MICRO-561

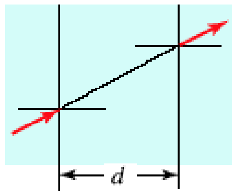
Fundamentals of Biomicroscopy

Syllabus (tentative)

Lecture 1	Introduction & Ray Optics-1
Lecture 2	Ray Optics-2 & Matrix Optics-1
Lecture 3	Matrix Optics-2
Lecture 4	Matrix Optics-3 & Microscopy Design-1
Lecture 5	Microscopy Design-2
Lecture 6	Microscopy Design-3
Lecture 7	Resolution-1
Lecture 8	Resolution-2
Lecture 9	Resolution-3 & Contrast
Lecture 10	Fluorescence-1
Lecture 11	Fluorescence-2
Lecture 12	Fluorescence-3, Sources, Filters
Lecture 13	Detectors
Lecture 14	Bio-application Examples

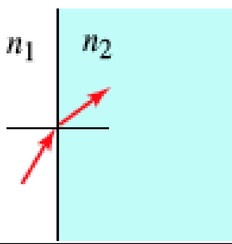
Reminder: Summary of matrix optics for basic functions & components

Propagation



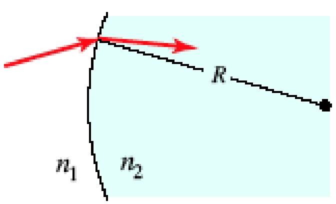
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

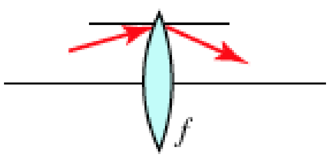
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

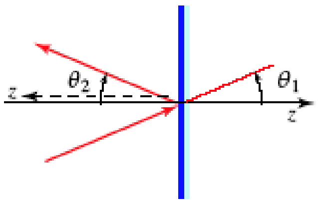
Lens



Convex, $f > 0$; concave, $f < 0$

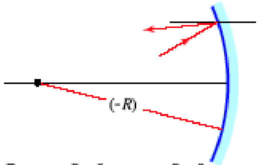
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Planar mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Spherical mirror



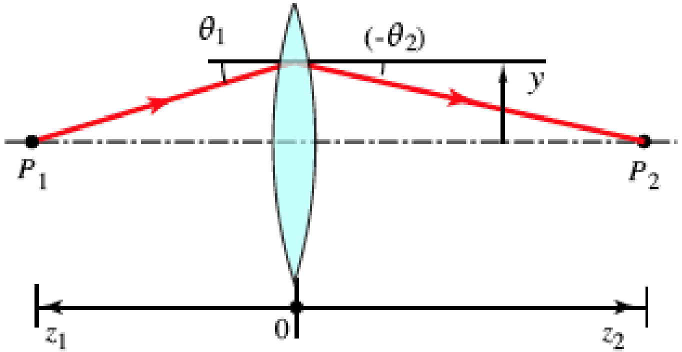
Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

How to link ray tracing & matrix representation?

THIN LENS:

1) Position is ~nearly same immediately after the ray exits a thin lens
2) Thin lens only **deflects** the ray (i.e., change the outgoing angle) according to the law of refraction



From Ray Tracing

Ray Deflection :

$$\theta_2 = \theta_1 - \frac{y}{f},$$

Focal length:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Imaging Condition:

$$\frac{1}{f} = \frac{1}{z_{obj}} + \frac{1}{z_{im}}$$

Magnification:

$$\text{Mag} = -\frac{z_{im}}{z_{obj}}$$

From Matrix Representation

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \rightarrow$$

Ray Deflection :

$$\theta_{out} = \theta_{in} - \frac{y_{in}}{f}$$

Where, f is :

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

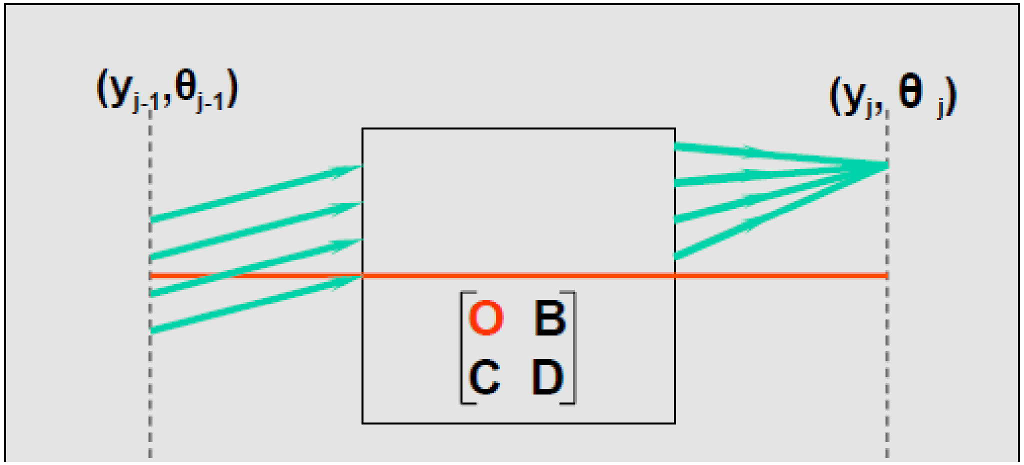
System Properties for "**A=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If A=0 ➔ $y_j = 0.\, y_{j-1} + B.\, \theta_{j-1}$

Under this condition the equation is **independent of positions** (y_{j-1})

➔It means all possible rays entering the system from **different positions** but with same angle (θ_{j-1}), will cross each other at the same point at the exit the system.



A=0 corresponds to FOCUSING

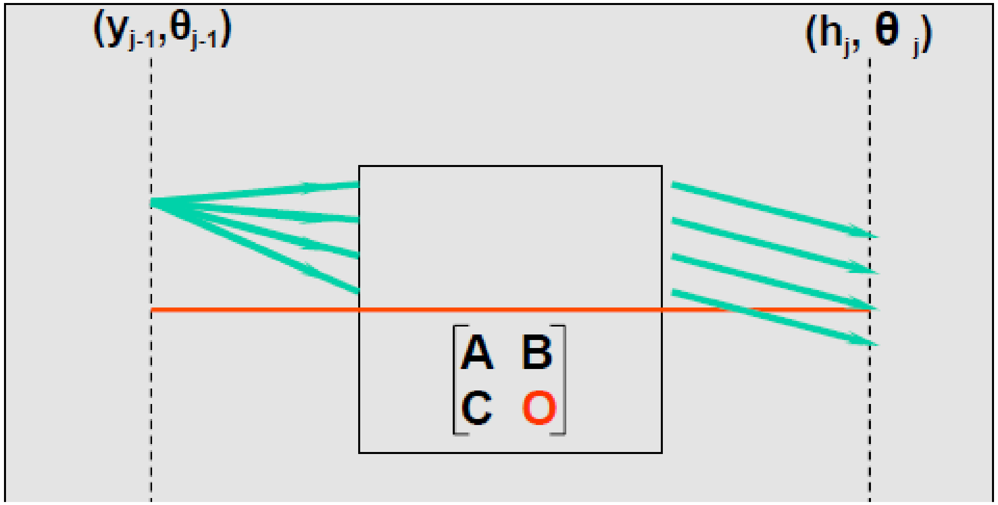
System Properties for "**D=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If D=0 ➔ $\theta_j = C.y_{j-1} + 0.\theta_{j-1}$

Under this condition the equation is **independent of angle** (θ_{j-1})

➔It means all possible rays entering the system from different angles but originated from the same position (y_{j-1}), will exit the system with the same angle.



D=0 represents COLLIMATING

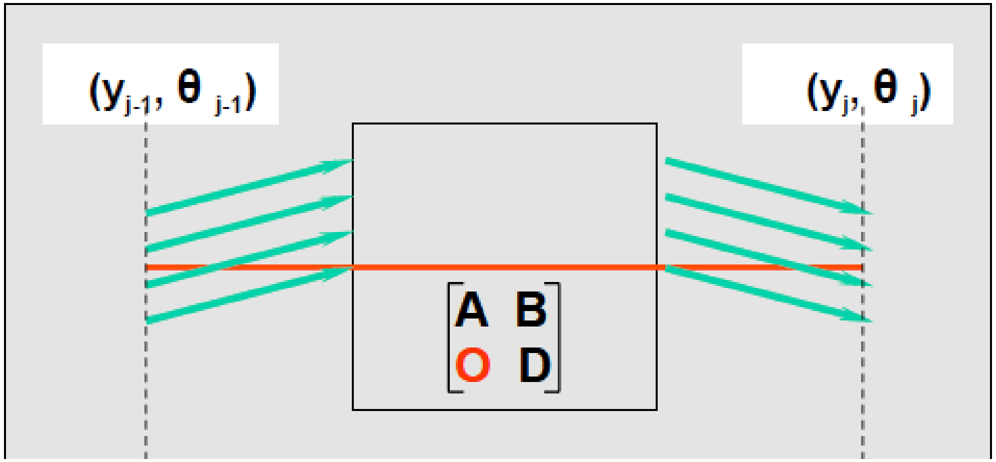
System Properties for "**C=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If C=0 ➔ $\theta_j = 0.\, y_{j-1} + D.\, \theta_{j-1}$

Under this condition the equation is **independent of position** (y_{j-1}).

➔It means all possible rays entering the system from different entrance points but with same entrance angle (θ_{j-1}), will exit the system with the same exit angle.



C=0 corresponds to deviation

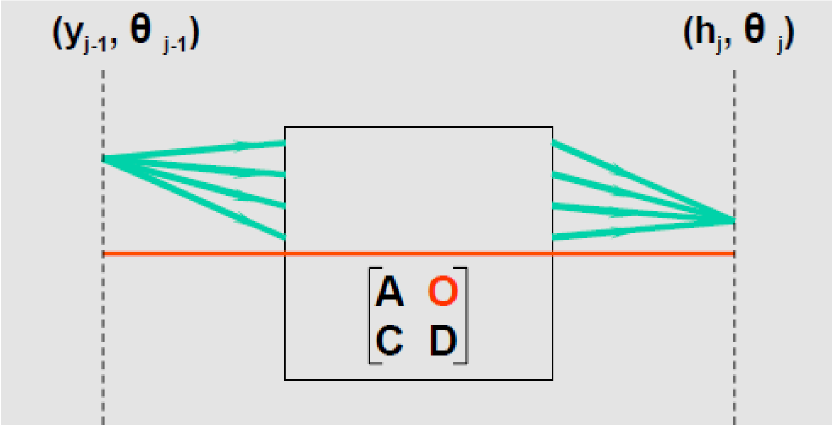
System Properties for "**B=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If B=0 ➔ $y_j = A.y_{j-1} + 0.\theta_{j-1}$

Under this condition the equation is **independent of angle** (θ_{j-1})

➔ It means all possible rays entering the system with different angles but from the same point (y_{j-1}), will cross each other at the same point at the exit of the system.



B=0 corresponds to IMAGING condition

It means these two planes (indicated by dashed vertical line) are conjugate

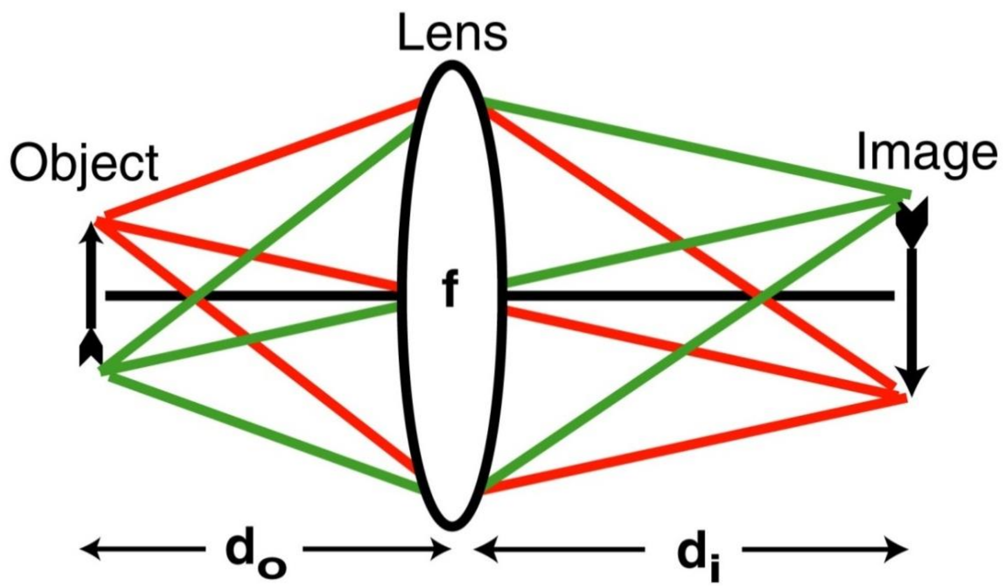
According to Matrix Representation:

An optical system forms an **image** of an object when "$\boldsymbol{B}$ **= 0**"

When $\boldsymbol{B} = \mathbf{0}$ ➔ $y_{out} = A\, y_{in}$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} A y_{in} \\ C y_{in} + D\theta_{in} \end{bmatrix}$$

Independent of their angle, all rays from a point y_{in} arrive at the same point y_{out}

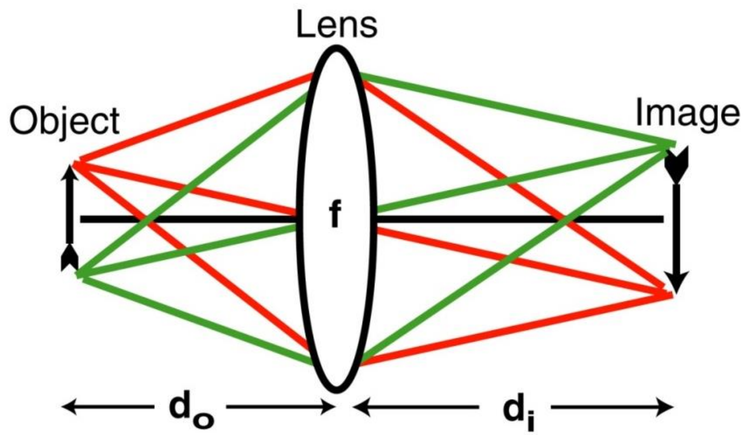


When $B = 0$, $y_{out} = A\, y_{in}$

A is the **magnification**.

$\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ ➔ this is called "**conjugate**" matrix ➔ it describes an "**imaging**" optical set-up

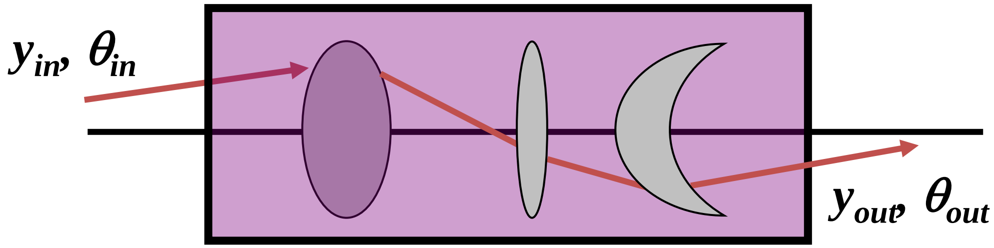
Example:
Find the conjugate matrix of an imaging system based on single thin-lens

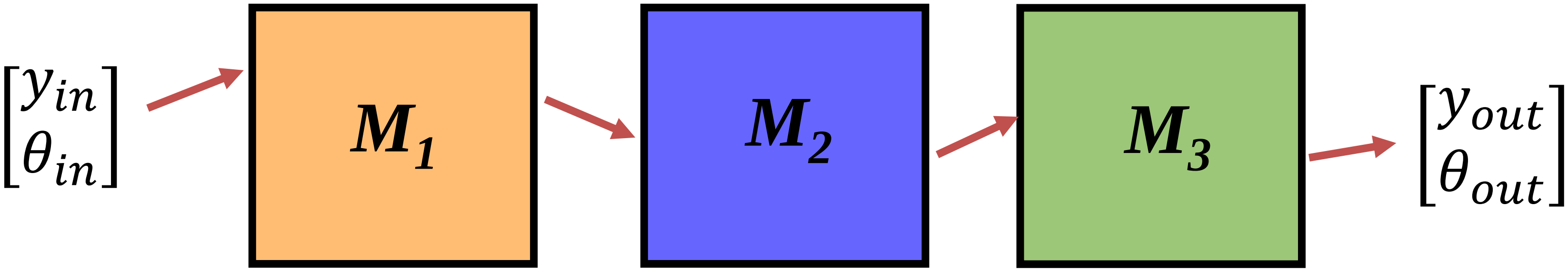


Q: How can we write the "conjugate matrix" of this one thin-lens imaging system?

