

MICRO-523: Optical Detectors

Week Four: Photoemissive detectors (Solutions Ex4)

Claudio Bruschini

Institute of Electrical and Micro Engineering (IEM), School of Engineering (STI)
Ecole polytechnique fédérale de Lausanne (EPFL), Neuchâtel, Switzerland

Based on MICRO-523, P.-A. Besse, 2023

TAs: Samuele Bisi, Kodai Kaneyatsu

The logo of the École Polytechnique Fédérale de Lausanne (EPFL), consisting of the letters 'EPFL' in a bold, red, sans-serif font.

Outline

4.1 Detection limit of PMT

Exercise 4.1: The Detection Limit of a PMT

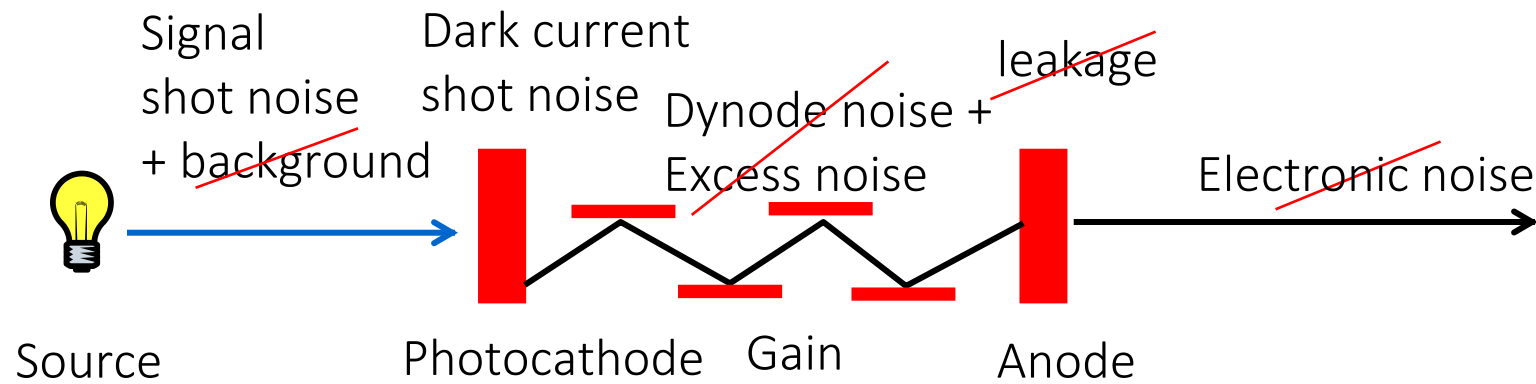
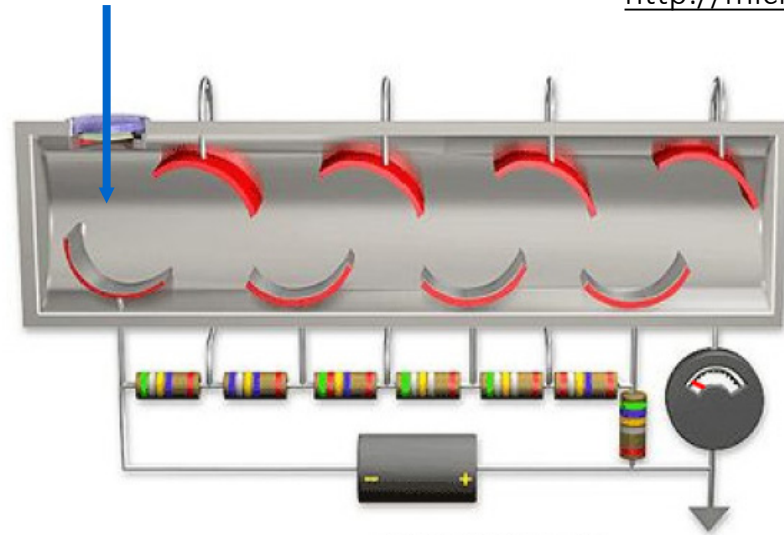
Consider a PMT with the following characteristics:

- Photocathode surface area: 2 cm^2
- Bandwidth: 100 Hz
- Temperature: 300K
- Work function: 1.25 eV
- Quantum Efficiency: 10% at 550nm
- Same sensitivity as the human eye.

