

MICRO-523: Optical Detectors

Week Five: Photodiodes (Solutions Ex5)

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Based on MICRO-523, P.-A. Besse, 2023

TAs: Samuele Bisi, Kodai Kaneyasu

The logo of the École polytechnique fédérale de Lausanne (EPFL), consisting of the letters 'EPFL' in a bold, red, sans-serif font.

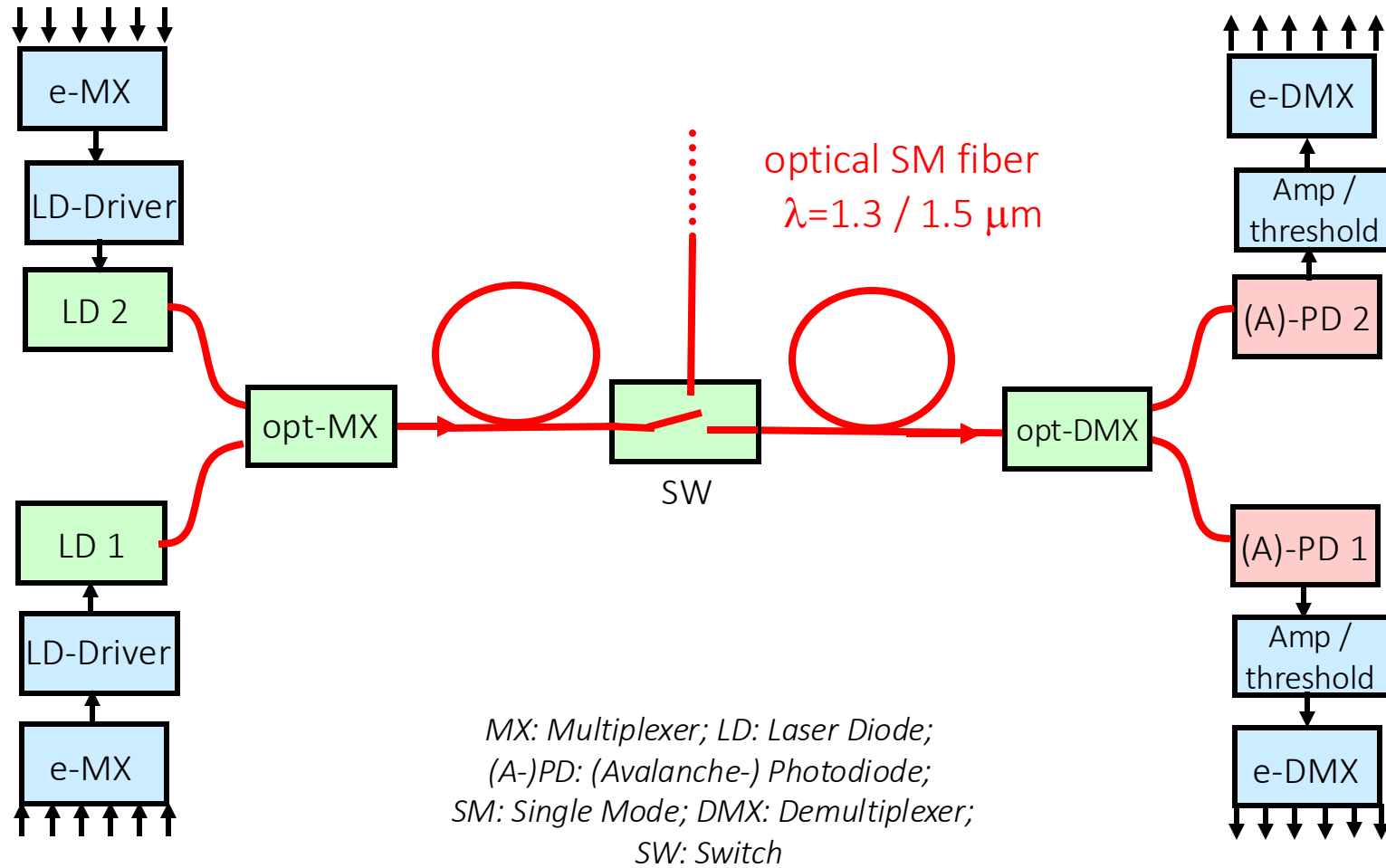
Outline

5.1 [High-speed photodiodes and BER](#)

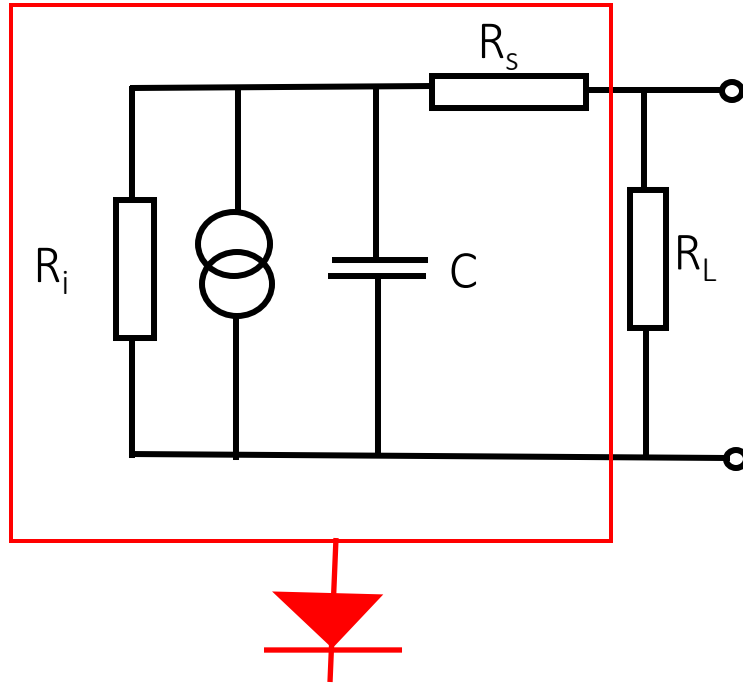
5.2 PIN Heterostructure photodiodes

Homework 5.1: Maximum Efficiency of Solar Cells

Exercise 5.1: Schematic Diagram of a Digital Optical Communication System



Exercise 5.1: Bandwidth Limits of PDs



1. RC-constant:
 $\tau_{RC} = 2(R_s + R_L)C$
($R_i \gg R_L$)

$$\text{but } C_j = S \cdot \frac{\epsilon_0 \epsilon}{W} = \pi \left(\frac{d}{2}\right)^2 \cdot \frac{\epsilon_0 \epsilon}{W}$$

2. Drift of carriers in the depletion layer:

$$\tau_{\text{drift}} = W/v_c$$

W : width of depletion layer

v_c : carrier velocity (10^7 cm/sec)

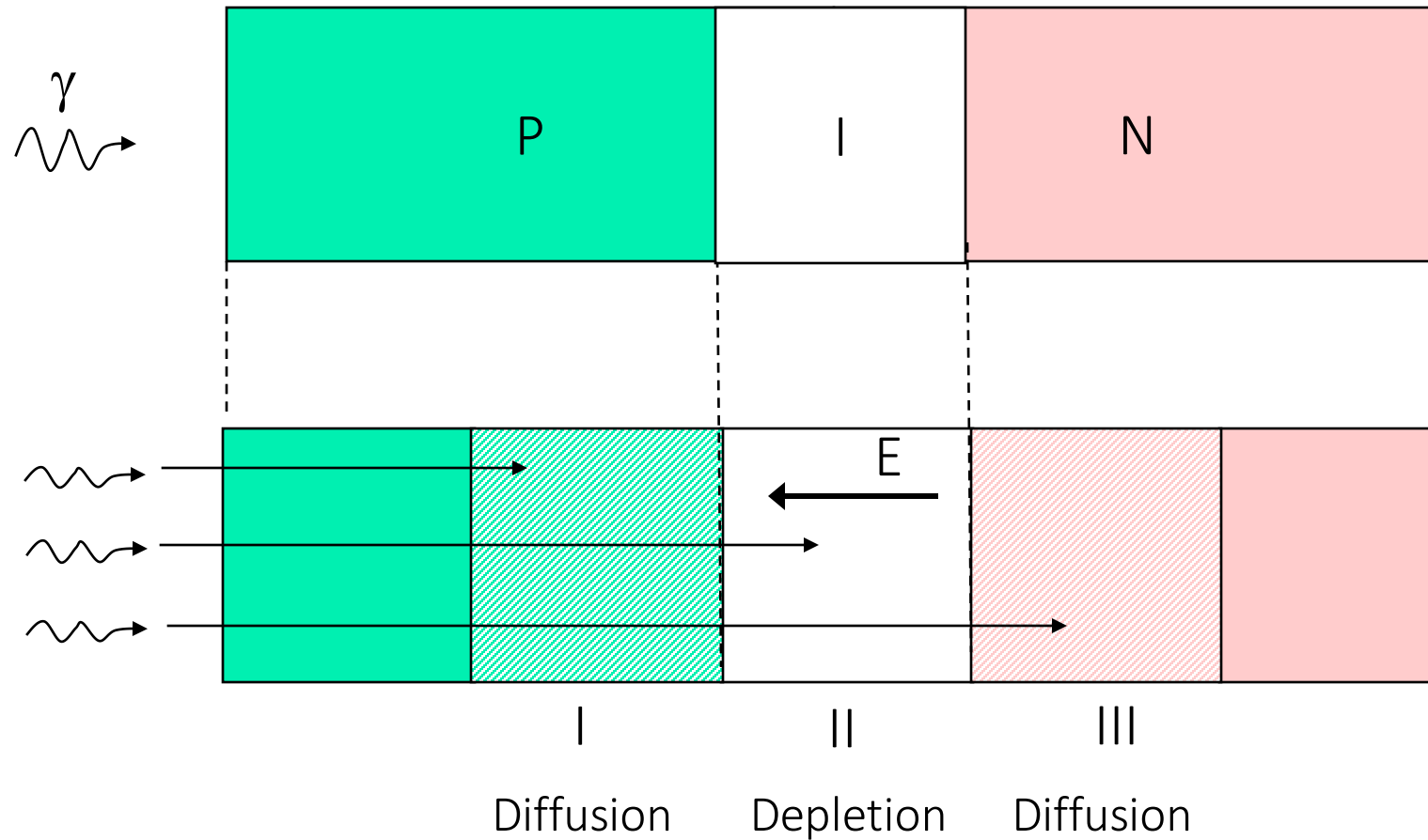
3. Diffusion of carriers to the p-n junction:

$$\tau_{\text{diff}} = d^2/D$$

d : distance to depletion layer

D : diffusion constant

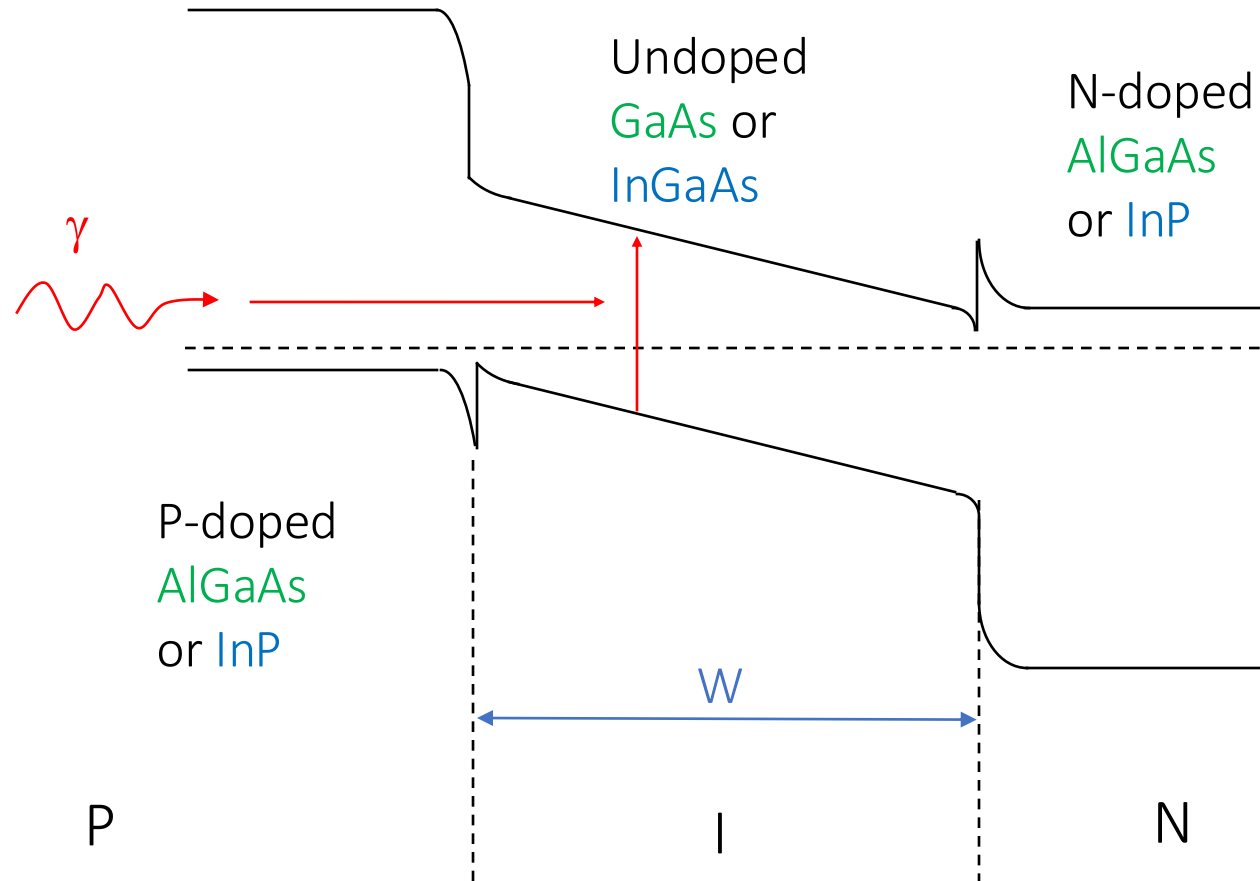
Exercise 5.1: PIN: Diagram of the Working Principle



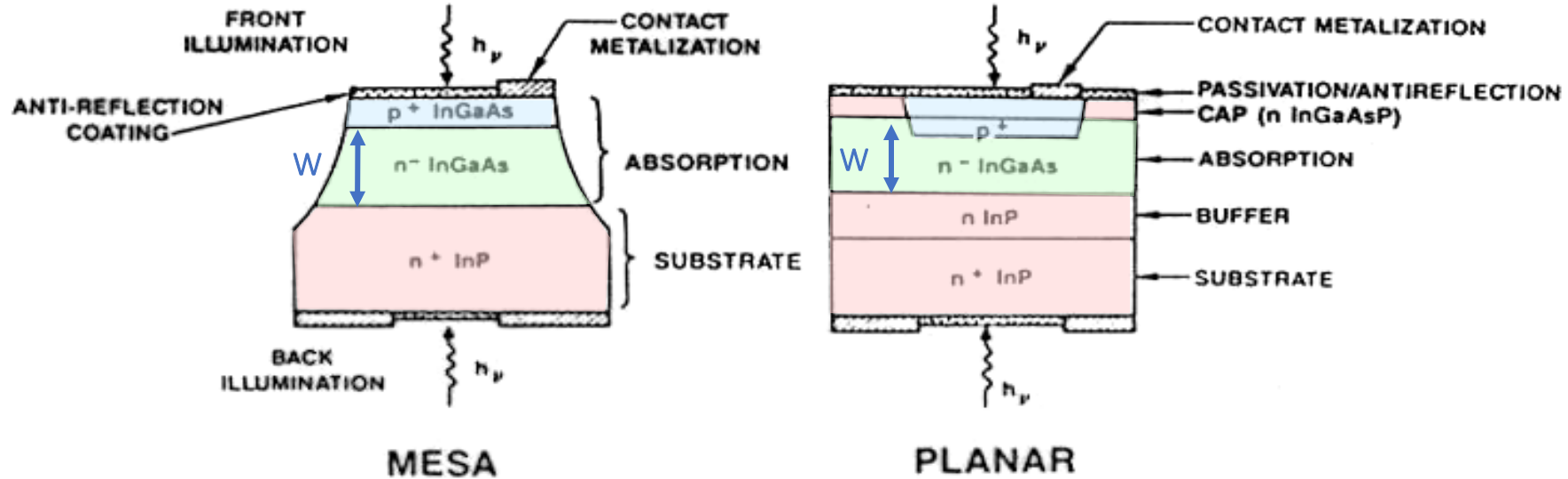
Exercise 5.1: PIN Heterojunction

$$\tau_{drift} \approx W$$

$$\tau_{RC} \approx \frac{1}{W}$$



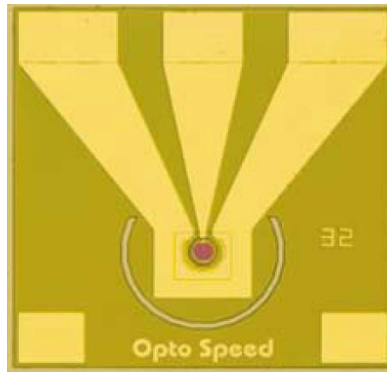
Exercise 5.1: InGaAs PIN Heterojunction Photodiode



$$\tau_{drift} \approx W$$

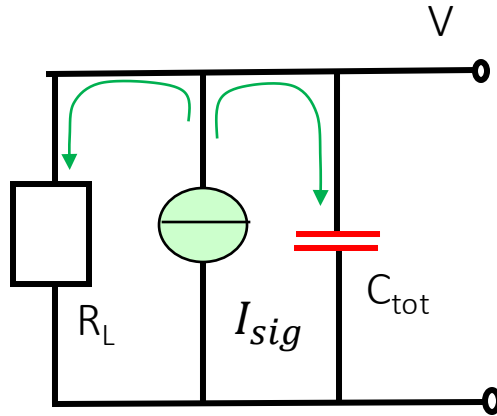
➡ Decreased W

➡ Decreased absorption



InGaAs/InP photodiode
Opto Speed

Exercise 5.1: Continuous Detection: High Speed Digital Transmission

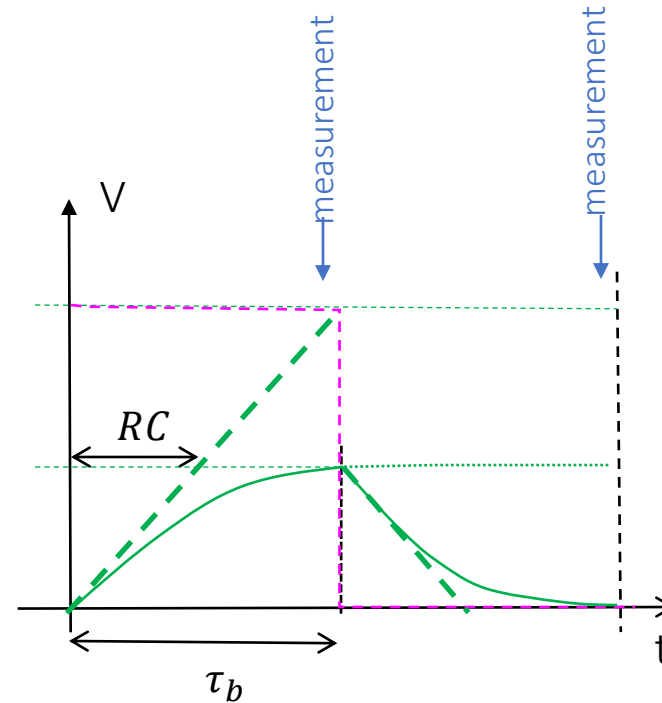


1) At the beginning of the bit:

The current loads the capacitor

2) Once the capacitor is charged:

The current flows through the resistor



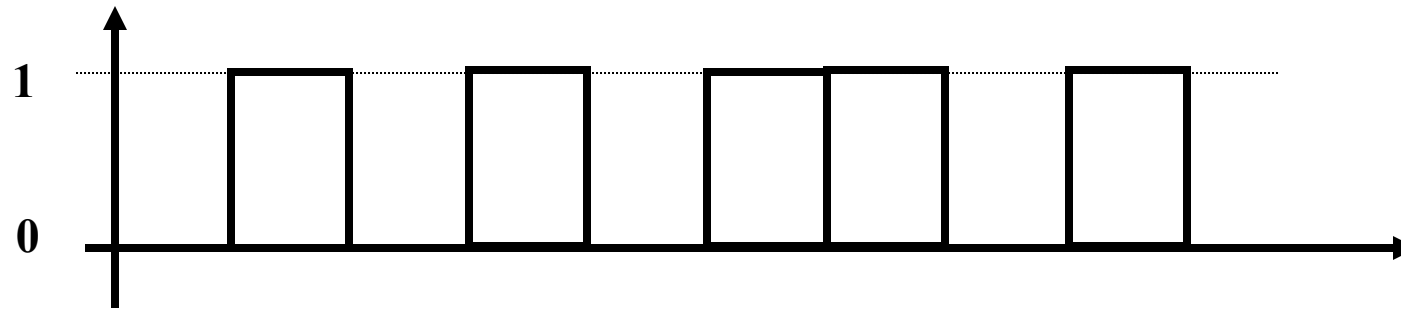
Number of electrons stored in the capacitor:

$$m_{c,sig} \cong \frac{RC}{\tau_b} mM \cong \frac{1}{2} mM$$

m : # of generated electrons per bit
 M : amplification before the electronics (e.g. avalanche or optical amplification)

Exercise 5.1: Bit Error Rate (BER)

- On-off keying system: bits “0” and “1”

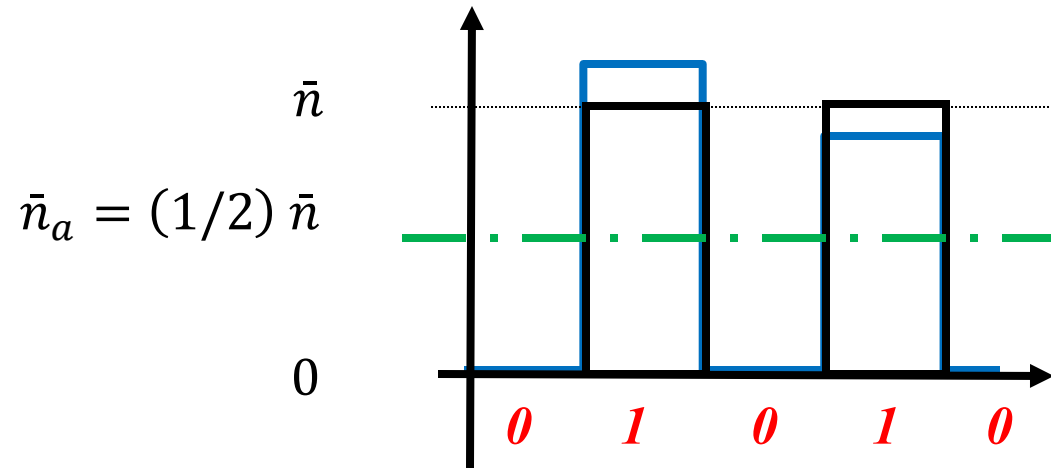


- BER: probability of error per bit
- If p_0 = probability of mistaking a “0” for a “1”
& p_1 = probability of mistaking a “1” for a “0”, then

$$\text{BER} = p_0 / 2 + p_1 / 2 \quad (\text{BER definition})$$

Exercise 5.1: Ideal BER (1)

Ideal = limited by the optical signal shot noise



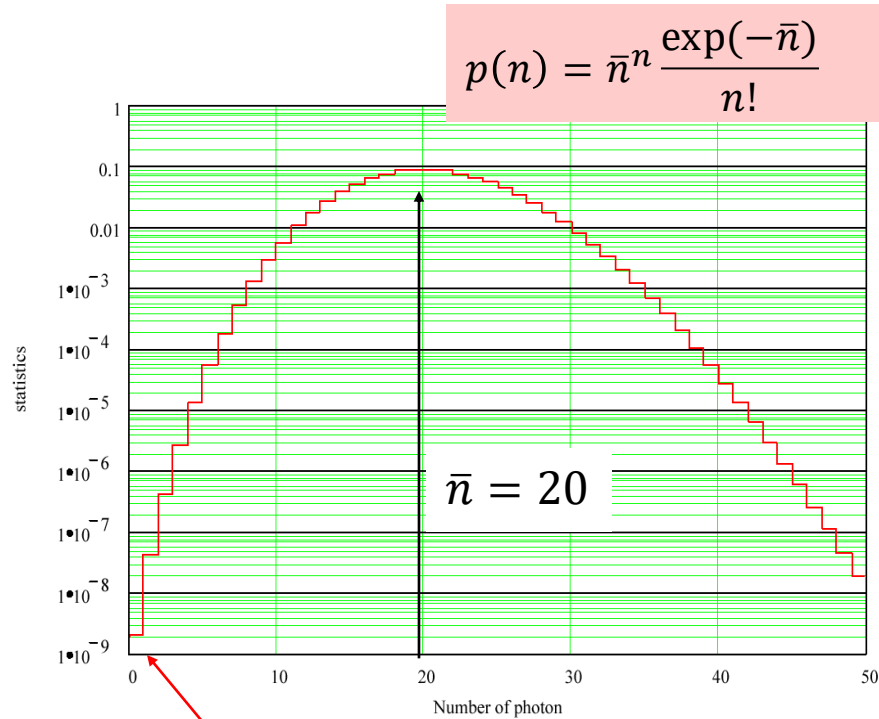
- If an average of \bar{n} photons is transmitted by a laser diode, the probability of detecting n photons is given by:

$$p(n) = \bar{n}^n \frac{\exp(-\bar{n})}{n!}$$

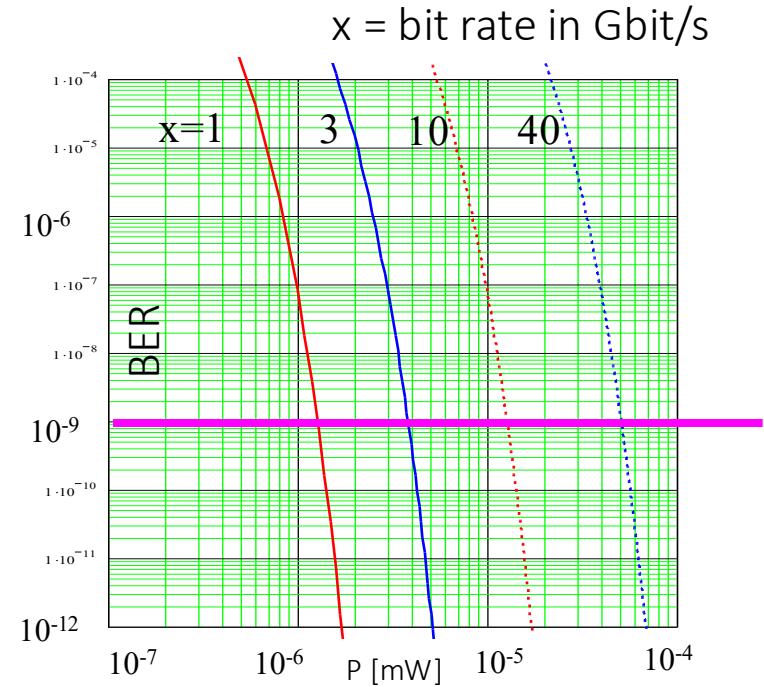
→ How many photons per “1” bit are needed to guarantee a BER of 10^{-9} ?

Exercise 5.1: Ideal BER: Example

Because of the shot noise on each “1” bit, it takes 20 photons per “1” to guarantee a BER of 10^{-9} .



