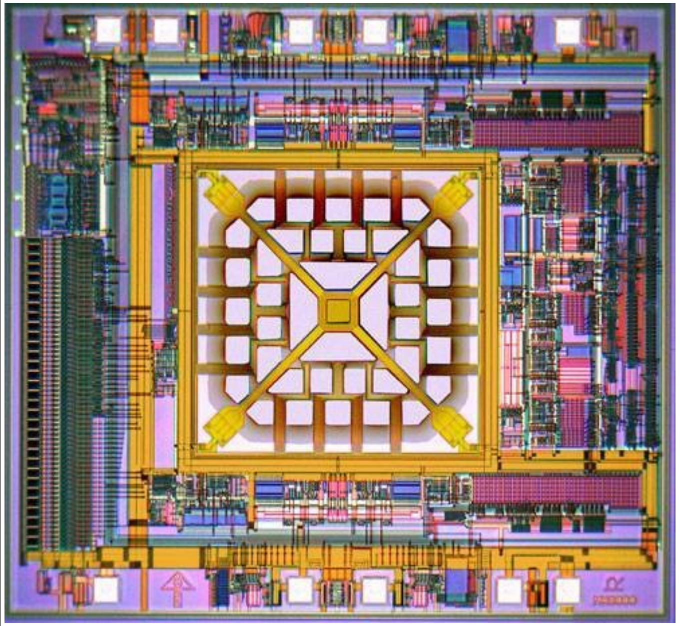
Low Power :
- need suspended heater
- low thermal conductance to the walls

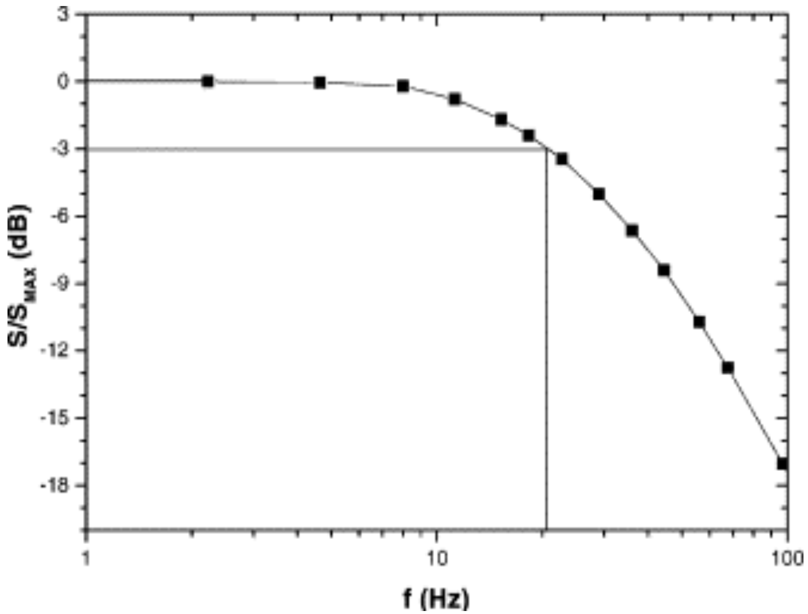
But for fast response :
- high thermal conductance of fluid
- low mass of heater

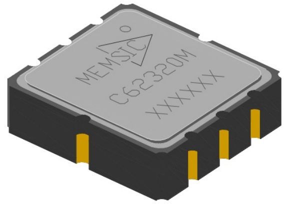
Typically 50 mW, 20 Hz, sensitivity 10^{-2} ms^{-2}

R. Mukherjee, J. Basu, P. Mandal, P. K. Guha, A review of micromachined thermal accelerometers.
J. Micromech. Microeng. **27**, 123002 (2017).

2-axis Thermal accelerometer







$$\tau_{convection} = RC \approx \frac{\rho c_v V}{Ah} = \frac{\rho c_v d}{h} \sim L$$

h: heat transfer coefficient
d: height of cavity

To improve the bandwidth:

i) the cavity size should be made small and
ii) thermal diffusivity of the working fluid must be high.

But this leads to higher power consumption and lower resolution.

MEMSIC developed this concept for commercial use. CMOS integration (excellent circuit, but limits materials choice, especially for sensors). Power consumption is rather high (several mW), long turn-on time (300 ms) – excellent shock resistance
https://www.memsic.com/thermal-accelerometer

	Thermal accelerometer	Capacitive accelerometer
Model number	MXR9500MZ	ADXL327
Sensitivity	500 mV g^{-1}	420 mV g^{-1}
Bandwidth	17 Hz	550/1600 Hz
Full scale range	$\pm$1.5 g	$\pm$2 g
Nonlinearity	0.5% of full scale	$\pm$0.2% of full scale
RMS noise density	0.6/0.9 mg $(\sqrt{\text{Hz}})^{-1}$	0.25 mg $(\sqrt{\text{Hz}})^{-1}$
Cross-axis sensitivity	$\pm$2%	$\pm$1%
Mechanical shock survival	50 000 g	10 000 g
Supply current	4.2mA at 3.0 V	350 μA at 3.0 V
Package dimension	7 mm × 7 mm × 1.8 mm	4 mm × 4 mm × 1.45 mm

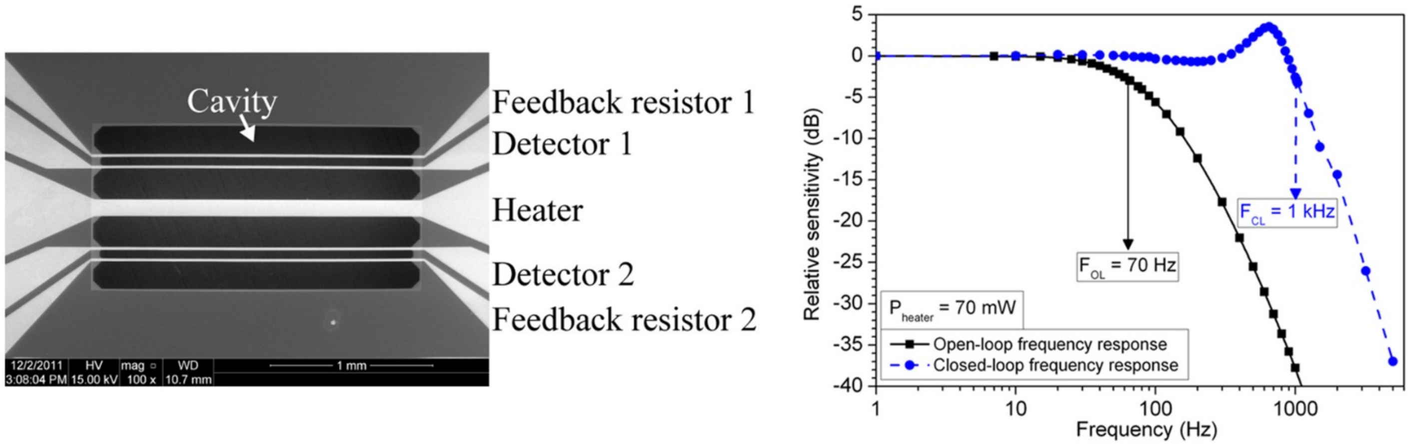


Figure 11. Scanning electron microscope image and measured frequency response of convective accelerometer with thermal feedback arrangement. Reproduced with permission from [91].

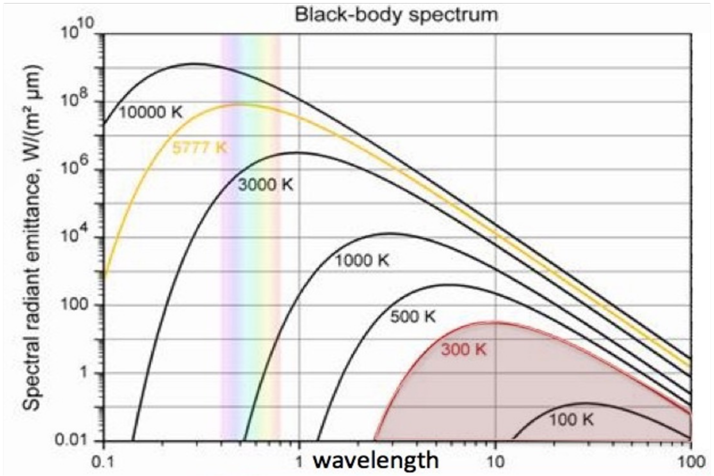
R. Mukherjee, J. Basu, P. Mandal, P. K. Guha, A review of micromachined thermal accelerometers. *J. Micromech. Microeng.* **27**, 123002 (2017).

Radiative heat transfer

Black body radiation

$$Q_{th} = A\varepsilon\sigma T^4 \quad [\mathrm{W}] \qquad with\ \sigma = 5.7\cdot 10^{-8}\ \frac{W}{m^2K^4}$$

ε: emissivity; A: Area of source
σ Stefan–Boltzmann constant



radiative conductance: $$G_{th} = \frac{dQ_{th}}{dT} = 4A\varepsilon\sigma T^3 = 4A\sigma\varepsilon(T_1^3 - T_2^3)$$ G_{th} [W/K]

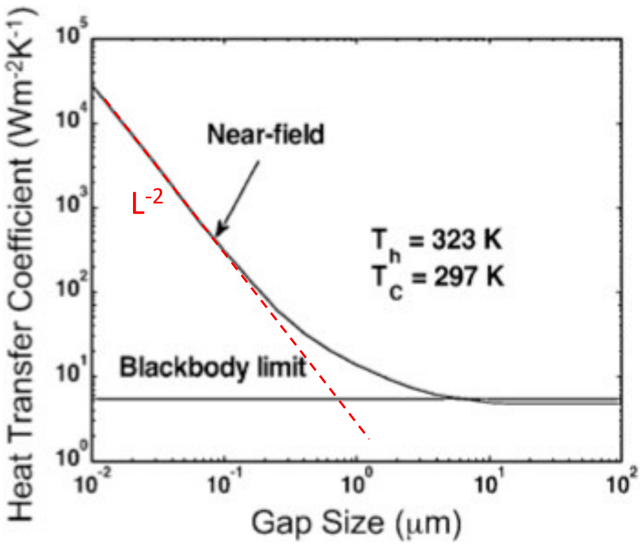
- heat flux is **independent of distance d for parallel plates**
- heat transfer coefficient is approximately **5 W/m² K** at ambient temperature (black body)
    - i.e. comparable to convection

