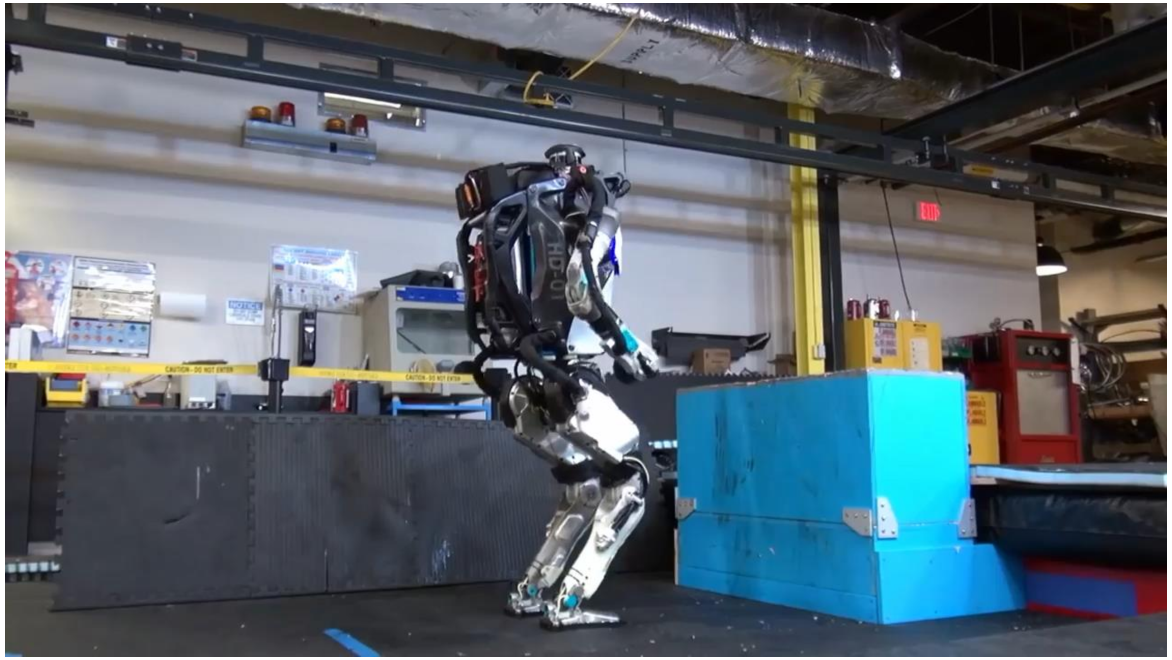
Legged Robots: Practice Sessions

Week 1



Plan

Fundamentals (ungraded)

- **W1-3** (09.09, 16.09, 23.09)
  - Introduction + double pendulum kinematics and dynamics
  - Jacobian (Cartesian PD + Force Control)
  - Inverse Kinematics (compare with force control)
  - Single-leg hopping

Mini-Project 1

- **W4-6** (30.09, 07.10, 14.10)
  - Model-based control of a Quadruped
    [MP1 Report – 15% of grade]

Mini-Project 2

- **W7-14** (28.10-16.12)
  - Quadruped CPG Trot
  - Quadruped Locomotion Project (CPGs, Deep RL)
    [MP2 Report – 35% of grade]

Mini-Project Preview

Amplitude:

$$\dot{r}_i = \alpha(\mu - r_i^2)r_i$$

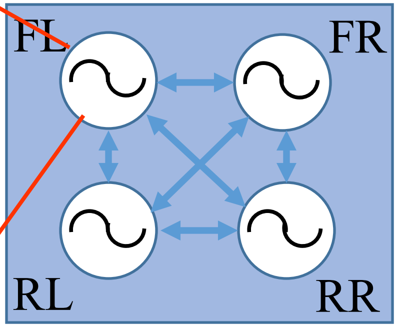
Phase:

