Vision-Aided Dynamic Quadrupedal Locomotion on Discrete Terrain

Agrawal, Chen, Rai & Sreenath — *ICRA 2022*

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Dynamic locomotion on discrete stepping-stone terrain

Main Idea

Studying the problem of dynamic locomotion for quadrupedal robots across discrete terrain, using visual feedback.

Aspect	Description
Robot	Quadruped : unitree A1
Control	Torque-based MPC Swing-leg PD control (bezier)
Design	Optimization + geometric control
Gait	Trotting (2-step periodic)
Sensors	Depth camera, IMU, foot contacts
Mapping	Probabilistic elevation map + EKF

Background - Hybrid Model for Trotting

Trotting :

- **Continuous dynamics** (legs in contact with ground)
- **Discrete impacts** (when swing legs touch down)

A ‘two-step’ periodic trotting gait : four DS and four QS phases

Phase	Legs on Ground	Purpose
Double Support (DS)	One diagonal pair	Body is supported; other two legs swing
Quadruple Support (QS)	All four legs on ground	Short stabilization phase before switching support

Captures the instant momentum change + helps the controller predict which feet are loaded

➔ foundation for the robot's motion library and control strategy.

Steps

Vision and Mapping → Gait Selection → Geometric MPC and PD → Robot

Vision and Mapping (Step 1)

Extended Kalman Filter : pose estimation of the robot

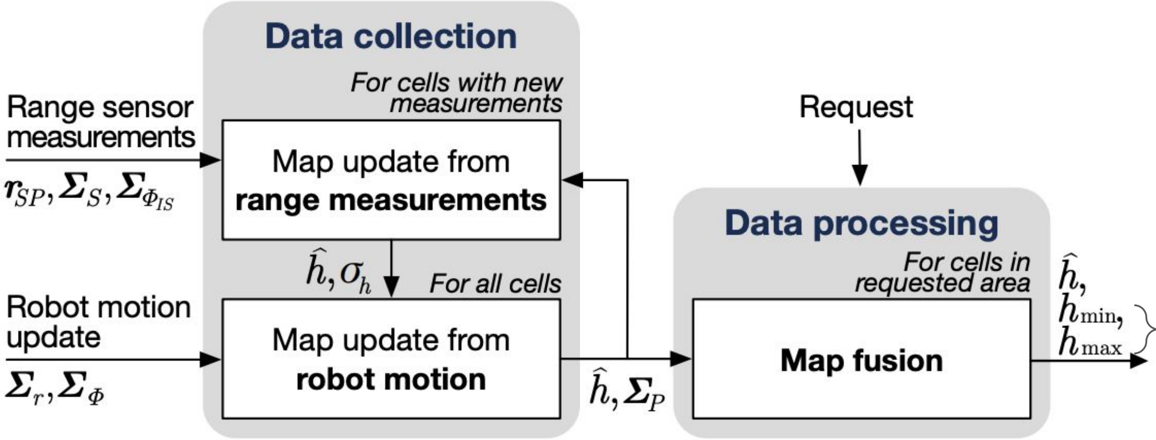
Algorithm 1 Update procedure for the IEKF with noise-free pseudo-measurements.

Compute K_k using (7)
$z_k^0 \leftarrow \hat{\chi}_{k|k-1}^{-1} y_k - d_k$
$\chi \leftarrow \hat{\chi}_{k|k-1} \exp(K_k z_k^0)$
$z_k^1 \leftarrow \chi^{-1} y_k - d_k$
$i \leftarrow 1$
while $|z_k^i - z_k^{i-1}| > \mathrm{tol}$ **do**
    $\chi \leftarrow \chi \exp(K_k z_k^i)$
    $z_k^{i+1} \leftarrow \chi^{-1} y_k - d_k$
    $i \leftarrow i + 1$
end while
$\hat{\chi}_{k|k} \leftarrow \chi$
$P_{k|k} \leftarrow (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T$

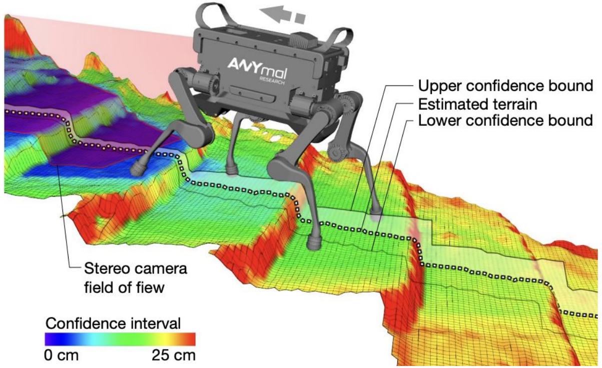
R. Hartley, M. Ghaffari, R. M. Eustice, and J. W. Grizzle, “Contactaided invariant extended kalman filtering for robot state estimation,” The International Journal of Robotics Research, vol. 39, no. 4, pp. 402–430, 2020.