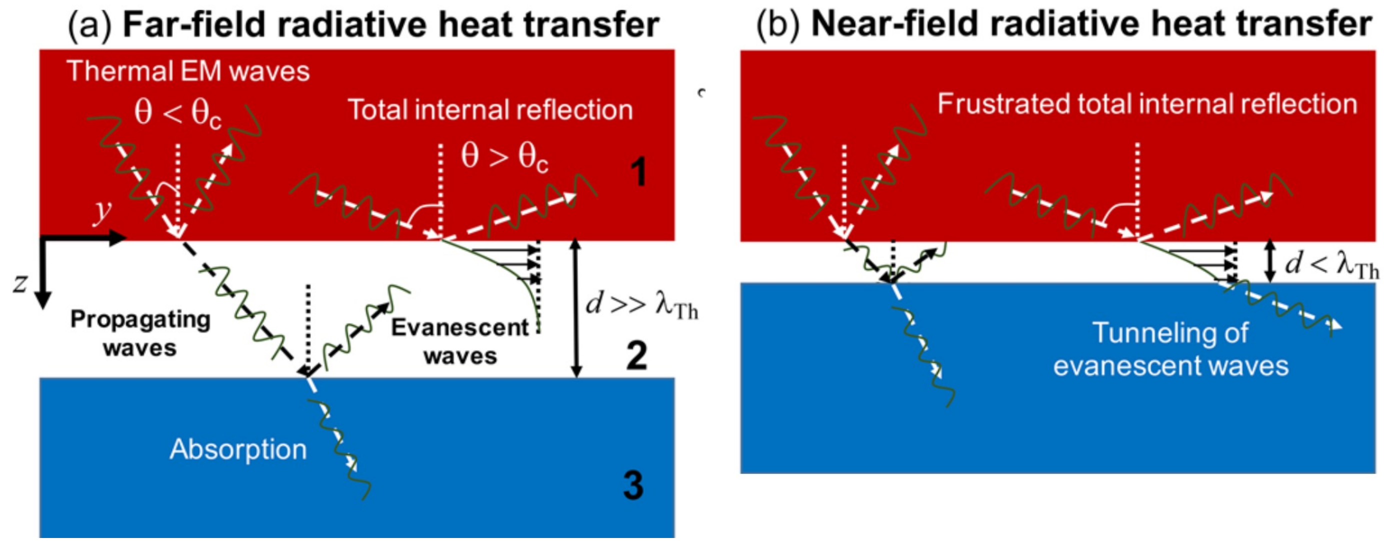
Planck's law assumes all dimensions > 10 µm !

When the distance *d* becomes comparable to the maximum **wavelength** of the Planck blackbody spectrum (approx 10 µm at room temperature), then the radiative transfer coefficient depends on *d*.

Radiative heat transfer in very small gaps

J. C. Cuevas, F. J. García-Vidal, Radiative Heat Transfer. *ACS Photonics*. **5**, 3896–3915 (2018)



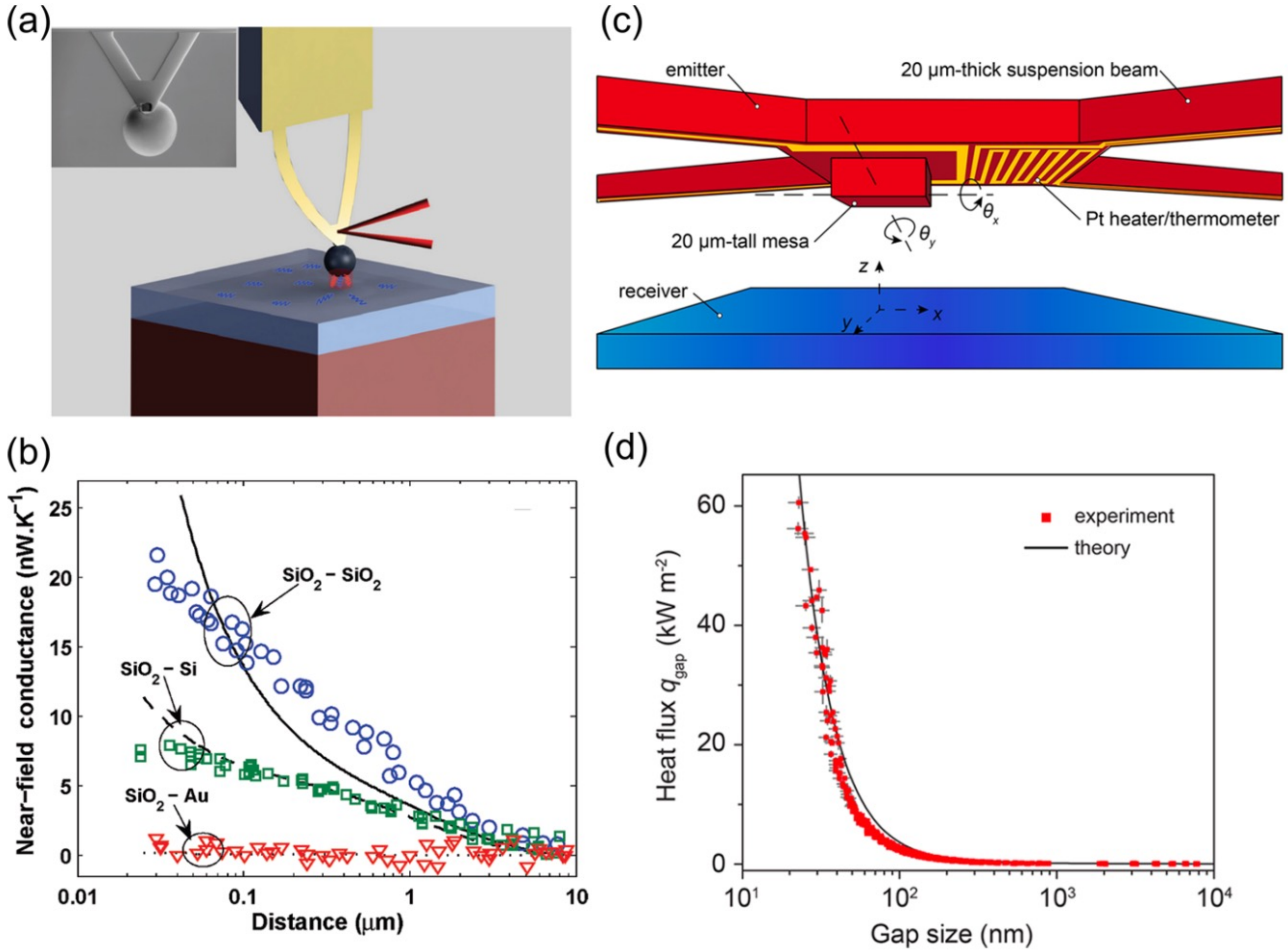


Increased radiation heat transfer at µm scale gaps is due to the **resonant tunnelling** of electromagnetic surface phonon polaritons between two closely spaced objects.

When d is lower than 1 µm, the radiative heat transfer :

$$Q_{th} \propto \frac{1}{d^2}$$

- *JP Mulet et al., Microscale Thermophysical Engineering, 6:209–222, 2002*
- *A. Narayanaswamy et al., Breakdown of the Planck blackbody radiation law at nanoscale gaps, Appl Phys A (2009) 96: 357–362*



J. C. Cuevas, F. J. García-Vidal, Radiative Heat Transfer.
ACS Photonics. **5**, 3896–3915 (2018)

Temperature fluctuations in microstructures

Temperature fluctuations $$\Delta T_{n,rms} = \sqrt{\frac{4k_B T}{G_{th}}}\sqrt{\Delta f}$$ G_{th}: thermal conductance

In radiation limit conditions (vacuum, no contact) $G_{th} = 4\mathrm{A}\sigma T^3$ *with* $\sigma = 5.7 \cdot 10^{-8} \quad W/m^2K^4$

at T=300 K $$\Delta T_{n,rms} = \sqrt{\frac{3 \cdot 10^{-21}}{A}}\sqrt{\Delta f} \qquad \propto L^{-1}$$

For a cube of side *x*, in vacuum, at 300K, for a bandwidth of 100 Hz:

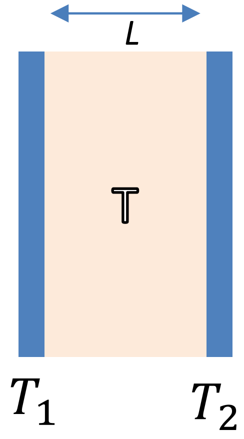
x= 1 mm	$A=6 \cdot 10^{-6}\ m^2$	ΔT=0.5 μK
x= 10 μm	$A= 6 \cdot 10^{-10}\ m^2$	ΔT=50 μK
x= 1 μm	$A= 6 \cdot 10^{-12}\ m^2$	ΔT=0.5 mK

Radiation conduction limit is the **ultimate resolution limitation** in MEMS **bolometers,** if all other heat transfer mechanism removed by design

Paul W. Kruse, "Can the 300-K radiating background noise limit be attained by uncooled thermal imagers?", Proc. SPIE 5406, Infrared Technology and Applications, 437 (August 30, 2004)

F. Niklaus et al., "MEMS-based uncooled infrared bolometer arrays: a review", Proc. SPIE 6836, MEMS/MOEMS (2008)

Dynamics of thermal systems (1D)



$$Q_{th} = G_{th}(T_1 - T_2)$$

$$Q_{th} = C_{th}\frac{dT}{dt} = c_v \rho V \frac{dT}{dt}$$

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c_v}{\kappa_{th}}\frac{\partial T}{\partial t}$$

$$T(t) = \frac{T_1 + T_2}{2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Time constant $$\tau_{th} = \frac{C_{th}}{G_{th}} = \frac{mc_v}{G_{th}}$$

- κ_{th} : thermal conductivity in [W/(m.K)]
- c_v : specific heat capacity [J/(kg.K)]
- C_{th} : heat capacity [J/K)]
- A : the area [m^2]
- ρ: Density (kg/m^3)
- Q: heat transferred per unit time [W]

$$\tau_{conduction} = \frac{L\, m\, c_v}{k_{th} A}$$

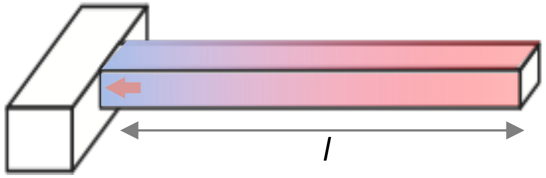
$$\tau_{conduc} = \frac{\rho c_v L.V}{2\kappa_{th} A} \propto L^2$$

Dissipated power per volume $$\frac{Q_{th}}{V} = \frac{1}{\tau_{th}} \propto \frac{1}{L^2}$$

fast systems -> requires a lot of energy per volume

Cooling time constant of a cantilever due to longitudinal conduction

Time constant cantilevers that are cooled by conduction along the material (no convection). Heated at the free end.



$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c_v}{\kappa_{th}}\frac{\partial T}{\partial t} = D_{th}\frac{\partial T}{\partial t}$$

Lumped model
Bi <<1

Thermal diffusivity $D_{th} = \frac{k_{th}}{\rho c_v}$

$$l_{th\_diff} = \sqrt{D_{th}\, t}$$

thermal diffusion length

t: time

Thermal time constant for this beam $\tau = \frac{\rho c_v l^2}{2\kappa_{th}}$

	D_{th} [10^{-6} m^2/s]	l=10 μm	l=100 μm	l=1000 μm
Si	102	0.5 µs	50 µs	5 ms or f=200Hz
Pt	25	2 µs	0.2 ms	20 ms or f=50Hz
SiO_2	0.6	83 µs	8.3 ms	830 ms or f=1Hz
Water	0.14	0.35 ms	35 ms	3.5 s

Can't propagate change in temperature faster than this!