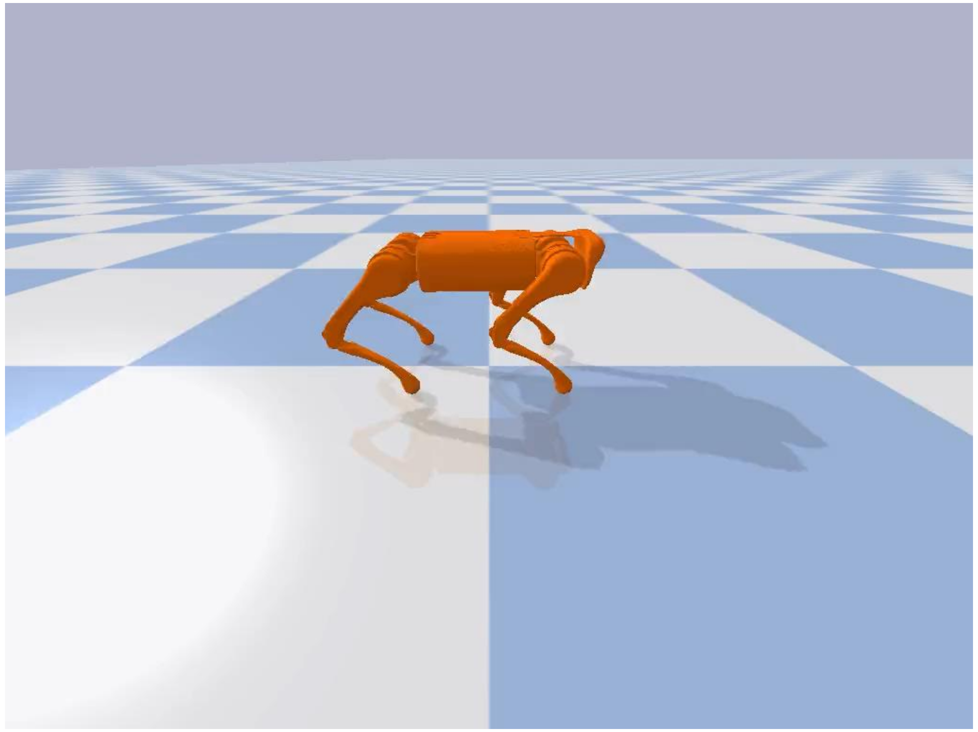
$$\dot{\theta}_i = \omega_i + \sum_{j=0}^{3} r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij})$$

Output:

$$x_{\text{foot}} = -d_{step} r_i \cos(\theta_i)$$

$$z_{\text{foot}} = \begin{cases} -h + g_c \sin(\theta_i) & \text{if } \sin(\theta_i) > 0 \\ -h + g_p \sin(\theta_i) & \text{otherwise} \end{cases}$$





Practical Information

- Students should be present during the practical session 13:15-15:00
- Course is given in Python
- First 3 weeks ungraded assignments (important foundations!)
- Mini-Projects: Groups of 3
- Two graded project deliverables:
  - Report
  - Code
  - Videos
- One quadruped locomotion "competition" on 16.12
- Please ask questions on Moodle!

AI Policy

- Use of AI for the projects is allowed (optional)
- Explicit declaration about AI use: (helpful insights for us!)
  - How you prompt it
  - Where it's been helpful
  - Where it hasn't been

- When using AI for coding: modify the individual functions (not the full codebase)
  - TA support might otherwise be limited

