Biomicroscopy I - Solutions Exercise Sheet 4

September 30, 2025

1 Two thin lenses forming a thick lens

A. Recall the Lensmaker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the first thin lens L_1, we have $n = 1.5$, $R_1 = 5\text{cm}$, $R_2 = -5\text{cm}$. Therefore, $f_1 = 5\text{cm}$. For the second one, we have $n = 1.5$, $R_1 = -5\text{cm}$, $R_2 = 5\text{cm}$. So, $f_2 = -5\text{cm}$.

B. We can get the corresponding image point by intersecting two rays which start at the same point of the object. The easiest way in this case is to intersect a ray leaving parallel to the optical axis (red) and a ray passing through the first focal point (blue). The magnification is $m = -1$. The image is located 5cm to the right of the second lens L_2.

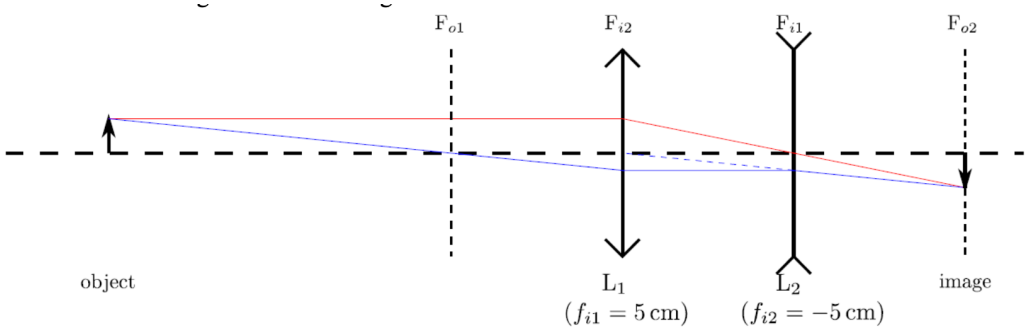

Figure 1: Ray tracing solution

C. The whole system can be represented with the conjugate matrix C:

$$C = T_2 L_2 T_1 L_1 T_0$$

Let's first compute the ABCD matrix associated to the first lens, L_1:

$$L_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.2 & 1 \end{bmatrix}$$

Similarly, for L_2, the ABCD matrix is

$$L_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix}$$

The translation matrices T_0, T_1 and T_2 are:

$$T_0 = \begin{bmatrix} 1 & d_0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}$$

where d_i denotes the image location. The whole conjugate matrix C is:

$$C = T_2 L_2 T_1 L_1 T_0 = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -0.2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 15 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.2d_i & 5 - d_i \\ -0.2 & -1 \end{bmatrix}$$

Imposing the imaging condition, we can find the image location d_i:

$$0 = C_{12} = 5 - d_i \iff d_i = 5cm$$

and the magnification is given by C_{11}, so $m = -0.2d_i = -1$.

D. Starting with a ray parallel to the optical axis, we trace the ray through the whole system. The intersection of the optical axis with the straight line defined by the ray segment after the last lens places the image side focal plane of the total system (F_i). The intersection of the straight line given by the incoming horizontal ray with the straight line given by the ray after the last lens defines the image side principal plane (H_i).In the same manner, starting from the right side with a ray parallel to the optical axis and passing through the whole system, but from the right to the left side, we get the focal plane (F_o) and the principal plane (H_o) on the object side.

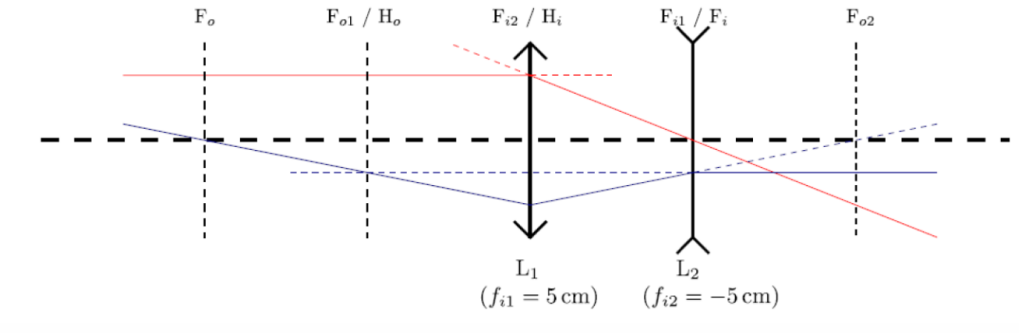

Figure 2: Cardinal planes by ray tracing

E. The focal length is the distance from the corresponding principal plane to the focal plane. Thus the object side focal plane is 10 cm to the left from the first lens and the image side focal plane is located on the second lens. The particularity of this system is that on the object side, the distance from the focal plane to the first lens is larger than the focal length. This can be useful in a microscope setup where you need enough space for your sample, but still a short focal length to get a high magnification.