Table: thermal diffusion times for various cantilever length (+water for comparison)

Bolometers

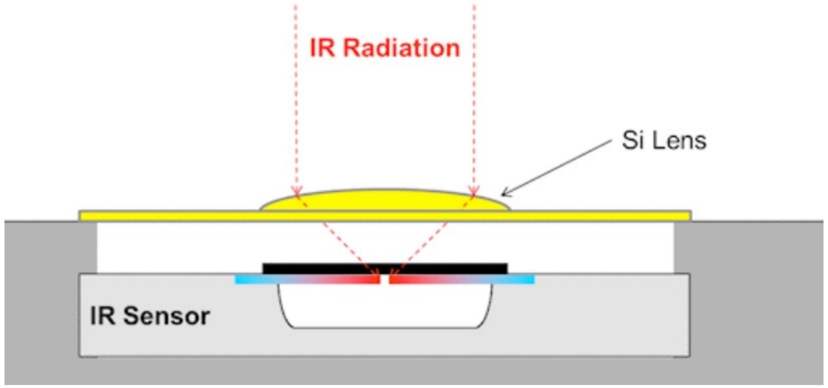
Principle: The IR radiation is absorbed on a thermally isolated absorber leading to a few mK increase in T.

The temperature rise $T - T_0$ is proportional to the intensity of the incident IR radiation Φ_{rad}

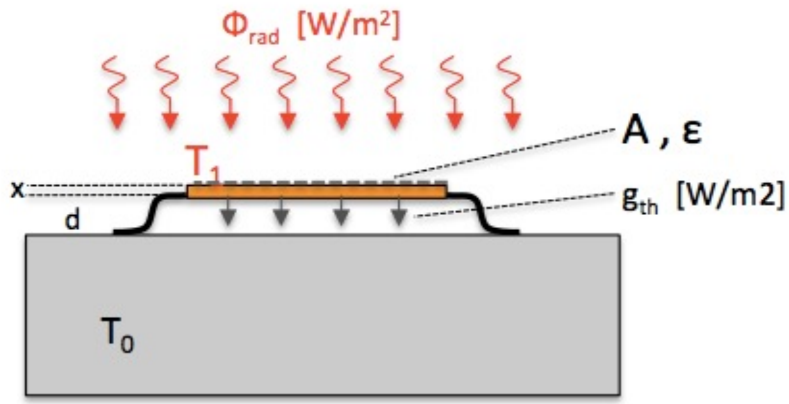
The temperature variation is measured by the change in resistance of the thermally insulated sensing pixel (membrane)

Sensor Temperature Balance:

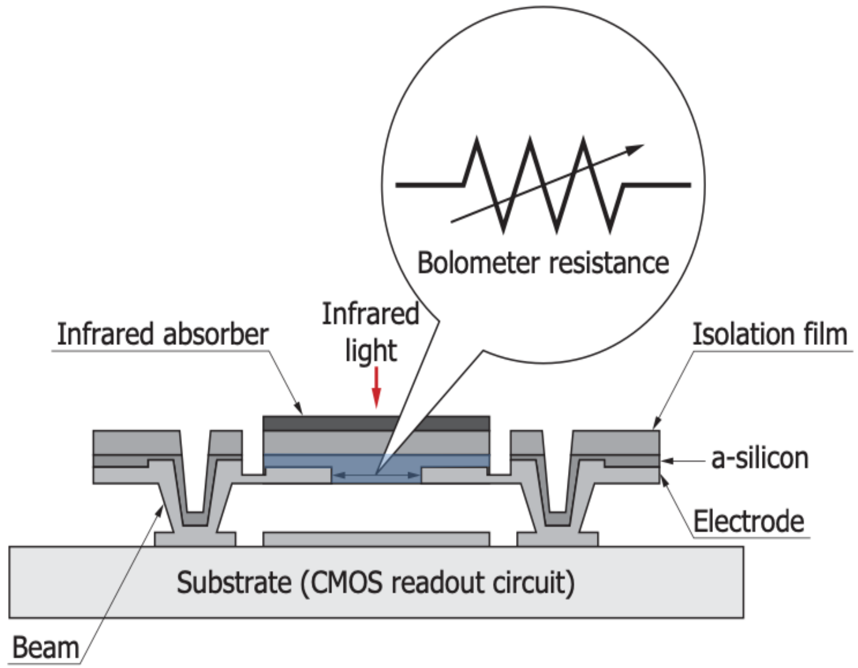
Incident IR radiation ↔ conduction to substrate

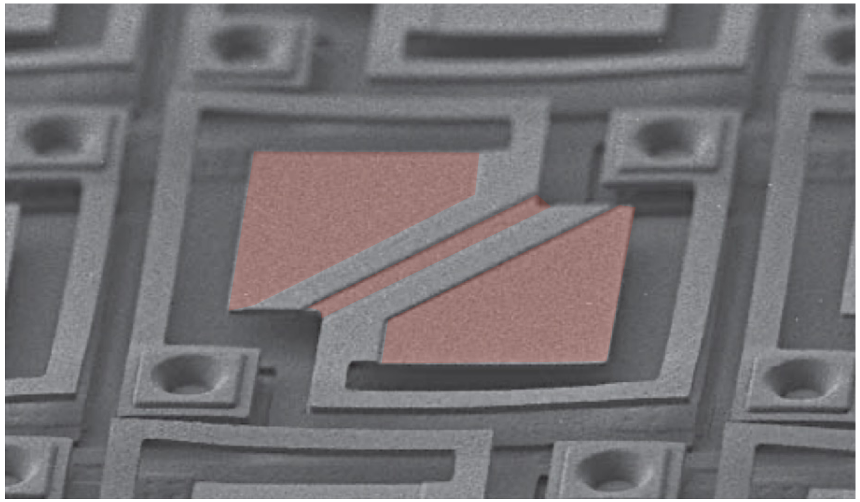


https://jsss.copernicus.org/articles/2/85/2013/jsss-2-85-2013.pdf



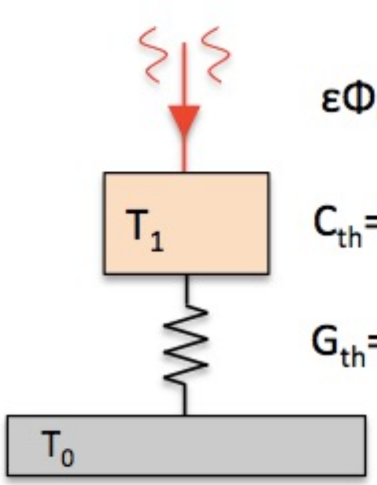
Bolomètre (matrix -> imaging)





- Typical pixel size is 10 µm to 50 µm
- pixel count anges from 80x60 to >HD
- CMOS below for ΔR measurement
- Air-dominated heat conduction (not suspension arms)

Bolometer: equivalent thermal model



$\varepsilon\Phi_{rad} \cdot A = \Phi_0$

$C_{th} = m \cdot c_v$

$G_{th} = \lambda_{air} \cdot A/d$

Time constant (general)

$$\tau_{th} = \frac{C_{th}}{G_{th}} = \frac{mc_v}{G_{th}}$$

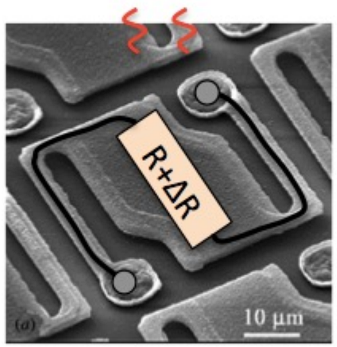
Time constant (conduction)

$$\tau_{th} = m\, c_V \frac{d}{\lambda_{air} A} \sim L^2$$

DC sensitivity: $$T - T_0 = \frac{\Phi_0}{G_{th}} = \Phi_0 \frac{d}{\lambda_{air} A} \sim L^{-1}$$

For d/A, trade-off speed vs sensitivity!

d: air gap (see next slide)
λ_{air}: thermal conductivity air
α_r: themal coeff of resistance
Φ_0: absorbed IR flux



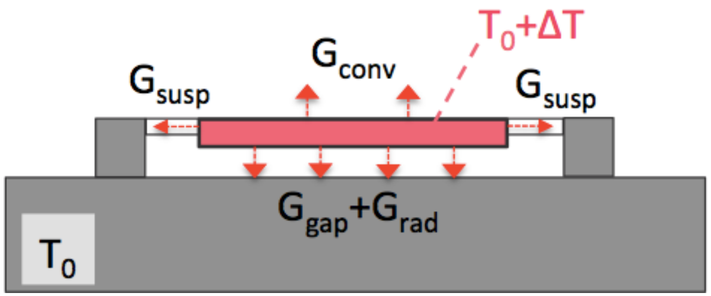
Change in sensor R with ΔT

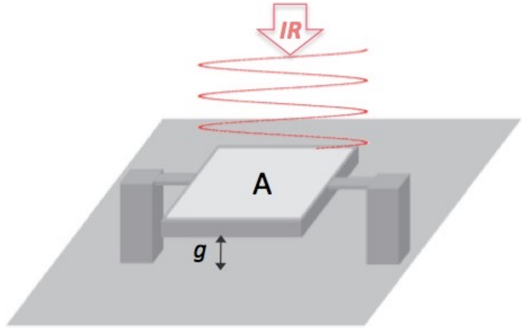
$$\frac{\Delta R}{R} = \alpha_R (T - T_0) = \frac{\Phi_0 \alpha_R}{G_{th}} = \frac{\varepsilon\Phi_{rad}\, \alpha_R}{\lambda_{air}} d$$

sensitivity depends on:

- α_R
- λ_{air}
- d

Heat losses in surface-micromachined bolometers





Num: plate of 200 µm x 200 µm gap=2 µm

Si suspension beams (2x) 200 x 15 x 1 µm

Conductance due to:

top surface convection	$G_{th} = h_k \cdot A = 4 \cdot 10^{-7} \quad WK^{-1}$
Si suspensions	$G_{th} = k_{Si} \cdot A/l = 6 \cdot 10^{-6} \quad WK^{-1}$
Radiation	$G_{th} = 4A \cdot \varepsilon\sigma T^3 = 4 \cdot 10^{-7} \quad WK^{-1}$
Air gap conduction	$G_{th} = \kappa_{air} \cdot A/d = 5 \cdot 10^{-4} \quad WK^{-1}$

- **Air gap conduction** is the **dominant** heat loss mechanism in most bolometers (which affects both the *sensitivity* and the *time constant*)

- Detail: Some groups modulate the thermal parameters: they control the gap (using electrostatic force) to trade-off sensitivity vs. bandwidth

J. Talghader,, J. Phys D: Appl. Phys. 37 (2004) R109

Micro-PCR for DNA amplification

PCR: DNA amplification by thermal cycling

Common method is 384 well plate that is heated and cooled at a few °C/s, dozens to hundreds of times.