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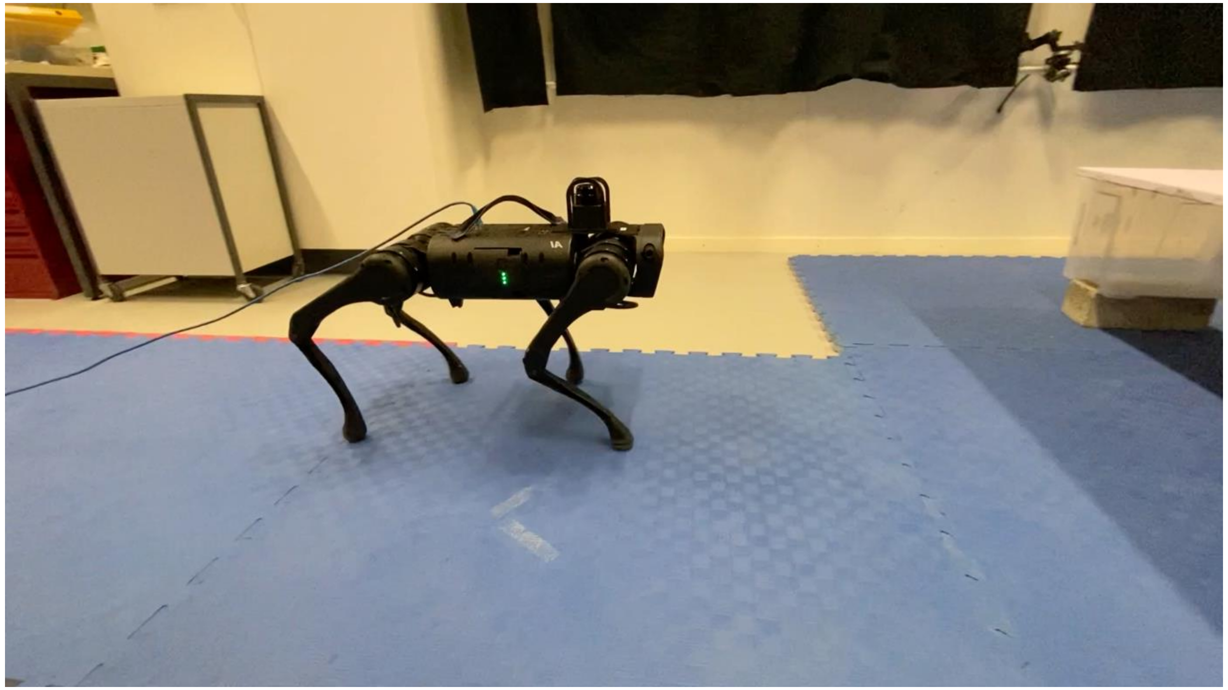
Questions?



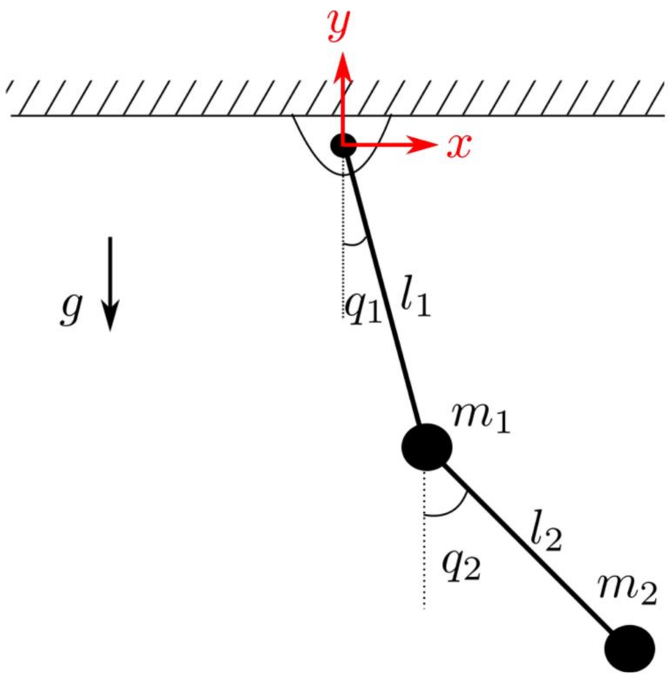
W1-3 Fundamentals

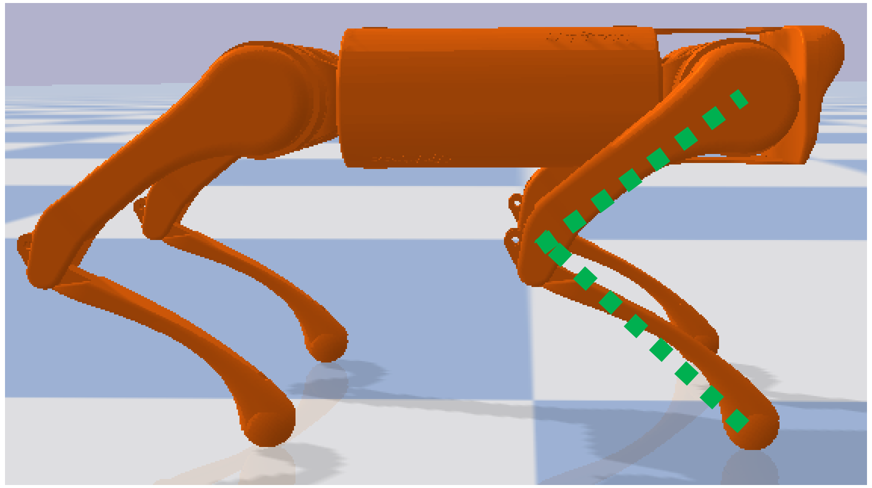
- Part 1: double pendulum kinematics and dynamics
- Part 2: Jacobian (Cartesian PD + Force Control)
- Part 3: Inverse Kinematics (compare with force control)
- Part 4: Single-leg hopping