- A) What is the NEP if the PMT is limited by the shot noise from the dark current?
(consider the excess noise factor to be equal to 1)
- B) At what distance L does a candle have to be placed in complete darkness to match this NEP ?

Exercise 4.1: Noise analysis

<http://micro.magnet.fsu.edu/primer/flash/photomultiplier>



Exercise 4.1: Detection Limit of a PMT

A) Dark current

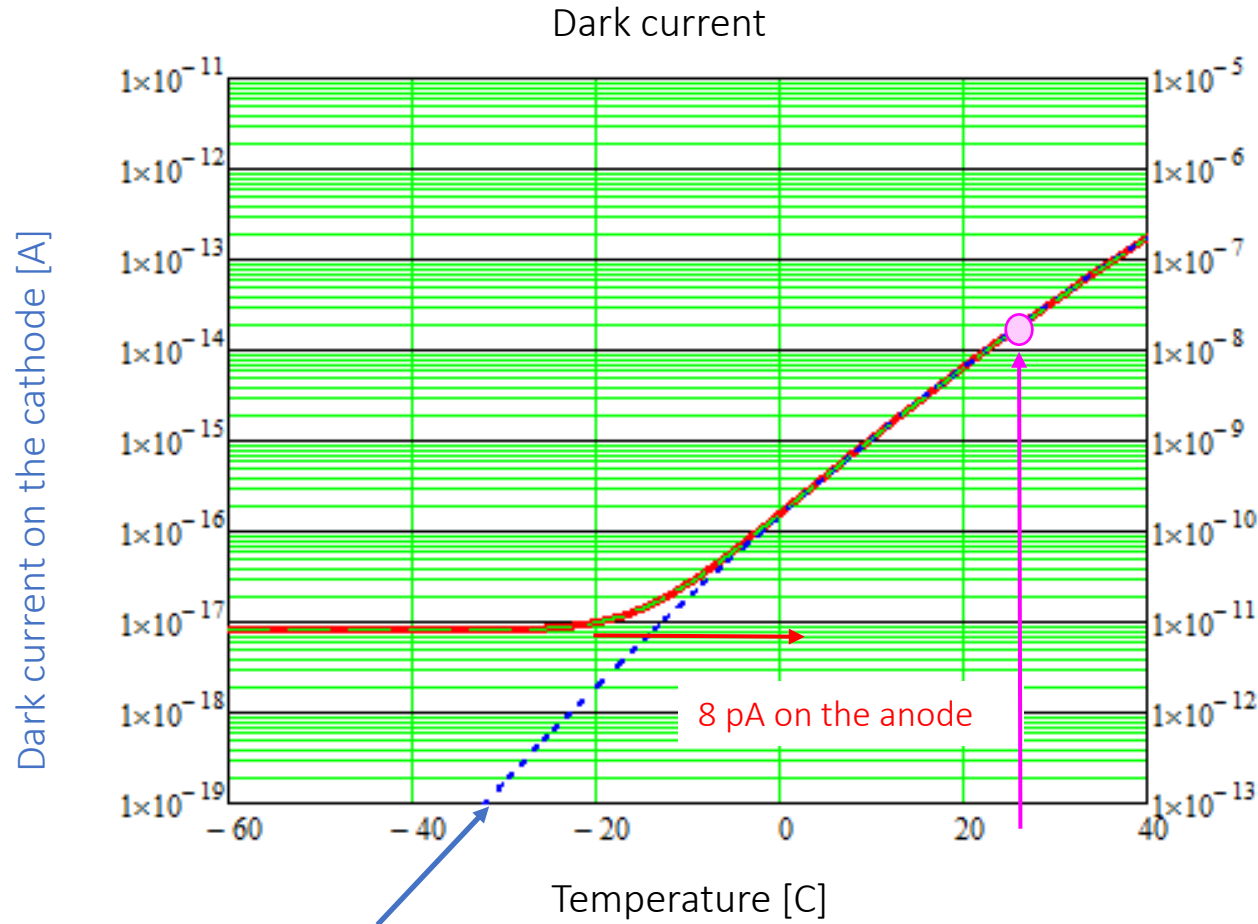
$$I_{dark,cat} = S_d \cdot A_0 T^2 e^{(-W/kT)} = 2 \cdot 120 \cdot 300^2 \cdot e^{\left(-\frac{1.25}{0.0258}\right)} [A] = 20 [fA]$$