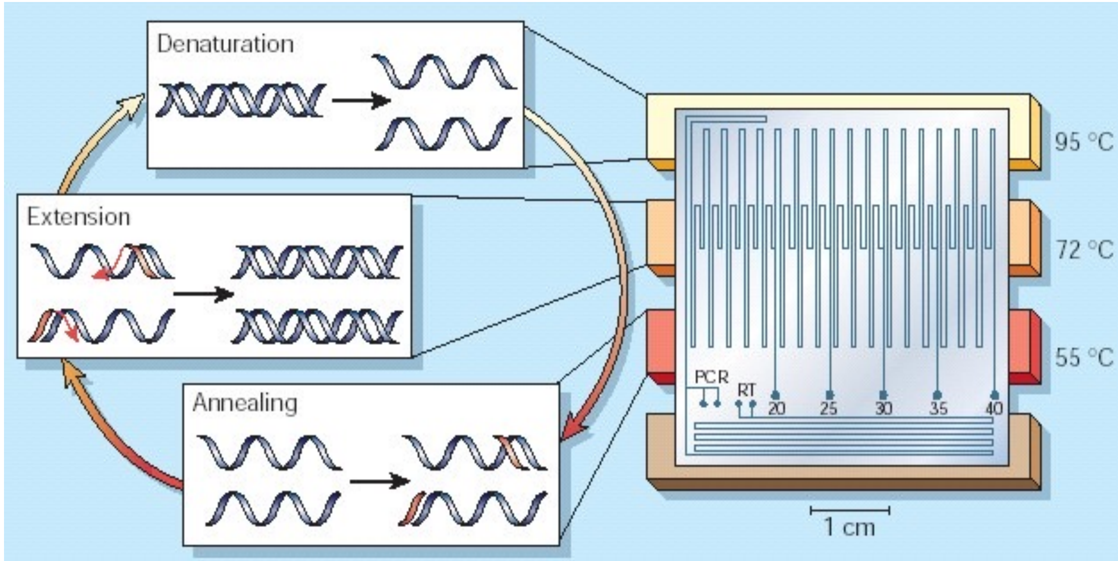
Using microfluidics (scaling) allows for very rapid thermal cycling

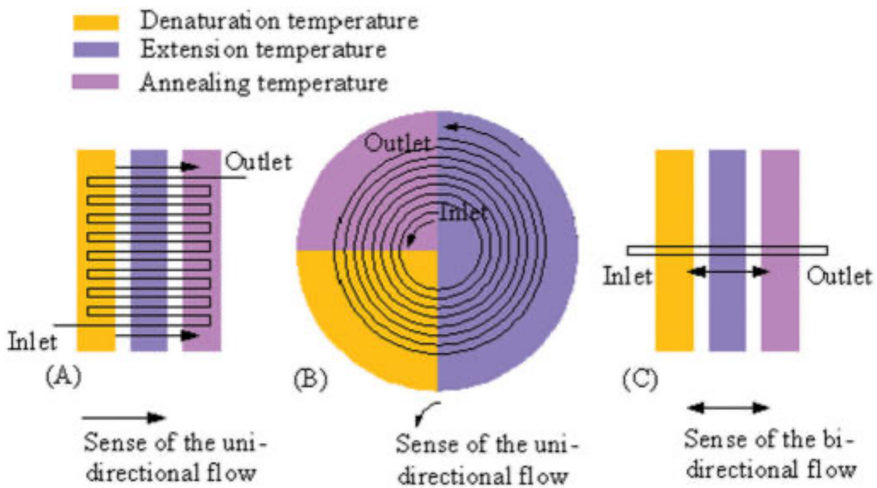
faster cycling ➔ faster amplification

Two approaches:

1. small volume chamber with integrated heater,
2. continuous flow PCR with thermal gradient on chip

P. Obeid, "Microfabricated Device for DNA and RNA Amplification by Continuous-Flow Polymerase Chain Reaction and Reverse Transcription-Polymerase Chain Reaction with Cycle Number Selection", Anal. Chem. 75 (2003) 288





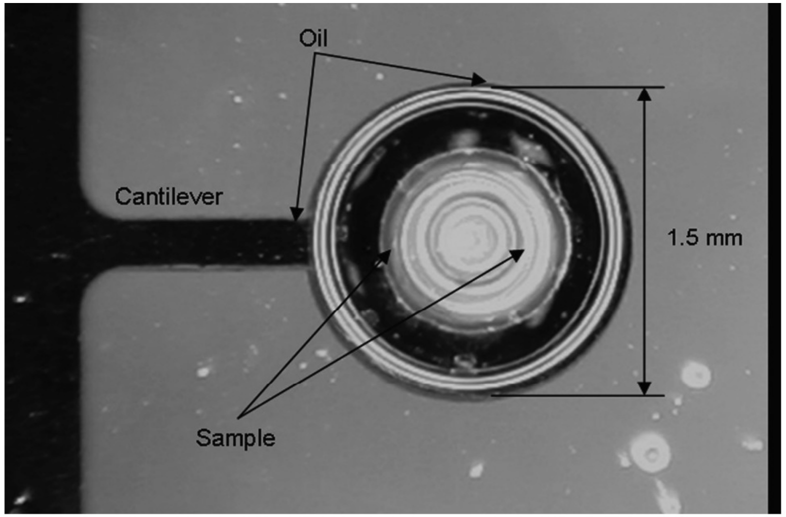
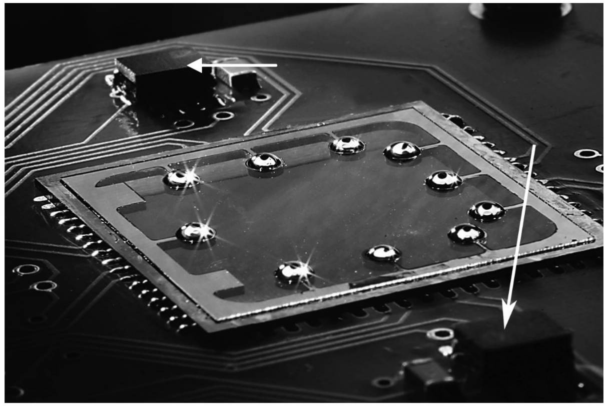


Figure 3. Optical photograph of the sample encapsulated by oil (edges pointed to by arrows) placed above the heater and separated from it by a microscope cover slip. The hemispherical oil shape formed a lens which magnified the sample.



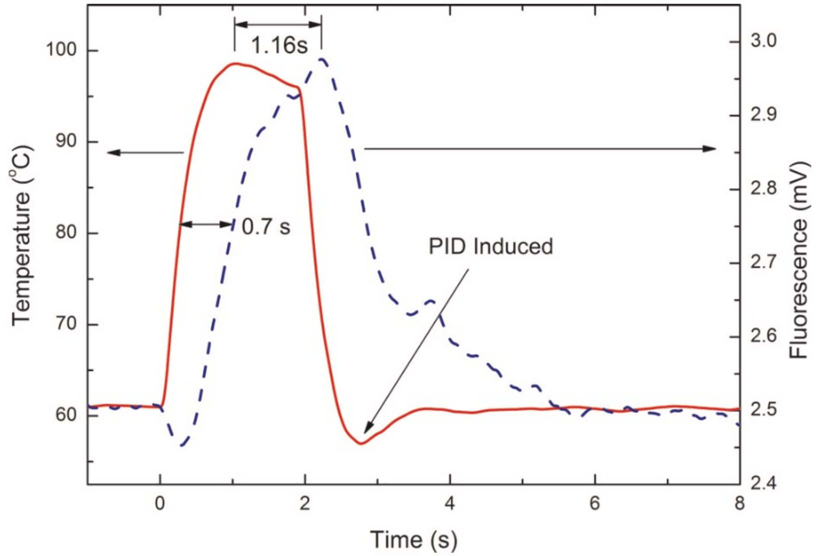
. Microphotograph of a fabricated chip 24.2 × 24.2 mm in size, soldered to a PCB. The chip consists of 10 individually controlled heaters,

P. Neuzil, C. Zhang, J. Pipper, S. Oh, L. Zhuo,
Ultra fast miniaturized real-time PCR: 40 cycles in less than six minutes.
Nucleic Acids Research. **34**, e77 (2006).

Table 1. Electrical and thermal parameters of the nanoPCR device itself

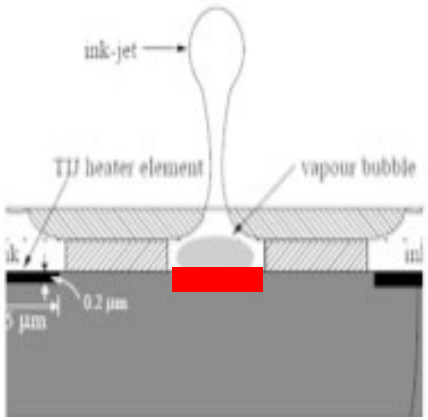
Sensor resistance (at 25°C)	427Ω
Heater resistance (at 25°C)	141Ω
System temperature response	11 mV/°C
PCR thermal conductance	0.42 mW/°C
PCR thermal capacitance	1.5 mJ/°C
PCR thermal time constant	0.28 Hz
Heating rate from 60 to 95°C	175°C/s
Cooling rate from 95 to 60°C	−125°C/s

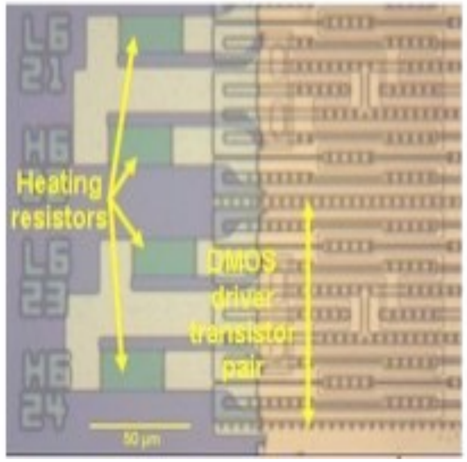
$\tau = RC$



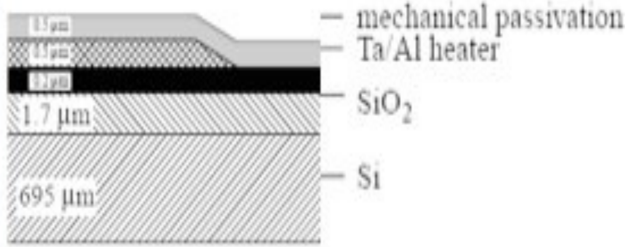
Thermal Inkjet (HP+Canon)

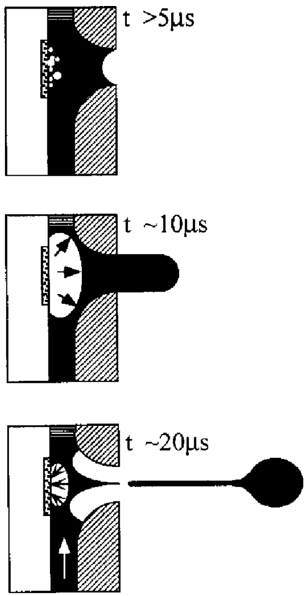
Use the µs time constant in micro-system





Heater on Si, hence high k_{th}





HP data :

Resistor : tantalum,	30 x 15 µm	30 nm thickness	R=8 Ω
Current pulse	250 mA	during 2.5µs	
Heating at	10^8 degree/second		
Surface power density	10^9 W/m^2		
Internal pressure	130 bar (cracks…)		
Ejection speed	15 m/s (aging, abrasion…)		
	32 pl droplets at rate of 6000 Hz		