W1-3 Fundamentals

- **Part 1: double pendulum kinematics and dynamics**
- Part 2: Jacobian (Cartesian PD + Force Control)
- Part 3: Inverse Kinematics (compare with force control)
- Part 4: Single-leg hopping

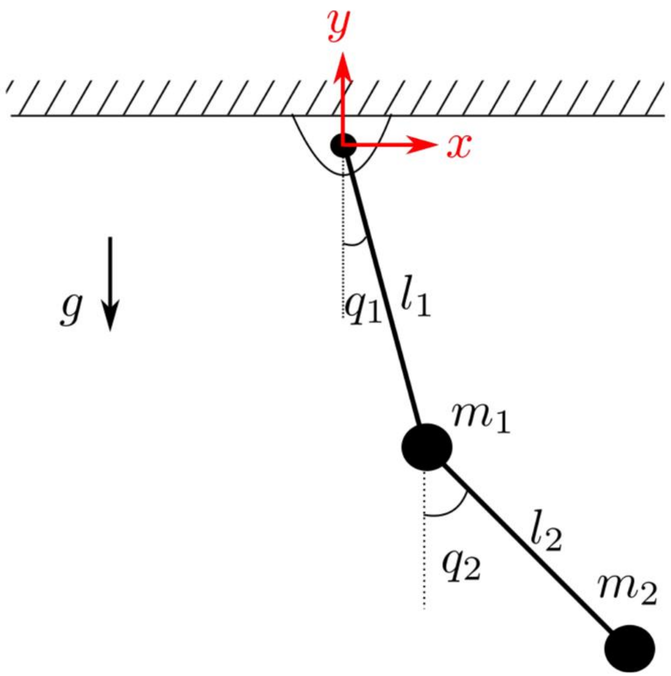
Why the double pendulum?

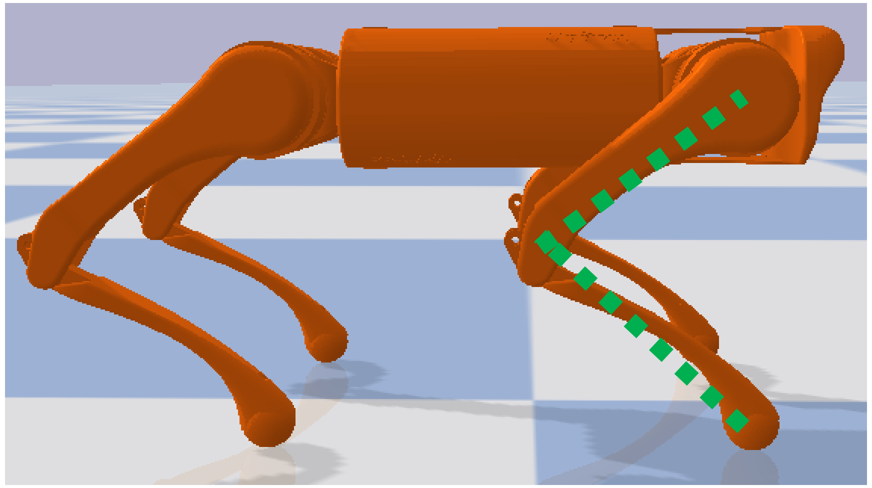




- Good approximation for a quadruped leg

Why derive the kinematics and dynamics?