Remark: a typical value for the gain is 10^6
→ the dark current on the anode is approximately: 20 [nA]

NEP

$$NEP_{dark} = \frac{\sqrt{\langle \Delta I_{dark,cat}^2 \rangle}}{R_I} \cdot \sqrt{F} = \frac{\sqrt{2qI_{dark,cat}\Delta f}}{\frac{\eta q \lambda}{hc}} \cdot \sqrt{F} = 18 [fW]$$

Exercise 4.1: Detection Limit of a PMT



$$I_{d,cat} = S_d \cdot A_0 T^2 e^{(-W/kT)}$$

$$A_0 = 120 \text{ [A/cm}^2 \text{ K}^2]$$

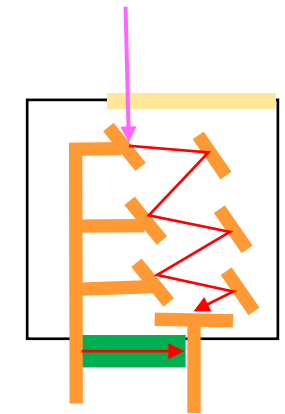
$$S_d = 2 \text{ cm}^2$$

$$W = 1.25 \text{ eV}$$

$$I_{leakage} = 8 \text{ [pA] on anode}$$

$$(G = 10^6)$$

Dark current on the anode [A]



$$2 \cdot Q \cdot I_d \cdot \Delta f$$

$$2 \cdot q \cdot I_d \cdot \Delta f$$

Exercise 4.1: Detection Limit of a PMT

B) Illuminance of a candle at 1 m: = 1 lux

Luminous flux of a candle at 1 m on the detector

$$P_0 [lm] = 1 [lux] \cdot S_d [m^2] \cong \frac{1}{683} \left[\frac{W}{m^2} \right] \cdot 2 \cdot 10^{-4} [m^2] = 290 [nW]$$

To compare with 18 [fW]

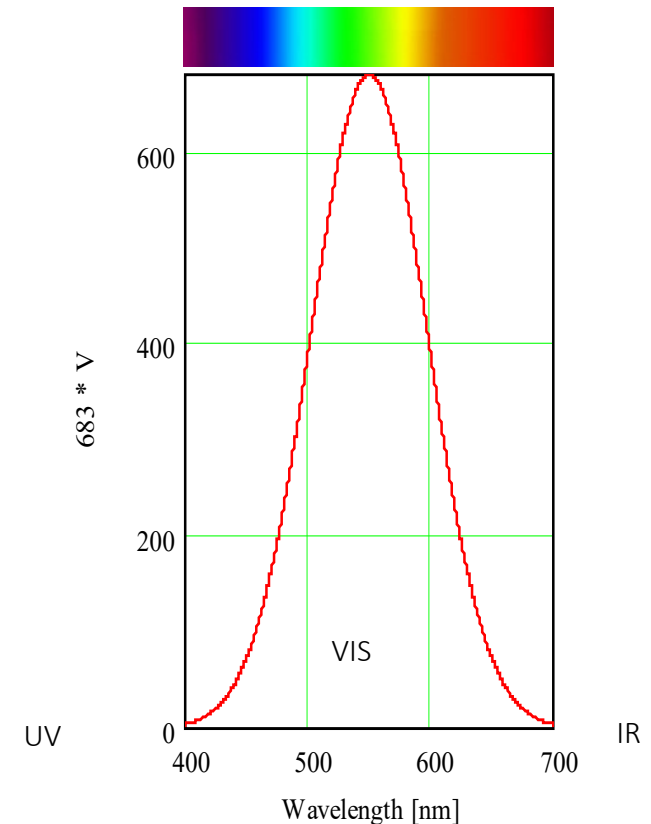
The intensity decreases with the square of the distance →