$$p(0) = \exp(-\bar{n}) = 2 \cdot 10^{-9}$$

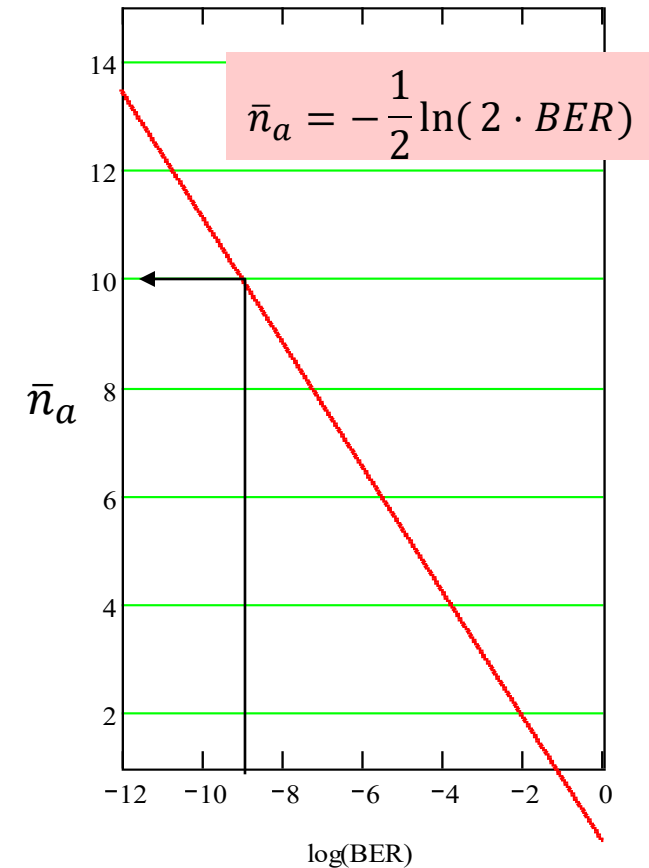


$$\bar{n}_a = \frac{P}{h\nu} T$$

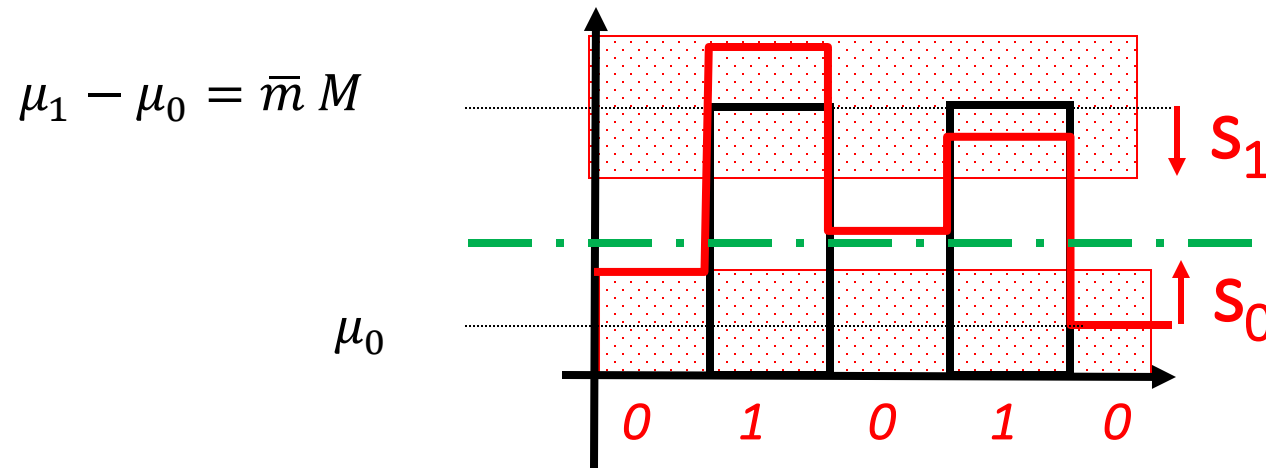
$$BER = \frac{1}{2} e^{-2\left(\frac{P}{h\nu}\right) \cdot \left(\frac{1}{x \cdot 10^9}\right)}$$

Exercise 5.1: Receiver Sensitivity of Ideal Digital Systems

- The receiver sensitivity of digital systems is defined as the average number of photons per bit required to achieve a certain BER (usually 10^{-9}).
- For an ideal receiver, 10 photons per bit are required to achieve a BER of 10^{-9} . (20 photons for “1” bits).

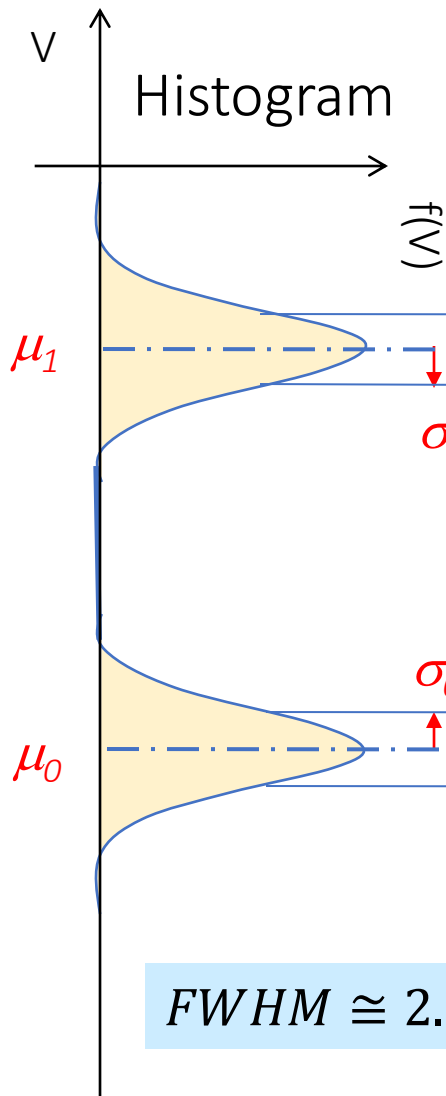


Exercise 5.1: Real Receiver: Sensitivity of a Receiver with Gaussian Noise



- Average number of photo-electrons:
(η_i : quantum efficiency) $\bar{m} = \eta_i \bar{n}$
- Gaussian noise of a photo-detector circuit characterized by a zero mean and an rms value of σ_q (noise parameter) within a bandwidth Δf .

Exercise 5.1: Eye Diagram

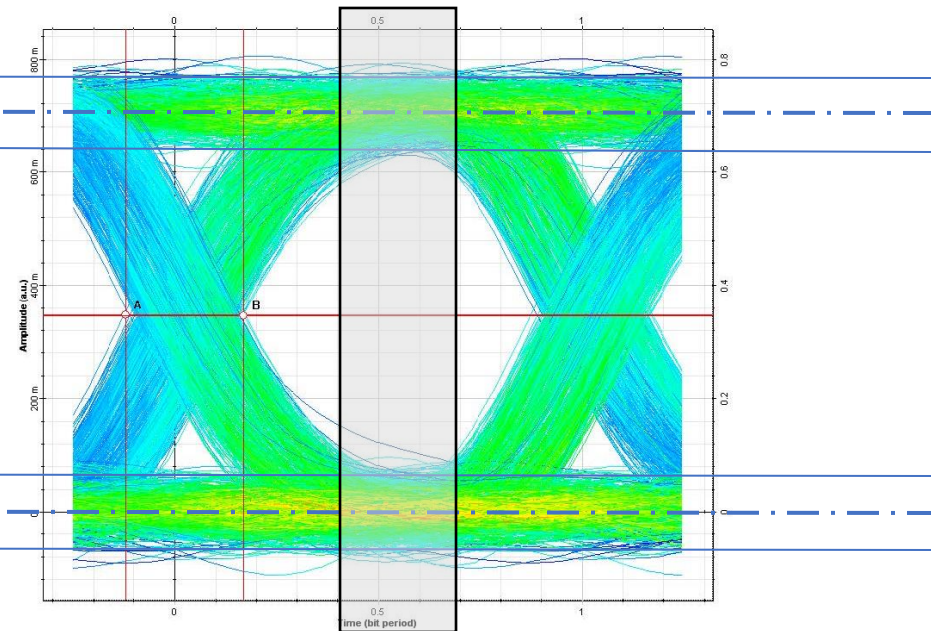


$$FWHM \cong 2.3 \cdot \sigma$$

«Signal degradation—Jitter», Optical System Tutorials

<http://optiwave.com/resources/applications-resources/optical-system-signal-degradation-jitter/>

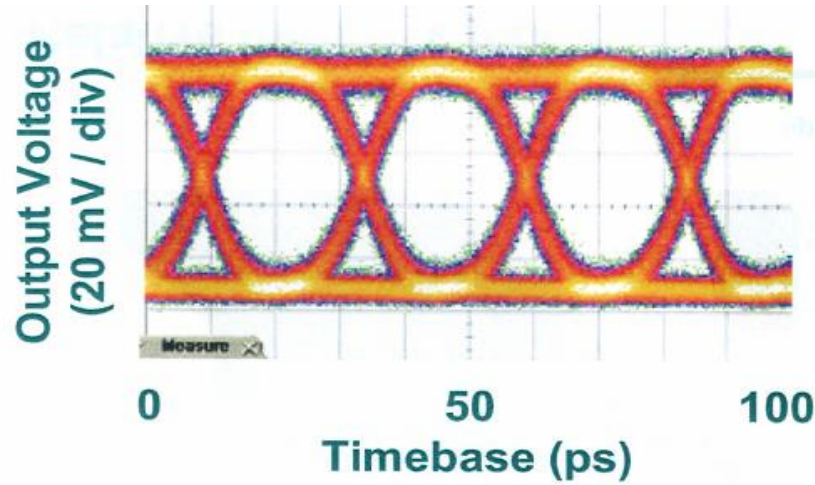
Eye diagram, NRZ (Non-Return to Zero)



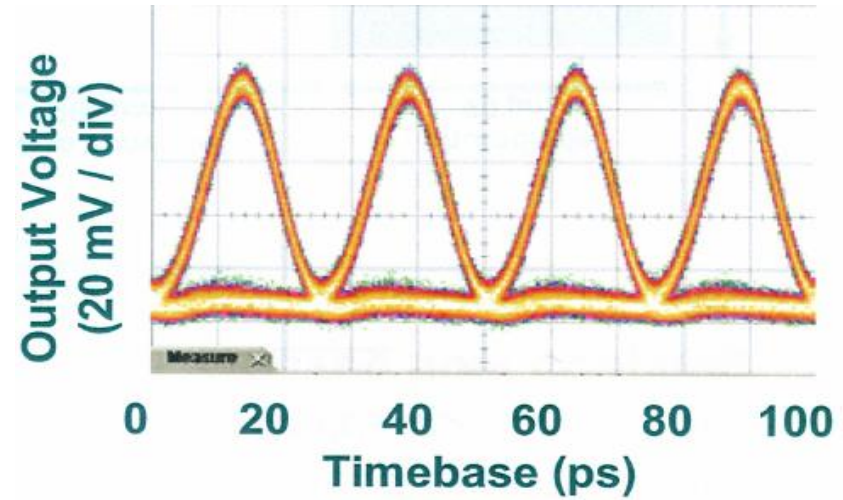
$$\rightarrow Q = (\mu_1 - \mu_0) / (\sigma_0 + \sigma_1) \rightarrow BER$$