From the object plane to the image plane, light rays :
1)**Propagate** over a distance of d_o
2) Then, **encounter a thin lens** of focal length f
3) Finally, **propagate** over a distance of d_i

Reminder: Cascaded Elements





we simply multiply ray matrices.

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_3 \left\{ M_2 \left(M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \right) \right\} = M_3 M_2 M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Notice the order !!

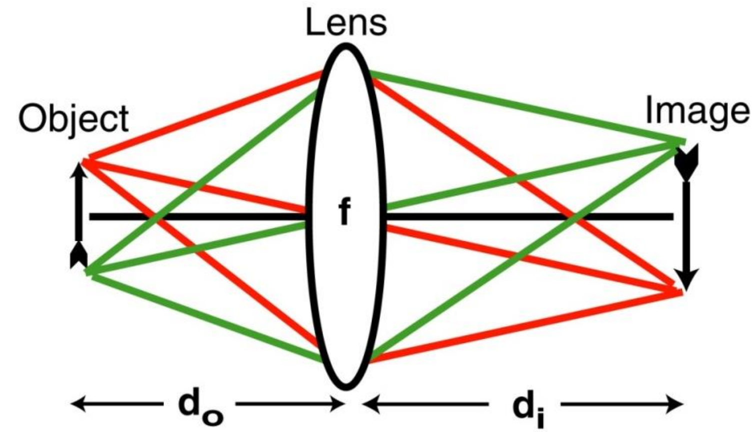
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1)**Propagate** over a distance of d_o

2) Then, encounter a thin lens of focal length f

3) Finally, propagate over a distance of d_i



$$M_1 = \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}$$

$$M_{system} = M_3 \times M_2 \times M_1$$

$$M_{system} = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d_o \\ -1/f & 1-d_o/f \end{bmatrix}$$

$$= \begin{bmatrix} 1-d_i/f & d_o+d_i-d_o d_i/f \\ -1/f & 1-d_o/f \end{bmatrix}$$

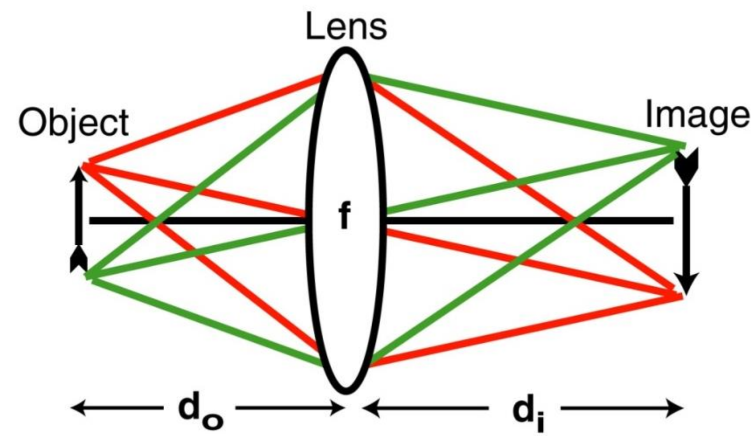
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1) **Propagate** over a distance of d_o

2) Then, encounter a thin lens of focal length f

3) Finally, propagate over a distance of d_i



$$M_1 = \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}$$

$$M_{system} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1-d_i/f & d_o + d_i - d_o d_i/f \\ -1/f & 1-d_o/f \end{bmatrix}$$

For imaging, set B as 0 ➔ $B = d_o + d_i - d_o d_i / f = 0$

It means, $d_o d_i \left[1/d_o + 1/d_i - 1/f\right] = 0$, and this happens only if :

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

This is the imaging condition (a.k.a. Lens law)

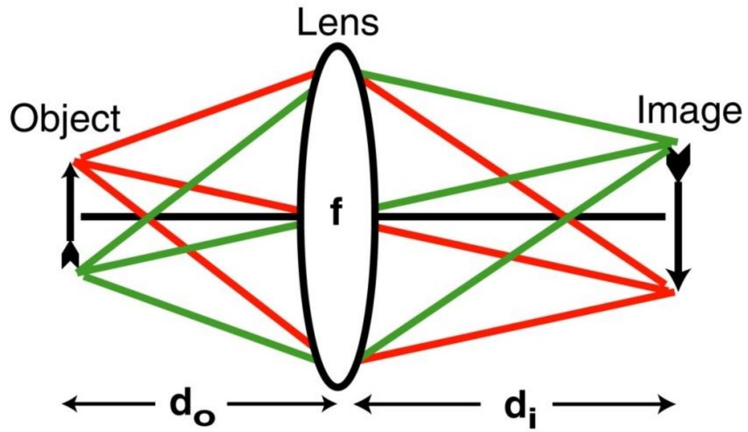
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1)**Propagate** over a distance of d_o

2) Then, **encounter a thin lens** of focal length f

3) Finally, **propagate** over a distance of d_i



At the imaging condition:

$$M_{system} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1 - d_i / f & 0 \\ -1/f & 1 - d_o / f \end{bmatrix}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

If the imaging condition is satisfied then:

$$A = 1 - d_i / f$$

$$= 1 - d_i \left[\frac{1}{d_o} + \frac{1}{d_i} \right]$$

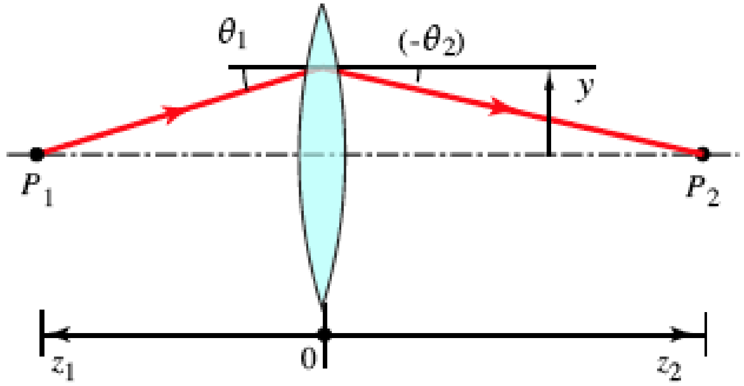
$$= -\frac{d_i}{d_o}$$

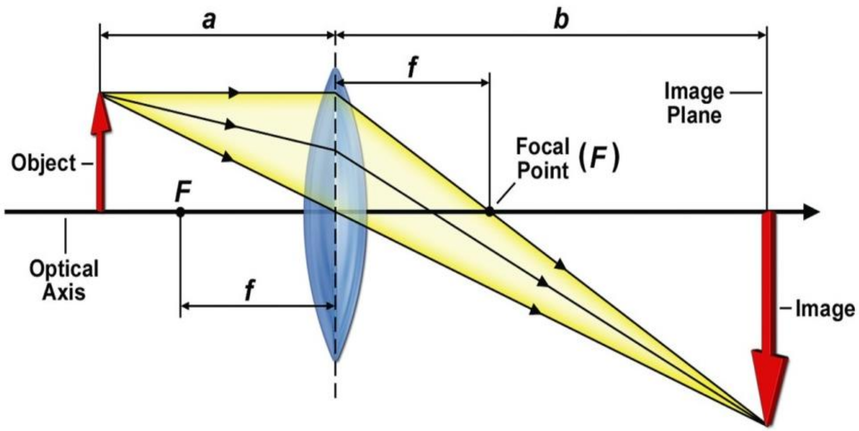
Magnification:

$$A = M$$

$$M = -\frac{d_i}{d_o}$$

How to link ray tracing & matrix representation?





From Ray Tracing

Ray Deflection :

$$\theta_2 = \theta_1 - \frac{y}{f},$$

Focal length: (lens makes formula)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Imaging Condition:

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

Magnification:

$$\text{Mag} = -\frac{b}{a}$$

From Matrix Representation

$$\text{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \rightarrow$$

Ray Deflection :

$$\theta_{out} = \theta_{in} - \frac{y_{in}}{f}$$

Where, f is :

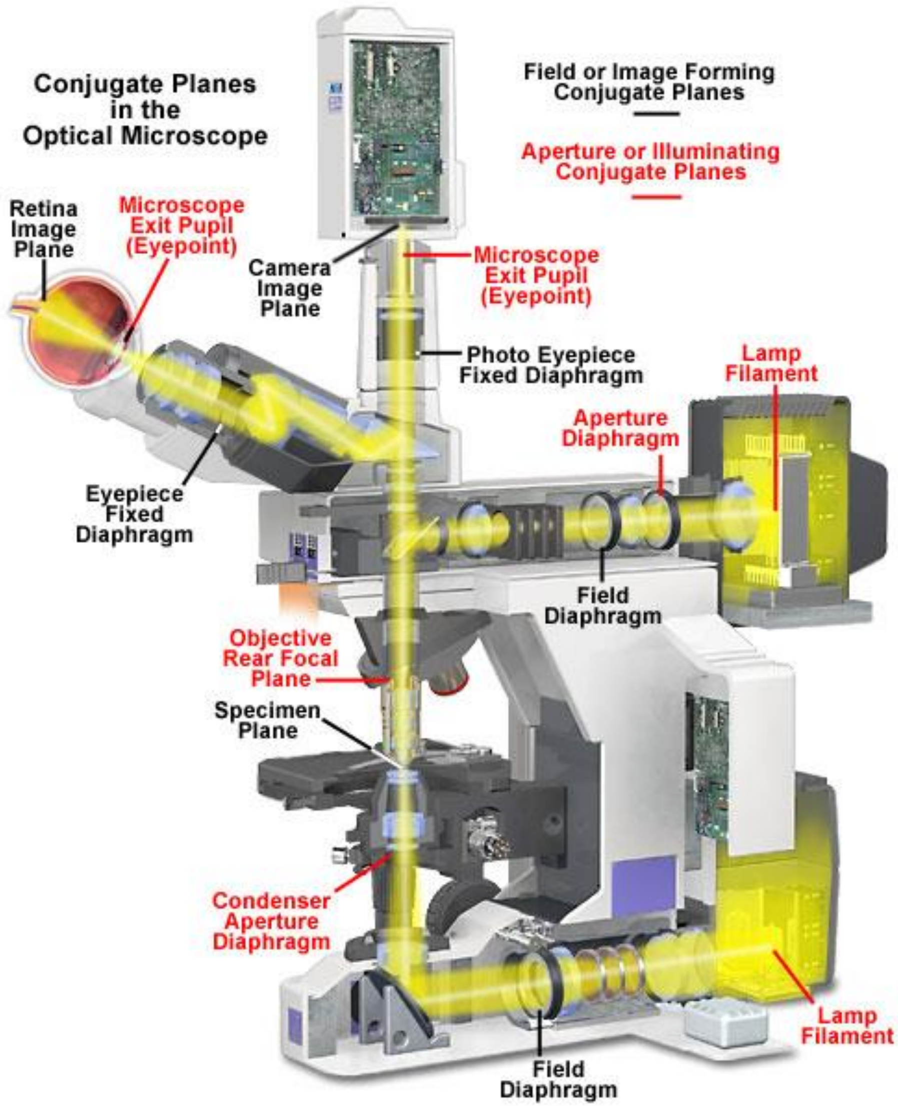
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

At imaging condition, B=0 and A = Mag:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$M = -\frac{d_i}{d_o}$$

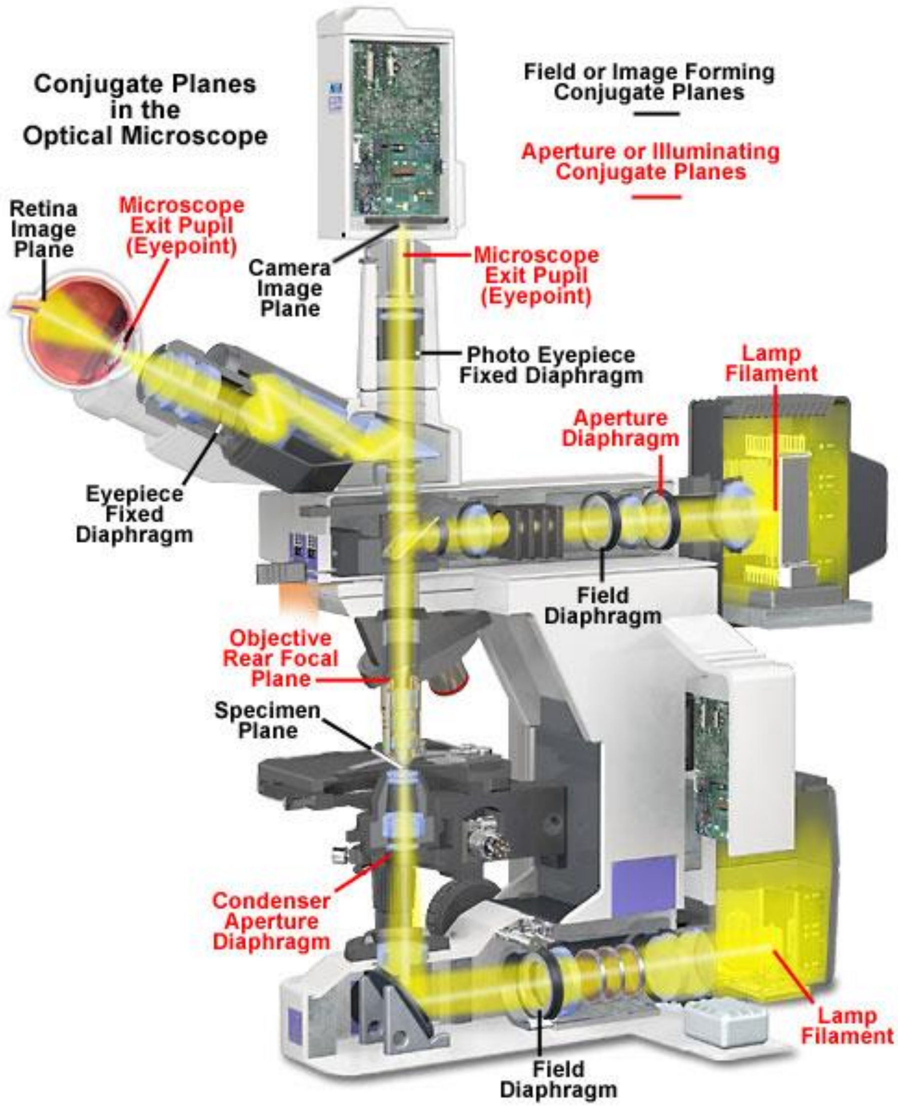
Generalization: How to trace an image in an optical system?



Optical systems (i.e. microscope) contain multiple lenses.