F. The principal planes and the focal planes are a simplifying abstraction of our total system. Those cardinal elements can almost be taken as a single thin lens, and one can forget about all the other lenses of the system. A ray parallel to the optical axis is "refracted" at the image side principal plane such that the ray passes through the image side focal point.

Analogically, a ray going through the object side focal point is "refracted" at the object side principal plane and exits the system parallel to the optical axis. By the way, a ray impinging in a certain angle on the center of the object side principal plane exits the system at the same angle starting from the center of the image side principal plane. Therefore, points can be considered as a single thin lens.

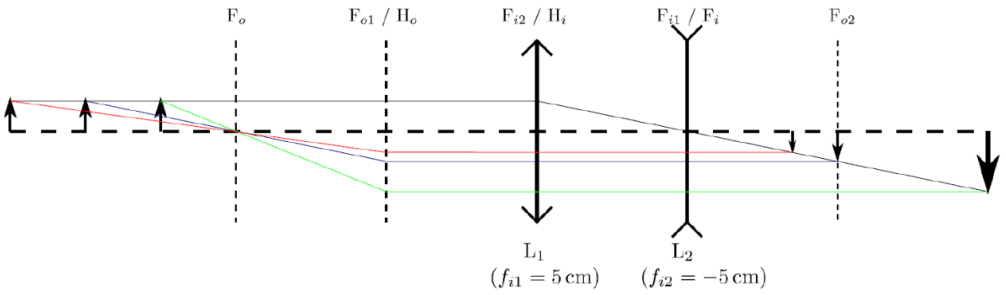

Figure 3: Ray tracing using cardinal planes

G. By introducing a different index material between two lenses, both lens matrices and translation matrix between the lenses are affected. As the refractive indices of the lenses and the intermediate medium are same, the previously found L_1 and L_2 lens matrices are no longer valid. Moreover, note that we cannot use the thin lens matrix to generate matrices for L_1 and L_2. Instead, the lenses can now be interpreted as spherical boundaries, as the refraction index between them is equal to the refraction index of the lenses. Recall the matrix formulation for spherical boundaries:

$$R = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 r} & \frac{n_1}{n_2} \end{bmatrix}$$

where n_1 and n_2 are the refractive indices and r is the radius of the surface. The ABCD matrix for both surfaces is then:

$$R_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1.5-1}{1.5 \cdot 5} & \frac{1}{1.5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{15} & \frac{2}{3} \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1-1.5}{1 \cdot 5} & \frac{1.5}{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{10} & 1.5 \end{bmatrix}$$

Therefore, the new conjugate matrix C' can be rewritten as:

$$C' = T_2' R_2 T_1' R_1 T_0' = \begin{bmatrix} 1 & d_i' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{10} & 1.5 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{15} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 & 15 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} - \frac{d_i'}{30} & \frac{40}{3} + \frac{5d_i}{6} \\ -\frac{1}{30} & \frac{5}{6} \end{bmatrix}$$

Imposing the imaging condition, we find that $d_i' = -16$cm from the right of the second lens. Therefore, the (virtual) image is located 11cm to the left of the thick lens. The magnification is $m = 1.2$.

2 Two thin lenses with extra optical components

A. The marginal rays start at the object plane on the optical axis and touch the diaphragm's boundaries (see figure 4). No other optical element is limiting the light cone starting from optical axis at the object point than the diaphragm. This is the aperture defining the entrance and the exit pupils. The entrance pupil is the image of the aperture through the optical system before the aperture. Since there is no lens before the aperture, the entrance pupil is the aperture. The exit pupil is the image of the aperture through the optical system behind the aperture. Since the aperture is located on the front focal plane of the total system, its image, i.e. the exit pupil, is at infinity. The pupils limit the amount of light (energy) that can enter the system.