$$\frac{NEP}{P_0} = \left(\frac{L_0}{L} \right)^2 \quad \text{with} \quad L_0 = 1 \text{ m}$$

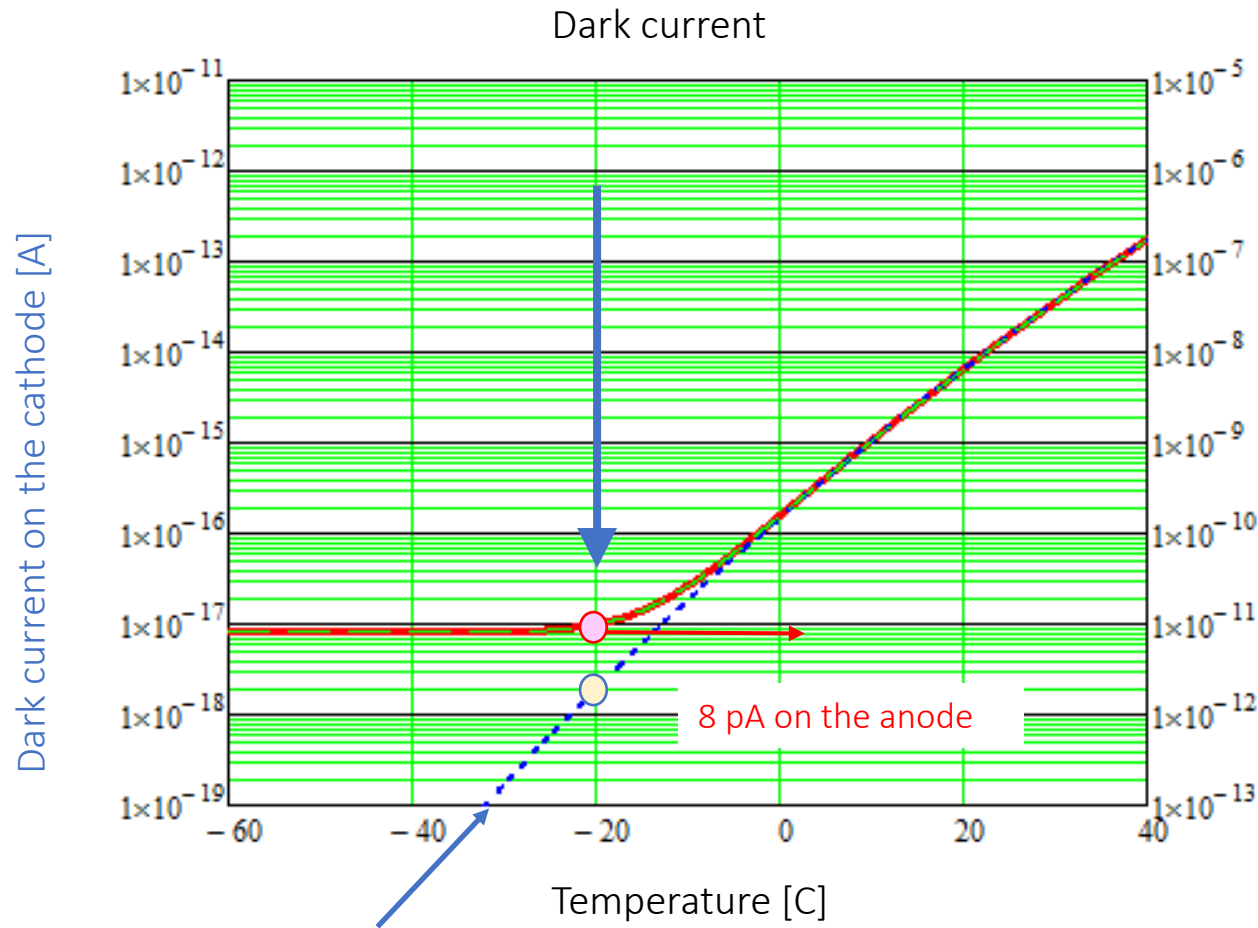


$$L = \sqrt{\frac{P_0}{NEP_{dark}}} [m] \cong 4 \text{ km}$$

Lux = visible intensity per detector surface



Exercise 4.1: Detection Limit of a PMT



$$I_{d,cat} = S_d \cdot A_0 T^2 e^{(-W/kT)}$$

$$A_0 = 120 \text{ [A/cm}^2 \text{ K}^2\text{]}$$

$$S_d = 2 \text{ cm}^2$$

$$W = 1.25 \text{ eV}$$

$$I_{leakage} = 8 \text{ [pA] on anode}$$

$$(G = 10^6)$$

Dark current on the anode [A]

Dark current on the cathode [A]

8 pA on the anode

Exercise 4.1: Detection Limit of a PMT

Shot noise of the signal:

$$\frac{(\eta P)^2}{2 \cdot h\nu \cdot (\eta P) \cdot \Delta f \cdot F} = 1 \quad \Rightarrow \quad NEP_{sig-shot} = \frac{1}{\eta} \cdot (2 \cdot h\nu \cdot \Delta f) \cdot F \cong 0.7 \text{ [fW]}$$

Dark current at T = -20C:

$$I_{dark,cat} = S_d \cdot A_0 T^2 e^{(-W/kT)} = 2 \text{ [aA]} \quad I_{dark,ano} = \langle G \rangle \cdot I_{dark,cat} \cong 2 \text{ [pA]}$$

$$I_{leak,ano} \cong 8 \text{ [pA]}$$

Dark current and NEP at T = -20C:

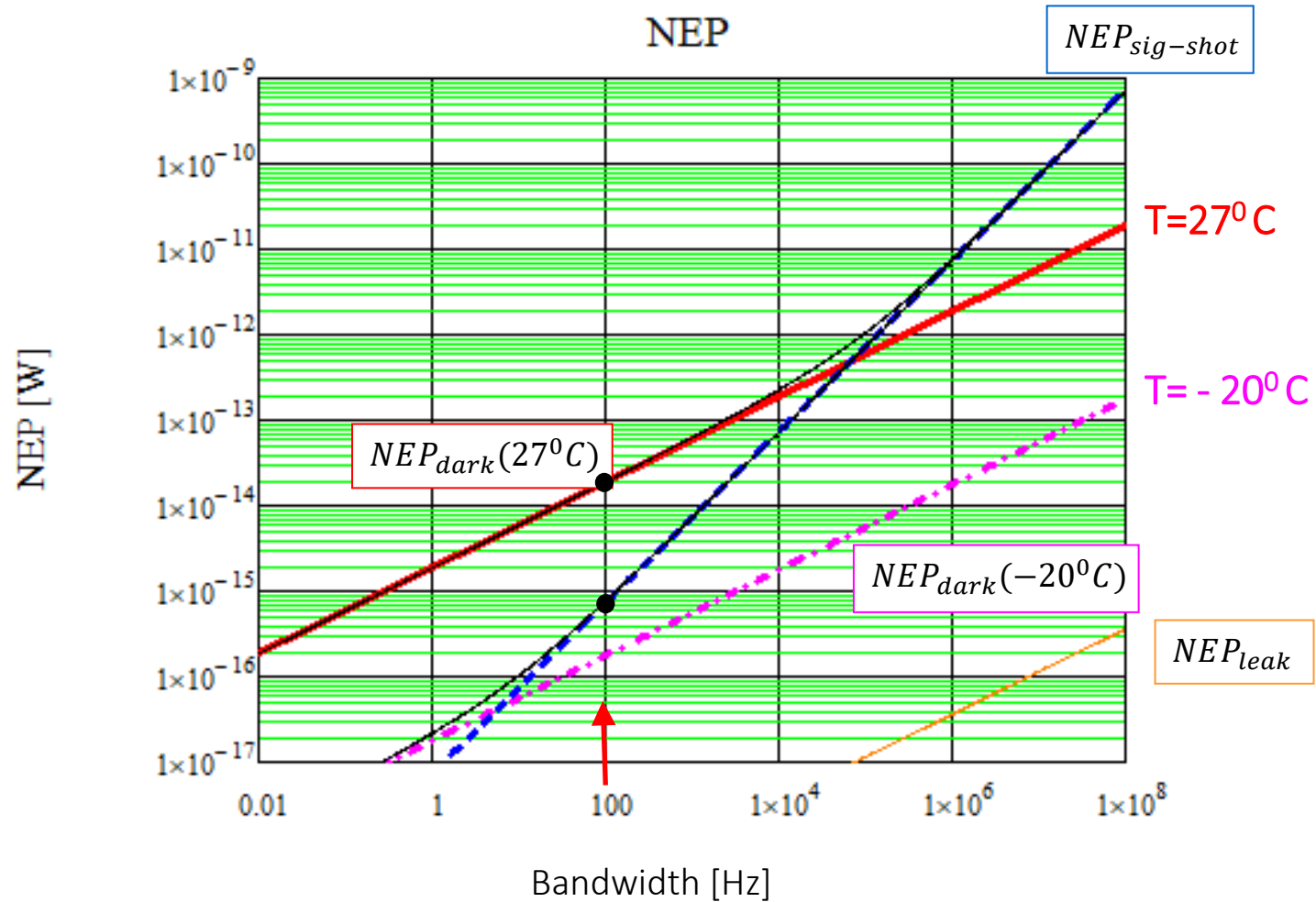
$$\frac{(R_I \cdot P \cdot \langle G \rangle)^2}{2q I_{dark,cat} \langle G \rangle^2 \Delta f \cdot F + 2q I_{leak,ano} \Delta f} = 1 \quad \Rightarrow \quad NEP_{dark} = \frac{\sqrt{2q \left(I_{dark,cat} \cdot F + \frac{I_{leak,ano}}{\langle G \rangle^2} \right) \cdot \Delta f}}{\frac{\eta q}{h\nu}} = 0.18 \text{ [fW]} < NEP_{sig-shot}$$

Detection distance (T = -20C):

$$L = \sqrt{\frac{P_0}{NEP_{sig-shot}}} \text{ [m]} = \sqrt{\frac{S_d \cdot \eta \cdot \lambda}{683 \cdot 2hc \cdot F \cdot \Delta f}} \text{ [m]} \cong 20 \text{ km}$$