Strategy: **Use <u>cardinal planes</u>** which connect matrix optics & ray tracing

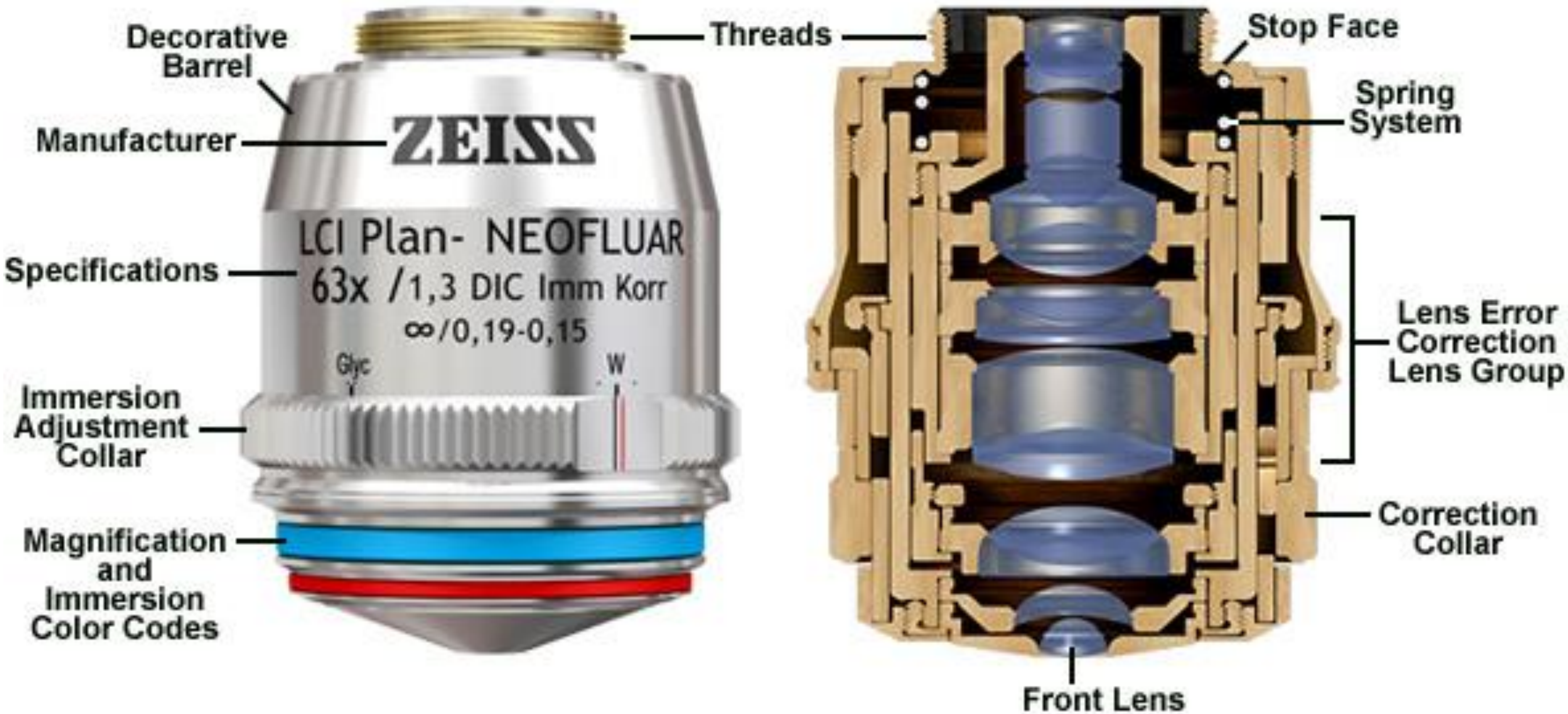
Microscope – An Optical System



Let's start with an important optical component:

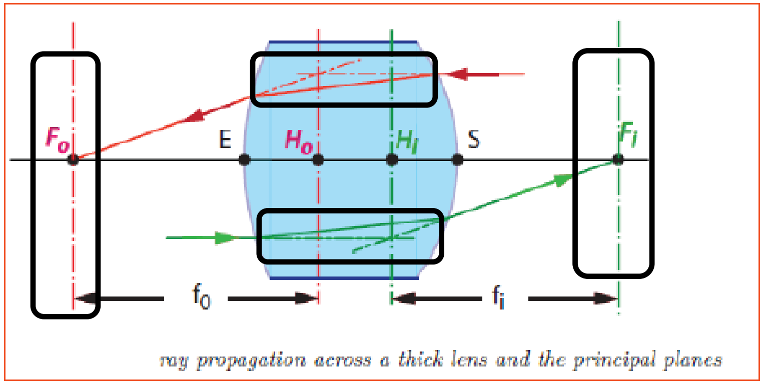
- The Objective

Anatomy of the Microscope Objective Lens



- It is not a single (and thin) lens
- It contains multiple THICK lenses
- Effectively this is an optical system itself

Thick lens



ray propagation across a thick lens and the principal planes

A thick lens is an "optical system":

- Refraction at entrance & exit surfaces
- Propagation inside the lens

The thick lens can be simplified by representing that **as if** the refraction is happening at **the principal planes**:

1) Two principal planes: $\mathbf{H_o}$ and $\mathbf{H_i}$ ➔ **virtual planes where the lens appears to bend the rays**

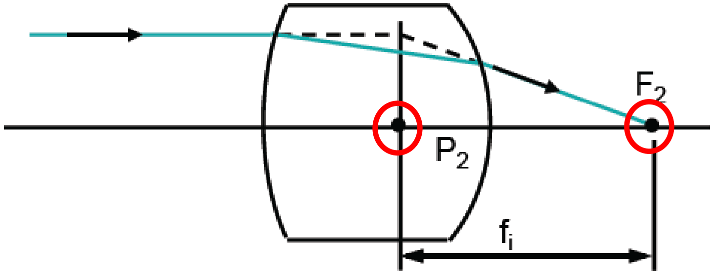
2) Two focal planes: $\mathbf{F_o}$ and $\mathbf{F_i}$

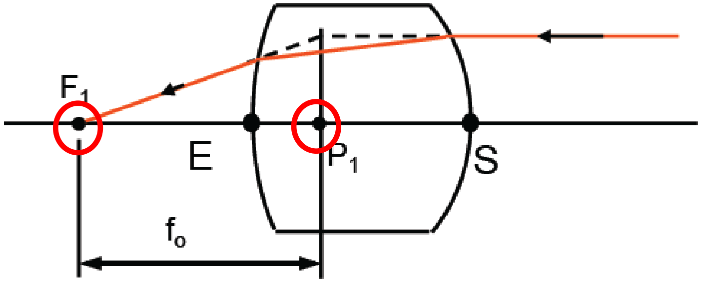
These are the FOUR CARDINAL PLANES.

There are also FOUR CARDINAL POINTS:

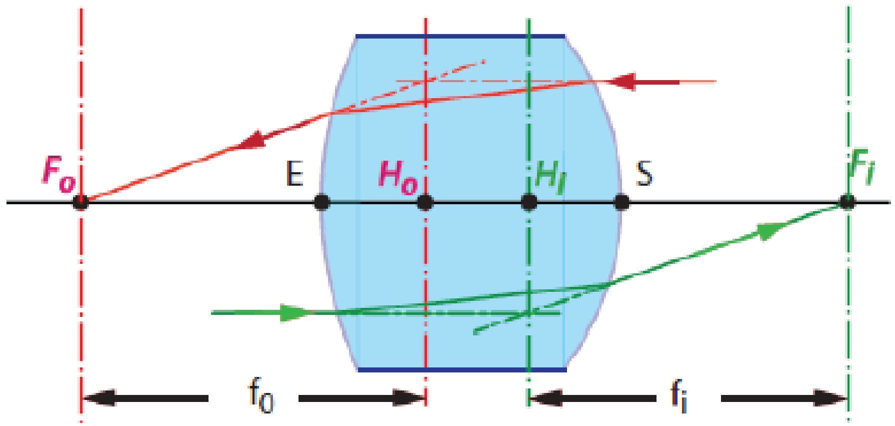
1) Two principal points: P_1 and P_2
which are the intersection points of the principal planes with the optical axis.

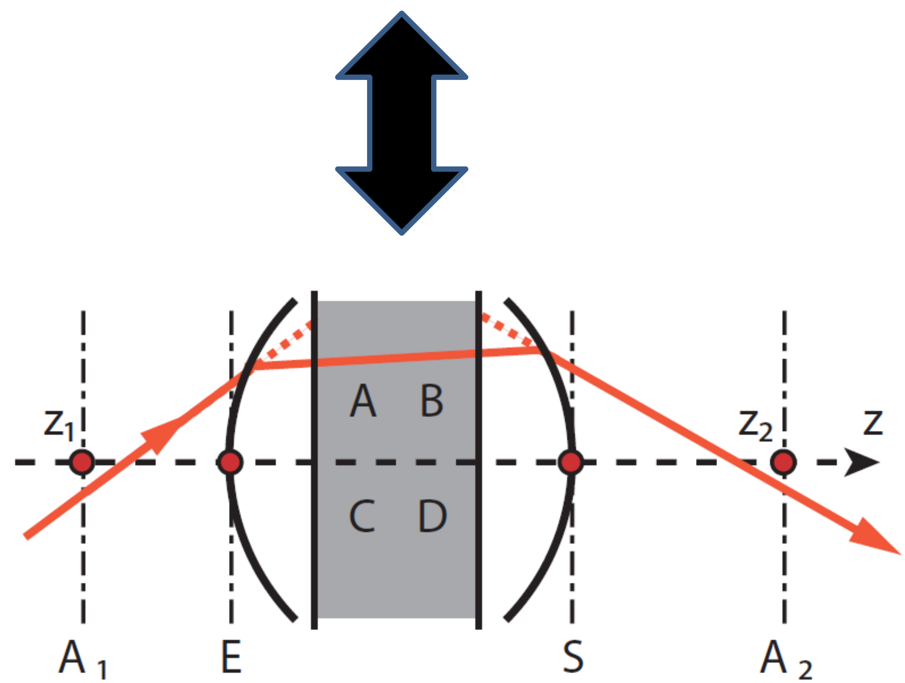
2) Two focal points: F_1 and F_2
which are the intersection points of the focal planes with the optical axis.





The thick lens between planes E & S is represented with [ABCD] matrix.



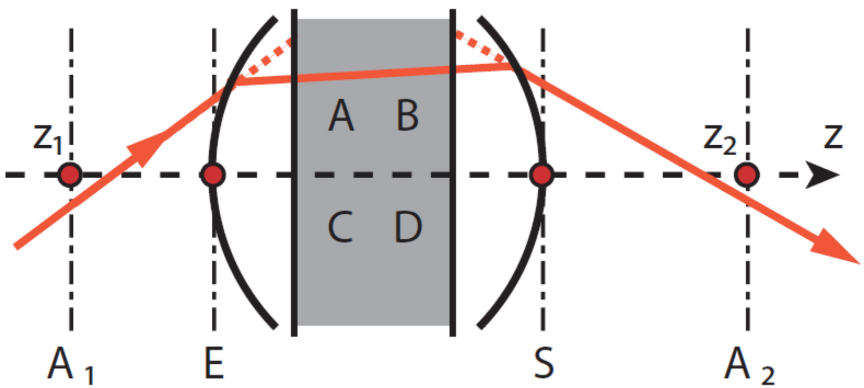


Generalization… beyond the thick lens case

Cardinal planes (two focal planes & two principal planes) are helpful concepts for ABCD analysis
➔ **They are not limited to thick lens and can be used <u>for any optical system in general</u>.**

To generalize, one needs to first realize that a system between the planes E & S is described with [ABCD].

<u>Next</u>, for the imaging condition, we need to find transfer from plane A_1 to plane A_2 by considering propagations and [ABCD] of the system between planes E & S, as it will be analyzed in the next slides.



$$z_1 = \overline{EA_1}$$

$$z_2 = \overline{SA_2}$$

<u>Some useful rules</u>

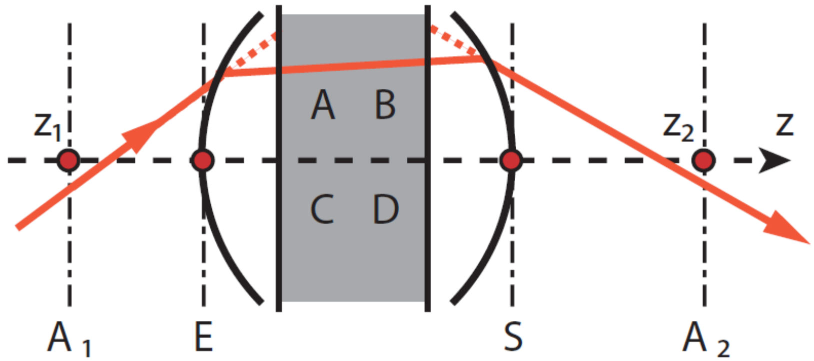
1. Light is traveling from left to right
2. Directed distance
   + if measured from reference plane to destination plane along propagation direction
   - if measured from reference plane to destination plane against propagation direction
3. Radius of curvature
   + if center of curvature after surface
   - if center of curvature before surface

Here, plane E is the origin (reference)

Generalized analysis with [ABCD] matrix & principal planes

We use the following three steps to transfer from plane A_1 to plane A_2 :

1) Transfer from $A_1 \rightarrow E$ by *propagation* over $z_1 = T(A_1E)$
2) Transfer from $E \rightarrow S$ by [ABCD] optical system = T(ES)
3) Transfer from $S \rightarrow A_2$ by *propagation* over $z_2 = T(SA_2)$



$z_1 = \overline{EA_1}$ & $z_2 = \overline{SA_2}$

$$T_{A_1E} = \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

$$T_{ES} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$T_{SA_2} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix}$$

$$T(A_{1\rightarrow}A_2) = T(SA_2) \times T(ES) \times T(A_1E)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

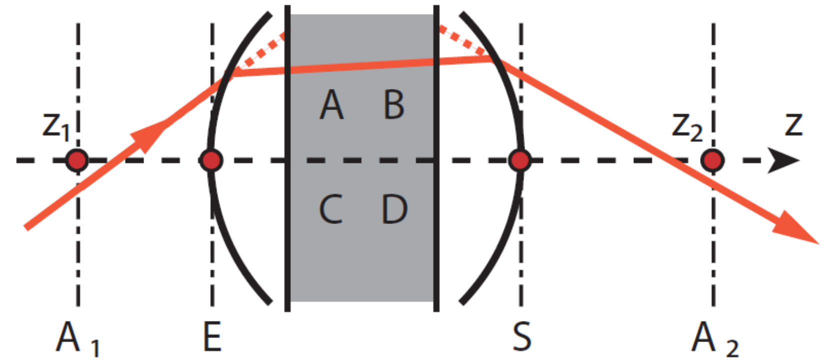
$$= \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B - z_1A \\ C & D - z_1C \end{bmatrix}$$

$$= \begin{bmatrix} A + Cz_2 & -Az_1 + B + z_2(-Cz_1 + D) \\ C & D - Cz_1 \end{bmatrix}$$

Generalized analysis with [ABCD] matrix & principal planes

We use the following three steps to transfer from plane A_1 to plane A_2 :

1) Transfer from A_1→E by *propagation* over z_1 = T(A_1E)
2) Transfer from E→S by [ABCD] optical system = T(ES)
3) Transfer from S→ A_2 by *propagation* over z_2 = T(SA_2)



$$T(A_{1\to}A_2) = T(SA_2) \times T(ES) \times T(A_1E)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

$$T_{11} = A + Cz_2$$

$$T_{12} = -Az_1 + B + z_2(-Cz_1 + D)$$

$$T_{21} = C$$

$$T_{22} = D - Cz_1$$

1st observation:

T_{21} is independent of A_1 and A_2

Therefore, it is only a system property

Vergence $\mathbf{V} = -\boldsymbol{C}$

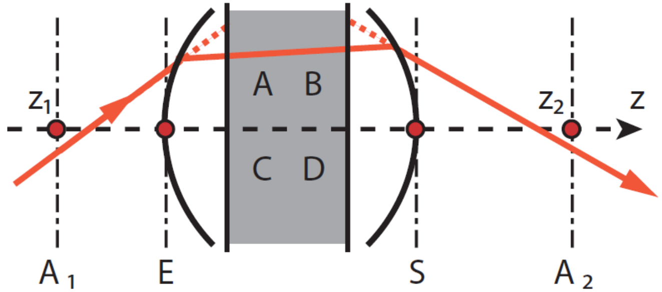
if V > 0 convergence or (+) system

if V < 0 divergence or (-) system

if V = 0 afocal system

Consider Imaging (Conjugation) Case:

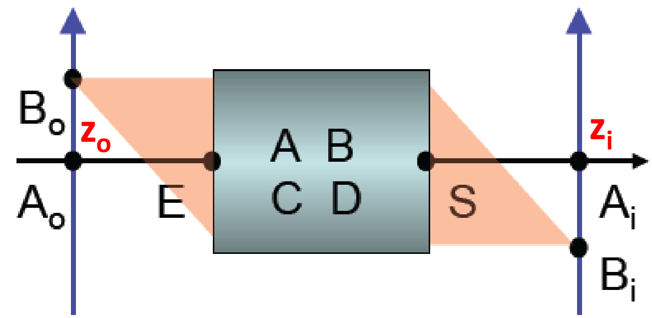
Transfer from plane A_1 to plane A_2



$$T(A_oA_i) = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

$$= \begin{bmatrix} A - Vz_i & (-Az_o + B) + z_i(Vz_o + D) \\ -V & D + Vz_o \end{bmatrix}$$

Now, lets consider its application to an imaging case:



$$z_o = \overline{EA_o}$$
$$z_i = \overline{SA_i}$$

- The corresponding notation in the above figure is as follows:

  $z_1 = z_o$

  $z_2 = z_i$

  $A_1 = (A_o, B_o, \ldots)$

  $A_2 = (A_i, B_i, \ldots.)$

- It maps **o**bject points (A_o,B_o...) to **i**mage points (A_i,B_i...)