Exercise 5.1: High-Speed Detectors (1)

u²t: 40 GBits/s



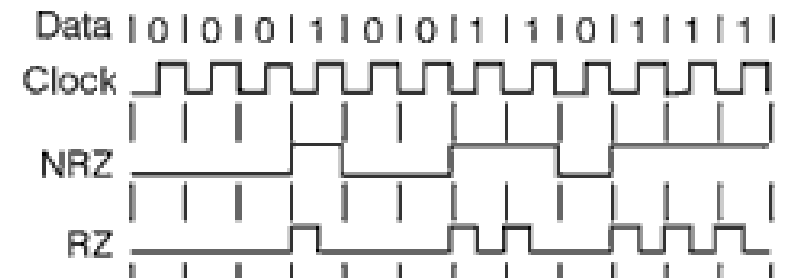
“NRZ: non-return to zero”



“RZ: return to zero”



Photoreceivers for DPSK, A.Umbach, May 2005. © u²t AG



Exercise 5.1: High-Speed Detectors (2)

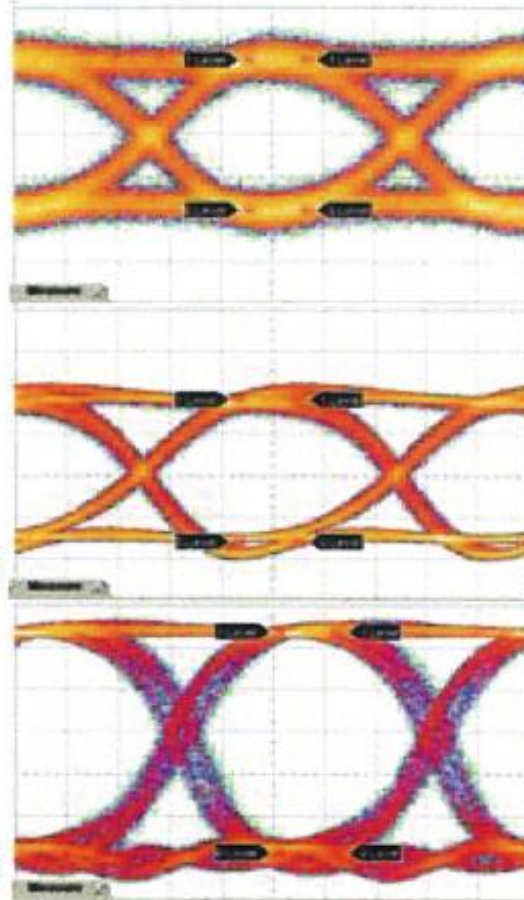
40 Gbit/s Eye Diagrams

Photoreceivers for DPSK, A.Umbach, May 2005. © u²t AG

-10 dBm, 0.1mW
35 mV eye ampl.
SNR = 7, RMS
jitter = 1.1 ps

0 dBm, 1mW
340 mV eye ampl.
SNR = 10,
RMS jitter = 0.7 ps

+5 dBm, 3mW
520 mV eye ampl.
SNR = 16,
RMS jitter = 1.1 ps

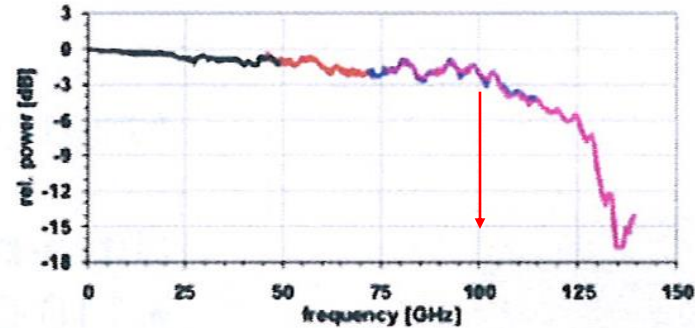


u²t: 40 GBits/s

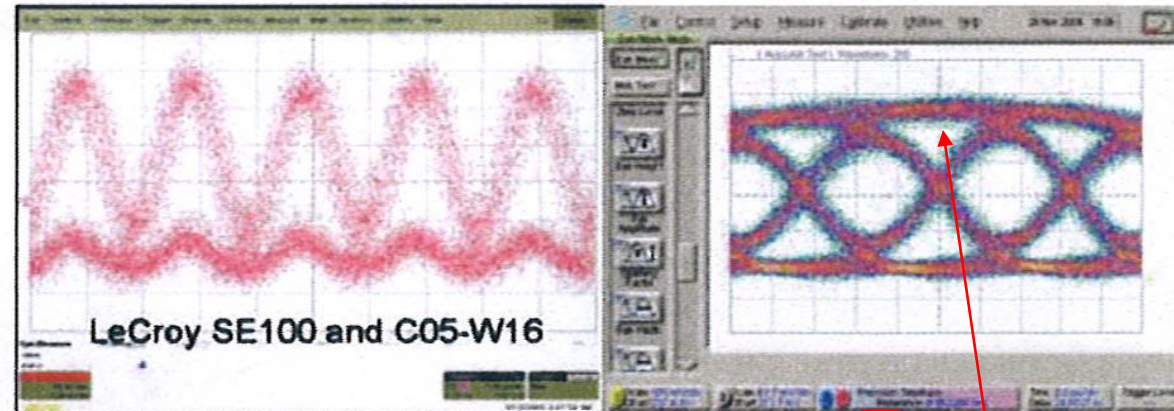


Increasing
input power
→
Better SNR
“open eyes”

Exercise 5.1: High Speed PIN Detectors (3)



frequency characteristics



100 Gbit/s RZ P_{opt} : +8dBm

160 Gbit/s RZ P_{opt} : +12dBm

NRZ

- DC responsivity: 0.73 A/W
- PDL: 0.41 dB at 1.55 μ m
- pulse response (FWHM): 7.4 ps (@250 mV_p output voltage (internally 50 Ω)).

Heinrich Hertz Institut (HHI): up to 160 GBit/s

Outline

5.1 High-speed photodiodes and BER

5.2 PIN Heterostructure photodiodes

Homework 5.1: Maximum Efficiency of Solar Cells

Exercise 5.2: PIN Heterostructure Photodiode

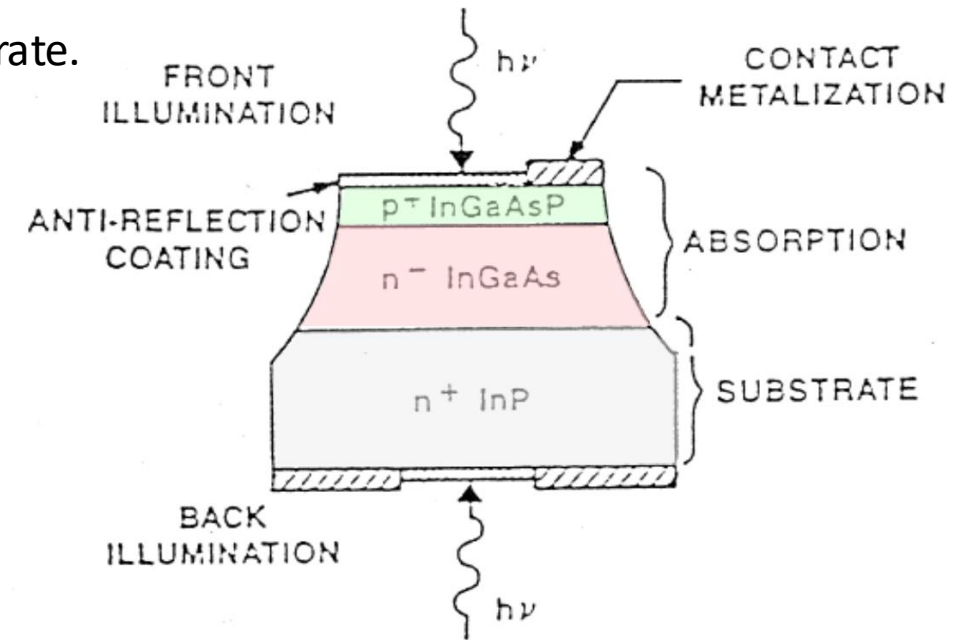
$\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ system on an InP substrate

- a) To grow a monocrystal, we have to preserve the lattice of the InP substrate.
This involves satisfying the following relationship:

$$x = \frac{0.4562 \cdot y}{1 - 0.031 \cdot y}$$

- b) In this case (“lattice matched to InP”) the bandgap can be changed according to:

$$E_g(y) = 1.35 - 0.72 \cdot y + 0.12 \cdot y^2 \quad [\text{eV}]$$



Exercise 5.2: PIN Heterostructure Photodiodes

This type of photodiode was designed for optical telecommunications and has to work at wavelengths between $1.50 \mu\text{m}$ and $1.60 \mu\text{m}$. Its diameter is $10 \mu\text{m}$, corresponding to that of a single mode fiber optic cable.

A) Considering a superficial layer with the following composition: $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$, with $y=0.84$.

