

# MICRO-523: Optical Detectors

## Week Three: Detector Formalism and Noise

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## Outline

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- 3.1 [Quantum efficiency](#)
- 3.2 Responsivity and depth
- 3.3 Optimal internal gain
- 3.4 Light source statistics

## Exercise 3.1: Quantum Efficiency and Detectivity

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Consider a semiconductor photodiode with a band gap  $E_g$  and an ideal quantum efficiency.

Sketch:

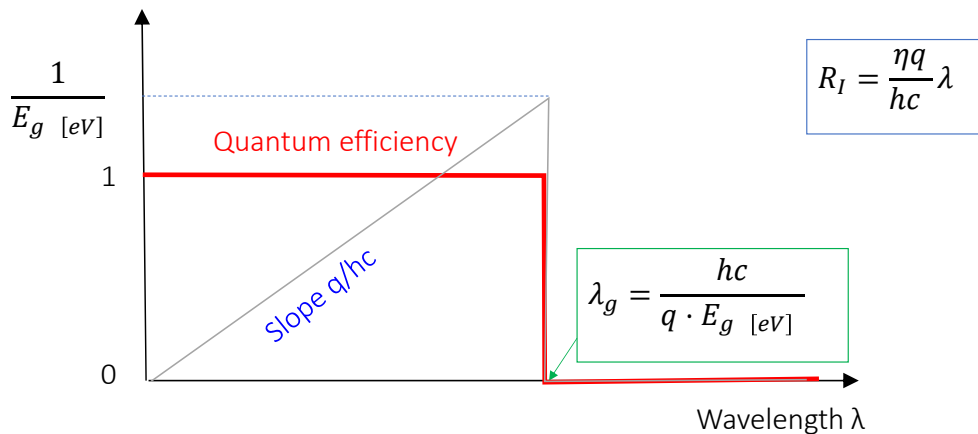
- its quantum efficiency  $\eta$  and
- its responsivity  $R_i$

as a function of the wavelength of the incident photons.

Consider the noise  $N$  to be independent of wavelength.

Sketch its detectivity as a function of its wavelength.

## Exercise 3.1: Quantum Efficiency



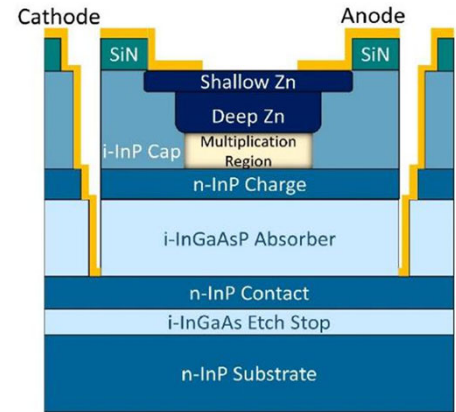
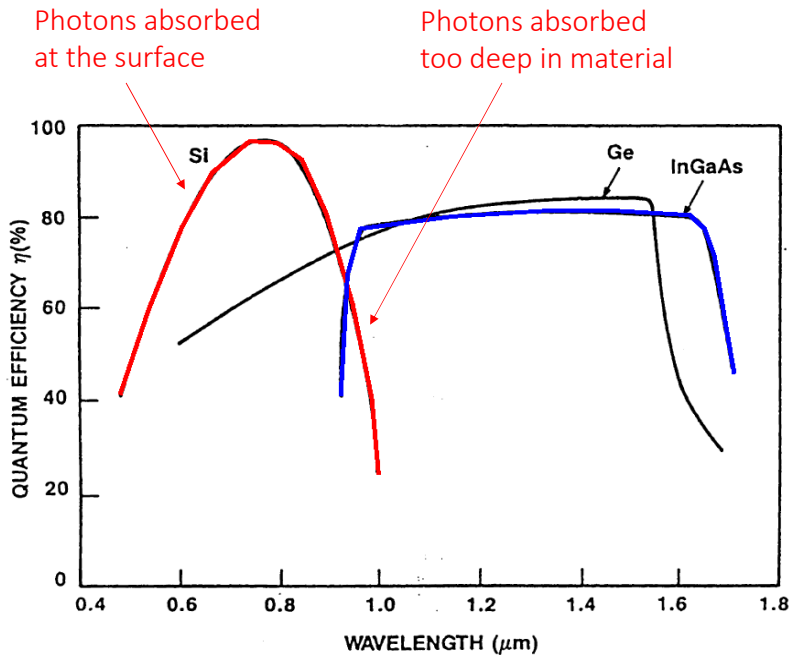
$$\frac{S}{N} = 1 \Rightarrow \frac{R_I \cdot NEP}{N} = 1 \Leftrightarrow D \equiv \frac{1}{NEP} = \frac{R_I}{N} \Leftrightarrow \text{The spectrum of } D \text{ is similar to that of the responsivity.}$$

- L'efficiency quantique idéale est égale à l'unité pour des longueurs d'onde inférieures à la cut-off donnée par  $\lambda_g = hc/E_g$ .
- Elle est nulle au-dessus.
- La responsivity est directement reliée à l'efficiency quantique par la formule  $R_I = \eta q \lambda / hc$ .
- La détectivité a un spectre similaire à la responsivity si le bruit est constant. (Voir, dans le script du cours, les résultats de mesure sur des détecteurs du commerce).