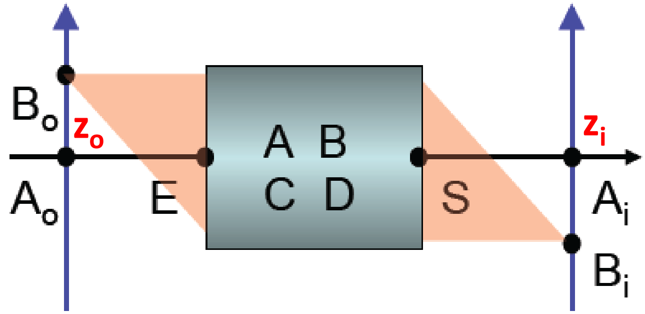
- If we define ray vectors as $\vec{X}_i$ & $\vec{X}_o$, we can write that:

$$\vec{X}_i = T(A_oA_i).\vec{X}_o$$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ -V & T_{22} \end{bmatrix} \begin{bmatrix} y_o \\ \theta_o \end{bmatrix}$$

$$= \begin{bmatrix} T_{11}y_o + T_{12}\theta_o \\ -Vy_o + T_{22}\theta_o \end{bmatrix}$$

Consider Imaging (Conjugation) Case:



- If we define ray vectors as $\vec{X}_i$ & $\vec{X}_o$, we can write that:

$$\vec{X}_i = \mathrm{T}(A_o A_i).\vec{X}_o$$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} y_o + T_{12}\theta_o \\ -V y_o + T_{22}\theta_o \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ -V & T_{22} \end{bmatrix} \begin{bmatrix} y_o \\ \theta_o \end{bmatrix}$$

Imaging requires that:

$T_{12} = 0$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} y_o \\ -V y_o + T_{22}\theta_o \end{bmatrix}$$

$T_{11} = M_t = y_i / y_o$

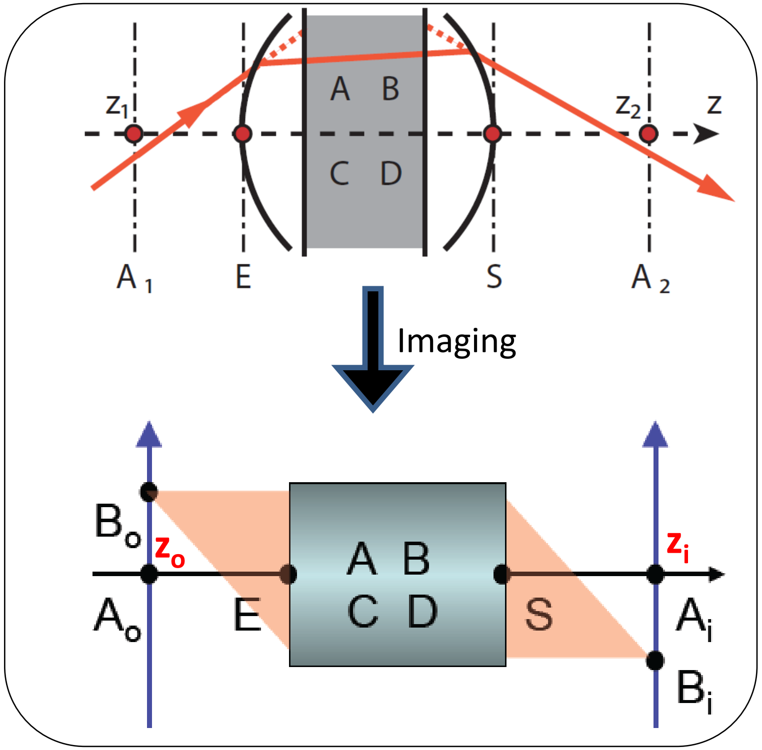
It corresponds to lateral magnification

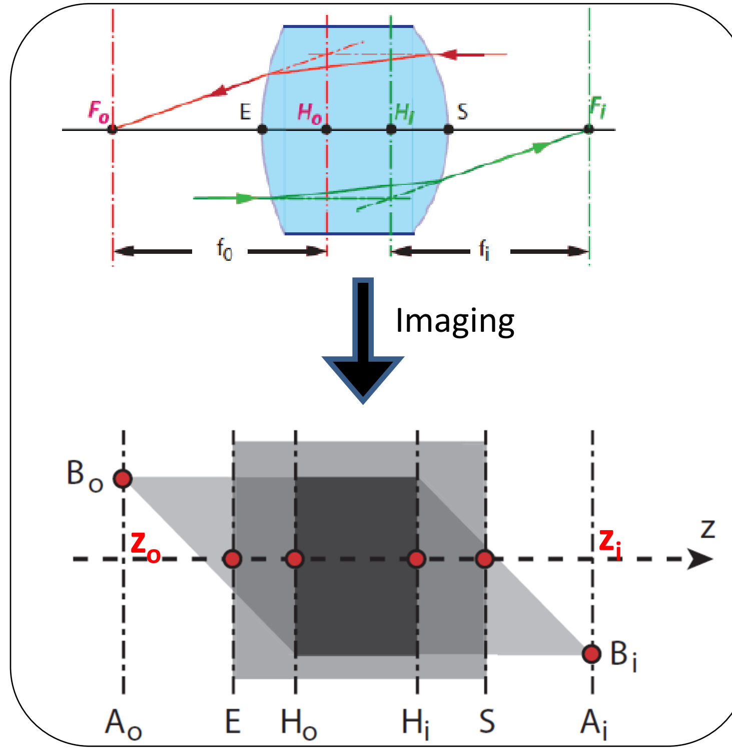
$T_{22} = M_\alpha = \theta_i / \theta_o$

It corresponds to angular magnification

$$\mathrm{T}(A_o A_i) = \begin{bmatrix} M_t & 0 \\ -V & M_\alpha \end{bmatrix}$$

Conjugated System: Connection to the **Principal Planes** (H_o & H_i)





The two principal planes (H_o and H_i) are conjugated:
➔Object-image relation holds between the planes H_o and H_i with the condition that M_t=M_α=1

➔ When we apply $\mathrm{T}(A_oA_i)$ for $\mathrm{H}(H_oH_i)$, then we get:

Recall:

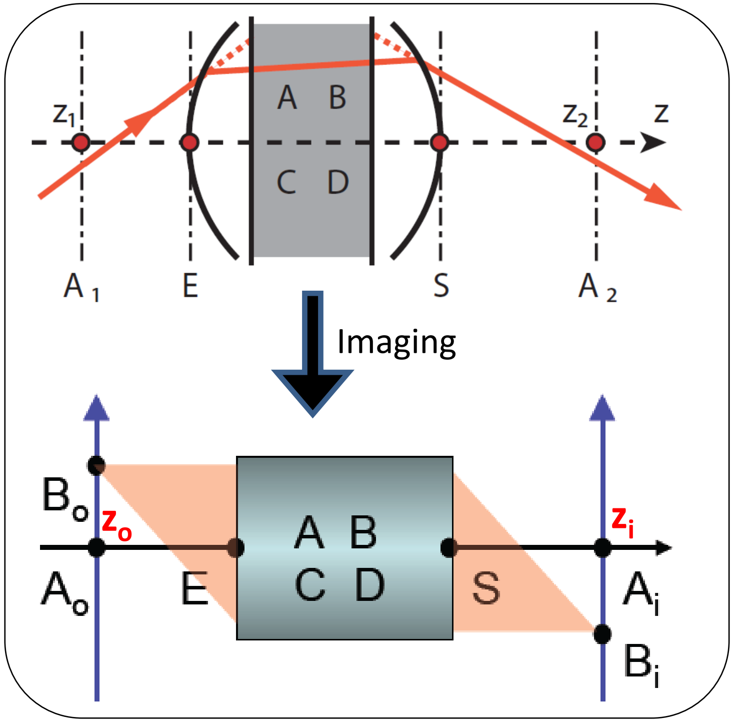
$$\mathrm{T}(A_oA_i) = \begin{bmatrix} M_t & 0 \\ -V & M_\alpha \end{bmatrix}$$

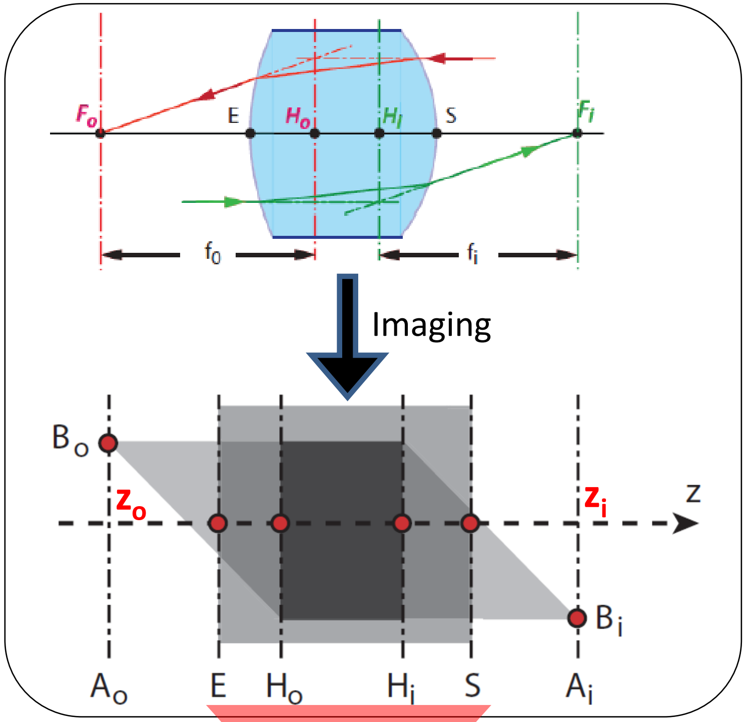
In this case, M = 1 ➔

Transfer matrix H between the principal planes H_o and H_i as:

$$H(H_oH_i) = \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix}$$

Conjugated System: Connection to the **Principal Planes** (H_o & H_i)





$$C = -V$$

For the region between planes E & S, the operation is represented by [ABCD]:

$$M_1 = M_{EH_o} = \begin{bmatrix} 1 & \overline{EH_o} \\ 0 & 1 \end{bmatrix}$$

$$M_2 = M_{H_oH_i} = \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix}$$

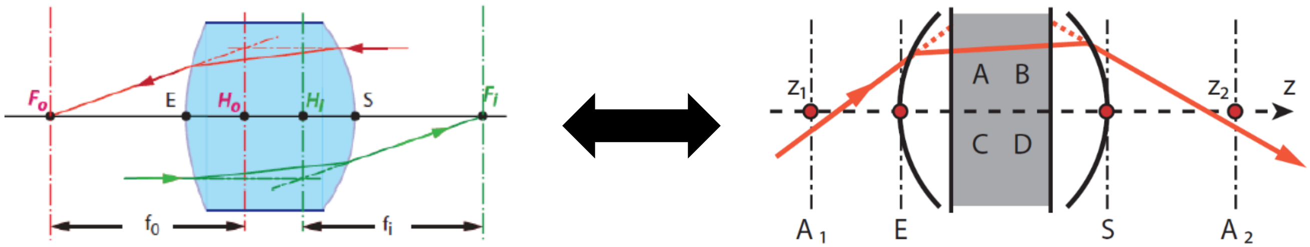
$$M_3 = M_{H_iS} = \begin{bmatrix} 1 & \overline{H_iS} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ -V & D \end{bmatrix} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1 & \overline{H_iS} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix} \times \begin{bmatrix} 1 & \overline{EH_o} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - V\overline{H_iS} & \overline{H_iS} + \overline{EH_o} - V\overline{EH_o}\,\overline{H_iS} \\ -V & 1 - V\overline{EH_o} \end{bmatrix}$$

Conjugated System: Object Principal Plane & Object Focal Plane



For the object side (indicated by the red line), the relevant components are H_o, F_o & f_o

For finding the location of object principle plane (H_o):

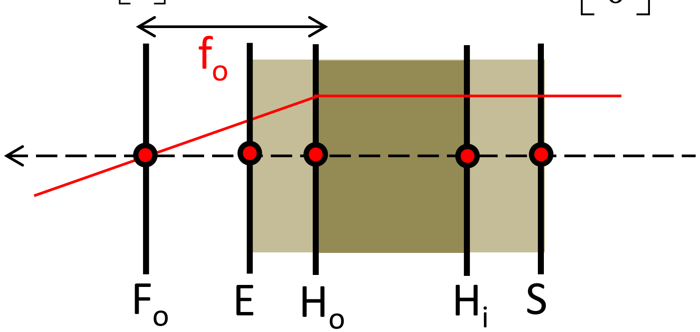
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1-V\overline{H_iS} & \overline{H_iS}+\overline{EH_o}-V\overline{EH_o}\,\overline{H_iS} \\ -V & 1-V\overline{EH_o} \end{bmatrix}$$

$$D = 1 - V\overline{EH_o} \Rightarrow \overline{EH_o} = \frac{1}{V}(1-D)$$

Object principle plane: $H_o = \overline{EH_o} = \frac{1}{C}(D-1)$

For finding the location of object focal plane (F_o) & object focal length (f_o):

$$X_{in} = \begin{bmatrix} 0 \\ \theta \end{bmatrix} \qquad X_{out} = \begin{bmatrix} y_0 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} y_o \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & \overline{F_oE} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ \theta \end{bmatrix}$$