Vision and Mapping (Step 1)

Probabilistic Mapping Framework: understand the shape of the ground around the robot



uncertainty limits



P. Fankhauser, M. Bloesch, and M. Hutter, “Probabilistic terrain mapping for mobile robots with uncertain localization,” IEEE Robotics and Automation Letters (RA-L), vol. 3, no. 4, pp. 3019–3026, 2018.

Gait & Foothold Selection (Offline library) (Step 2)

Goal:

Compute energy-efficient periodic gaits for different step lengths (l_0, l_1), used later during online motion planning.

The desired gaits are two-step periodic trot gaits, where diagonal leg pairs move in alternation.

Key ideas:

- **Direct Collocation**: time-discretization of continuous dynamics into NNN nodes.
- **Objective**: Minimizes total torque effort → smooth and energy-efficient gait.
- **Constraints** : enforce dynamics, friction limits, step periodicity, and feasible joint torques.

Method: Direct Collocation Optimization

Each gait is obtained by solving a trajectory optimization problem that minimizes the total torque effort over time:

$$(x^*(\cdot), \tau^*(\cdot)) = \underset{x(t),\tau(t)}{\arg\min} \quad \Sigma_i \int_0^T ||\tau(t)||_2^2 \, dt \tag{4}$$

$$\text{st. } x(t) = \int_0^T f_i(x(t)) + g_i(x(t))\tau(t)dt,$$

$$c_i(x(t), \tau(t)) \leq 0, \quad 0 \leq t \leq T, \ \forall i \in \mathcal{I}.$$

Geometric Variational MPC (Step 3)

Motivation:

- Standard Euler-based discretization → loses energy & momentum consistency.
- Small-angle linearization → invalid on steep slopes or large rotations.

GV-MPC preserves the rotation matrix geometry & energy invariants by construction.

➡ Dynamics are discretized using a **Geometric Variational Integrator (GVI)**, then linearized around the reference gait.

From Lagrangian dynamics to discrete-time model

1. Discretization via Geometric Variational Integrator (GVI)

GVI integrates the physical principles of motion directly:

$$L_d(q_k, q_{k+1}) \approx \int_{t_k}^{t_{k+1}} L(q, \dot{q})\, dt$$

2. Linearization on the SO(3) manifold

Classical linearization:
Flattens the full SE(3) space (position + rotation) → loses geometric structure, causes energy drift and orientation errors.

This approach:
Linearizes directly on the SO(3) manifold → keeps true rotation geometry, avoids singularities, and preserves physical consistency.

Quadratic Program (MPC Formulation)

Given the desired CoM trajectory dξd, compute contact forces λc* by minimizing error over a prediction horizon N:

$$\lambda_c^* = \arg \min_{\lambda_c, \delta\xi_k, \delta F_k} \left(||\delta\xi_N||_P + \sum_{k=0}^{N} (||\delta\xi_k||_Q + ||\delta F_k||_R) \right)$$