### EN:

- The ideal quantum efficiency is equal to unity for wavelengths below the cut-off given by  $\lambda_g = hc/E_g$ .
- It is zero above.
- Responsivity is directly related to quantum efficiency by the formula  $R_I = \eta q \lambda / hc$ .
- Detectivity has a spectrum similar to responsivity if noise is constant (see course script for measurement results on commercial detectors).

## Exercise 3.1: Quantum Efficiency: Example



Voici quelques exemples d'efficiency quantique pour des photodiodes.

- 1) En silicium (Si), l'efficiency quantique est centrée sur le visible et le proche infrarouge. Aux longueurs d'onde élevées, les photons sont mal ou pas du tout absorbés. L'efficiency quantique diminue et s'annule.

Aux basses longueurs d'onde, les rayons UV sont absorbés en surface, avant de pénétrer la zone de déplétion de la jonction p/n. Les paires électrons/trous générées ne sont pas séparées par un champ électrique et ne provoquent donc pas de courant. L'efficiency quantique disparaît. Au maximum, l'efficiency quantique approche l'unité.

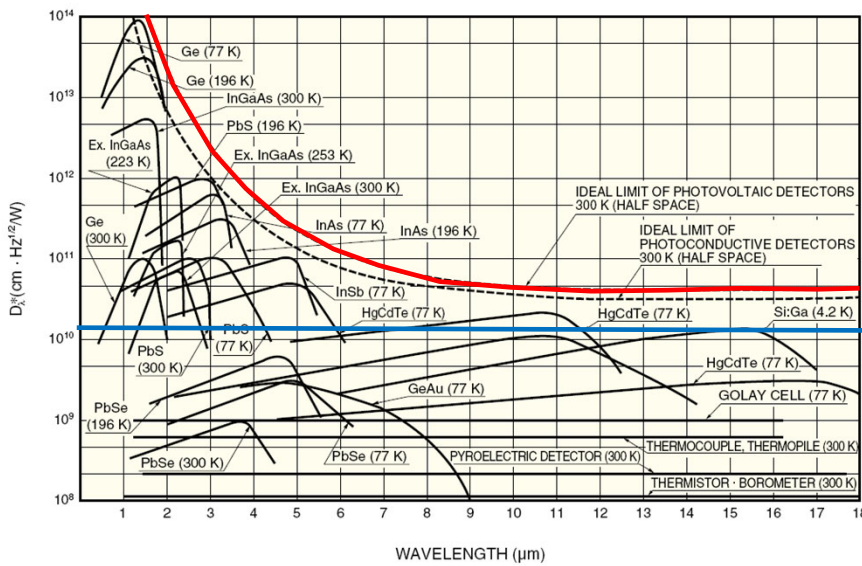
- 2) Le germanium (Ge) et le InGaAs ont les mêmes propriétés, mais ils sont optimaux pour le proche infrarouge (longueurs d'onde « télécom »: 1.3μm et 1.55μm).

### EN:

Here are a few examples of quantum efficiency for photodiodes.

- 1) In silicon (Si), quantum efficiency is centered on the visible and near-infrared. At longer wavelengths, photons are poorly or not at all absorbed. Quantum efficiency decreases and becomes zero. At low wavelengths, UV rays are absorbed at the surface, before penetrating the depletion zone of the p/n junction. The electron/hole pairs generated are not separated by an electric field, and therefore do not generate a current. Quantum efficiency disappears. At maximum, quantum efficiency approaches unity.
- 2) Germanium (Ge) and InGaAs have the same properties, but are optimal for the near infrared (telecom wavelengths: 1.3μm and 1.55μm).

## Exercise 3.1: Spectral Dependence of the Responsivity



Thermocouple (300 K)

Hamamatsu Catalog

Voici quelques exemples de détectivité normée.

On distingue clairement les deux grandes sortes de détecteurs optiques

-Les détecteurs thermiques traduisent directement la puissance reçue en signal, indépendamment de la longueur d'onde. Leur détectivité est donc constante en longueur d'onde. Elle est généralement nettement plus faible que la détectivité des détecteurs photoniques. En infrarouge, les détecteurs thermiques n'ont pas besoin d'être refroidis pour fonctionner. C'est pour cela qu'ils sont fréquemment utilisés dans cette région.