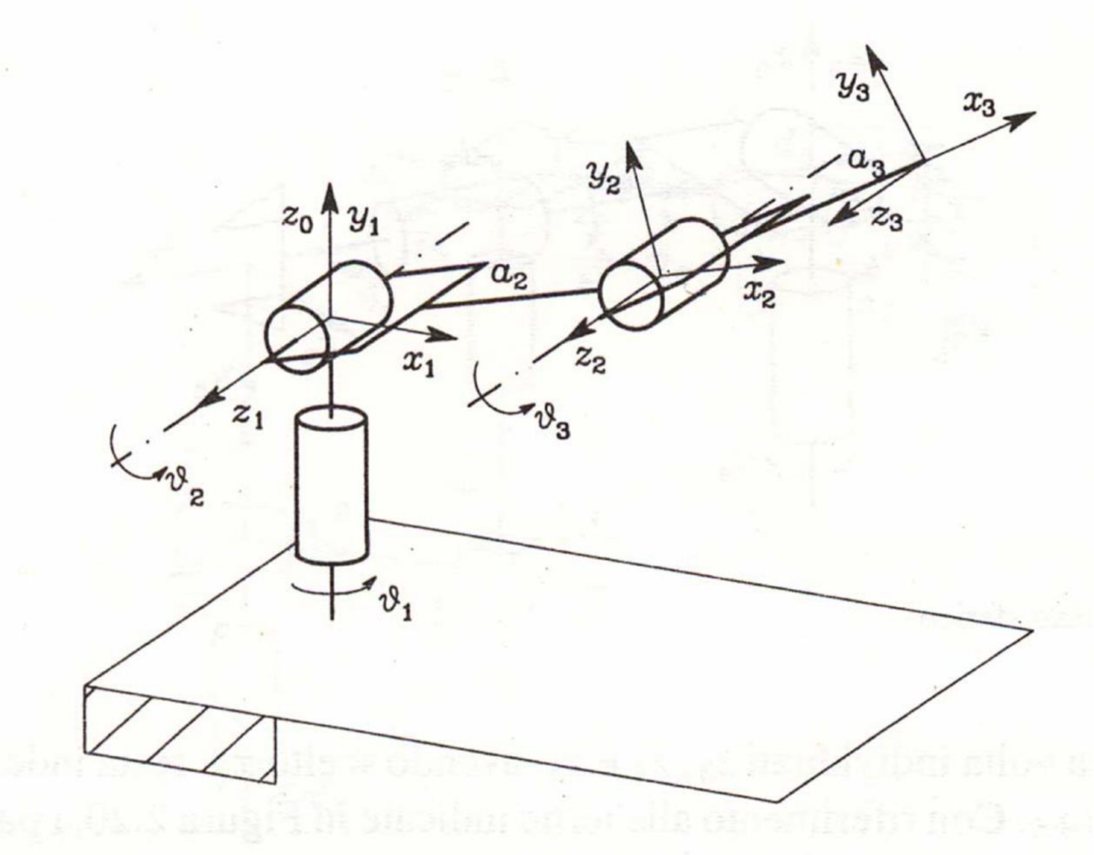




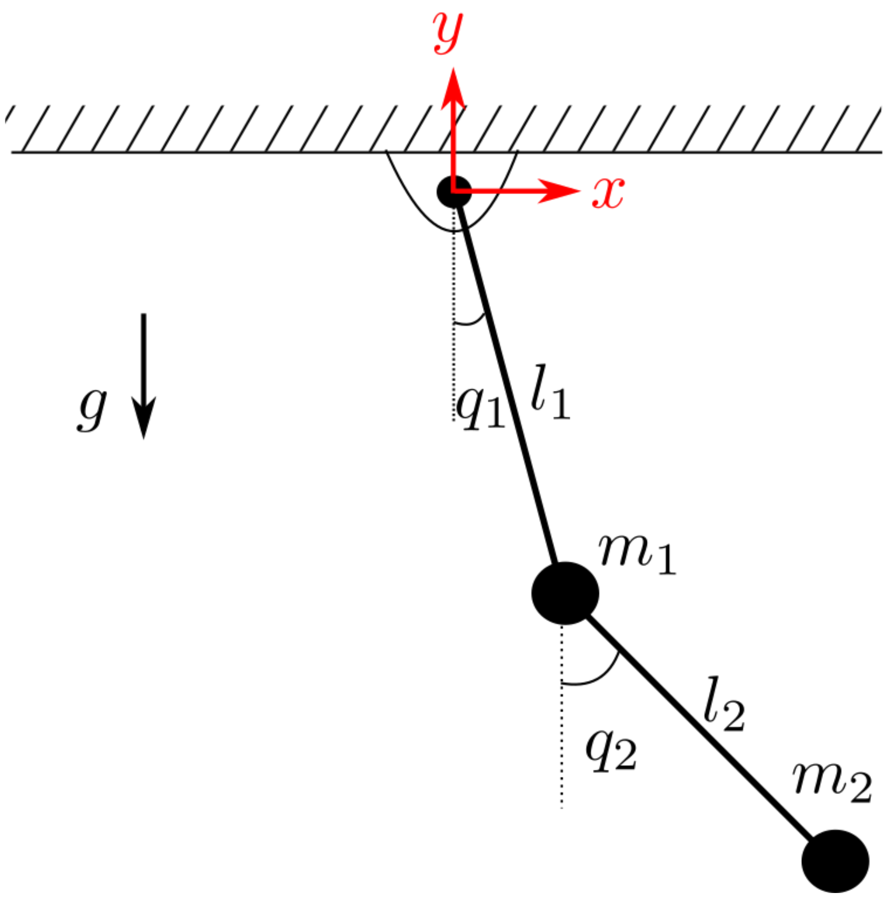
- To effectively control the system!
- The methodology will also be useful for other systems

Kinematics

- **Forward Kinematics**:
  - Input: joints' angles
  - Output: position and orientation of the end effector
- **Inverse Kinematics**:
  - Input: position and orientation of the end effector
  - Output: all the possible joints’ angles combinations



Forward Kinematics



Invert these relationships to obtain the inverse kinematics model (unique?)

$$x_1(q_1) = l_1 sin\big(q_1(t)\big)$$

$$y_1(q_1) = -l_1 cos\big(q_1(t)\big)$$

$$x_2(q_1, q_2) = l_1 sin\big(q_1(t)\big) + l_2 sin\big(q_2(t)\big)$$

$$y_2(q_1, q_2) = -l_1 cos\big(q_1(t)\big) - l_2 cos\big(q_2(t)\big)$$

$$\dot{x}_1(q_1, \dot{q}_1) = l_1 cos\big(q_1(t)\big)\dot{q}_1(t) \qquad \dot{x} = \frac{dx}{dt}$$

$$\dot{y}_1(q_1, \dot{q}_1) = l_1 sin\big(q_1(t)\big)\dot{q}_1(t)$$

$$\dot{x}_2(\ldots) = l_1 cos\big(q_1(t)\big)\dot{q}_1(t) + l_2 cos\big(q_2(t)\big)\dot{q}_2(t)$$

$$\dot{y}_2(\ldots) = l_1 sin\big(q_1(t)\big)\dot{q}_1(t) + l_2 sin\big(q_2(t)\big)\dot{q}_2(t)$$

Dynamics

- Dynamics studies the relation between the **joint actuator torques** and the **resulting motion**

- **Inverse Dynamics (used for designing controllers):**

$$\boldsymbol{f} = \boldsymbol{m}\ddot{\boldsymbol{x}}$$

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

- **Direct (Forward) Dynamics (used for simulation)**

$$\ddot{\boldsymbol{x}} = \frac{\boldsymbol{f}}{\boldsymbol{m}}$$

$$\ddot{\boldsymbol{q}} = \boldsymbol{M}^{-1}(\boldsymbol{q})(\boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\boldsymbol{q},\ \dot{\boldsymbol{q}}))$$

