Biomicroscopy I - Exercise Sheet 9

November 11, 2025

1 Fourier transform with a lens

As you know, a single lens performs the Fourier transform of the object in its back focal plane. For each of the apertures (object plane) shown in Figure 1 (b) define:

A. The mathematical representation of the input function $f(x)$ of the apertures in the object plane.

B. The image formed at the Fourier plane.

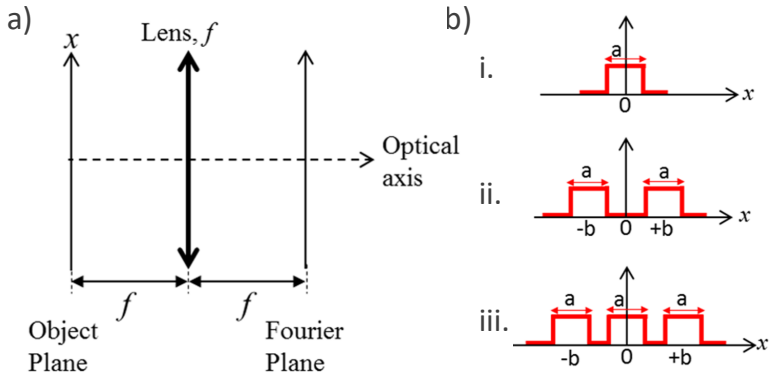

Figure 1: Simple lens system. a) Object and Fourier planes of a lens. b) Appertures located at the object plane.

2 Resolution in $4f$ scheme

Consider a classical $4f$ scheme consisting of 2 thin lenses in air with transversal sizes (full height) $h_1 = h_2 = 6$ cm. The focal distance is $f = 9.54$ cm. In the object plane there are two monochromatic sources radiating at $\lambda = 600$ nm (Fig. 2).

A. Define the minimum distance d_{min} between these sources, so that they are still resolved by introduced optical scheme, using Abbe and Rayleigh criteria.

B. How would the resolution change if the refractive index is changed to $\tilde{n} = 1.5$ (suppose that the lenses are substituted with the ones having the same focal distance f in this medium). How would the resolution change if the medium is still air with $n = 1$ but the wavelength is now $\tilde{\lambda} = 480$ nm?

C. How would the resolution change if the full height of the second lens from initial problem formulation would be $h_2' = 12$ cm? How would the resolution change if the full height of the second lens from initial problem formulation would be $h_2'' = 4$ cm?

Hint: consider the total angular opening of the system.

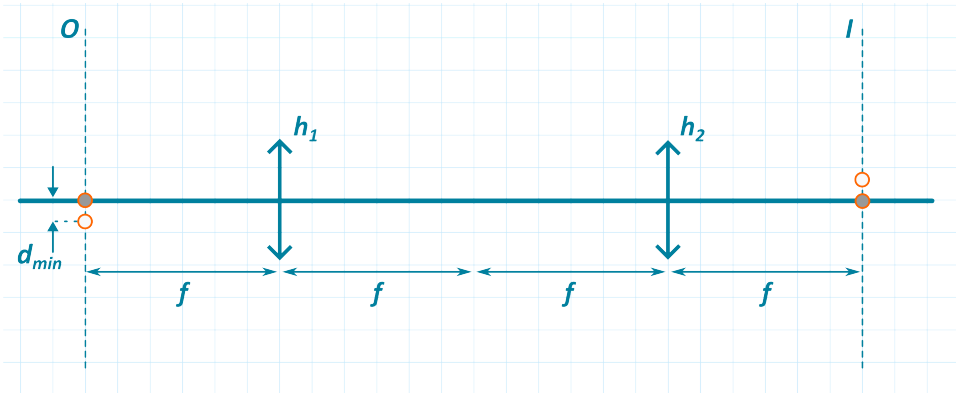

Figure 2: The $4f$ scheme consisting of two thin lenses. Two monochromatic sources in the object planes radiate at wavelength $\lambda = 600$ nm. *Note:* the distance between the sources d_{min} is drawn not up to scale.

3 Resolution of a human eye

A. In the human eye, the iris (which is circular) acts as an aperture whose size changes under light intensity. The focal length is approximately determined by the size of eye ball. For an adult, let us assume the iris diameter (D) is 2mm and eye-ball size (thus, f) is 20 mm. If we use a beam of laser beam of $\lambda = 500$nm and focus it at the back focal plane of our eye, what will be the diffraction limited spot size? Use Rayleigh's criteria.

B. If we had a square shaped iris (instead of a circular one), in which the side size of the square is equal to the diameter, will our image resolution improve? Explain your reasoning using Rayleigh's criteria.