-La détectivité des détecteurs photoniques dépend fortement de la longueur d'onde. Premièrement, au-dessus de la longueur d'onde de cut-off  $\lambda_g$ , un semi-conducteur n'absorbe plus la lumière. Deuxièmement, à même puissance optique un signal bleu contient moins de photons qu'un signal rouge. Il génère donc moins de porteurs. La responsivity et la détectivité baisse en diminuant la longueur d'onde. Le maximum de responsivity et de détectivité est en général proche de la cut-off  $\lambda_g$ . En refroidissant le détecteur on diminue souvent le bruit et on augmente ainsi la détectivité (voir par exemple les courbes PbSe à 25°C et à -77°C).

### EN:

Here are a few examples of standardized detectivity.

There are two main types of optical detectors

- Thermal detectors translate received power directly into a signal, irrespective of wavelength. Their detectivity is therefore constant over wavelength. They are generally much weaker than photonic detectors. In infrared, thermal detectors do not need to be cooled to operate. This is why they are frequently used in this region.

- The detectivity of photonic detectors is highly wavelength-dependent. Firstly, above the cut-off wavelength  $\lambda_g$ , a semiconductor no longer absorbs light. Secondly, at the same optical power, a blue signal contains fewer photons than a red signal. It therefore generates fewer carriers. Responsivity and detectivity decrease with decreasing wavelength. Maximum responsivity and detectivity are generally close to the  $\lambda_g$  cut-off. Cooling the detector often reduces noise and increases detectivity (see, for example, the PbSe curves at 25°C and -77°C).

## Exercise 3.2: Responsivity and Depth

The absorption coefficient of silicon can be approximated as  $\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$

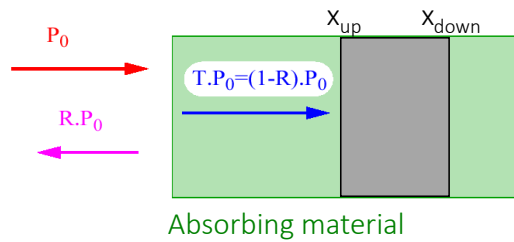
Consider two detectors and two wavelengths ( $\lambda=450\text{nm}$  and  $\lambda=600\text{nm}$ ).

The first detector is sensitive between  $x_{1up} = 0.05\mu\text{m}$  and  $x_{1down} = 0.3\mu\text{m}$ .

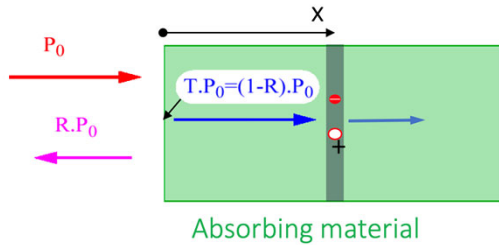
The second detector is sensitive between  $x_{2up} = 0.9\mu\text{m}$  and  $x_{2down} = 4\mu\text{m}$ .

The reflection coefficient is 10%.

Calculate the responsivity  $R_i$  and the quantum efficiency of both detectors at the abovementioned wavelengths.



## Exercise 3.2: Absorption and Generation Rate



R= reflection coefficient

T= transmission coefficient

$g(x)$ = generation rate of carriers

$$g(x) = \frac{P(x)}{h\nu} \cdot \alpha = \frac{P_0}{h\nu} \cdot (1 - R) \cdot e^{-\alpha x} \cdot \alpha \quad \left[ \frac{1}{\text{cm} \cdot \text{s}} \right]$$

Considérons un rayon lumineux de puissance  $P_0$ . En pénétrant la matière, une partie de cette puissance est réfléchiée ( $R \cdot P_0$ ). Le reste est transmis et absorbé dans le matériel. Lors de sa propagation, l'intensité diminue suivant une courbe exponentielle (voir page précédente).

$$P(x) = (1 - R) \cdot P_0 \cdot e^{-\alpha x}$$

Chaque photon absorbé génère une paire de porteurs (électron/trou). À la profondeur  $x$ , le **taux de génération des porteurs** par seconde et par distance est donné par le produit du nombre de photons par seconde à cette profondeur ( $P(x)/h\nu$ ) avec la probabilité  $\alpha$  qu'ils soient absorbés par centimètre de propagation.

### EN:

Consider a light beam of power  $P_0$ . As it penetrates matter, part of this power is reflected ( $R \cdot P_0$ ). The rest is transmitted and absorbed by the material. As it propagates, the intensity decreases following an exponential curve (see previous page).

$$P(x) = (1 - R) \cdot P_0 \cdot e^{-\alpha x}$$

Each absorbed photon generates a pair of carriers (electron/hole). At depth  $x$ , the rate of carrier generation per second and per distance is given by the product of the number of photons per second at that depth ( $P(x)/h\nu$ ) and the probability  $\alpha$  that they will be absorbed per centimeter of propagation.