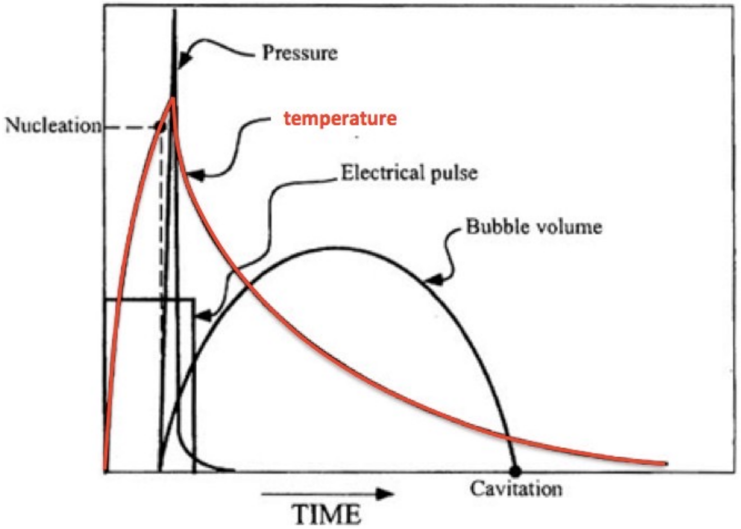
Calculated power : $p_{in} = 8 \cdot 0.25^2 = 500mW$ $\quad p_{in} / A = 1.1 \cdot 10^9 \quad W/m^2$

need very high heating rate (>10^6 K/s) to achieve supercritical heating to ensure rapid bubble collapse

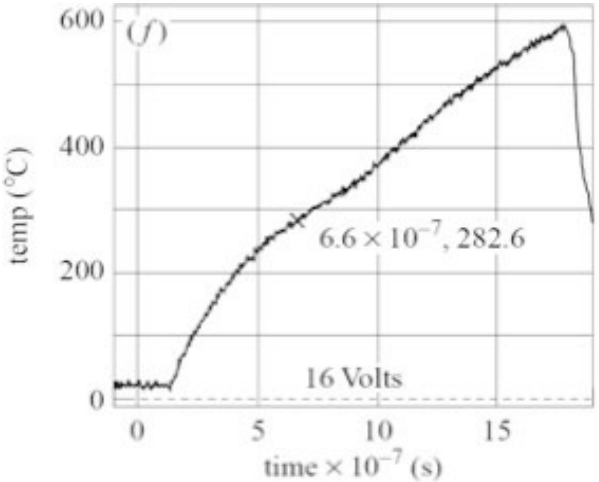
Need very high cooling to enable rapid repeat: ie need high ternal conductivity from heater to substrate

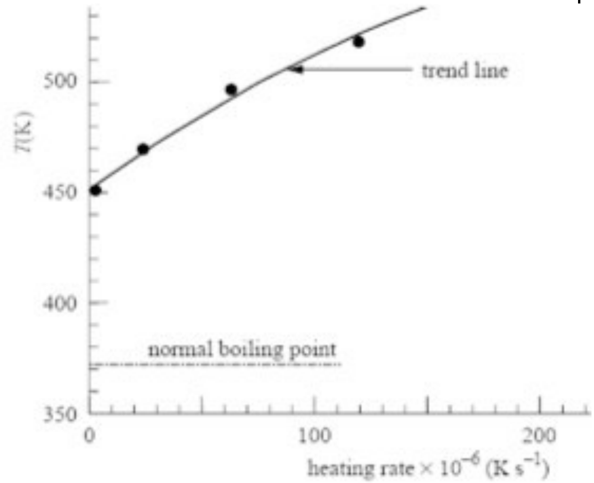
https://www.imaging.org/IST/IST/Resources/Tutorials/Inkjet.aspx

Model as Planar heater to determine the time constant



https://www.imaging.org/IST/IST/Resources/Tutorials/Inkjet.aspx





Heating rate and its effect on the nucleation temperature (15 µs pulse)

$T = R_{th}\ C_{th} = 5\ \mu s$

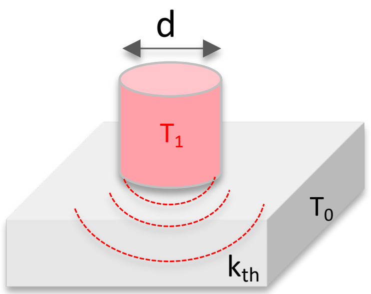
Energy efficacy on thermal inkjets:

- Electrical energy input to eject one drop : $I^2\ R{\cdot}t = 3{\cdot}10^{-7}$ J
- Kinetic energy (assuming 40 µm diameter) = $0.5\ m.v^2 = 3.10^{-12}$ J

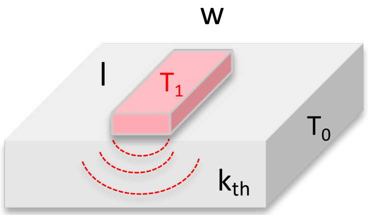
=> $E_{in}/\ E_{out}$ ratio is 10^{-5} 0.001 % thermal to kinetic energy conversion !

Model as Planar heater to determine the time constant

Very approximate…



$$R_{th} = \frac{1}{2d\,k_{th}}$$

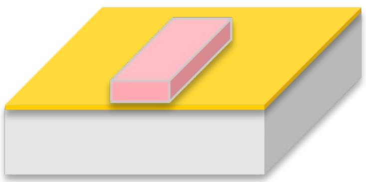


$$R_{th} \approx \frac{1}{4\sqrt{lw}\,k_{th}}$$

$$\Delta T = R_{th}\,P_{in} = \frac{P_{in}}{4\sqrt{lw}\,k_{sio2}}$$

on SiO_2 ΔT = 4500 °C ??
on Si ΔT = 35 °C ??

Both solutions are wrong because the surface is 1.7 µm of SiO_2 on a Si substrate



$$\Delta T \approx P_{in}\left(\frac{h}{w\,l\,k_{SiO2}} + \frac{1}{4\sqrt{lw}\,k_{Si}}\right)$$

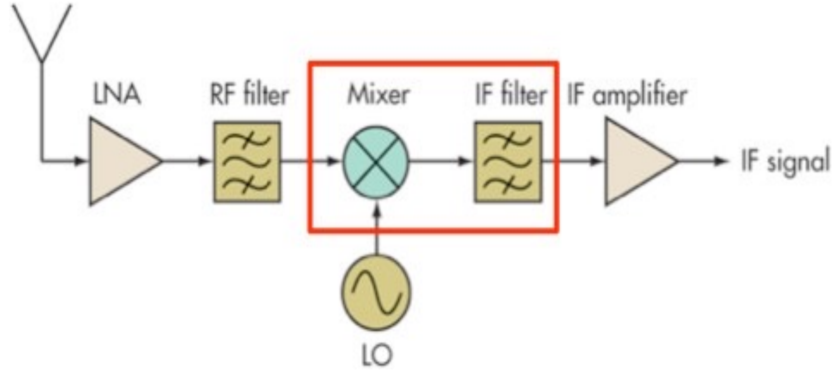
ΔT = 1500 °C

P_{in}=500mW l=30µm w=15µm h=1.7µm

High frequency thermal mixing. How fast can a thermal system be?

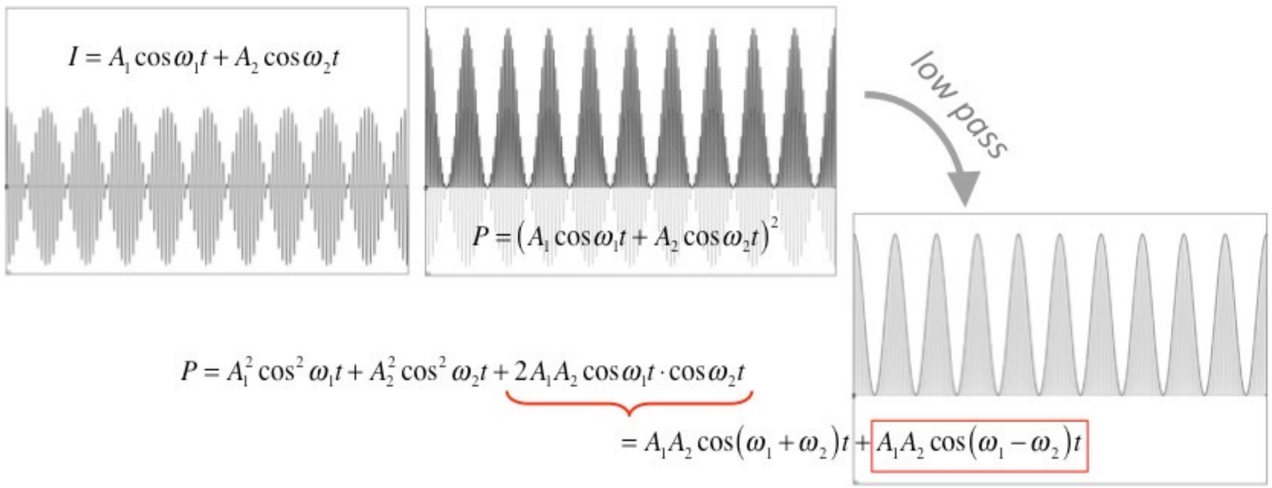
Concept: use a thermo-mechanical device as IF (intermediate frequency) stage for RF radio.

The mixer multiplies the RF frequency with the reference RF oscillator (LO) in order to extract the IF signal.
Typically GHz RF frequencies, and MHz IF freq



Use an **electro-thermal actuator** that is driven by the **sum** of high frequency signal currents, **RF + LO**

As the **power** is proportional to the **square of the current**, the power modulation contains both a **f_1+f_2** and **f_1-f_2** terms



$$P = A_1^2 \cos^2 \omega_1 t + A_2^2 \cos^2 \omega_2 t + 2A_1 A_2 \cos\omega_1 t \cdot \cos\omega_2 t$$

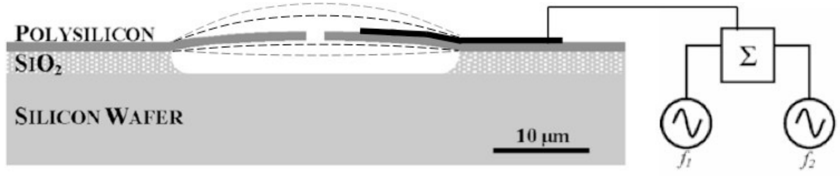
$$= A_1 A_2 \cos(\omega_1 + \omega_2)t + A_1 A_2 \cos(\omega_1 - \omega_2)t$$

High frequency thermal mixing

$$P(t) = RI^2 = R(I_1 \cos\omega_1 t + I_2 \cos\omega_2 t)^2 = I_1^2 \cos^2\omega_1 t + 2I_1 I_2 \cos\omega_1 t \cos\omega_2 t + I_2^2 \cos^2$$

with $$2I_1 I_2 \cos\omega_1 t \cos\omega_2 t = I_1 I_2 \cos(\omega_1 + \omega_2)t + I_1 I_2 \cos(\omega_1 - \omega_2)t$$

The device is a small **bimorph membrane** consisting of a polysilicon layer and a metal layer.
Current in the metal strip generates local heating and a **downward** deflection of the membrane
Finally, capacitive detection of the movement