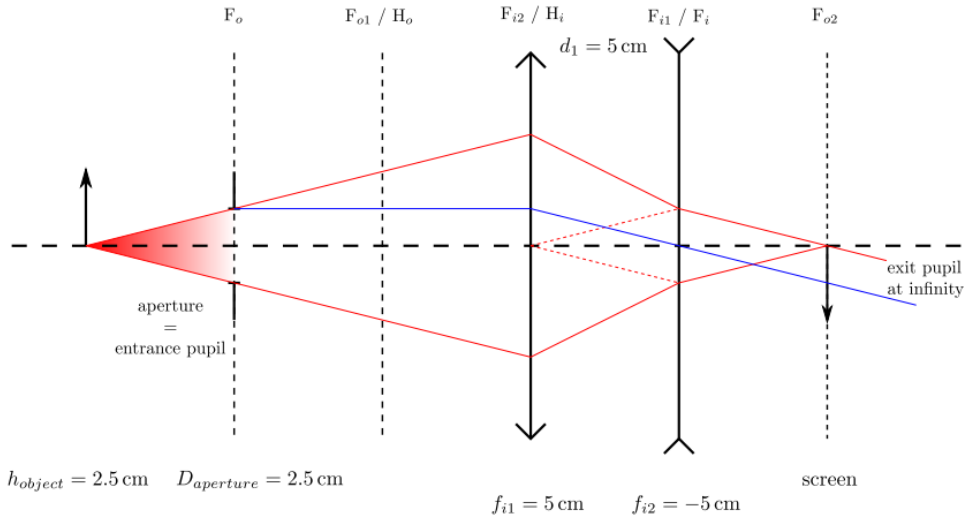

Figure 4: Marginal rays and aperture.

B. Consider a ray (red line) that starts from an off axis point in the object plane and crosses the center of the entrance pupil (and thus also the center of the aperture and the exit pupil, see figure 5). Such a ray with the highest possible angle with the optical axis

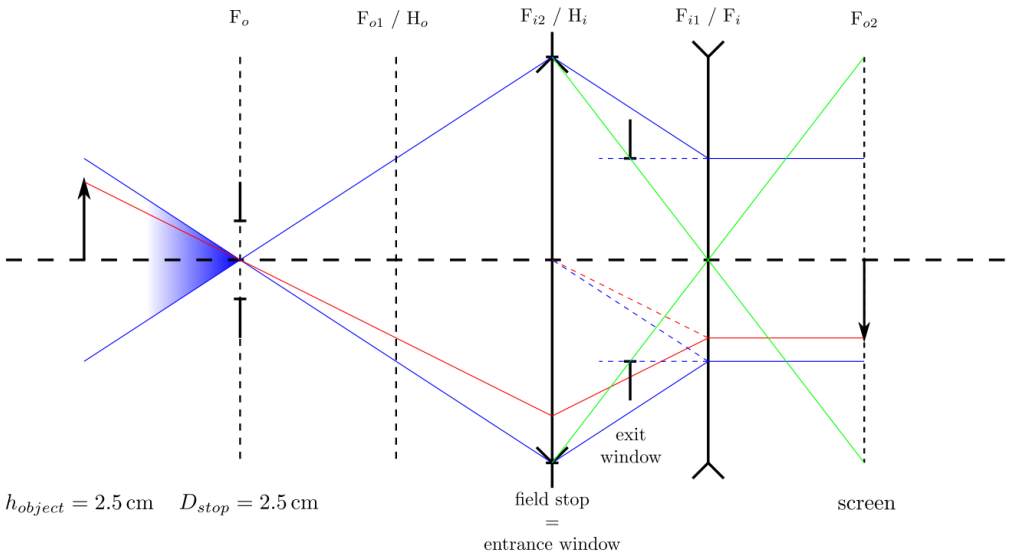

Figure 5: Chief ray, the field stop, the entrance and exit windows.

(farthest off axis point) that passes the whole system defines the field stop (blue line). The entrance window is the image of the field stop through the optical system before the field stop. Here the entrance window is equal to the field stop because it is the first lens.

The exit window is the image of the field stop through the optical system after the field stop. In this case it is the virtual image generated by the second lens: seeing Figure 5, we reuse the chief ray (blue) and find the exit window by using another ray leaving the object from the position where the chief ray intersects the field stop (green rays). The exit window is given by the intersection of these blue and green rays, as it represents the image .

The field stop and the windows respectively define the field of view, i.e. the farthest off axis points that can be imaged through the optical system.

C. In this configuration, the exit pupil is at infinity (because the aperture is on the front focal plane). Therefore, the chief ray (which passes through the center of all pupils) is parallel to the optical axes at the end of the system (see figure 6). Suppose a point light source at the extremity of the object. The light cone entering the system is illustrated by the two rays touching the aperture boundaries. In the image plane, the light cone is formed symmetrically around the chief ray. If you now move the screen forth and back, the imaged point gets blurred with approximately the diameter of the light cone at that position. However, the image size (the image point height) and thus the magnification remains the same. This is because the chief ray is parallel to the optical axis (and exit pupil at infinity).

Such a system is called *image side telecentric*. Remarkably, this behavior is caused by a non- diffracting element which is the aperture!