## Exercise 3.2: Quantum Efficiency

For photodetectors the quantum efficiency is defined as follows:

$$\eta = \frac{\text{number of optically generated \& collected electrons}}{\text{number of incident photons}}$$

It takes into account:

- reflection
- absorption
- recombination
- and electron scattering

It does not consider:

- internal gain
- avalanche phenomena, ...

Relationship with responsivity

$$\eta = \frac{I_s/q}{P_s/h\nu}$$



$$R_I = \frac{I_s}{P_s} = \frac{\eta q}{h \nu}$$

Les détecteurs photoniques transforment un photon en une paire électron/trou. Il est donc judicieux de définir l'efficacité de cette transformation.

L'efficacité quantique est définie comme le nombre de paires électrons/trous générées et récoltées sous forme de signal électrique divisé par le nombre de photons contenu dans le signal optique. Le nombre de paires électrons/trous est relié au courant électrique  $I_s$ , le nombre de photons incidents est relié à la puissance du signal optique  $P_s$ , donc l'efficacité quantique est proportionnelle à la responsivité en courant  $R_I$ .

Le nombre de photons est calculé avant l'entrée dans le détecteur, donc la réflexion en surface ou l'absorption dans des couches protectrices réduisent l'efficacité quantique en réduisant la puissance optique tombant réellement sur la zone sensible. De même, le nombre de paires électrons/trous est calculé à partir du courant récolté, donc la recombinaison ou des pertes par scattering réduisent l'efficacité quantique.

Certains détecteurs ont un gain interne (effet avalanche ou photoconduction), l'efficacité quantique exclut ces phénomènes d'amplification. Elle se base sur la génération de paires électrons/trous primaires. Avec cette définition elle est donc toujours inférieure à l'unité. L'efficacité quantique est un paramètre qui permet de juger de la qualité intrinsèque d'un détecteur. Idéalement elle tend vers l'unité. C'est une limite physique pour les détecteurs photoniques.

### EN:

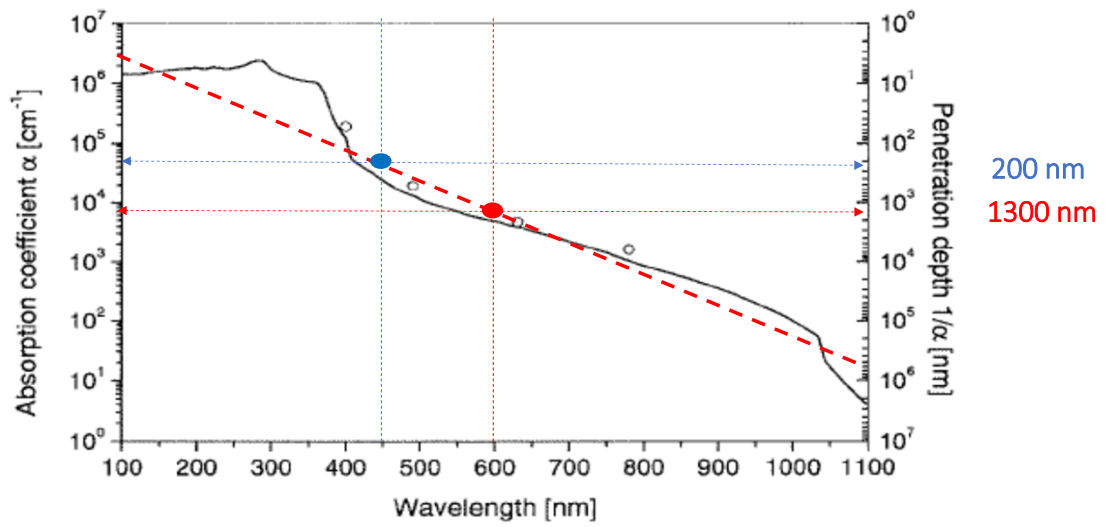
Photonic detectors transform a photon into an electron/hole pair. It therefore makes sense to define the efficiency of this transformation.

Quantum efficiency is defined as the number of electron/hole pairs generated and collected as an electrical signal, divided by the number of photons in the optical signal. The number of electron/hole pairs is related to the electric current  $I_s$ , the number of incident photons is related to the power of the optical signal  $P_s$ , so quantum efficiency is proportional to current responsivity  $R_I$ .

The number of photons is calculated before entering the detector, so surface reflection or absorption in protective layers reduce quantum efficiency by reducing the optical power actually falling on the sensitive area. Similarly, the number of electron/hole pairs is calculated from the collected current, so recombination or scattering losses reduce quantum efficiency.

Some detectors have an internal gain (avalanche effect or photoconduction), but quantum efficiency excludes these amplification phenomena. It is based on the generation of primary electron/hole pairs. With this definition, it is always less than unity. Quantum efficiency is a parameter used to judge the intrinsic quality of a detector. Ideally, it tends towards unity. This is a physical limit for photonic detectors.