Equations of motion

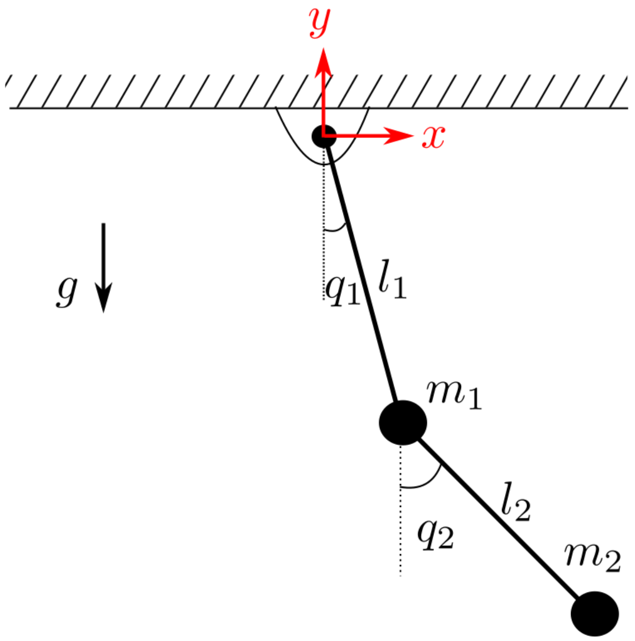
- **Lagrangian method:**
  - Variational method based on kinetics and potential energy

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

- **Newton-Euler recursive method**
  - Relies on $F = ma$ applied to each individual link of the robot

Dynamics



Potential Energy

$$\boldsymbol{q} = (\boldsymbol{q_1}, \boldsymbol{q_2})$$

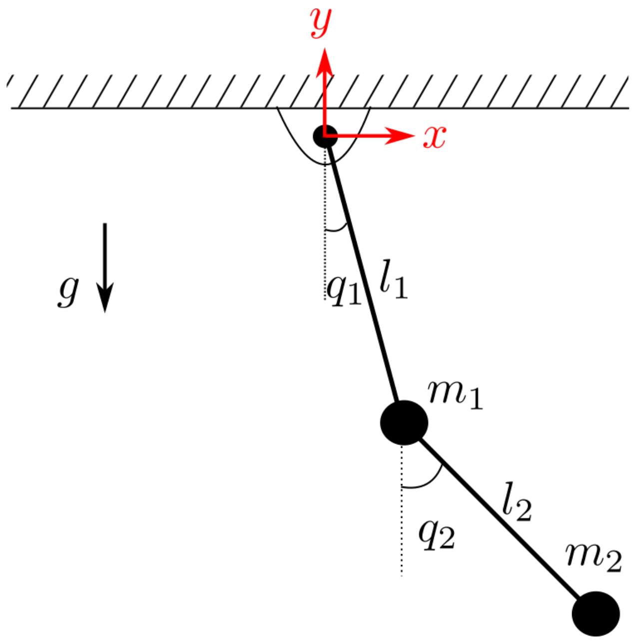
$$\boldsymbol{V(q) = V_1(q) + V_2(q)}$$

$$\boldsymbol{V_1(q) = m_1 g y_1 = -m_1 g l_1 cos(q_1)}$$

$$\boldsymbol{V_2(q) = m_2 g y_2 = \cdots}$$

Potential energy is always a function of q and not $\dot{q}$!

Dynamics



Kinetic Energy

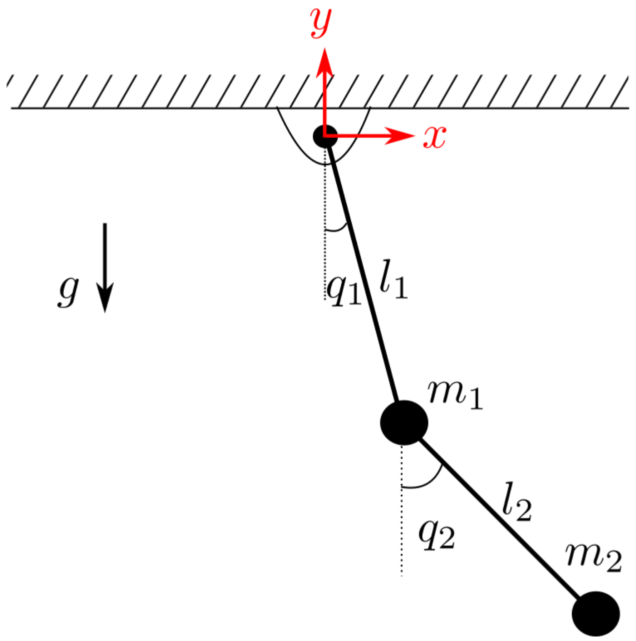
$$T(q,\dot{q}) = T_1(q,\dot{q}) + T_2(q,\dot{q})$$

$$T_1(q,\dot{q}) = \frac{1}{2} m_1 ({\dot{x}_1}^2 + {\dot{y}_1}^2) = \frac{1}{2} m_1 l_1^2 \, \dot{q}_1^2$$

$$T_2(q,\dot{q}) = \frac{1}{2} m_2 ({\dot{x}_2}^2 + {\dot{y}_2}^2) = \cdots$$

Kinetic energy is always a function of q and $\dot{q}$!