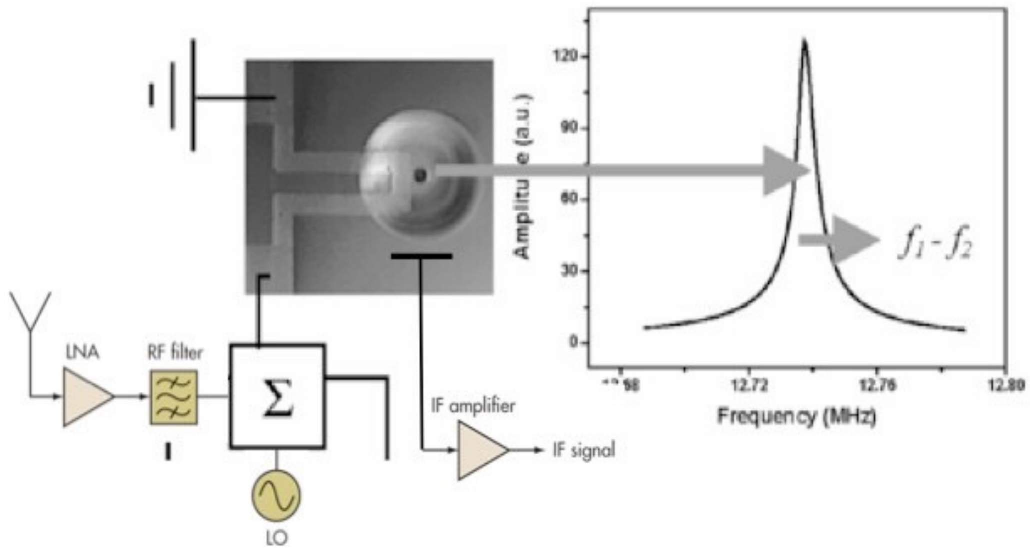
Thermal -> deflection -> change in capacitance

- **MHz thermal bandwidth** can be reached because of **downscaling** to 10 µm

f1= 1 GHz

f2= 1 GHz + 10 Mhz

Appl. Phys. Lett., 83, 18 (2003)



Electro-Thermal Actuators

- Low Voltage
- High force
- No magnetic field
- No high electric fields

- Slow!
- Very low efficiency

3 main types

- Bimorph
- Hot/cold arm
- Chevron

Review article: A. Potekhina and C. Wang, "Review of Electrothermal Actuators and Applications". *Actuators*. **8**, 69 (2019).

Electro-thermal actuators: Thermal bimorph

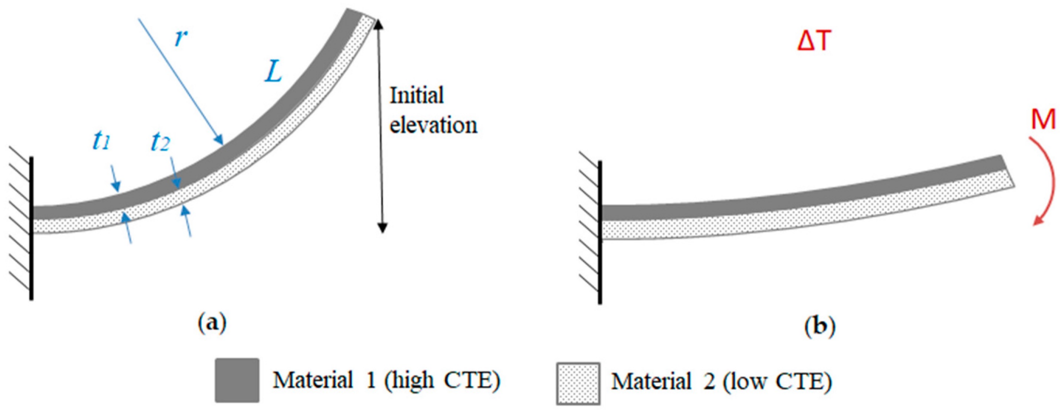
- 2 materials with different coefficients of thermal expansion α_1 and α_2
- One serves as the heater
- What ideal thickness? And thickness ratio?

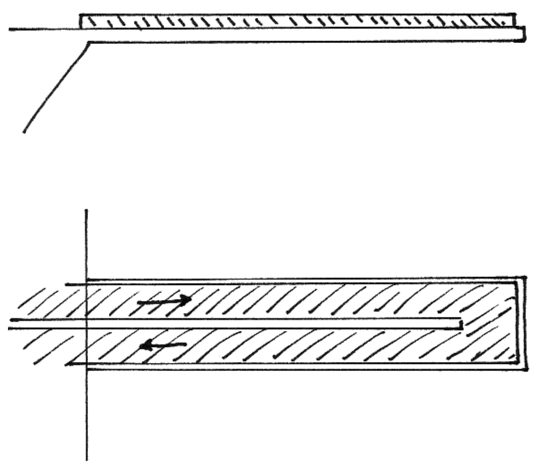
Curvature Γ of a bimorph of thickness h_1 and h_2, with Young's modulus E_1 and E_2:
Change temperature ΔT
Thermal expansion coefficient difference: $\Delta\alpha = \alpha_1 - \alpha_2$

$$\Gamma = \frac{1}{r} = \frac{6(h_1 + h_2)\cdot\Delta\alpha\cdot\Delta T}{4h_1^2 + 4h_2^2 + 6h_1h_2 + \frac{h_1^3E_1}{h_2E_2} + \frac{h_2^3E_2}{h_1E_1}}$$

Largest curvature when: $h_1E_1 = h_2E_2$

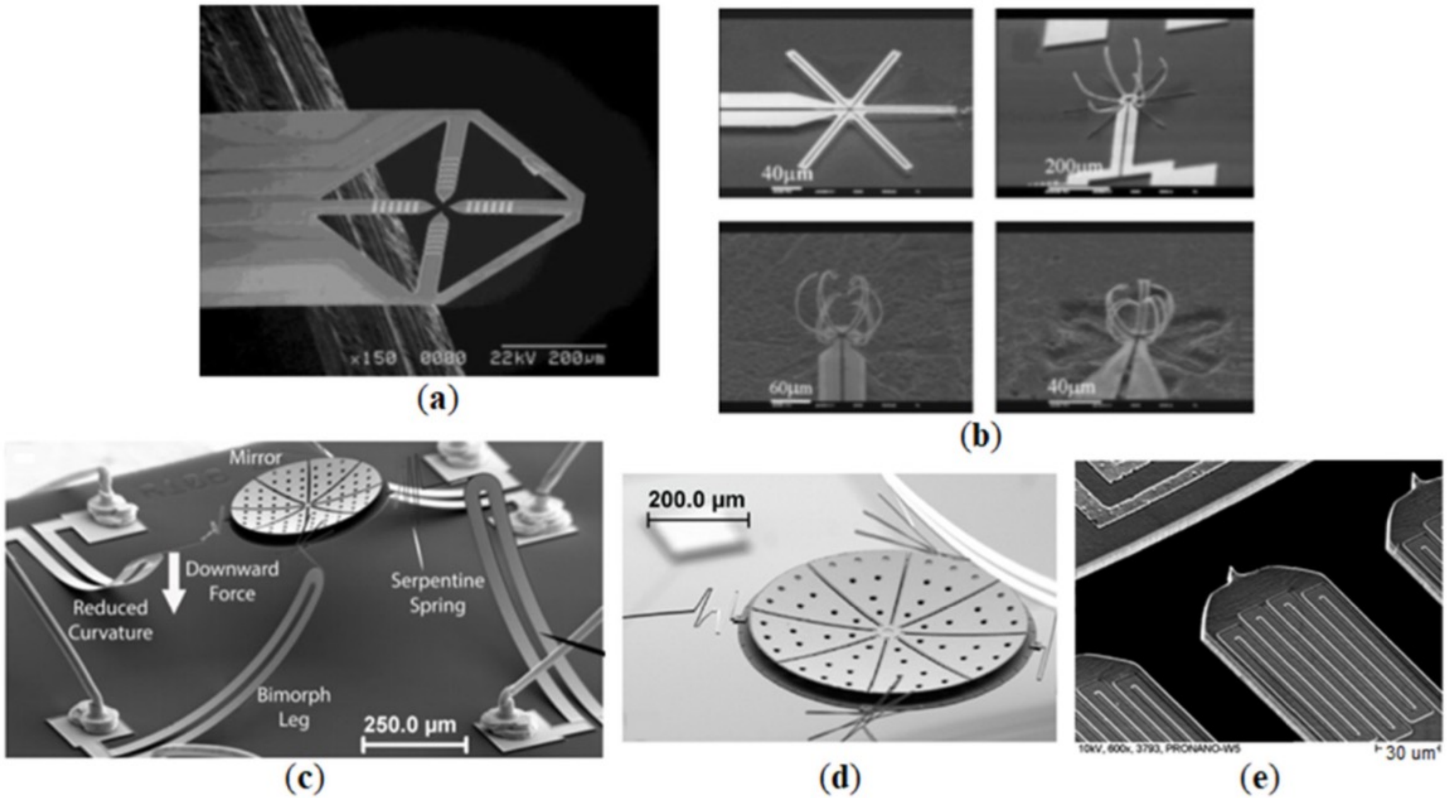
$$\frac{h_1}{h_2} = \frac{E_2}{E_1} = k$$





Side view and top view of a cantilever bimorph

Electro-thermal actuators:
Thermal bimorphs



If optimize thickness, get:

$$\Gamma(\Delta T) = \frac{0.7 \cdot \Delta\alpha}{h_{tot}^2} \Delta T$$

$$\Gamma \propto \frac{1}{h^2}$$

Bimorph bending angle scales favorably to small dimensions
(but force does not!)