## Exercise 3.2: Absorption by silicon



Approximation:  $\alpha_{\text{cm}^{-1}}(\lambda_{\mu\text{m}}) \cong 10^{7.2-5.5\lambda}$

## Exercise 3.2: Absorption, Responsivity and Quantum Efficiency

Absorption:  $\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$

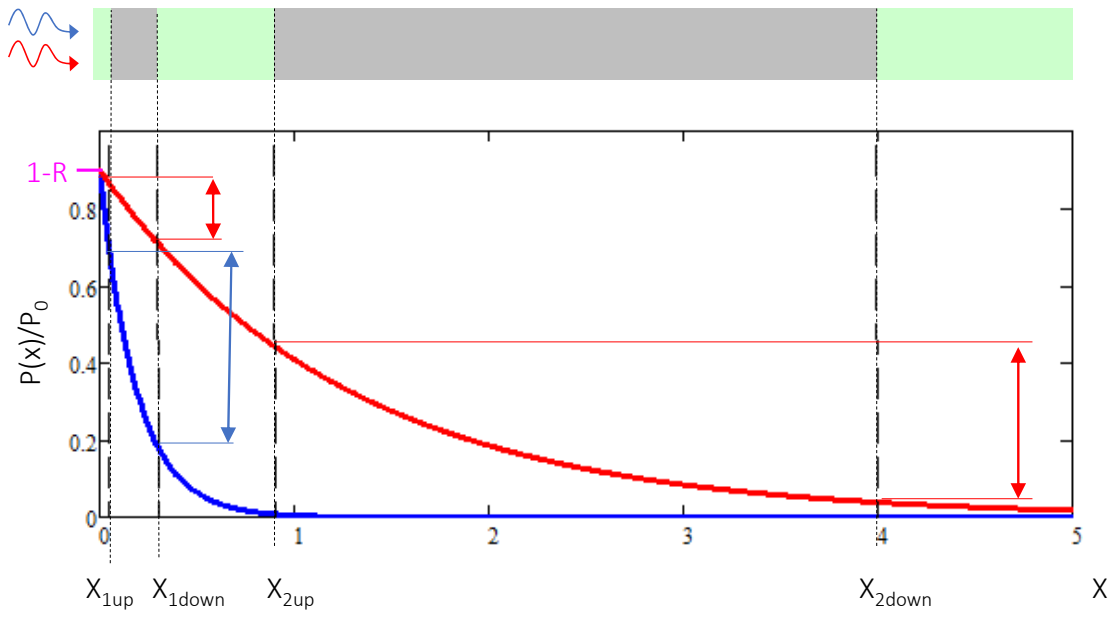
Generation rate:  $g(x) = \frac{P(x)}{h\nu} \cdot \alpha = \frac{P_0}{h\nu} \cdot (1-R) \cdot e^{-\alpha x} \cdot \alpha \quad \left[ \frac{1}{cm \cdot s} \right]$

Photocurrent:  $I = q \cdot \int_{x_{up}}^{x_{down}} g(x) \cdot dx = \frac{q}{h\nu} \cdot (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}}) \cdot P_0$

Responsivity:  $R_I = \frac{I}{P_0} = \frac{q}{h\nu} \cdot (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}})$

Quantum efficiency  $\eta = R_I \cdot \frac{h\nu}{q} = (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}})$

