In communication theory, the functions $f(t)$ and $F(\nu)$ represent a signal, with $f(t)$ its time-domain representation and $F(\nu)$ its frequency-domain representation. The squared-absolute value $|f(t)|^2$ is called the **signal power**, and $|F(\nu)|^2$ is the energy spectral density. If $|F(\nu)|^2$ extends over a wide frequency range, the signal is said to have a wide bandwidth.

Properties of the Fourier Transform

Some important properties of the Fourier transform are provided below. These properties can be proved by direct application of the definitions (A.1-1) and (A.1-2) (see any of the books in the reading list).

- *Linearity.* The Fourier transform of the sum of two functions is the sum of their Fourier transforms.
- *Scaling.* If $f(t)$ has a Fourier transform $F(\nu)$, and τ is a real scaling factor, then $f(t/\tau)$ has a Fourier transform $|\tau|F(\tau\nu)$. This means that if $f(t)$ is scaled by a factor τ, its Fourier transform is scaled by a factor $1/\tau$. For example, if $\tau > 1$, then $f(t/\tau)$ is a stretched version of $f(t)$, whereas $F(\tau\nu)$ is a compressed version of $F(\nu)$. The Fourier transform of $f(-t)$ is $F(-\nu)$.
- *Time Translation.* If $f(t)$ has a Fourier transform $F(\nu)$, the Fourier transform of $f(t-\tau)$ is $\exp(-j2\pi\nu\tau)F(\nu)$. Thus delay by time τ is equivalent to multiplication of the Fourier transform by a phase factor $\exp(-j2\pi\nu\tau)$.
- *Frequency Translation.* If $F(\nu)$ is the Fourier transform of $f(t)$, the Fourier transform of $f(t)\exp(j2\pi\nu_0 t)$ is $F(\nu-\nu_0)$. Thus multiplication by a harmonic function of frequency ν_0 is equivalent to shifting the Fourier transform to a higher frequency ν_0.
- *Symmetry.* If $f(t)$ is real, then $F(\nu)$ has Hermitian symmetry [i.e., $F(-\nu) = F^*(\nu)$]. If $f(t)$ is real and symmetric, then $F(\nu)$ is also real and symmetric.
- *Convolution Theorem.* If the Fourier transforms of $f_1(t)$ and $f_2(t)$ are $F_1(\nu)$ and $F_2(\nu)$, respectively, the inverse Fourier transform of the product

$$F(\nu) = F_1(\nu)F_2(\nu) \tag{A.1-3}$$

is

$$f(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)\,d\tau. \tag{A.1-4}$$

Convolution

The operation defined in (A.1-4) is known as the convolution of $f_1(t)$ with $f_2(t)$. Convolution in the time domain is therefore equivalent to multiplication in the Fourier domain.

- *Correlation Theorem.* The correlation between two complex functions is defined as

$$f(t) = \int_{-\infty}^{\infty} f_1^*(\tau)f_2(t+\tau)\,d\tau. \tag{A.1-5}$$

Correlation

The Fourier transforms of $f_1(t)$, $f_2(t)$, and $f(t)$ are related by

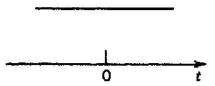
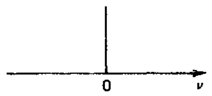
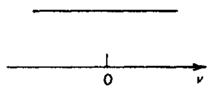
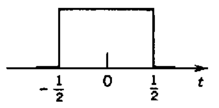
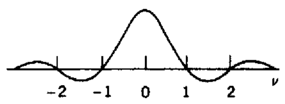
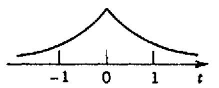
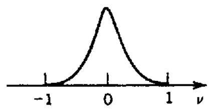
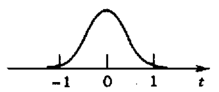
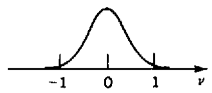
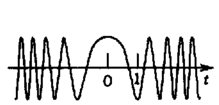
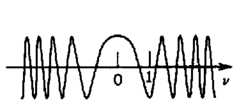
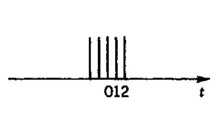
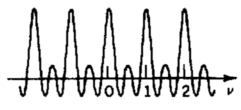
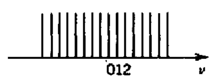
$$F(\nu) = F_1^*(\nu)F_2(\nu). \tag{A.1-6}$$

- *Parseval's Theorem.* The signal energy, which is the integral of the signal power $|f(t)|^2$, equals the integral of the energy spectral density $|F(\nu)|^2$, so that

$$\int_{-\infty}^{\infty} |f(t)|^2\, dt = \int_{-\infty}^{\infty} |F(\nu)|^2\, d\nu. \qquad \text{(A.1-7)}$$

Parseval's Theorem

TABLE A.1-1 Selected Functions and Their Fourier Transforms

Function		$f(t)$	$F(\nu)$	
Uniform		1	$\delta(\nu)$	
Impulse		$\delta(t)$	1	
Rectangular		rect (t)	sinc(ν)	
Exponential[a]		$\exp(-\lvert t\rvert)$	$\frac{2}{1+(2\pi\nu)^2}$	
Gaussian		$\exp(-\pi t^2)$	$\exp(-\pi\nu^2)$	
Chirp[b]		$\exp(j\pi t^2)$	$e^{j\pi/4}\exp(-j\pi\nu^2)$	
Sum of $M=2S+1$ impulses		$\sum_{n=-S}^{S} \delta(t-n)$	$\frac{\sin(M\pi\nu)}{\sin(\pi\nu)}$	
Infinite sum of impulses		$\sum_{n=-\infty}^{\infty} \delta(t-n)$	$\sum_{n=-\infty}^{\infty} \delta(\nu-n)$	

[a]The double-sided exponential function is shown. The Fourier transform of the single-sided exponential, $f(t) = \exp(-t)$ with $t \geq 0$, is $F(\nu) = 1/[1 + j2\pi\nu]$. Its magnitude is $1/[1 + (2\pi\nu)^2]^{1/2}$.
[b]The functions $\cos(\pi t^2)$ and $\cos(\pi\nu^2)$ are shown. The function $\sin(\pi t^2)$ is shown in Fig. 4.3-6.