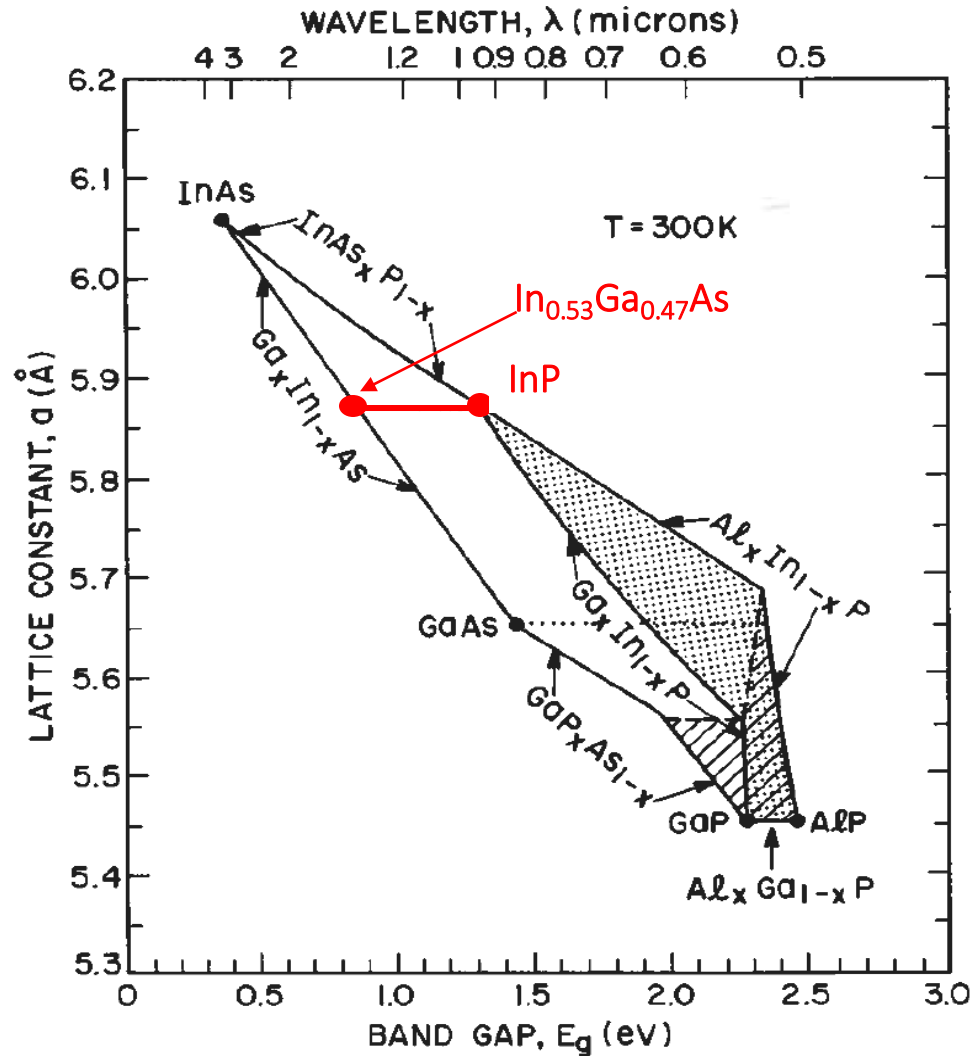
Sketch the quantum efficiency for front and back illumination. What is the main difference?

B) Estimate the width W of the intrinsic InGaAs region to optimize the bandwidth using a load resistance of $R_L = 50 \Omega$.

(use $\epsilon = 12$ and $v_{\text{sat}} = 10^5 \text{ m/s}$)

Does the diode have to be polarized, and if so, why?

Exercise 5.2: Complement: $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ Heterostructures



InGaAsP
matched to InP

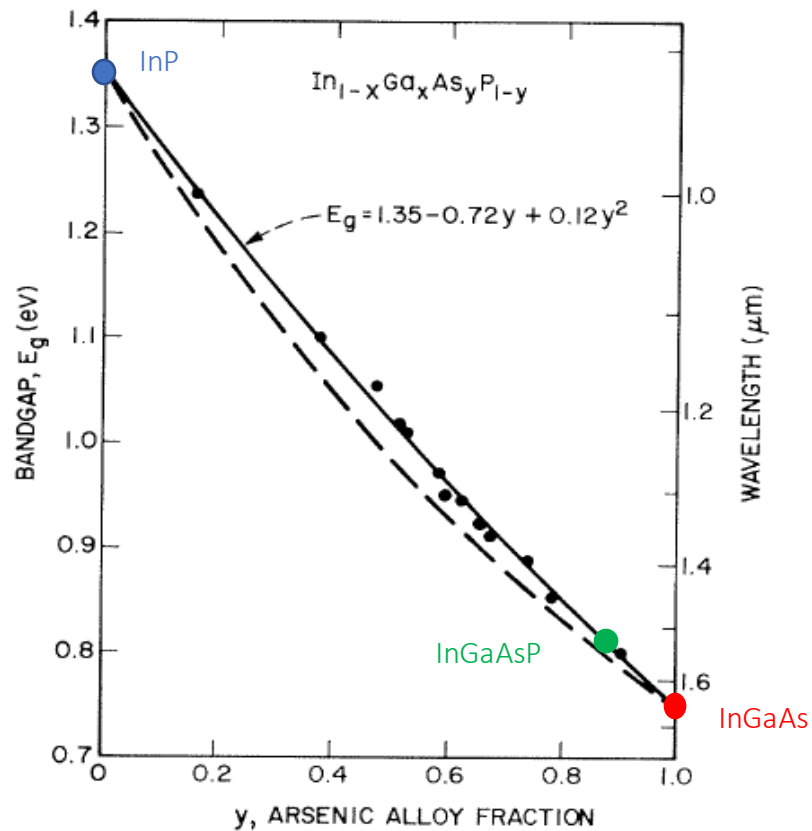
$$x = \frac{0.4562 \cdot y}{1 - 0.031 \cdot y} \cong 0.45 \cdot y$$

Agrawal, « Long-wavelength semiconductor lasers », VNR

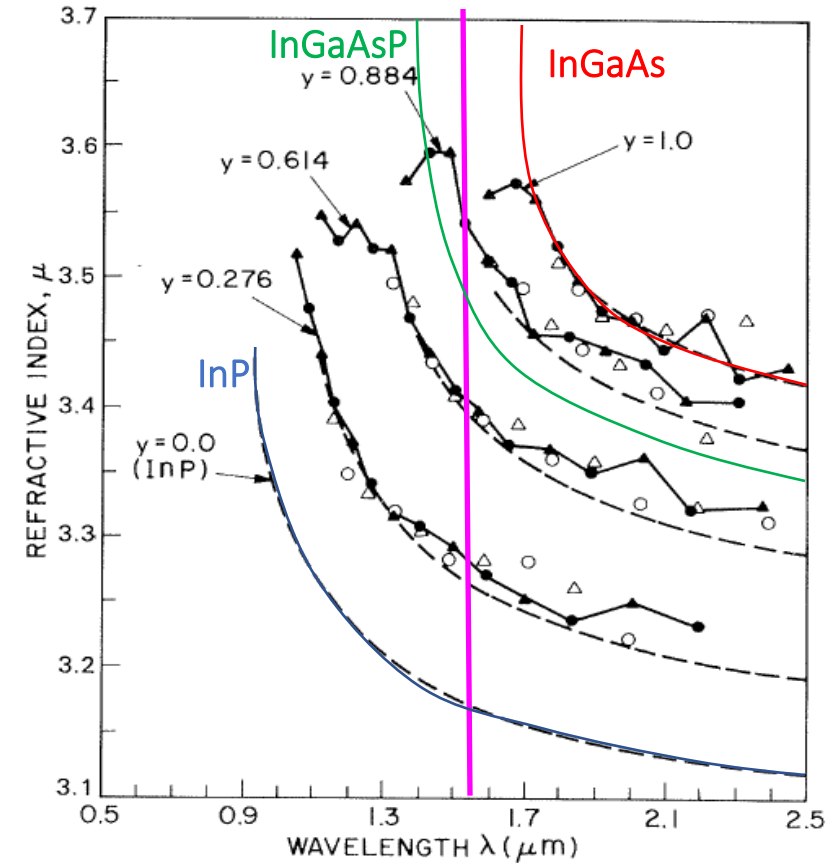
Exercise 5.2: $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ Matched to InP