## Exercise 3.2: Interpretation: Quantum Efficiency



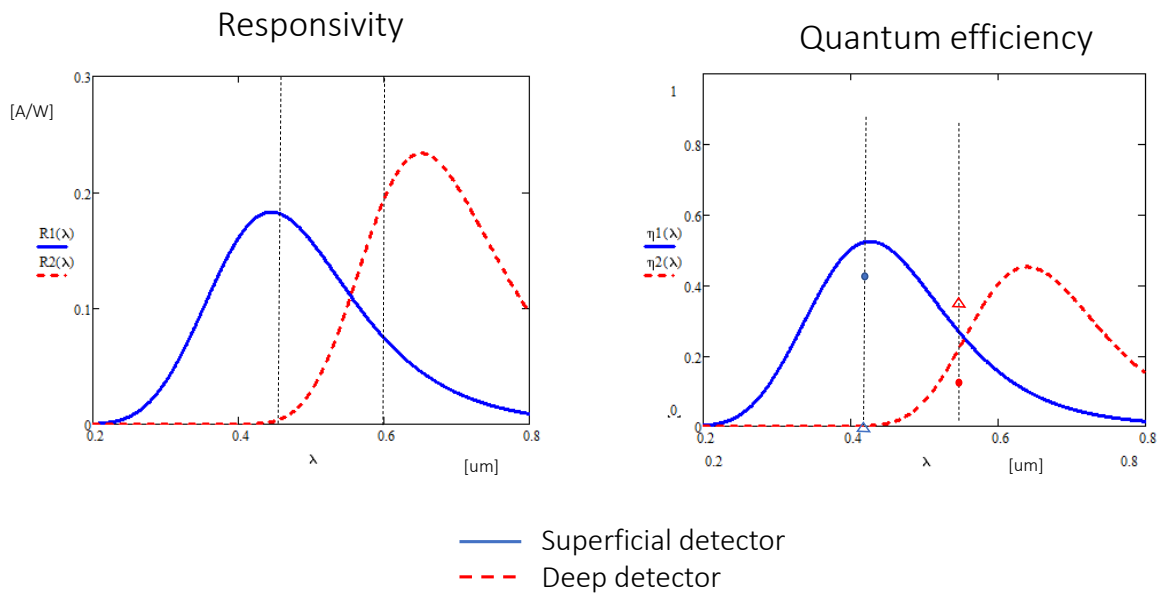
## Exercise 3.2: Numerical Values

$$\alpha(450) = 5.3 \cdot 10^4 \text{ cm}^{-1}$$

$$\alpha(600) = 7.9 \cdot 10^3 \text{ cm}^{-1}$$

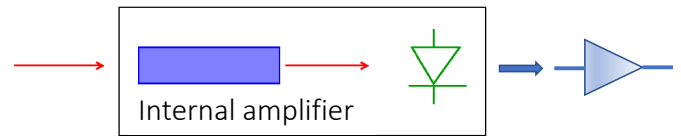
	"Superficial" detector		"Deep" detector	
	$R_i$ [A/W]	$\eta$ %	$R_i$ [A/W]	$\eta$ %
$\lambda=450\text{nm}$	0.183	51 %	0.003	0.8 %
$\lambda=600\text{nm}$	0.074	15.5 %	0.194	40 %
$R_{450}/R_{600}$ $\eta_{450}/\eta_{600}$	2.5	3.3	0.014	0.02

## Exercise 3.2: Responsivity and Quantum Efficiency



## Exercise 3.3: Optimal Internal Gain

We would like to detect an optical signal with  $P_0=25$  pW using a detector with a variable internal gain  $G$  (for example an avalanche photodiode). Its responsivity is  $R_i=0.4$  A/W, its bandwidth is  $\Delta f=1$  MHz, and its excess noise factor is  $F=G^{0.3}$ .



Determine as a function of its gain  $G$ :

- The amplified photocurrent  $I_{\text{sig}}(G)$ .
- The shot noise of this current.

The electronics generate an rms noise of  $\Delta I_{\text{el}}=100$  pA.

- Calculate the total noise  $\Delta I_{\text{tot}}$  as a function of the gain.
- Determine the signal to noise ratio as a function of gain.
- What is the optimal gain  $G_{\text{opt}}$ ?

## Exercise 3.3: Internal Gain (1)

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Primary photocurrent:  $I_0 = R_I \cdot P_0 = 10 \text{ pA}$

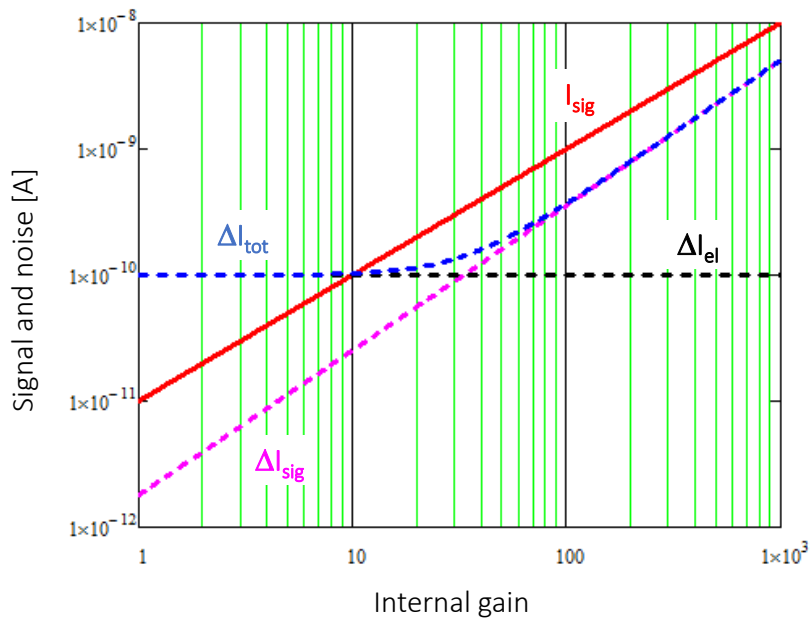
Amplified photocurrent:  $I_{sig}(G) = G \cdot I_0$