See also slide 4.29

Exercise 4.1: Detection Limit of a PMT



$$A = 120 \text{ [A/cm}^2 \text{ K}^2]$$

$$S = 2 \text{ cm}^2$$

$$W = 1.25 \text{ eV}$$

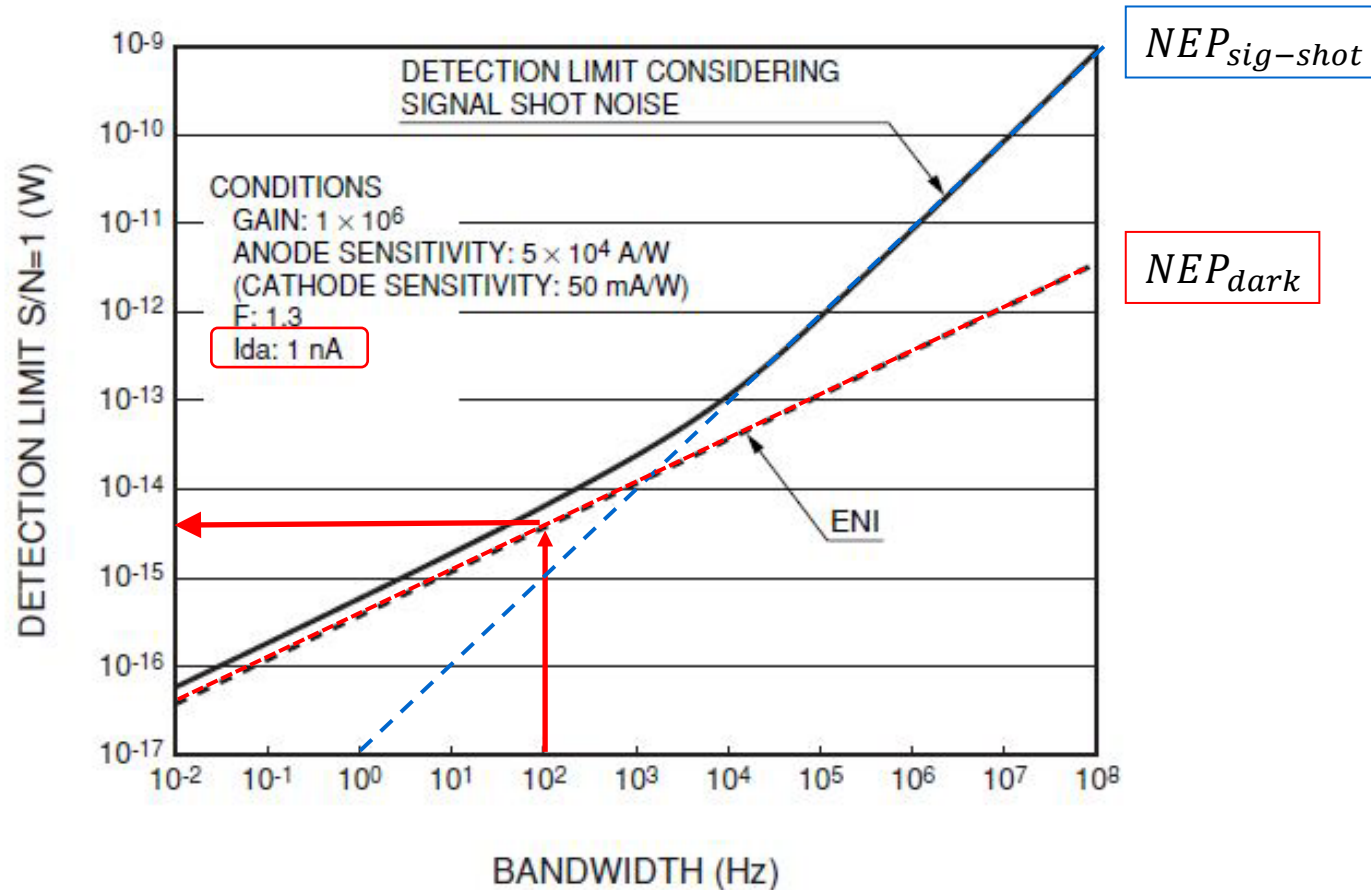
$$I_{leakage} = 8 \text{ [pA] on anode}$$

$$F = 1$$

$$\eta = 10\%$$

$$\lambda = 550 \text{ nm}$$

Exercise 4.1: Hamamatsu Handbook: NEP



Adapted from the Hamamatsu catalog

ENI= "Equivalent Noise Input" = NEP_{dark}