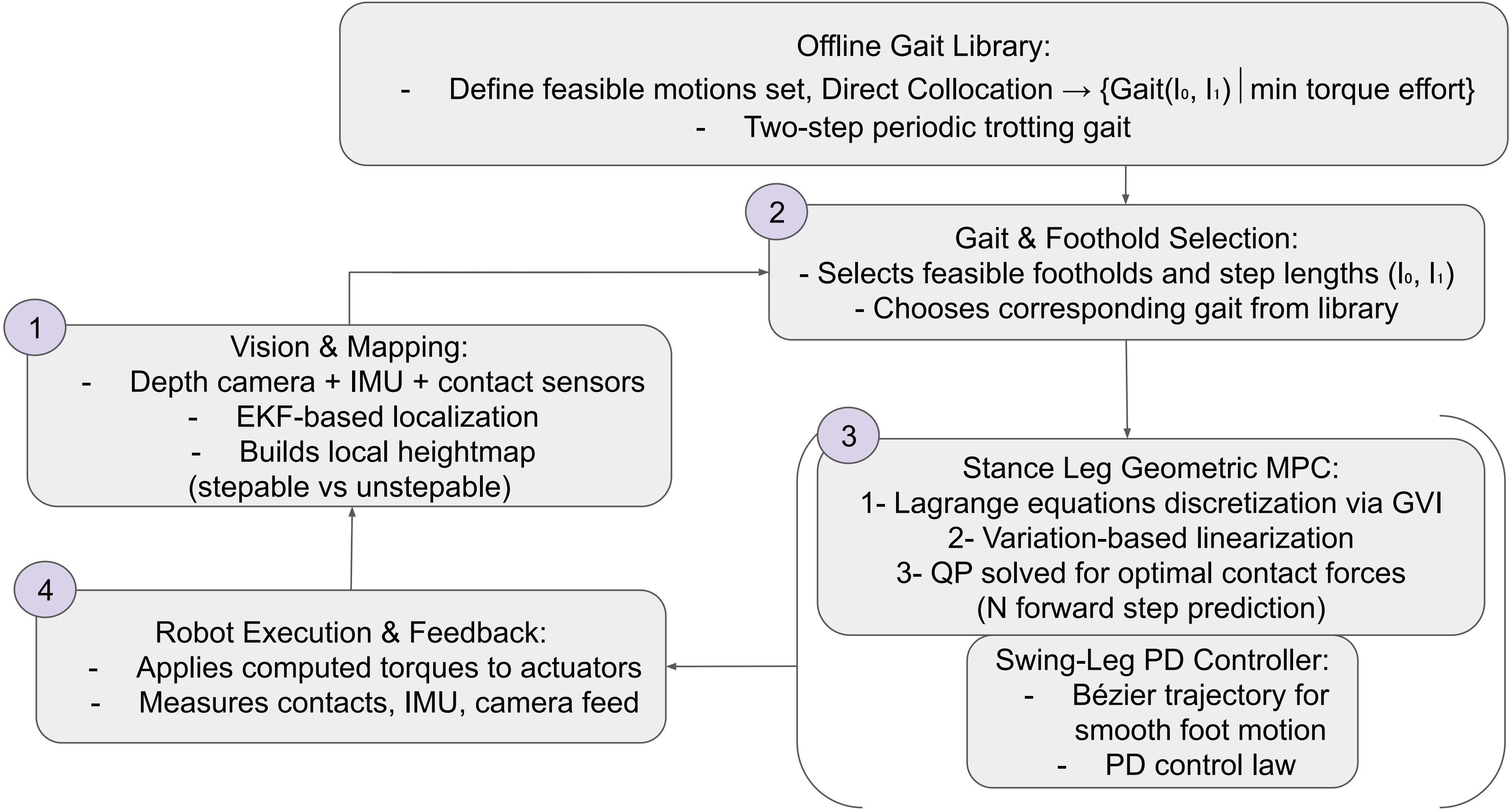
Subject to:

$$\delta\xi_{k+1} = A_k(\xi_k^d)\delta\xi_k + B_k(\xi_k^d)\delta F_k$$

$$0 \leq \lambda_c^i \leq c_i\bar{\lambda}, \quad i = 1, 2, 3$$

$$G_c\lambda_c = \delta F_0 + \begin{bmatrix} mg \\ 0_{3\times1} \end{bmatrix}$$

- δξ: deviation of state from reference gait
- λc: ground reaction/contact forces
- Q,R,P: weighting matrices for tracking, effort, terminal error


Offline Gait Library:
- Define feasible motions set, Direct Collocation → {Gait(l₀, l₁) | min torque effort}
- Two-step periodic trotting gait
2
Gait & Foothold Selection:
- Selects feasible footholds and step lengths (l₀, l₁)
- Chooses corresponding gait from library
1
Vision & Mapping:
- Depth camera + IMU + contact sensors
- EKF-based localization
- Builds local heightmap
(stepable vs unstepable)
3
Stance Leg Geometric MPC:
1- Lagrange equations discretization via GVI
2- Variation-based linearization
3- QP solved for optimal contact forces
(N forward step prediction)
Swing-Leg PD Controller:
- Bézier trajectory for smooth foot motion
- PD control law
4
Robot Execution & Feedback:
- Applies computed torques to actuators
- Measures contacts, IMU, camera feed

Used also in

Guillaume Bellegarda, Milad Shafiee, Auke Ijspeert, "Visual CPG-RL: Learning Central Pattern Generators for Visually-Guided Quadruped Locomotion", *2024 IEEE International Conference on Robotics and Automation (ICRA)*, pp.1420-1427, 2024.

Juntao He, Baxi Chong, Jianfeng Lin, Zhaochen Xu, Hosain Bagheri, Esteban Flores, Daniel I. Goldman, "Probabilistic Approach to Feedback Control Enhances Multilegged Locomotion on Rugged Landscapes", *IEEE Transactions on Robotics*, vol.41, pp.4776-4793, 2025.

Shaoting Zhu, Derun Li, Linzhan Mou, Yong Liu, Ningyi Xu, Hang Zhao, "SARO: Space-Aware Robot System for Terrain Crossing via Vision-Language Model", *2025 IEEE International Conference on Robotics and Automation (ICRA)*, pp.14820-14827, 2025.

Pros	Cons
• Works on real hardware (not just simulation) • Combines planning + control + vision • Robust to unknown terrain and variable gap sizes • Geometric MPC handles large orientation changes safely	• Requires accurate height maps and EKF localization • Motion library limits gaits to predefined trotting patterns • Not suited for soft, deformable, or slippery surfaces • Computationally heavier than heuristic approaches

THANK YOU