Bandgap

$$E_g(y) = 1.35 - 0.72 \cdot y + 0.12 \cdot y^2 \quad [eV]$$

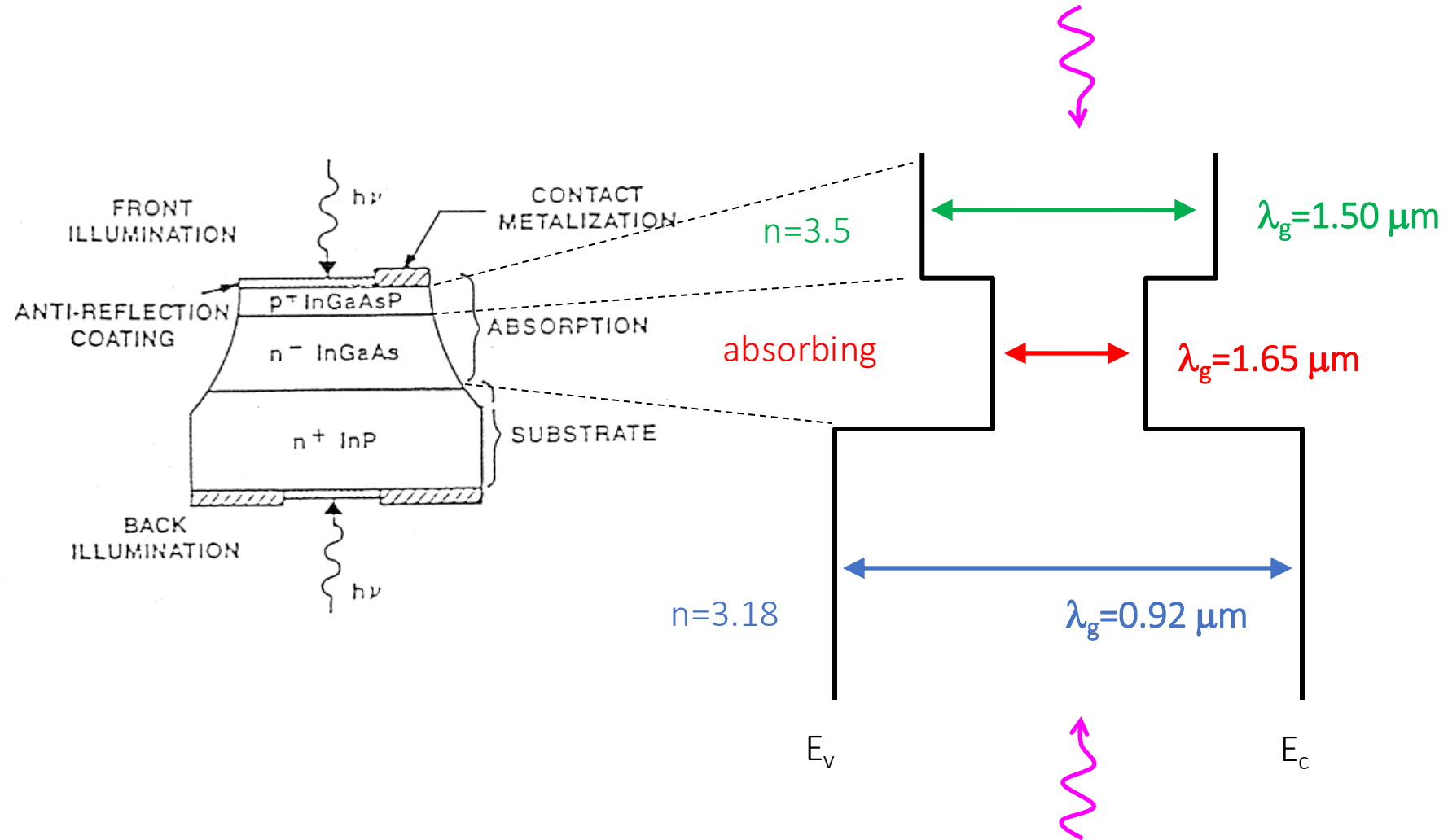


Refractive index



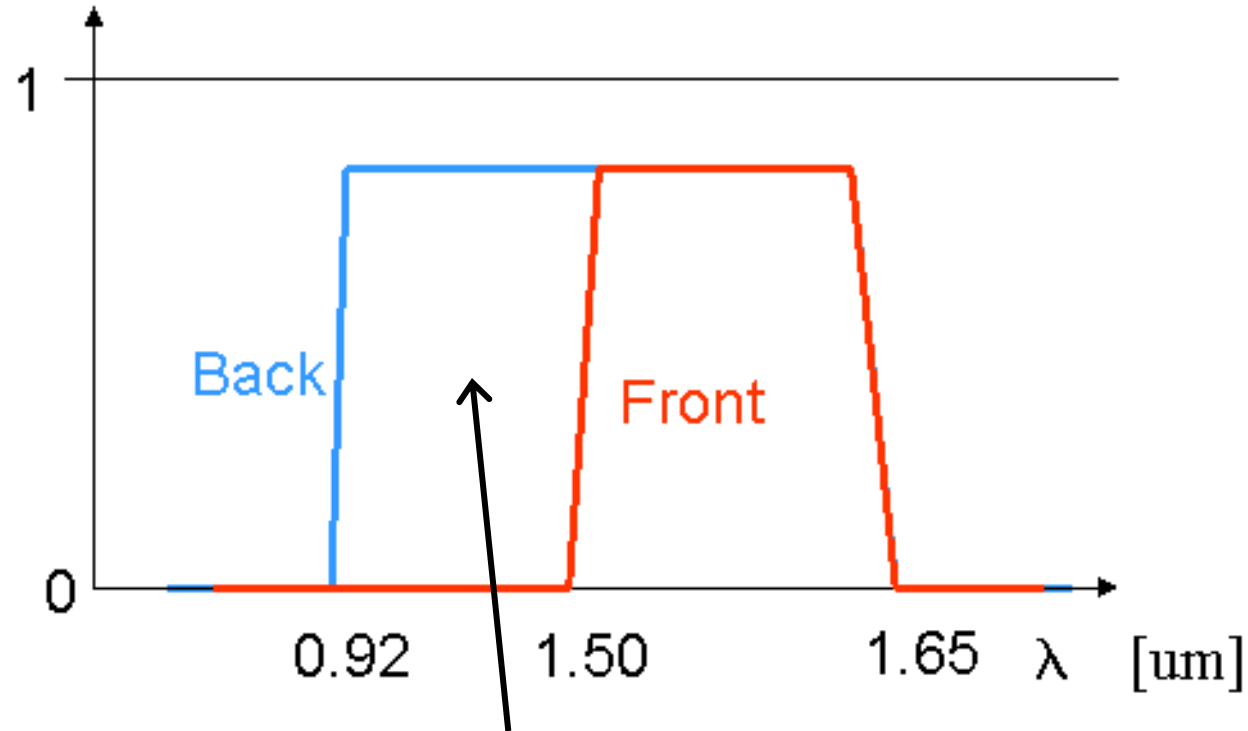
Agrawal, « Long-wavelength semiconductor lasers », VNR

Exercise 5.2: PIN Structure and Bands



Exercise 5.2: PIN Heterostructure Photodiodes

A) Quantum Efficiency



With front illumination, the diode is sensitive between 1.50 and 1.65 μm

With back illumination, the diode is sensitive over a wider region between 0.92 and 1.65 μm

Exercise 5.2: PIN Heterostructure Photodiodes

a) The capacity of the depletion zone:

$$C_j = S \cdot \frac{\epsilon_0 \epsilon}{W} = \pi \left(\frac{d}{2} \right)^2 \cdot \frac{\epsilon_0 \epsilon}{W}$$

b) The cutoff frequency due to RC time:

$$f_2 = \frac{1}{4R_L C_j} = \frac{W}{R_L \pi d^2 \epsilon_0 \epsilon}$$

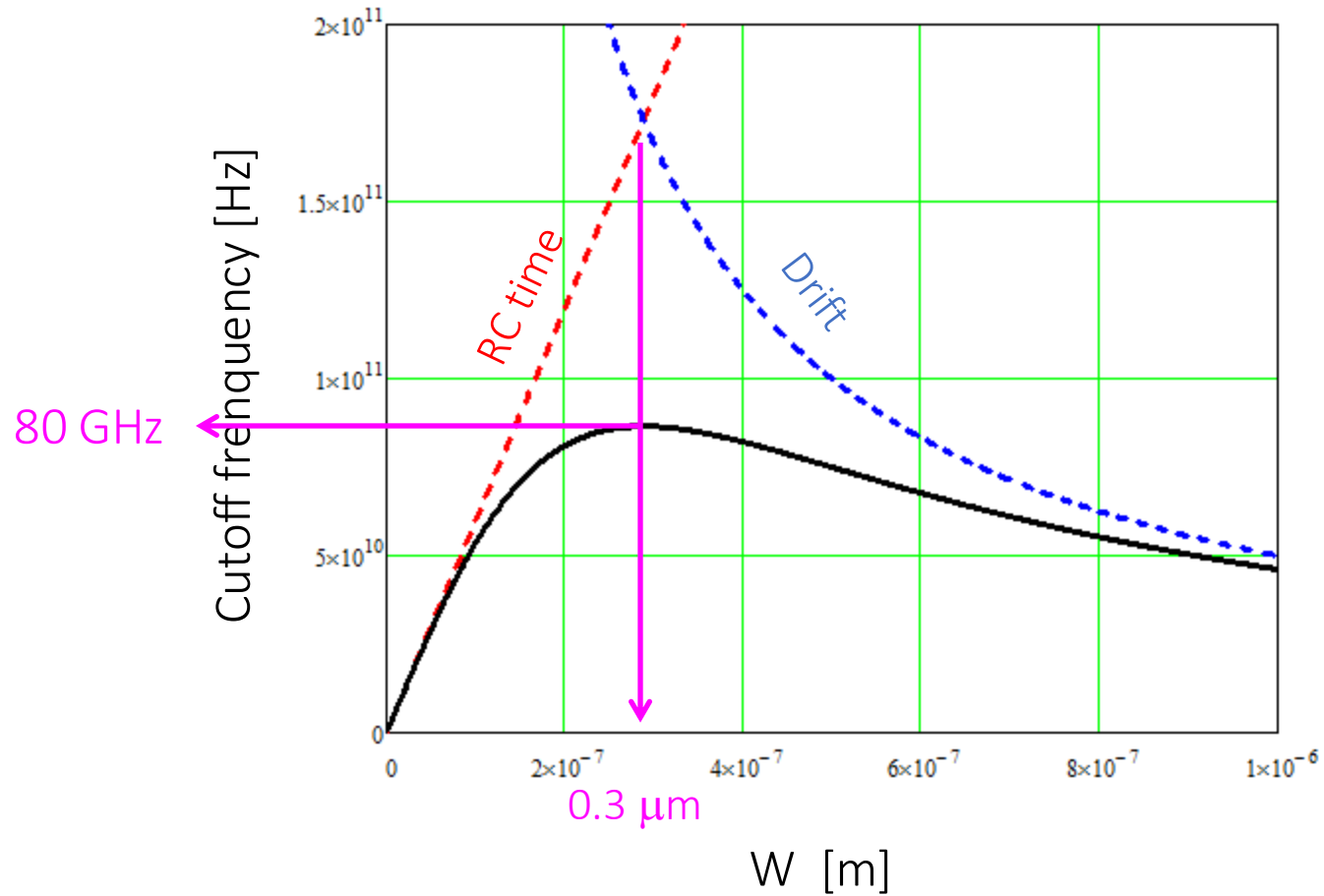
c) The cutoff frequency due to the drift time in the intrinsic zone:

$$f_{2d} = \frac{1}{2\tau_{drift}} \approx \frac{v_{sat}}{2W}$$

Optimal:

$$f_2 = f_{2d} \quad \rightarrow \quad W_{opt} = d \cdot \sqrt{\frac{\pi}{2} v_{sat} R_L \epsilon_0 \epsilon} \cong 0.3 \quad [\mu m]$$