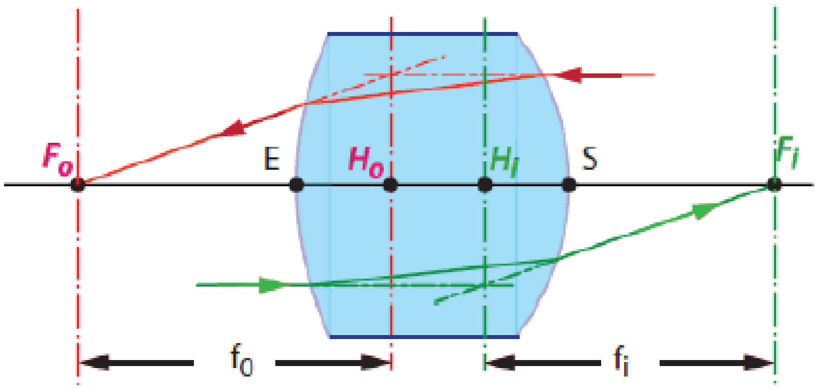
$$\begin{bmatrix} y_o \\ 0 \end{bmatrix} = \begin{bmatrix} \theta(B + \overline{F_oE}A) \\ \theta(D + \overline{F_oE}C) \end{bmatrix}$$

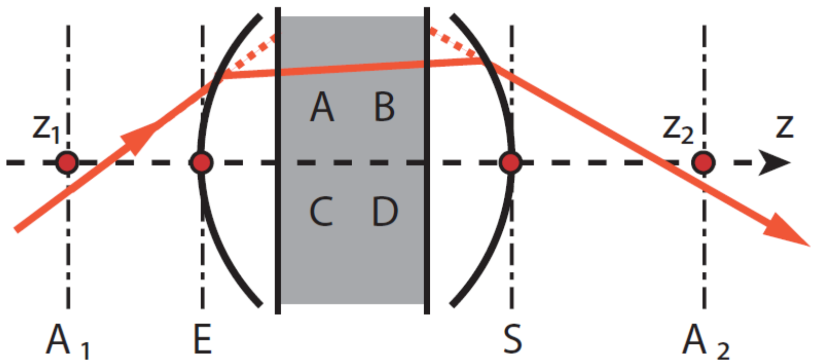
This can hold only if: $D + C\overline{F_oE} = 0$

Object focal point: $F_o = \overline{EF_o} = D/C$

Object focal length: $f_o = \overline{H_oF_o} = \overline{H_oE} + \overline{EF_o} = -\frac{1}{C}(D-1) + \frac{D}{C} = +\frac{1}{C}$

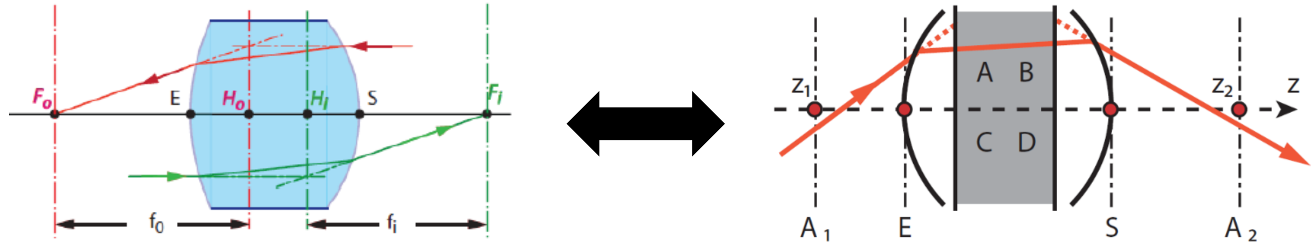
Summary: Cardinal Points & Planes for Object





Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$

Conjugated System: Image Principal Plane & Image Focal Plane



For the image side (indicated by the green line), the relevant terms are H_i, F_i & f_i

For finding the location of image principle plane (H_i):

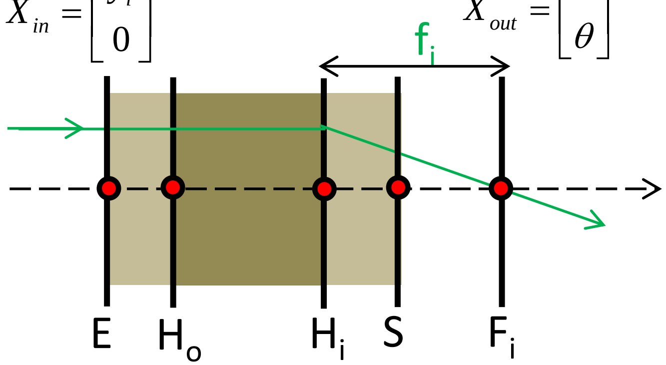
$$A = 1 - V\overline{H_iS} \Rightarrow \overline{H_iS} = \frac{1}{V}(1-A)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - V\overline{H_iS} & \overline{H_iS} + \overline{EH_o} - V\overline{EH_o}\,\overline{H_iS} \\ -V & 1 - V\overline{EH_o} \end{bmatrix}$$

Image principle plane: $H_i = \overline{SH_i} = \frac{1}{C}(1-A)$

For finding the location of image focal plane (F_i) & image focal length (f_i):

$$X_{in} = \begin{bmatrix} y_i \\ 0 \end{bmatrix} \qquad X_{out} = \begin{bmatrix} 0 \\ \theta \end{bmatrix}$$



$$\begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & \overline{SF_i} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} y_i \\ 0 \end{bmatrix}$$

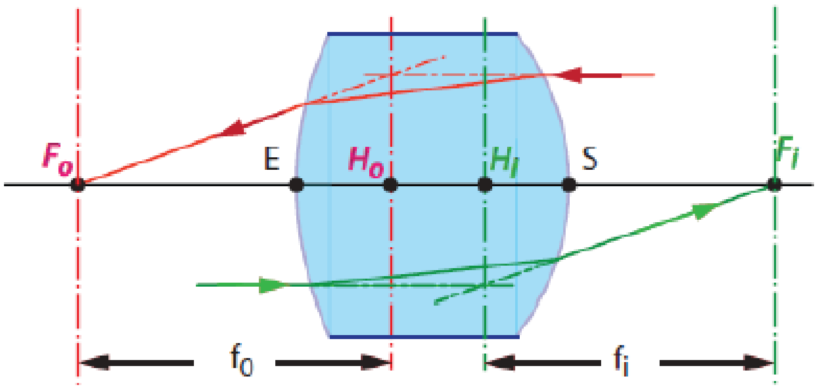
$$\begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} (A + C\overline{SF_i})y_i \\ Cy_i \end{bmatrix}$$

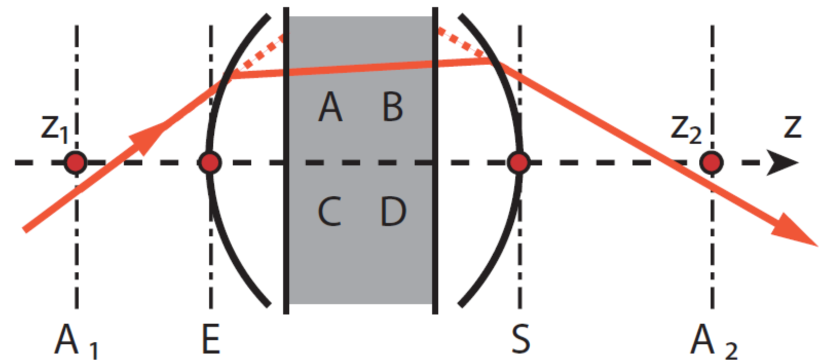
This can hold only: $A + C\overline{SF_i} = 0$

Image focal point: $F_i = \overline{SF_i} = -A/C$

Image focal length: $f_i = \overline{H_iF_i} = \overline{H_iS} + \overline{SF_i} = -\frac{1}{C}(1-A) - \frac{A}{C} = -\frac{1}{C}$

Summary: Cardinal Points & Planes for Image

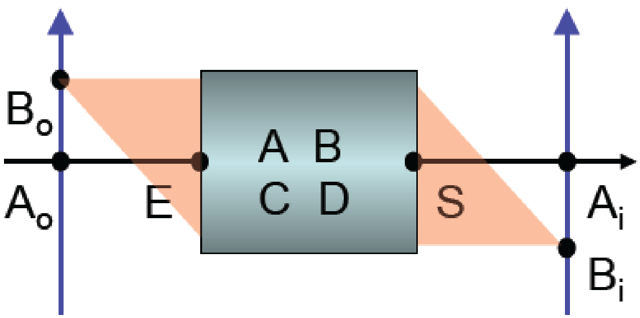


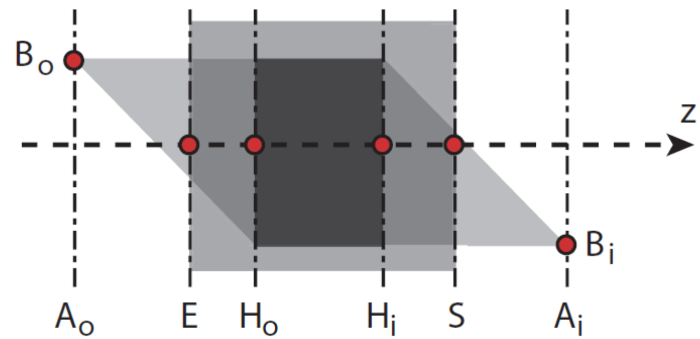


Distances	Notation	Directed Distances	ABCD elements
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

The analysis (and the table) is also valid for **generalized** optical systems

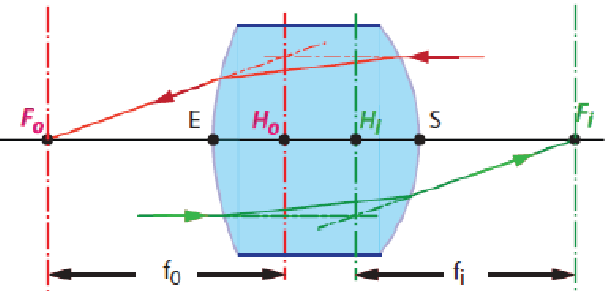
Cardinal points/planes can be used to find the image of the complex (i.e. cascaded) optical systems and the corresponding rays ➔ This is an alternative to ray tracing method.





Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

Example: Cardinal Points & Planes of a Thick lens

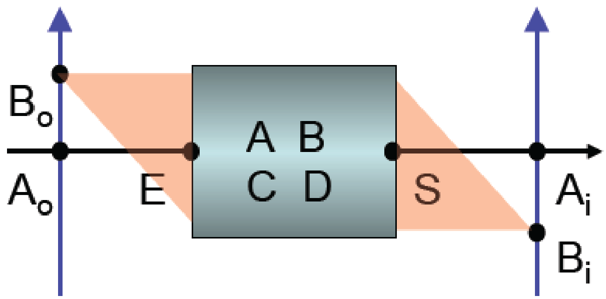


Lens thickness = $\overline{ES} = e$

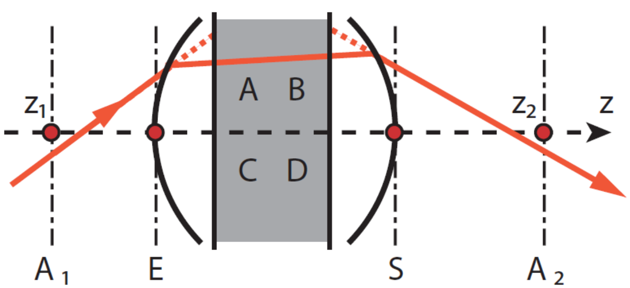
Lens index = n Outside index =1

Opt. Power of 1st Surface $\Phi_1 = (n-1)/nR_1$

Opt. Power of 2nd Surface $\Phi_2 = (1-n)/R_2$

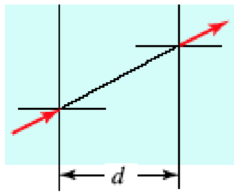


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = ?$$



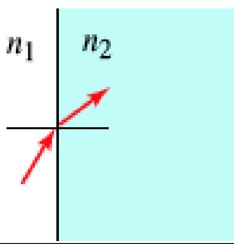
Remember: Matrix optics description of basic functions & components

Propagation



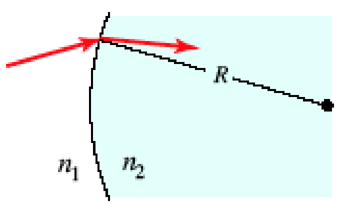
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

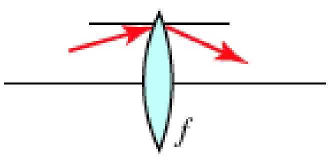
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

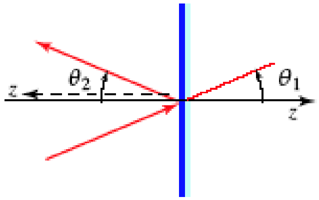
Lens



Convex, $f > 0$; concave, $f < 0$

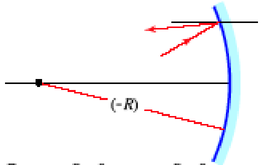
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Planar mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

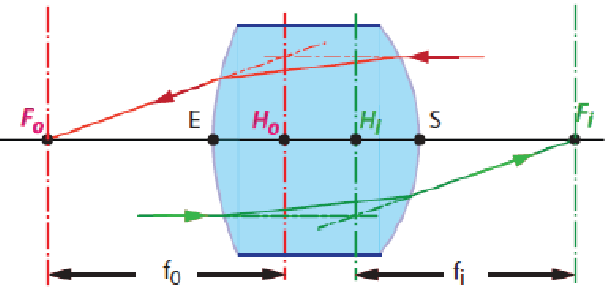
Spherical mirror



Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Example: Cardinal Points & Planes of a Thick lens

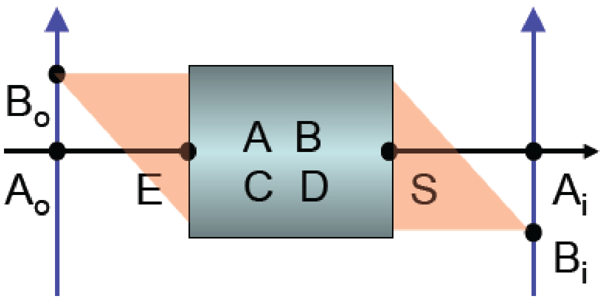


Lens thickness = $\overline{ES} = e$

Lens index = n Outside index =1

Opt. Power of 1st Surface $\Phi_1 = (n-1)/nR_1$

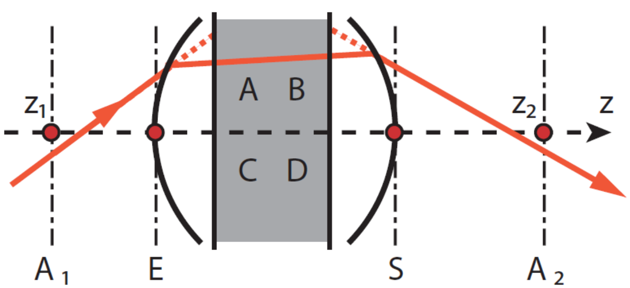
Opt. Power of 2nd Surface $\Phi_2 = (1-n)/R_2$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} =$$

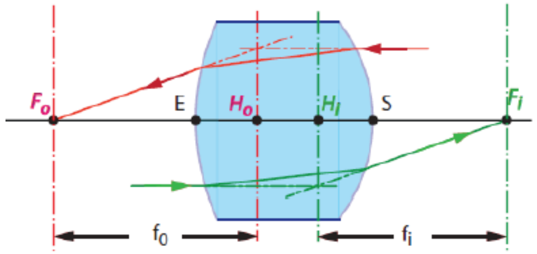
$$= \begin{bmatrix} 1 & 0 \\ -\Phi_2 & n \end{bmatrix} \times \begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\Phi_1 & 1/n \end{bmatrix}$$