Dynamics



Equations of Motion

$$\frac{d}{dt}\left(\frac{\partial L(q,\dot{q})}{\partial \dot{q}_1}\right) - \frac{\partial L(q,\dot{q})}{\partial q_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L(q,\dot{q})}{\partial \dot{q}_2}\right) - \frac{\partial L(q,\dot{q})}{\partial q_2} = 0$$

$$\Downarrow$$

$$\frac{d}{dt}\left(\frac{\partial T(q,\dot{q})}{\partial \dot{q}_1}\right) - \frac{\partial T(q,\dot{q})}{\partial q_1} + \frac{\partial V(q)}{\partial q_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T(q,\dot{q})}{\partial \dot{q}_2}\right) - \frac{\partial T(q,\dot{q})}{\partial q_2} + \frac{\partial V(q)}{\partial q_2} = 0$$

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}_1}) - \frac{\partial T}{\partial \theta_1} + \frac{\partial V}{\partial \theta_1} = 0$$

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}_2}) - \frac{\partial T}{\partial \theta_2} + \frac{\partial V}{\partial \theta_2} = 0$$

Dynamics

$\boldsymbol{q = (\theta_1, \theta_2)}$

$$\frac{\partial T_1}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1$$

$$\frac{\partial T_2}{\partial \dot{\theta}_1} = m_2 l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial T_1}{\partial \theta_1} = 0$$

$$\frac{\partial T_2}{\partial \theta_1} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)$$

$$\frac{\partial V_1}{\partial \theta_1} = m_1 g l_1 \sin(\theta_1)$$

$$\frac{\partial V_2}{\partial \theta_1} = m_2 g l_1 \sin(\theta_1)$$

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}_1}) - \frac{\partial T}{\partial \theta_1} + \frac{\partial V}{\partial \theta_1} = 0$$

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}_2}) - \frac{\partial T}{\partial \theta_2} + \frac{\partial V}{\partial \theta_2} = 0$$

Dynamics

$$\frac{\partial T_1}{\partial \dot{\theta}_2} = 0$$

$$\frac{\partial T_2}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial T_1}{\partial \theta_2} = 0$$

$$\frac{\partial T_2}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)$$

$$\frac{\partial V_1}{\partial \theta_2} = 0$$

$$\frac{\partial V_2}{\partial \theta_2} = m_2 g l_2 \sin(\theta_1)$$

Dynamics

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 - m_2 l_1 l_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) = 0$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin(\theta_2) = 0$$

Dynamics

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 - m_2 l_1 l_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) = 0$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin(\theta_2) = 0$$

$$\tau = B(q)\ddot{q} + C(q, \dot{q}) + g(q)$$

Dynamics

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2 - m_2 l_1 l_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) = 0$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin(\theta_2) = 0$$

$$\tau = B(q)\ddot{q} + C(q, \dot{q}) + g(q)$$

Dynamics

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 - m_2 l_1 l_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) = 0$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin(\theta_2) = 0$$

$C(q, \dot{q})\, \dot{q}$

$$\tau = B(q)\ddot{q} + C(q, \dot{q}) + g(q)$$

Dynamics

$$m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 - m_2 l_1 l_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) = 0$$

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 g l_2 \sin(\theta_2) = 0$$

$$C(q, \dot{q})\,\dot{q}$$

$$\tau = B(q)\ddot{q} + C(q, \dot{q}) + g(q)$$

W1-3 Fundamentals

- **Part 1: double pendulum kinematics and dynamics**
- Part 2: Jacobian (Cartesian PD + Force Control)
- Part 3: Inverse Kinematics (compare with force control)
- Part 4: Single-leg hopping

Assignment

- Start from https://gitlab.epfl.ch/lgevers/lr-practicals
- Instructions are on Moodle
- This week: part 1 only
- The assignment for weeks 1-3 are not graded