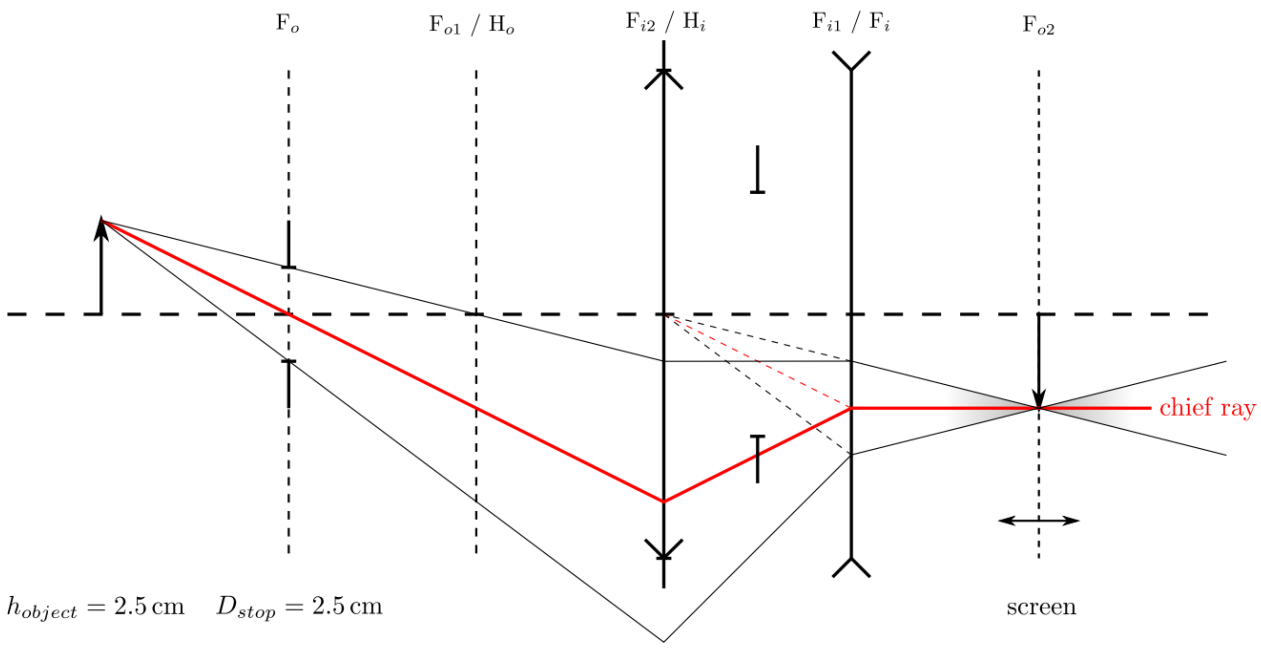

Figure 6: An image side tele-centric system.

D. In order simplify the drawing, the principal planes were used to trace the rays. If the object is further away, a smaller image is formed before the screen. The light cone from this object expands until it reaches the screen (see figure 7). This generates a blurred image on the screen, but still at the same smaller size, since the chief ray is parallel to the

optical axis. A larger image of is formed after the screen when the object is closer to the system. The light cone has not yet fully converged when it reaches the screen. Thus, the image on the screen is also blurred, with the same size as at the correct image position.

This shows *the perspective*: An object further away generates a smaller, an object closer to the optical system generates a larger blurred image.

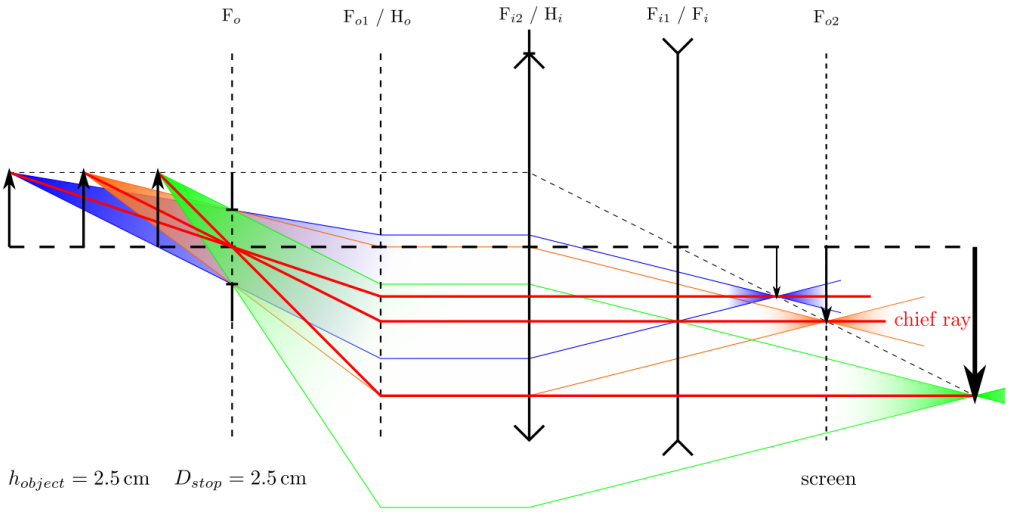

Figure 7: Perspective with an optical system.