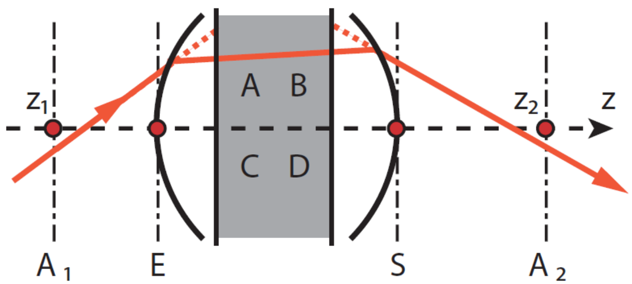
$$= \begin{bmatrix} 1 - e\Phi_1 & e/n \\ -\Phi_{tot} & 1 - e\Phi_2/n \end{bmatrix}$$



$$\Phi_{tot} = n\Phi_1 + \Phi_2 - e\Phi_1\Phi_2$$
$$= (\mathrm{n}-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{n-1}{n}\frac{e}{R_1 R_2}\right)$$

Example: Cardinal Points & Planes of a Thick lens





Distances:	Notation:	Directed Distances:	ABCD elements:
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

$$A = 1 - e\Phi_1$$

$$B = e/n$$

$$C = -\Phi_{tot}$$

$$D = 1 - e\Phi_2/n$$

Object Focal Point Position

$$F_o = \overline{EF_o} = \frac{D}{C} = \frac{\frac{e\Phi_2}{n} - 1}{\Phi_{tot}}$$

Image Focal Point Position

$$F_i = \overline{SF_i} = \frac{-A}{C} = \frac{e\Phi_1 - 1}{\Phi_{tot}}$$

Object Principle Plane Position

$$H_o = \overline{EH_o} = \frac{1}{C}(D-1) = \frac{1}{-\Phi_{tot}}\left(1 - \frac{e\Phi_2}{n} - 1\right) = \frac{e\Phi_1}{n\Phi_{tot}}$$

Object Focal & Image Focal Lengths

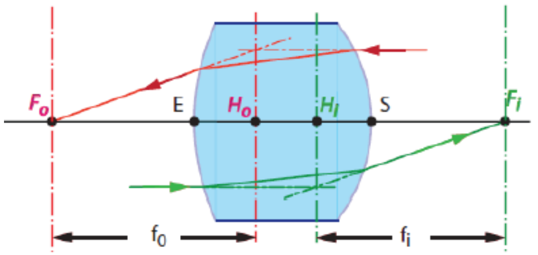
$$f_i = -f_o = -\frac{1}{C} = \frac{1}{\Phi_{tot}}$$

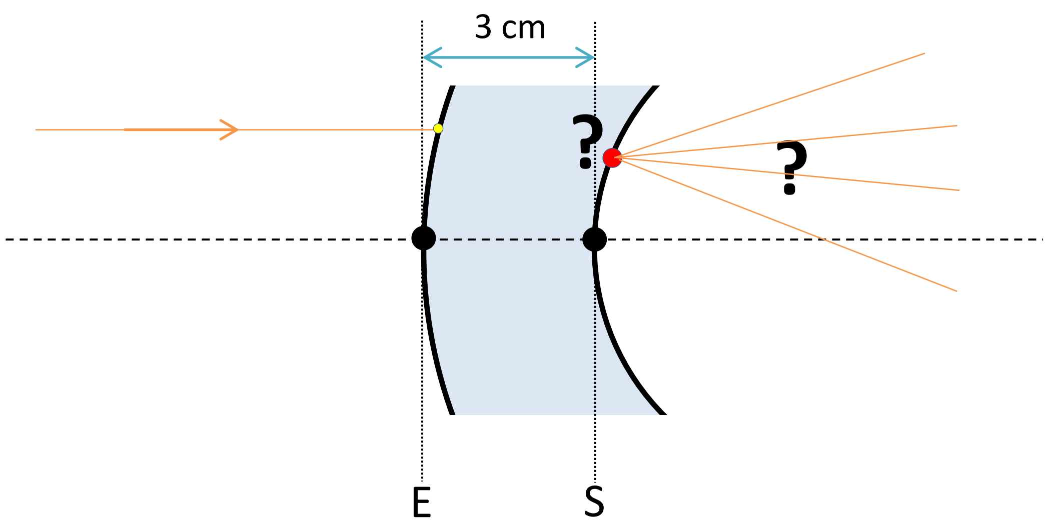
Image Principle Plane Position

$$H_i = \overline{SH_i} = \frac{1}{C}(1-A) = \frac{1}{-\Phi_{tot}}(1 - 1 + e\Phi_1) = -\frac{e\Phi_1}{\Phi_{tot}}$$

Example: if we use ray tracing for a thick lens

Lens parameters	Num. value
1^{st} interface curvature (R_1)	+5 cm
2^{nd} interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5





Can you determine the path of the ray after the thick lens?

This means, identifying:

- The point that the ray exits the lens?
- The exit angle?

➔ **Instead of ray optics, lets use the approach of cardinal points and planes!**

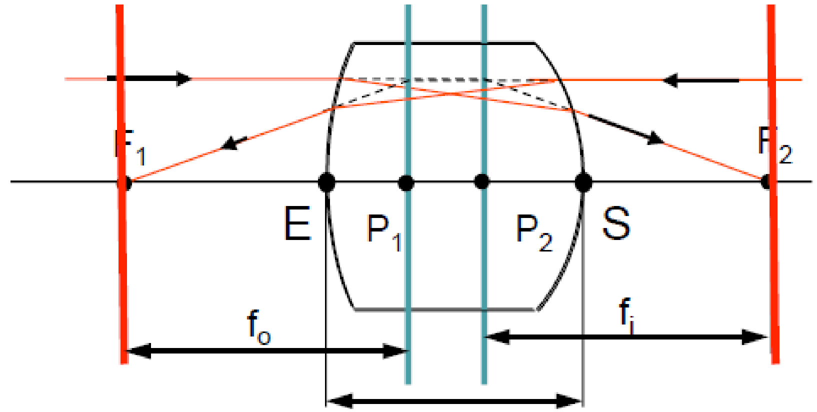
Example: if we use cardinal planes for a thick lens

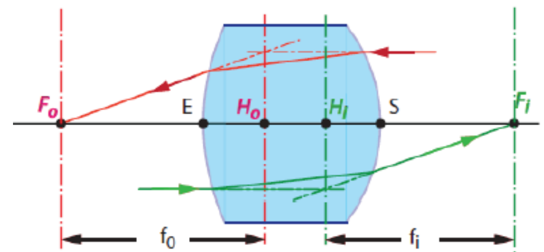
Lens thickness = $\overline{ES} = e$

Lens index = n   Outside index =1

Optical Power of 1st curvature $\Phi_1 = (n-1)/nR_1$

Optical Power of 2nd curvature $\Phi_2 = (1-n)/R_2$





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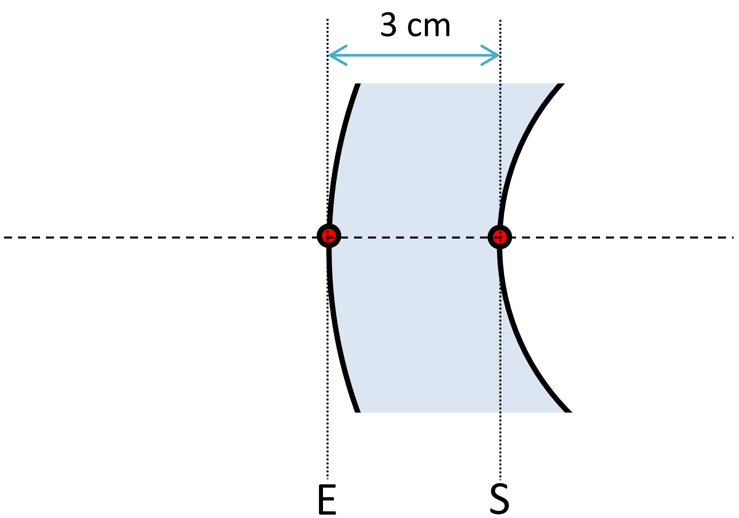
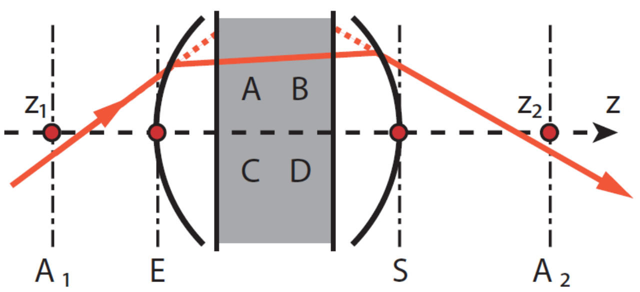
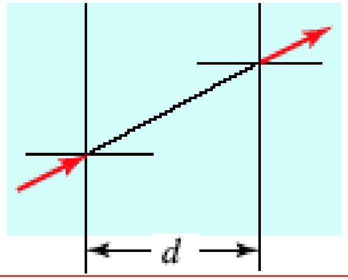




$$T(E \to S) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = T_{2nd\ spherical\ curvature} \times T_{propagation} \times T_{1st\ spherical\ curvature}$$

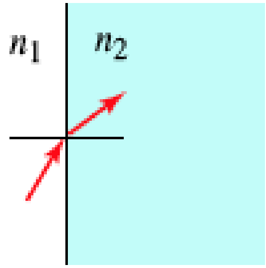
Reminder: Simple Optical Components

Propagation



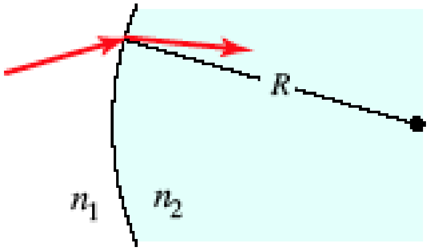
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

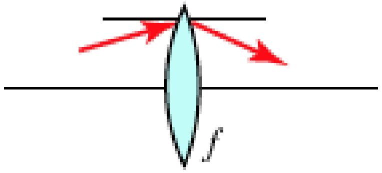
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

Thin Lens



Convex, $f > 0$; concave, $f < 0$

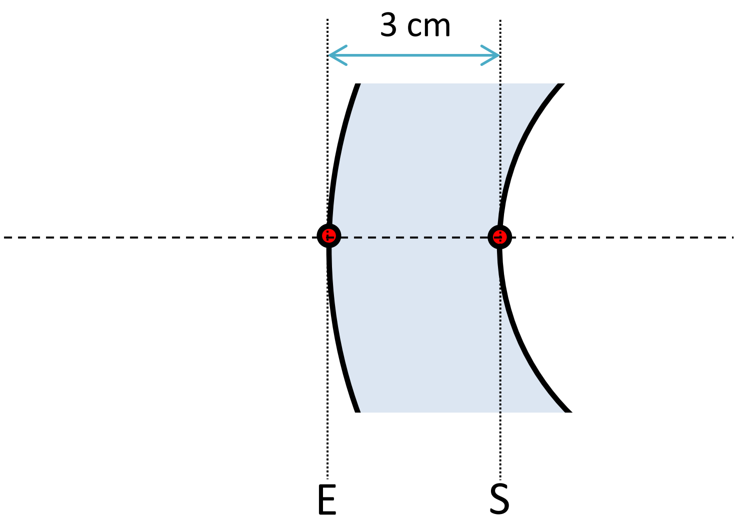
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

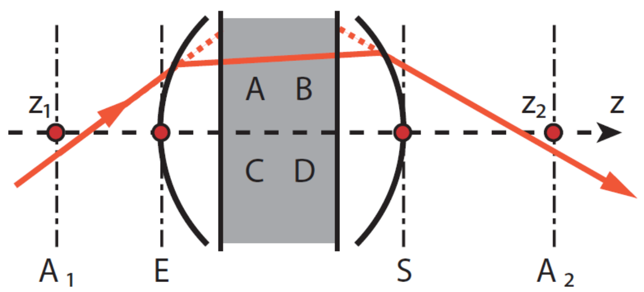
Example: Ray Tracing Thick lens

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$\Phi_1 = (n-1)/nR_1 = (1.5-1)/1.5x5 = 2/30$$

$$\Phi_2 = (1-n)/R_2 = (1-1.5)/2 = -1/4$$





$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = T_{2nd\ spherical\ curvature} \times T_{propagation} \times T_{1st\ spherical\ curvature}$$

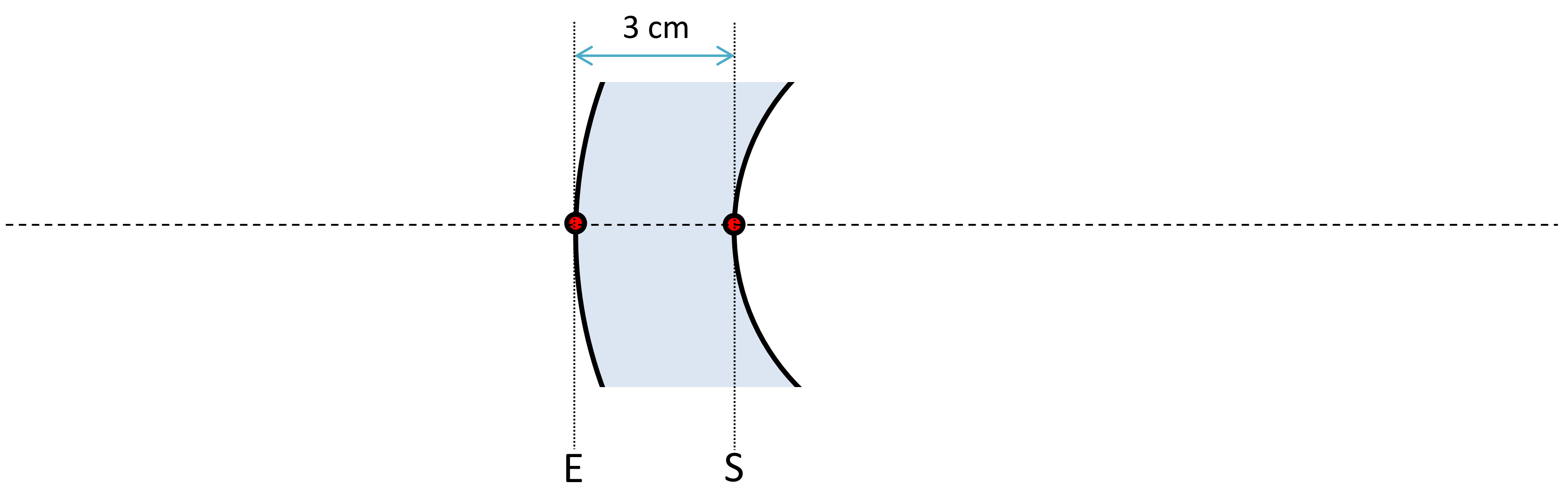
$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\Phi_2 & n \end{bmatrix} \times \begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\Phi_1 & 1/n \end{bmatrix}$$

