Biomicroscopy I - Exercise Sheet 2

September 16, 2025

1 Cartesian oval

The shape of the interface pictured in Figure 1 is known as Cartesian oval. It's the perfect configuration to carry any ray from S to the interface to P. Prove that the defining equation is

$$l_o n_1 + l_i n_2 = const$$

Show that it is equivalent to:

$$n_1(x^2 + y^2)^{1/2} + n_2(y^2 + (s_0 + s_i - x)^2)^{1/2} = const$$

where x and y are coordinates of point A.

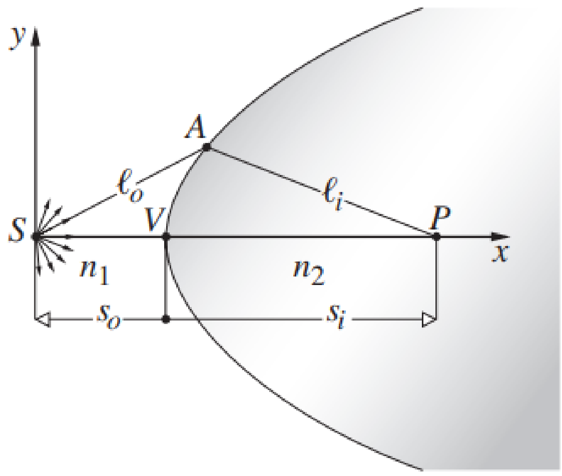

Figure 1: Cartesian oval

2 Thin lenses: Ray tracing

Consider a lens configuration shown in Figure 2. One lens is divergent, another is convergent.

1. Draw the system scaled correctly. Place a 1 cm tall object 15 cm before the first lens (see Fig. 2). By ray tracing through the two lenses, find out where the image is located. What is the magnification?

2. Now move the object 2.5 cm closer to the optical system and find its image by ray tracing. What magnification do you find here? Where is the image located?

3. Repeat the same calculations but moving the object 2.5 cm farther away from the optical system.

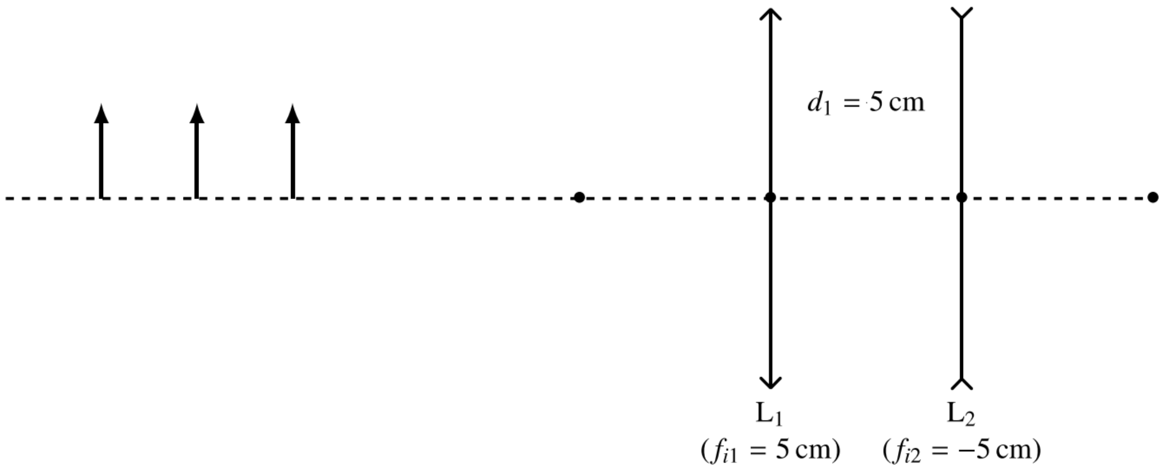

Figure 2: Two thin lenses and set of objects

3 Ray matrix of a thin lens

Using the basic ray-transfer (ABCD) matrices for optical interfaces that were derived in the lecture, obtain the ray matrix of a thin lens.

- Assume the lens is thin enough that propagation inside the lens material can be neglected.
- Start from the general interface matrices ($n_{medium} \to n_{lens}$, $n_{lens} \to n_{medium}$) and combine them appropriately.
- Express the final result as the overall ABCD matrix of the thin lens.

4 Thin lenses: ABCD-matrix

Consider the system of two lenses L_1 and L_2 with focal distances f_1 and f_2 correspondingly and separated by distance d in between (see Fig. 3). Calculate the ABCD-matrix for such a system using the multiplication rule for the transfer matrices. The ABCD-matrix for a thin lens is given by:

$$M = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

5 Consecutive thin lenses

Consider a system of two thin lenses L_1 and L_2 which are not separated, *i.e.* $d = 0$cm. Using ABCD matrices, prove that this system is equivalent to having no lenses if and only if either L_1 is convergent with focal distance f and L_2 is divergent with focal distance $-f$ or L_2 is convergent with focal distance f and L_1 is divergent with focal distance $-f$.

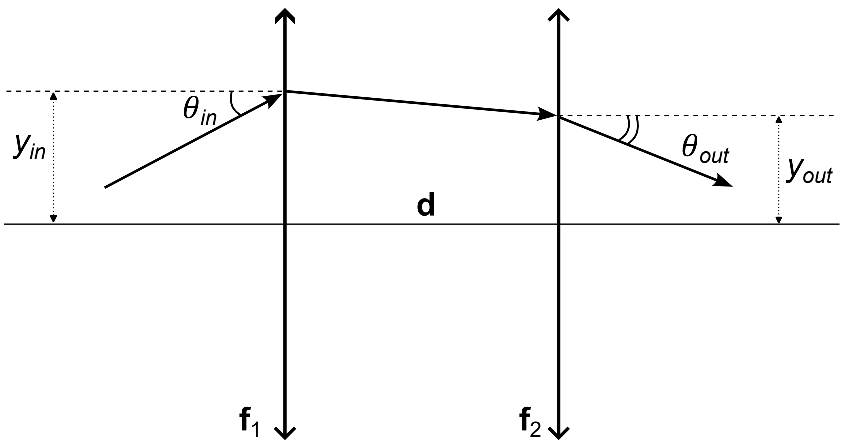

Figure 3: Two thin lenses with a free space distance in between