A. Potekhina, Review of Electrothermal Actuators and Applications. *Actuators*. **8**, 69 (2019).

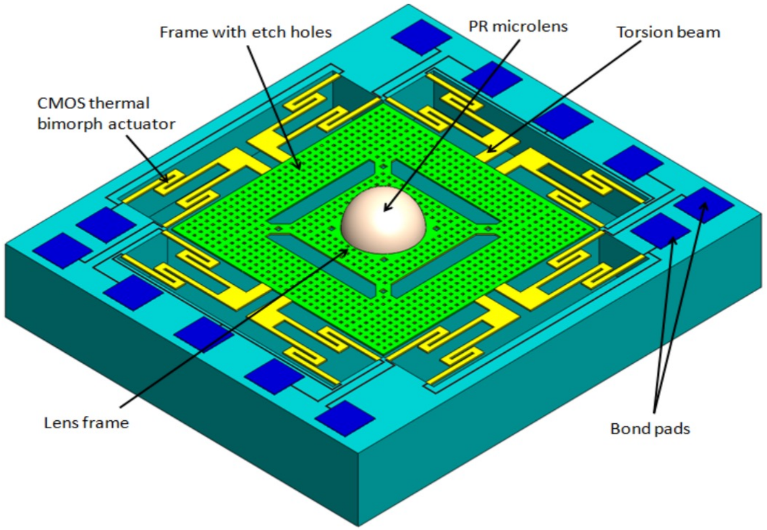
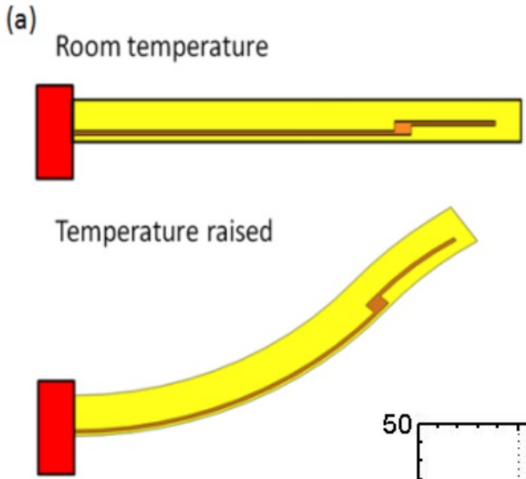
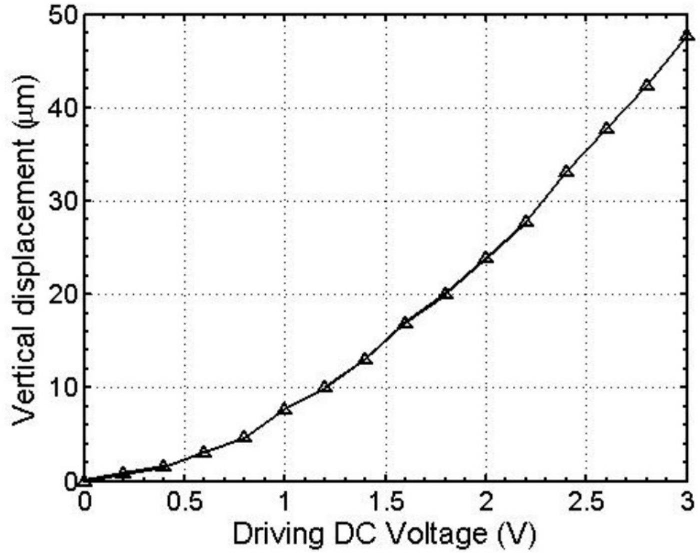


Fig. 1. Schematic diagram of the proposed 3-D scanning microlens, where the lens frame is actuated by 4 sets of CMOS thermal bimorph actuators.

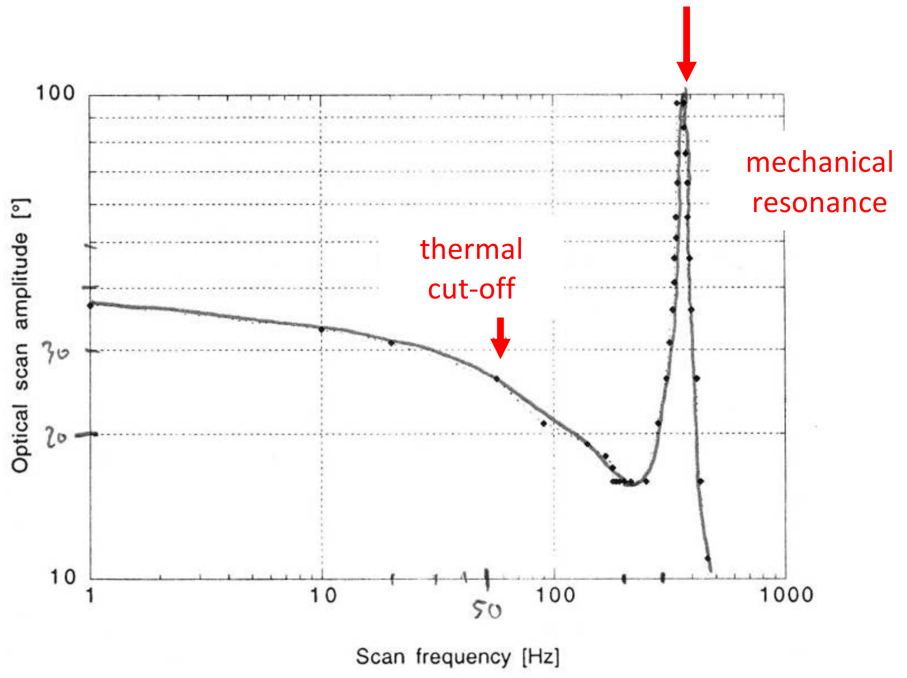


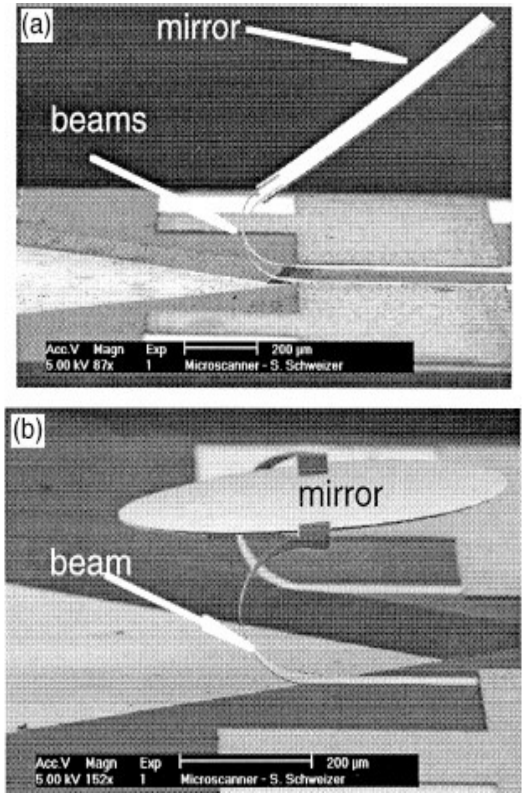


Vertical displacement of 47 µm and power consumption of 139 mW at 3 V dc.

K. H. Koh, C. Lee, J.-H. Lu, C.-C. Chen, "Development of CMOS MEMS thermal bimorph actuator for driving microlens" in *16th International Conference on Optical MEMS and Nanophotonics* (2011), pp. 153–154.

Thermomechanical transfer function



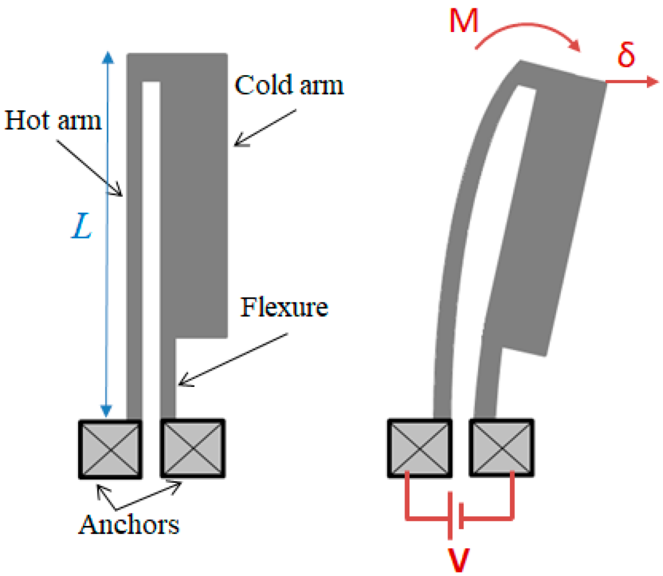


Dynamical response of bimorph thermal actuator. Thermal cut-off is 1st order function is in this case at about 50 Hz and there is a 2nd order mechanical resonance at about 400 Hz (from Schweizer paper).

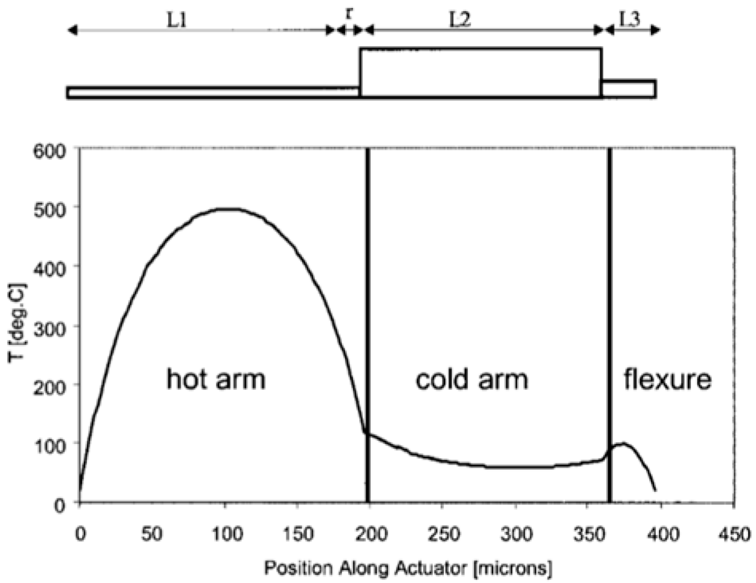
Thermally actuated optical microscanner with large angle and low consumption, S. Schweizer et al., Sensors and Actuators (1999)

Horizontal thermal actuator - U-shaped actuator (hot/cold arm)

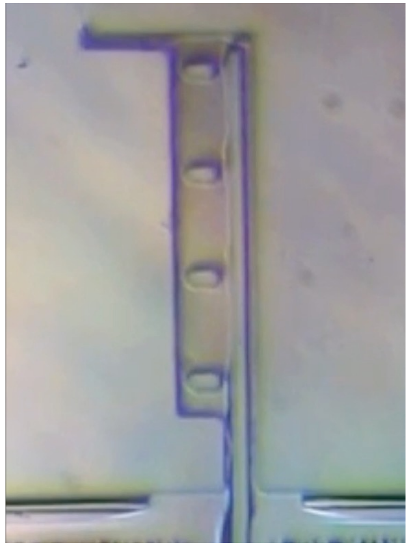
Principle: asymmetric beam size => asymmetric temperature (due to difference in heating and in losses)



(a)



(b)



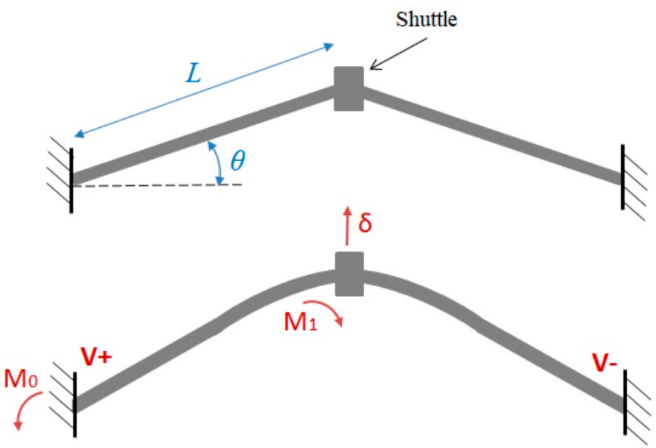
The arm with large area has lower electrical resistance and dissipates more heat and thus remains cooler than the other «hot» arm.