Example: Thick lens

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$



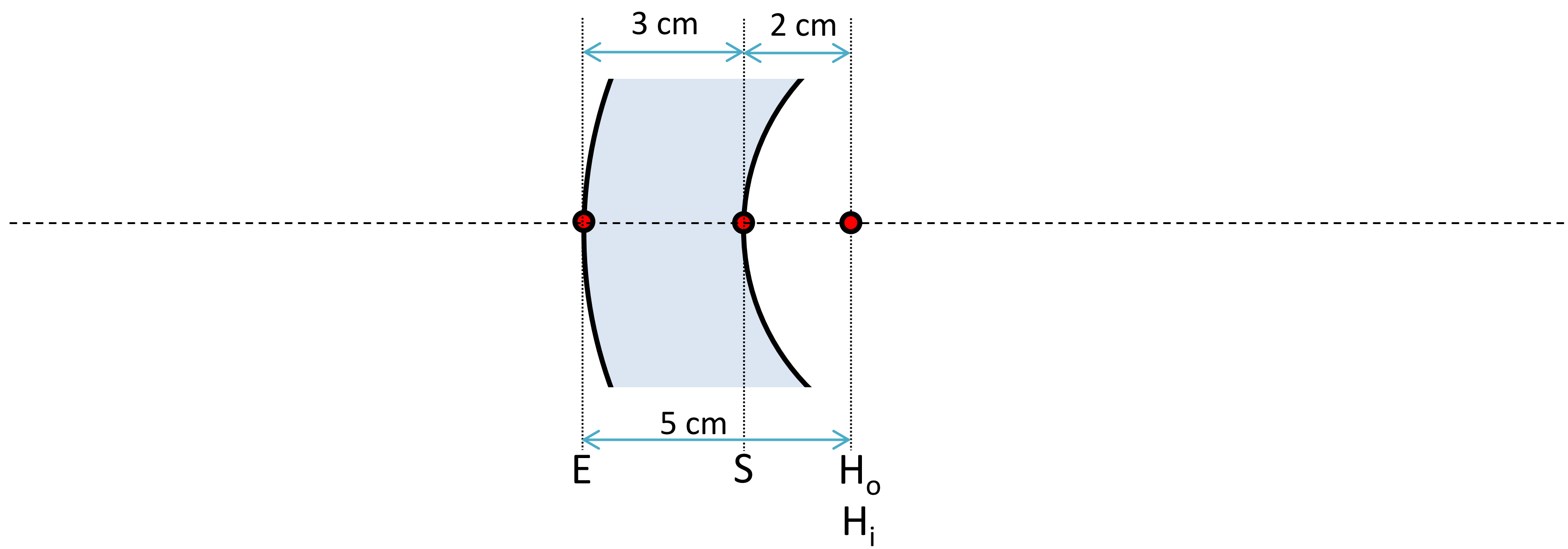
Example: Thick lens → Position H_o & H_i

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	F_o	$\overline{EF_o}$	D/C	+15 cm
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	$\boldsymbol{H_o}$	$\overline{\boldsymbol{EH_o}}$	$\boldsymbol{(D-1)/C}$	**+5 cm**
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$	-8 cm
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	$\boldsymbol{H_i}$	$\overline{\boldsymbol{SH_i}}$	$\boldsymbol{(1-A)/C}$	**+2 cm**

Position H_o & H_i:



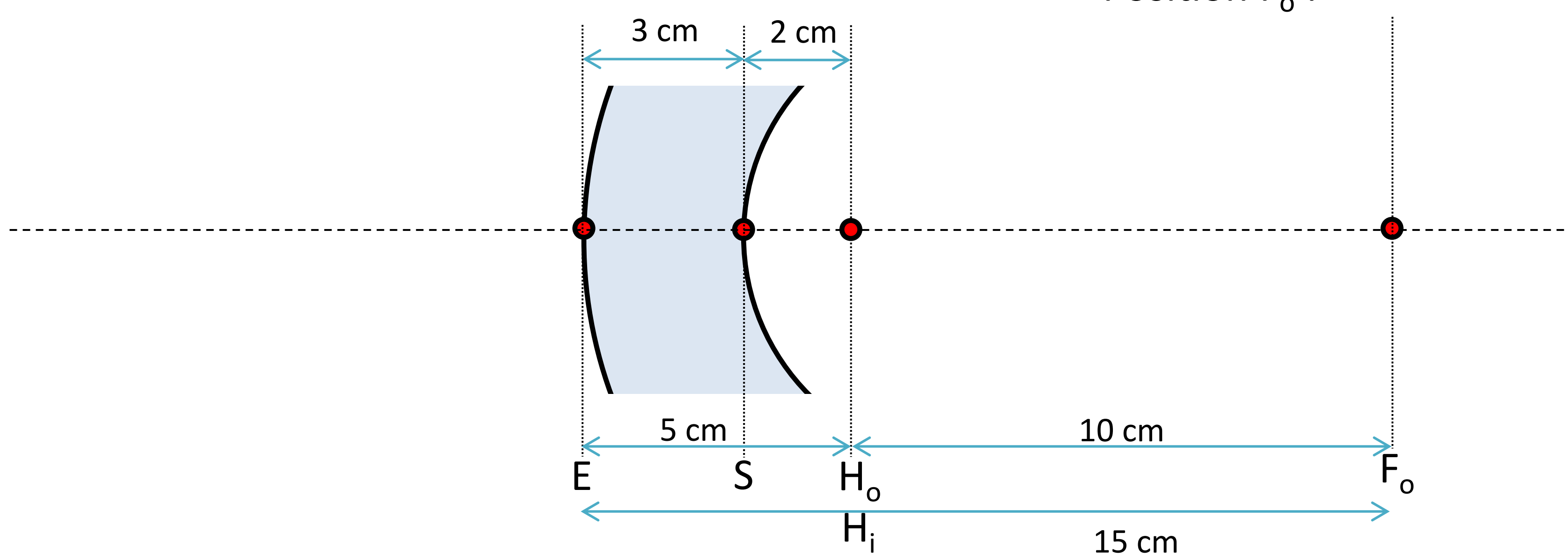
Example: Thick lens ➔ Position F_o

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	$\boldsymbol{F_o}$	$\overline{\boldsymbol{EF_o}}$	$\boldsymbol{D/C}$	**+15 cm**
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	$\boldsymbol{H_o}$	$\overline{\boldsymbol{EH_o}}$	$\boldsymbol{(D-1)/C}$	**+5 cm**
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$	-8 cm
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	$\boldsymbol{H_i}$	$\overline{\boldsymbol{SH_i}}$	$\boldsymbol{(1-A)/C}$	**+2 cm**

Position F_o :



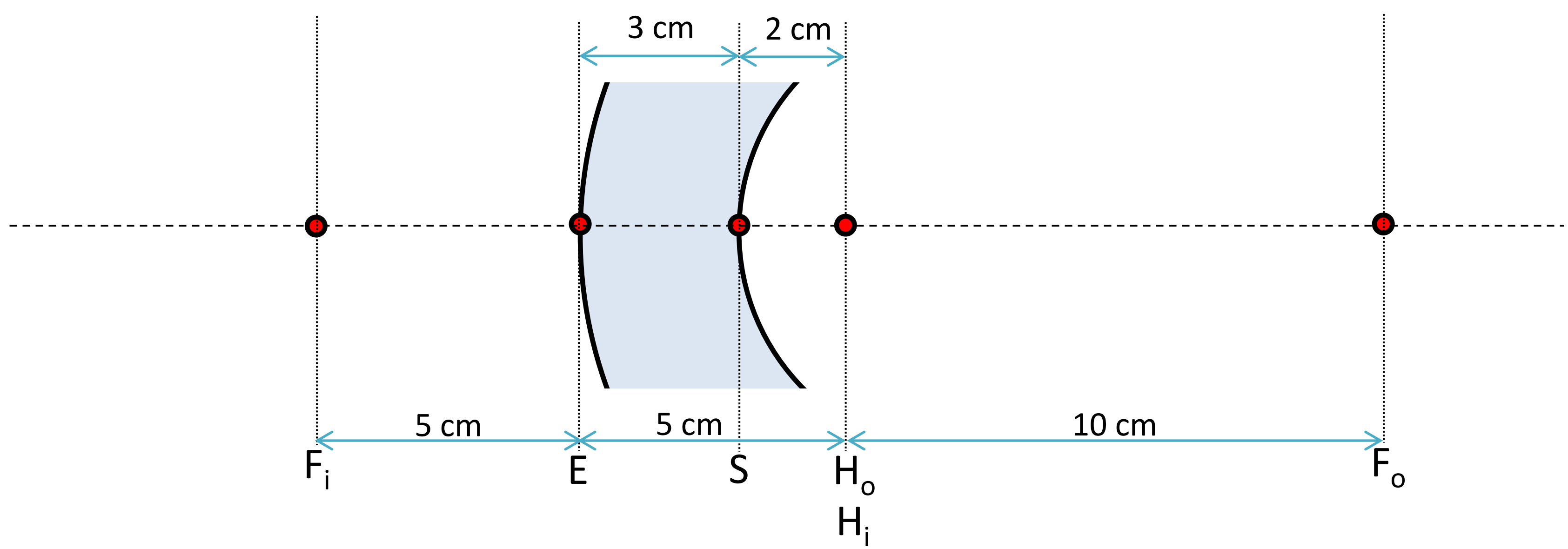
Example: Thick lens → Position F_i

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	$\boldsymbol{F_o}$	$\overline{\boldsymbol{EF_o}}$	$\boldsymbol{D/C}$	**+15 cm**
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	$\boldsymbol{H_o}$	$\overline{\boldsymbol{EH_o}}$	$\boldsymbol{(D-1)/C}$	**+5 cm**
Image Focal Point	$\boldsymbol{F_i}$	$\overline{\boldsymbol{SF_i}}$	$\boldsymbol{-A/C}$	**-8 cm**
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	$\boldsymbol{H_i}$	$\overline{\boldsymbol{SH_i}}$	$\boldsymbol{(1-A)/C}$	**+2 cm**

Position F_i :



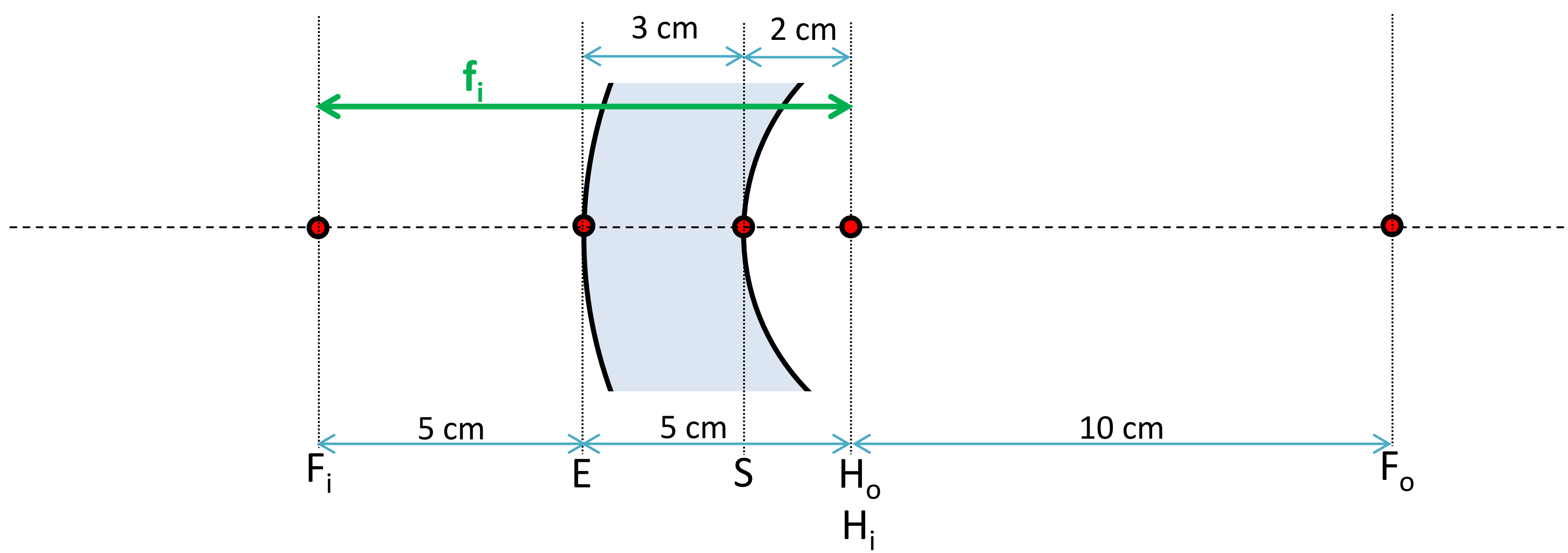
Example: Thick lens → Image Focal Length f_i

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	$\boldsymbol{F_o}$	$\overline{\boldsymbol{EF_o}}$	$\boldsymbol{D/C}$	**+15 cm**
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	$\boldsymbol{H_o}$	$\overline{\boldsymbol{EH_o}}$	$\boldsymbol{(D-1)/C}$	**+5 cm**
Image Focal Point	$\boldsymbol{F_i}$	$\overline{\boldsymbol{SF_i}}$	$\boldsymbol{-A/C}$	**-8 cm**
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	$\boldsymbol{H_i}$	$\overline{\boldsymbol{SH_i}}$	$\boldsymbol{(1-A)/C}$	**+2 cm**

Image Focal Length, f_i



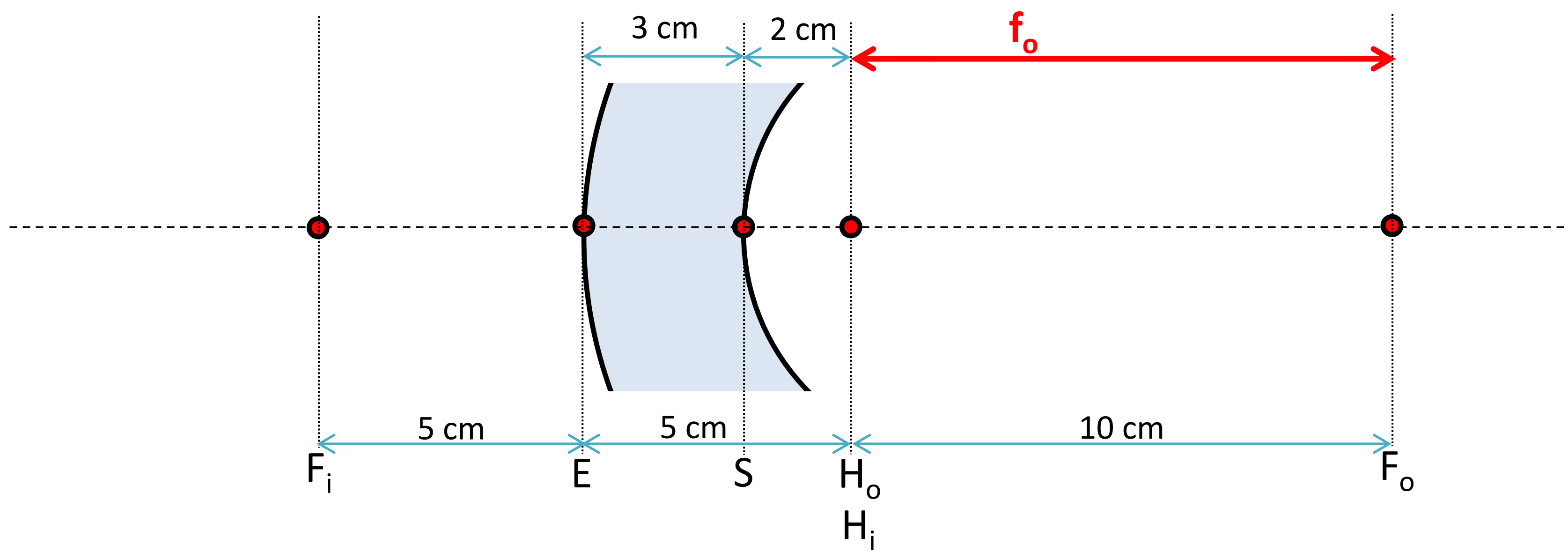
Example: Thick lens ➔ Object Focal Length f_o