Shot noise of the amplified signal:  $\Delta I_{sig}(G) = \sqrt{2qI_0\Delta f \cdot G^2 \cdot F(G)}$

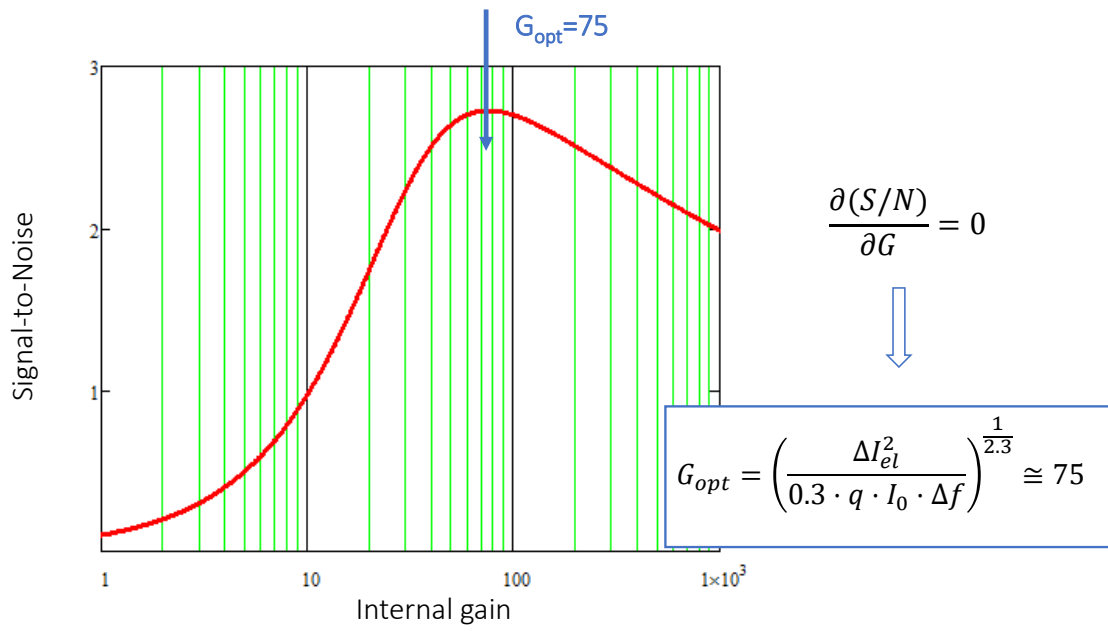
Total noise:  $\Delta I_{tot} = \sqrt{\Delta I_{el}^2 + \Delta I_{sig}^2(G)}$

Signal-to-Noise ratio: 
$$S/N = \frac{G \cdot I_0}{\sqrt{\Delta I_{el}^2 + 2qI_0\Delta f \cdot G^2 \cdot G^{0.3}}}$$

### Exercise 3.3: Internal Gain (2)



### Exercise 3.3: Internal Gain (3)



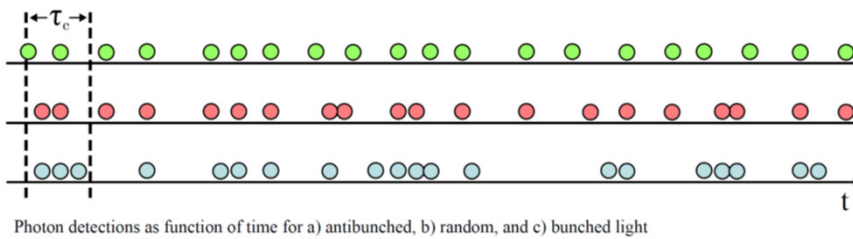
## Exercise 3.4: Light sources statistics

### Questions

- Which kind of light sources exist?
- How are their statistical emission properties?
- *Recap: shot noise*

$$N^2(f) = \frac{\langle \Delta I_{shot}^2 \rangle}{\Delta f} = 2q \cdot |I|$$

$$N^2(f) = \frac{\langle \Delta P_{shot}^2 \rangle}{\Delta f} = 2h\nu \cdot \langle P \rangle$$



By Ajbura - Vectorised version of File:Photon bunching.png, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=73299604>

Richard P. Feynman, Physics Nobel Prize:

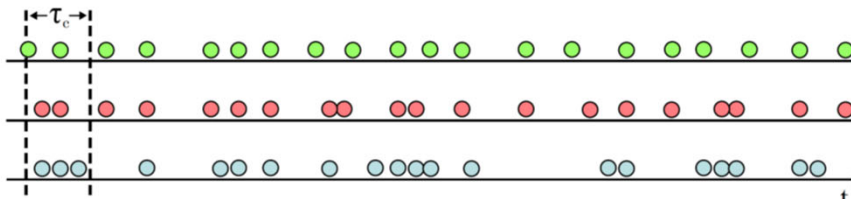
*“If you want to master something, teach it.”*

## Exercise 3.4: Poisson Distribution vs. Light Sources

- **Non-classical light: Sub-Poissonian -> antibunched (anticorrelated)**
- **Coherent light source (Laser): Poissonian, random spacing (uncorrelated)**
- **Thermal Light: Super-Poissonian, Bose-Einstein distribution with zero counts as most probable count (bunched, positively correlated)**

However, in practice it defaults to Gaussian due to the very low coherence time,  $O(\text{ps})$ , and the corresponding experimental difficulties. Experimentally one can use pseudothermal light\*.