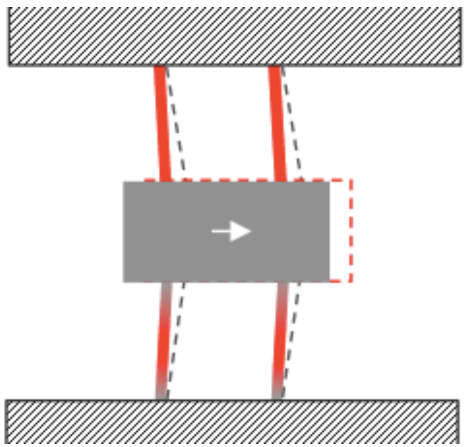
- *R. Syms, Long-travel electrothermally driven resonant cantilever microactuators, J. Micromech. Microeng. 12 (2002) 211–218*
- *N. Mankame et al., Comprehensive thermal modelling and characterization of an electro-thermal- compliant microactuator, J. Micromech. Microeng. 11 (2001) 452–462*

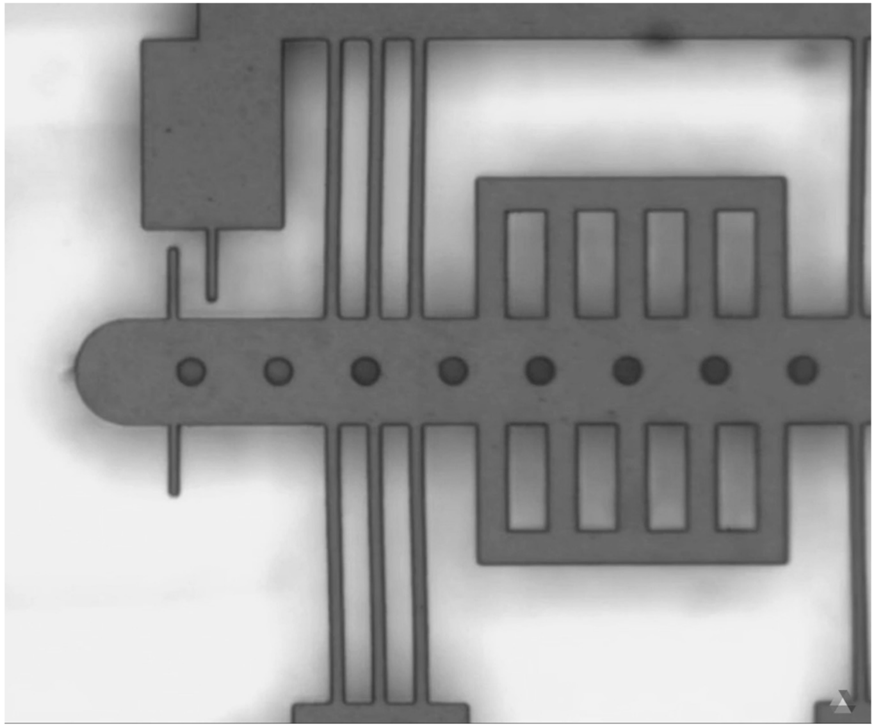
Heat flow dominated by conduction in the 1-2 μm gap between substrate and actuator: the cold arm is well cooled.

Horizontal thermal actuators - V-beam (or Chevron) actuator

- Doubly clamped beam with a "shuttle" at the middle
- Expansion make the shuttle moving laterally
- Relatively large displacement obtained





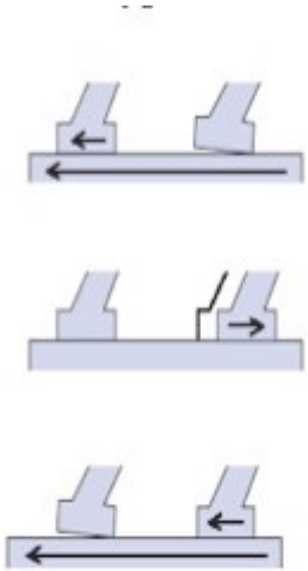


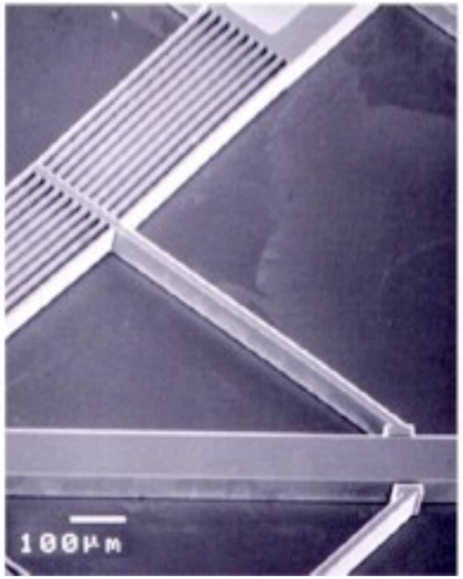
Small displacement, high force

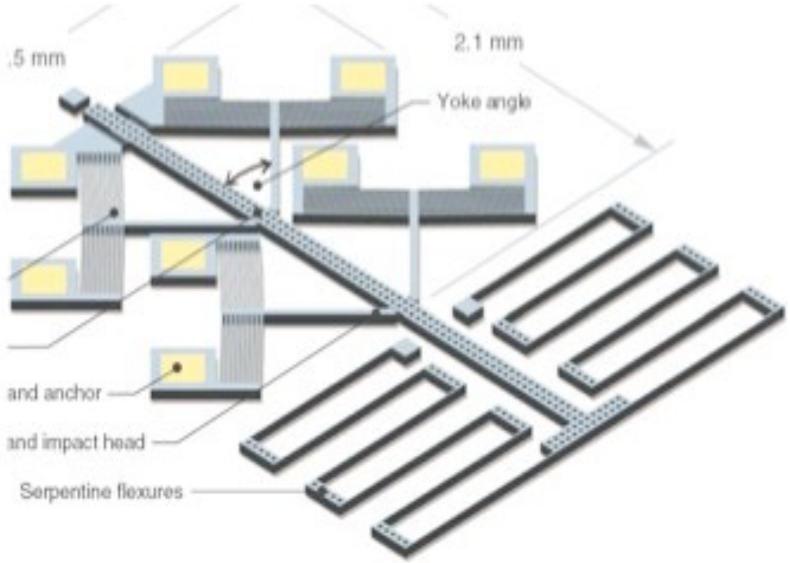
Lott et al., "Modeling the thermal behavior of a surface-micromachined linear-displacement thermomechanical microactuator", Sensors and Actuators A: Physical, Volume 101, 2002, Pages 239-250

Horizontal thermal actuators

Inchworm type:

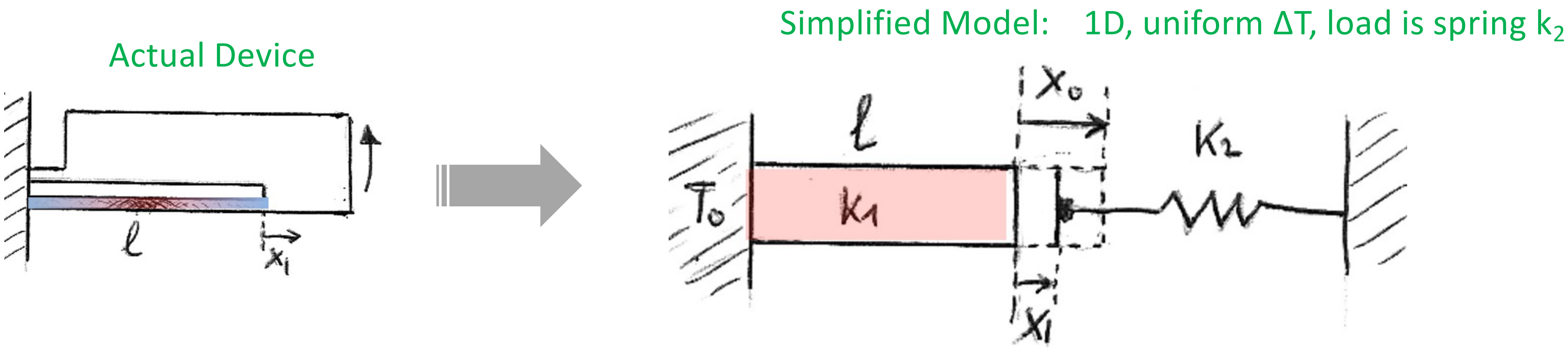






- J. Maloney, J. Micromech. Microeng. 14 (2004) 226
- D. de Voe, “Thermal issues in MEMS and microscale systems”, IEEE Trans. On Comp. and Pack Tech. 25 (2003) 576

Energy density in thermo-mechanical actuators



Free beam thermal expansion $$x_0 = l \cdot \alpha \cdot \Delta T_{average}$$

Constrained beam expansion $$x_1 = \frac{k_1}{k_1 + k_2} x_0$$

Thermomechanical work energy on k_2 $$W_{tmec} = \frac{1}{2} k_2 x_1^2 = \frac{1}{2} \frac{k_1 k_2}{k_1 + k_2} x_0^2$$

when $k_2 \to 0$ $\quad W_{tmec} \to 0$: *no work if there is no load*

when $k_2 \to k_1$ $\quad W_{tmec} \to \frac{1}{4} k_1 x^2$: ***maximal efficacy when $k_1 = k_2$***

when $k_2 \to \infty$ $\quad W_{tmec} \to 0$: *no work done if the actuator displacement is completely blocked*

Energy density in thermo-mechanical actuators

Our model assumption: $k_2 = k_1$ (i.e. maximal efficacy)

- thermomechanical energy $W_{tmec} = \frac{1}{4} k_1 x_0^2 = \frac{1}{4} k_1 l^2 \left(\alpha \cdot \Delta T_{average}\right)^2$
- thermomechanical energy density $w_{tmec} = \frac{1}{4} E \left(\alpha \cdot \Delta T_{average}\right)^2$ E: elastic modulus

Numerical value for Silicon beam: $\alpha = 2.7 \cdot 10^{-6}$ $\Delta T_{max} = 300°C$ $E = 190\ GPa$

For an actuator beam of dimensions 20x2x300µm³

$\Delta T_{average} = \frac{2}{3} \cdot \Delta T = 200°C$ => $w_{tmec} \cong 10^5\ J/m^3$ $W_{tmec} = 2.10^{-11} J$

=1 bar

At 100 Hz actuation frequency $P_{mec} = f_0 \cdot W_{tmec} \cong 2 \cdot 10^{-9}\ Watt$ of mechanical output

Efficiency of thermo-mechanical actuators

Let's compute the dissipated thermal power (to reach T_{max} in clamped beam):

$$P_{therm} = \frac{2\kappa_{th} w\, h}{l} \Delta T_{max} \qquad P_{therm} = 10 \text{ mW}$$

=> Power efficiency of the electrothermal actuator

$$\frac{P_{mec}}{P_{therm}} = 10^{-6}$$

Compare with the mechanical energy stored in a resonator beam:

Example: 0.1 Nm, x= 50 µm $E_{mec}=10^{-10}$ J

Now calculate mechanical dissipation and then compare with thermal energy needed to maintain oscillation:

$$Q = 2\pi \frac{E_{mec}}{E_{diss}} \qquad E_{diss} = \frac{2\pi E_{mec}}{Q}$$

Q=100, f=500 Hz P_{diss}=14 nW $P_{therm} = 10 mW$

ie need 10 mW thermal power to sustain 13 nW mechanical power, ridiculous solution…