Lens parameters	Num. value
1^{st} interface curvature (R_1)	+5 cm
2^{nd} interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

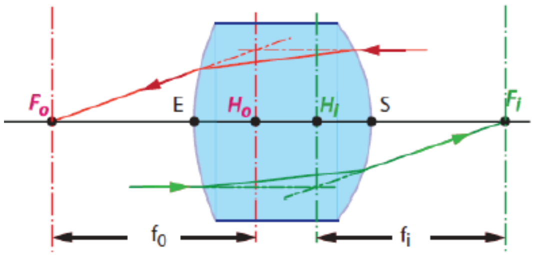
Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	$\boldsymbol{F_o}$	$\overline{\boldsymbol{EF_o}}$	$\boldsymbol{D/C}$	**+15 cm**
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	$\boldsymbol{H_o}$	$\overline{\boldsymbol{EH_o}}$	$\boldsymbol{(D-1)/C}$	**+5 cm**
Image Focal Point	$\boldsymbol{F_i}$	$\overline{\boldsymbol{SF_i}}$	$\boldsymbol{-A/C}$	**-8 cm**
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	$\boldsymbol{H_i}$	$\overline{\boldsymbol{SH_i}}$	$\boldsymbol{(1-A)/C}$	**+2 cm**

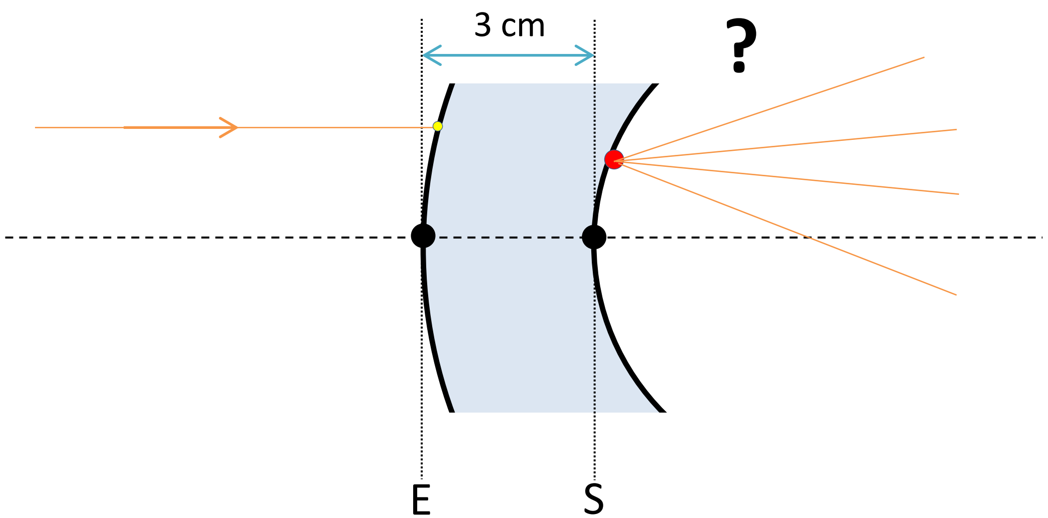
Object Focal Length, f_o



Remember the question:

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5



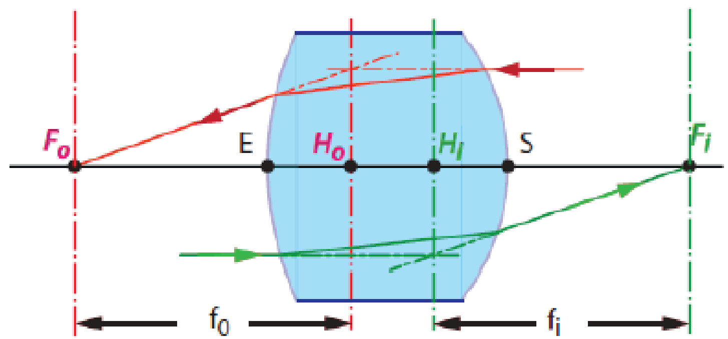


Can you determine the path of the ray after the thick lens?

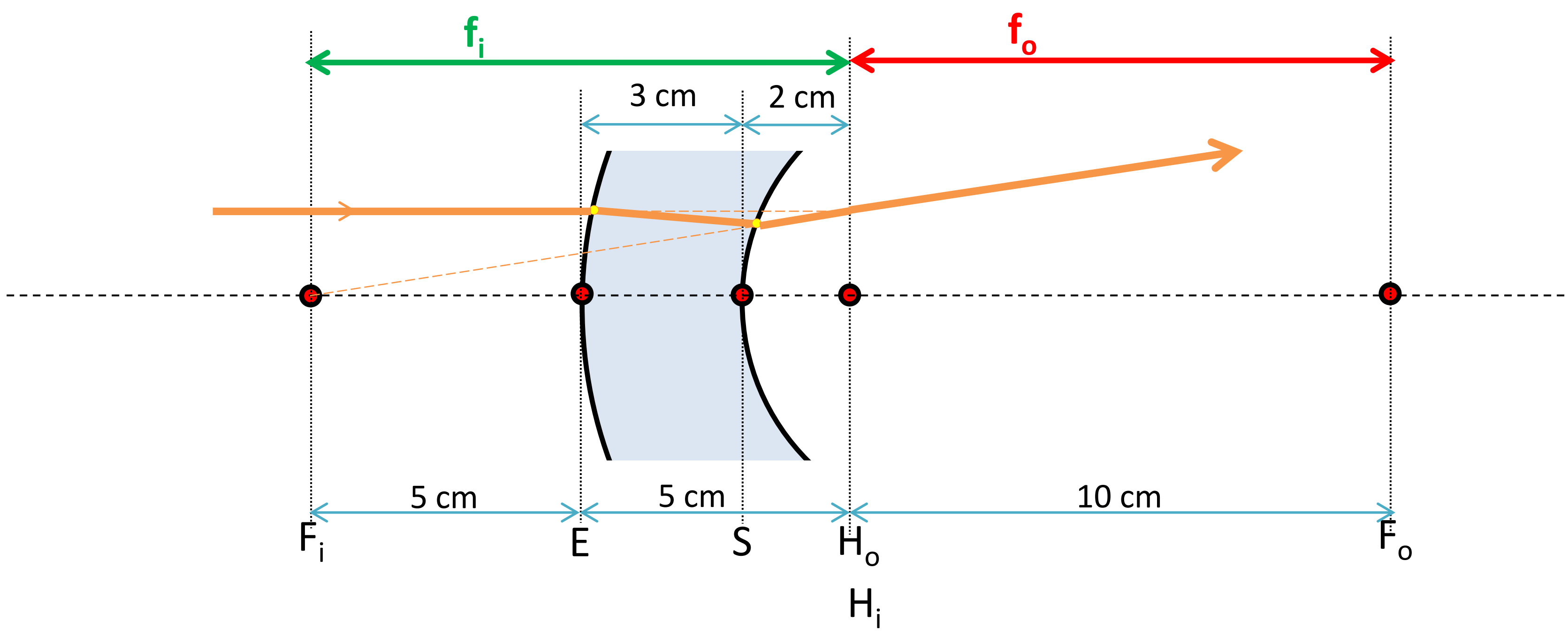
This means, finding :

- the point that the ray exits the lens?
- the exit angle?

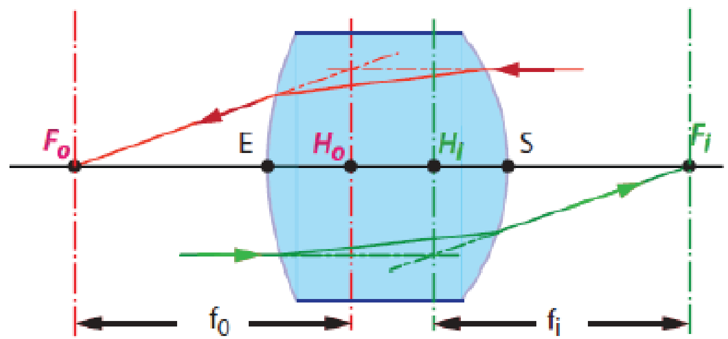
Thick lens example: ray tracing with cardinal planes



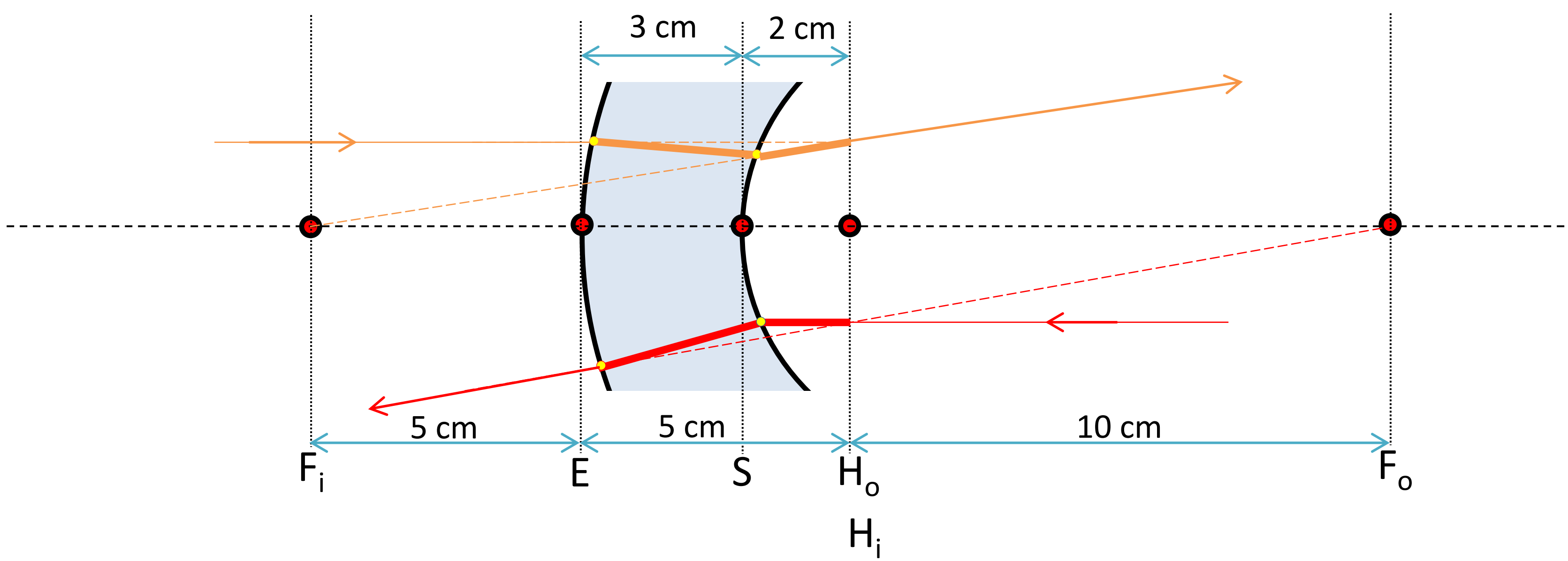
Ray tracing is simpler to implement with the cardinal points & planes:



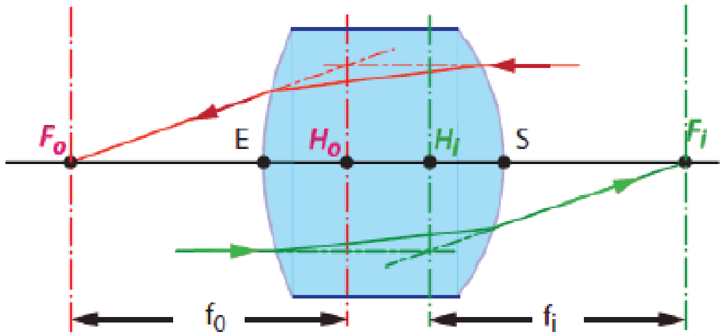
Thick Lens Example: Ray Tracing with Cardinal Points



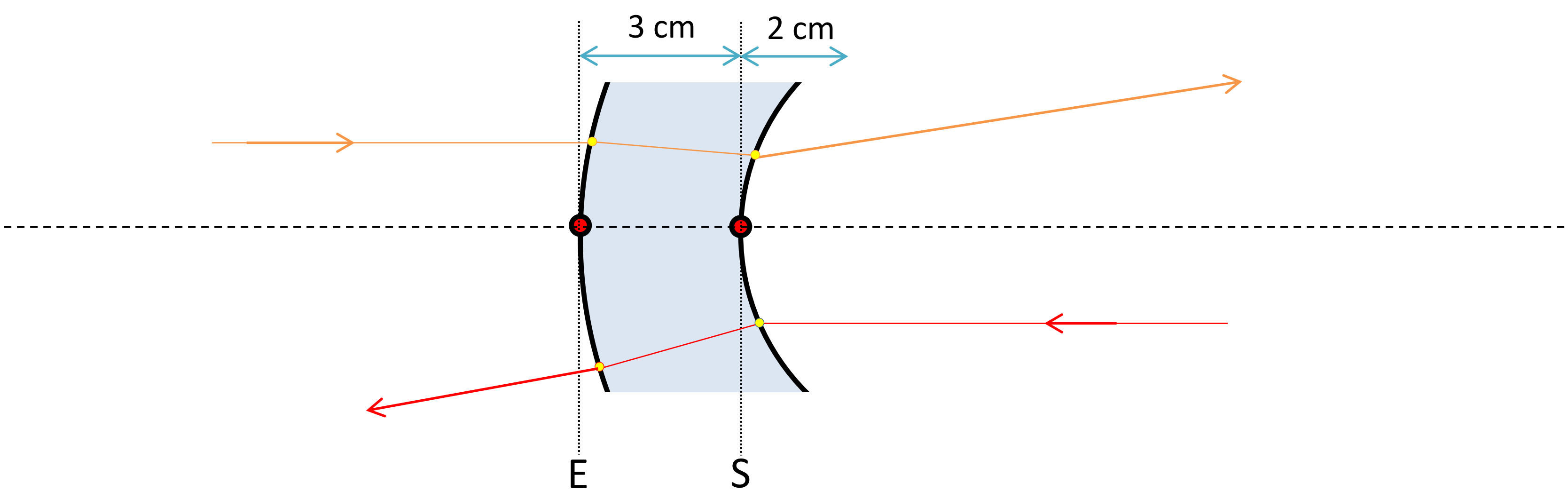
Ray tracing is simpler to implement with the cardinal points & planes:



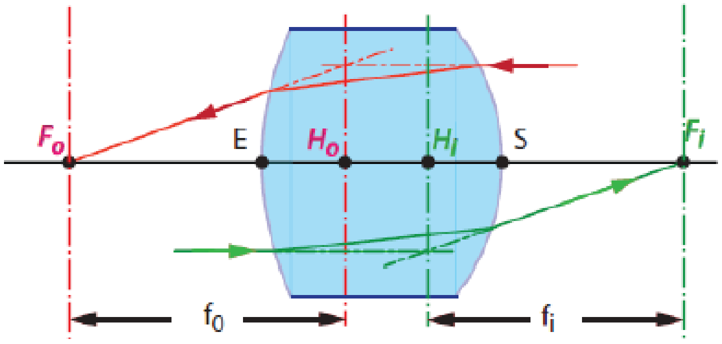
Thick Lens Example Summary: Ray Tracing With Cardinal Points



Ray tracing is simpler to implement with the cardinal points & planes.



Example summary: Thick lens



Cardinal plane(s)

- ***Principal planes H_o , H_i***
- ***Not always inside the thick lens***
- ***The refraction happens at the virtual/fictious principal plane***
- ***Always magnification M = 1 between principal planes***
- ***Focal plane(s) F_o , F_i***
- ***Common point for rays parallel to axis***