Exercise 5.2: Example



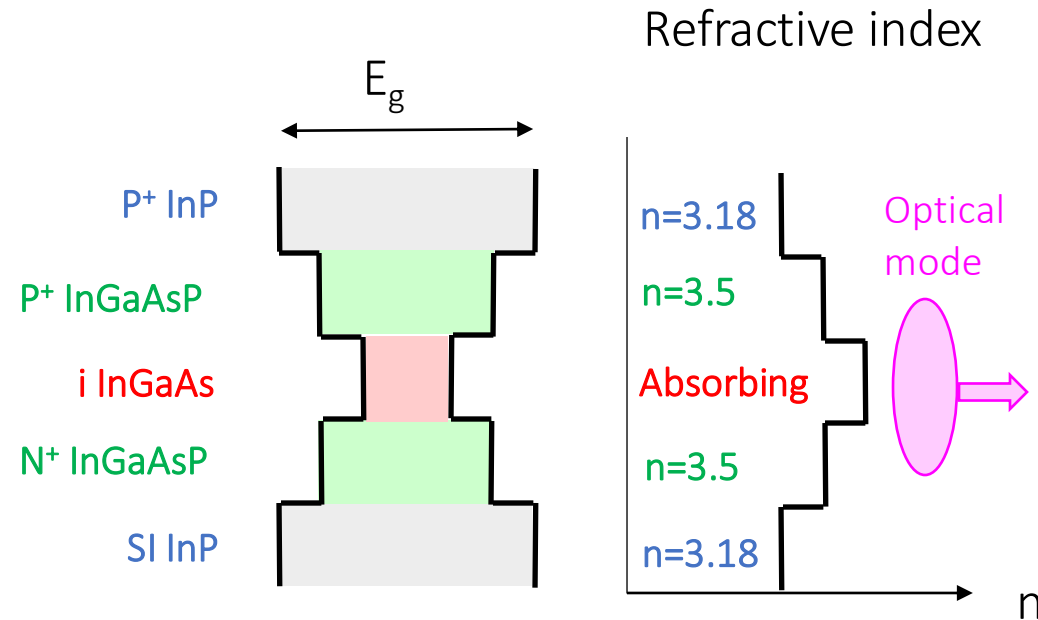
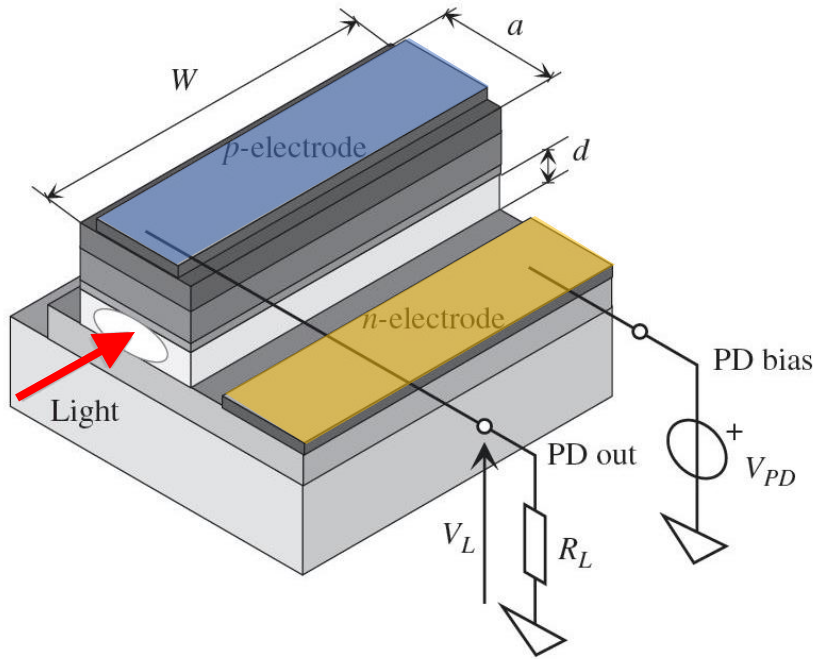
80 GHz

0.3 μm

Thin absorbing layer !!

$$\frac{1}{f_{tot}} = \frac{1}{f_2} + \frac{1}{f_{2d}}$$

Exercise 5.2: Waveguide Photodiode (WG-PD)



G. Ghione, «Semiconductor devices for high-speed optoelectronics», Chap. 4, Cambridge

Large gap \rightarrow lower refractive index \rightarrow waveguide

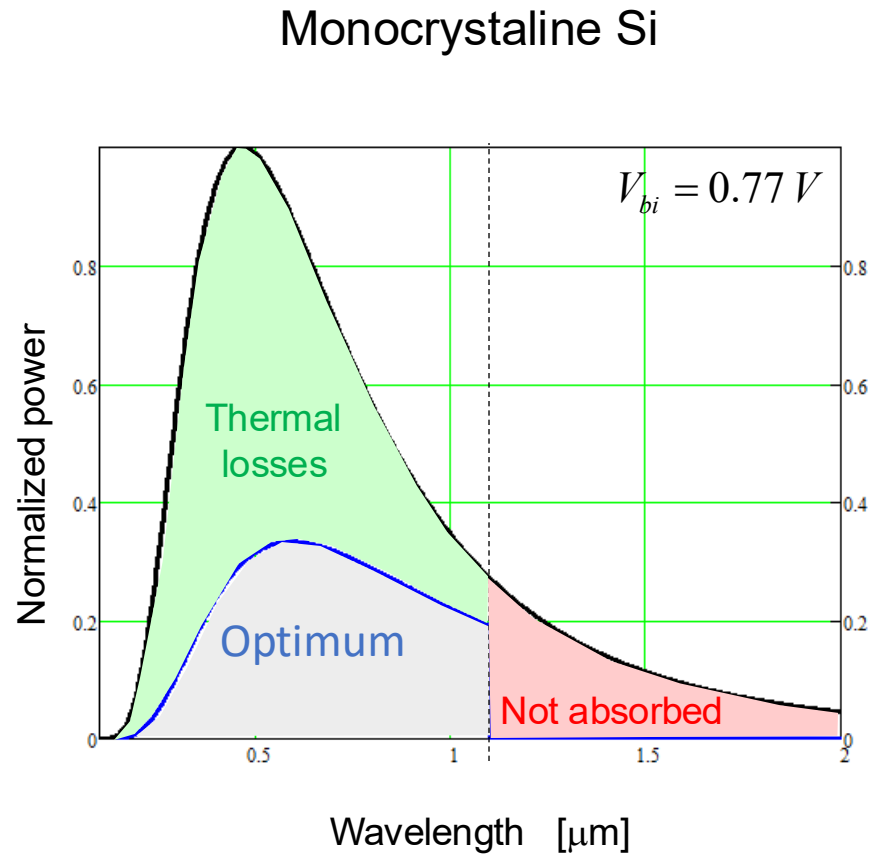
Outline

5.1 High-speed photodiodes and BER

5.2 PIN Heterostructure photodiodes

Homework 5.1: Maximum Efficiency of Solar Cells

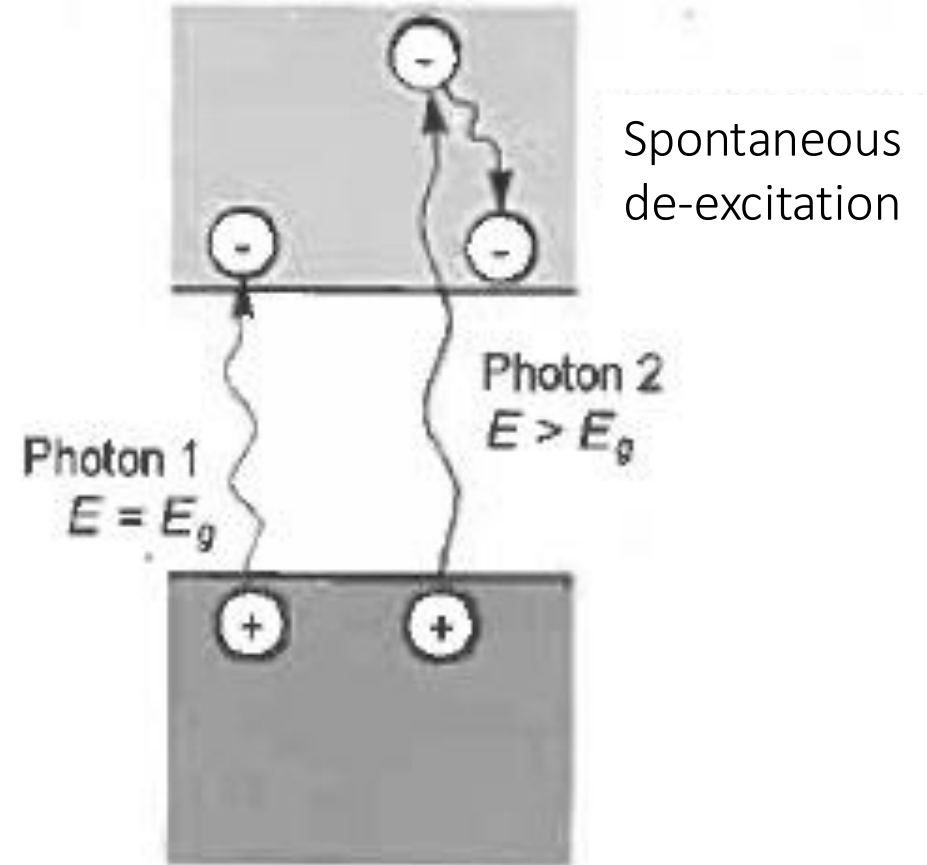
A5.2 Maximum Efficiency of Solar Cells: *which is its origin?*



$\eta_{\text{tot}} = 31\%$

($E_g = 1.1\text{ eV}$)

Thermal losses: we generate «only»
the bandgap energy



A. Labouret, M. Villoz,
« installations photovoltaïques »,
Dunod, 2012

A5.2 Maximum Efficiency of Solar Cells

$$\eta_{sol} \equiv \frac{P_{el}}{P_{opt}} \leq \frac{I_{ph} V_{bi}}{P_{opt}} \cong \frac{\eta q}{h\nu} \cdot V_{bi} \cong \eta \cdot \frac{qV_{bi}}{h(c/\bar{\lambda})}$$

Silicon: $qV_{bi} = 0.77 \text{ eV}$, $\eta = 1$

Sun: $\lambda = 500 \text{ nm}$ (2.5 eV), $(P_{0sun} = 1 \text{ kW/m}^2)$

$$\eta_{sun} \leq 31\% \cong \frac{0.77 \text{ eV}}{2.5 \text{ eV}}$$

$\eta_{sun, typical} \cong 25\%$