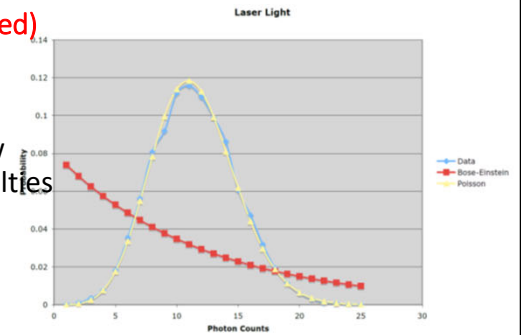
<https://demonstrations.wolfram.com/PhotonNumberDistributions/>



Photon detections as function of time for a) antibunched, b) random, and c) bunched light

[http://physics.gu.se/~tfkhj/lecture\\_X\\_differential\\_transmission-2.pdf](http://physics.gu.se/~tfkhj/lecture_X_differential_transmission-2.pdf)

[https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport\\_0.pdf](https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport_0.pdf)



By Ajbura - Vectorised version of File:Photon bunching.png, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=73299604>

\*E.g. scattering of a laser beam on a rotating ground glass disc

You might remember that some sources have non-Poissonian distributions, such as thermal ones (super-Poissonian -> bunched photon arrival times, resulting in a Bose-Einstein distribution with zero counts as most probable value, but in practice difficult to observe due to the very low coherence times and the corresponding experimental difficulties). In the case of coherent light sources (e.g. laser), the resulting Poissonian distribution can be derived directly from first principles.

## Exercise 3.4: Light sources statistics

$\bar{n}$  = average photon number

Non-classical light: Sub-Poissonian  $< \sqrt{\bar{n}}$

Coherent light source (Laser): Poissonian

$$P(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \sigma = \sqrt{\bar{n}}$$

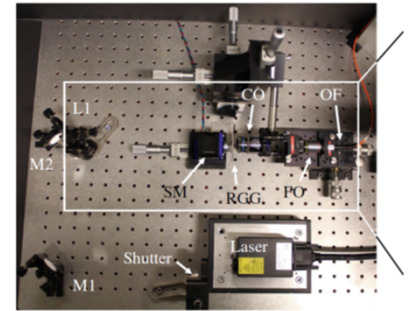
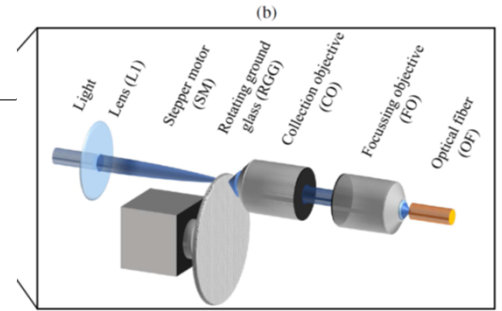
For large photon numbers, the relative fluctuations  $\sigma/\bar{n}$  tend to 0

Thermal Light: Super-Poissonian, Bose-Einstein distribution

$$P(n) = (1 - e^{-\hbar\omega/k_B T}) e^{-n\hbar\omega/k_B T} = \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}}, \bar{n} = (e^{\hbar\omega/k_B T} - 1)^{-1},$$

$$\sigma = \sqrt{\bar{n}^2 + \bar{n}} \text{ (for } T \ll \tau_c) > \sqrt{\bar{n}}$$

For large photon numbers, the relative fluctuations  $\sigma/\bar{n}$  tend to 1



Pseudothermal light source

- Advanced Lab Course (F-Praktikum), Exp. 45, *Photon Statistics*, v. Aug. 21 2017
- [http://physics.gu.se/~tfkhj/lecture\\_X\\_differential\\_transmission-2.pdf](http://physics.gu.se/~tfkhj/lecture_X_differential_transmission-2.pdf)
- [https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport\\_0.pdf](https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport_0.pdf)

- T. Stagner et al., *Step-by-step guide to reduce spatial Coherence of laser light using a rotating ground glass diffuser*, OSA Applied Optics 56 (2017).

Thermal light: (B45E.pdf): For large average photon numbers  $\bar{n}$ , the quantum mechanical Bose-Einstein distribution becomes identical to the Boltzmann distribution (classical limit) (= exponential like in the previous slide...).