EPFL

# Swing-Leg Retraction for Limit Cycle Walkers

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# Introduction

## Limit Cycle Walkers

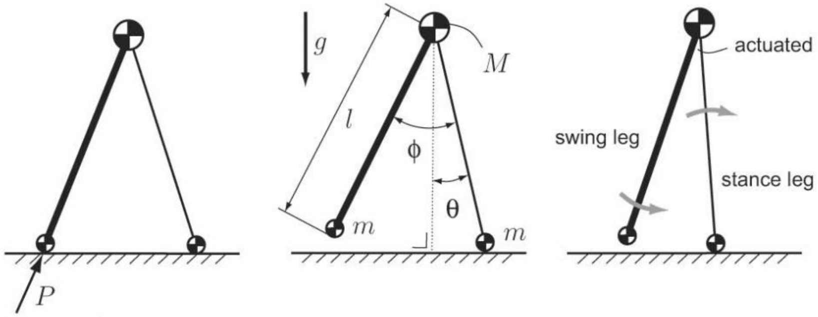
- Example: Passive Dynamic Walkers
- Not requiring local controllability
- Emphasized Stabilization Mechanism:
  - step time ↓
  - step length ↑
  - velocity ↑ (swing foot ground hitting velocity ↑)
  - dissipation of energy ↑
- It is suggested that dissipation of energy is the main gait stabilizing mechanism.
- every passive dynamic walker has a nominal velocity that it is stable.

# Introduction

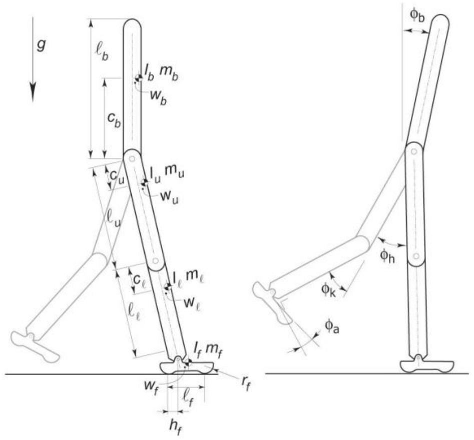
## What is controlled here?

- The quantity indirectly controlled is the **swing-foot impact velocity**,
- The quantity directly controlled is the **inter-leg angular velocity** during the swing-retraction phase,
- A hip-joint actuator is used to achieve this control,
- Additional actuators are used to control foot alignment during walking.

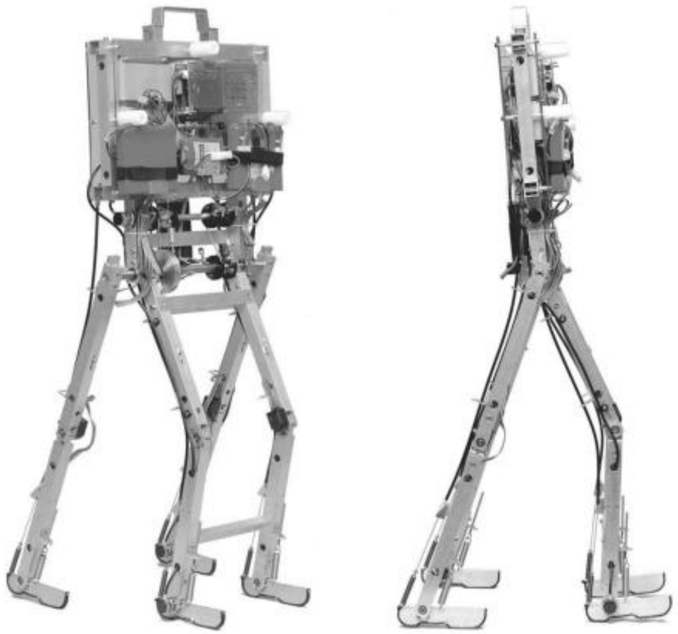
# Simple point mass model

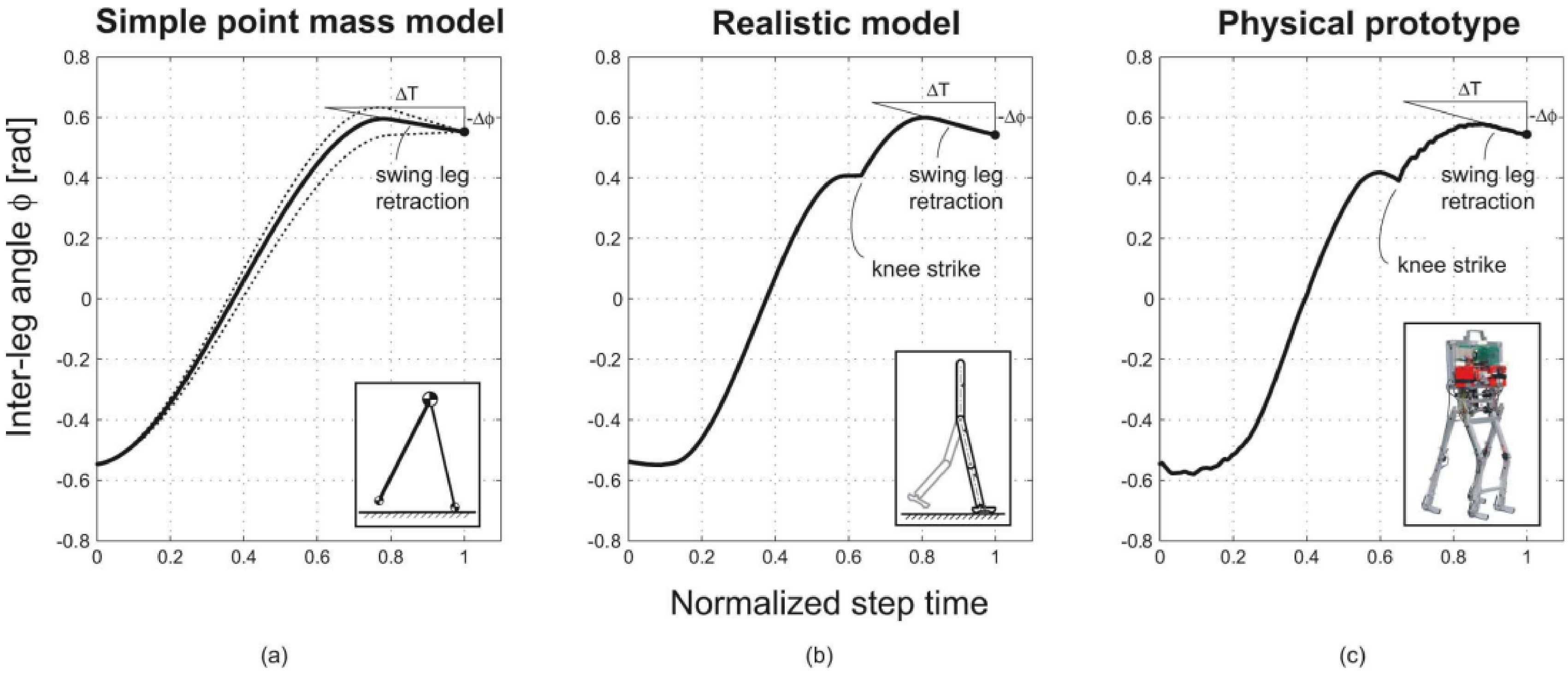



# Realistic Model



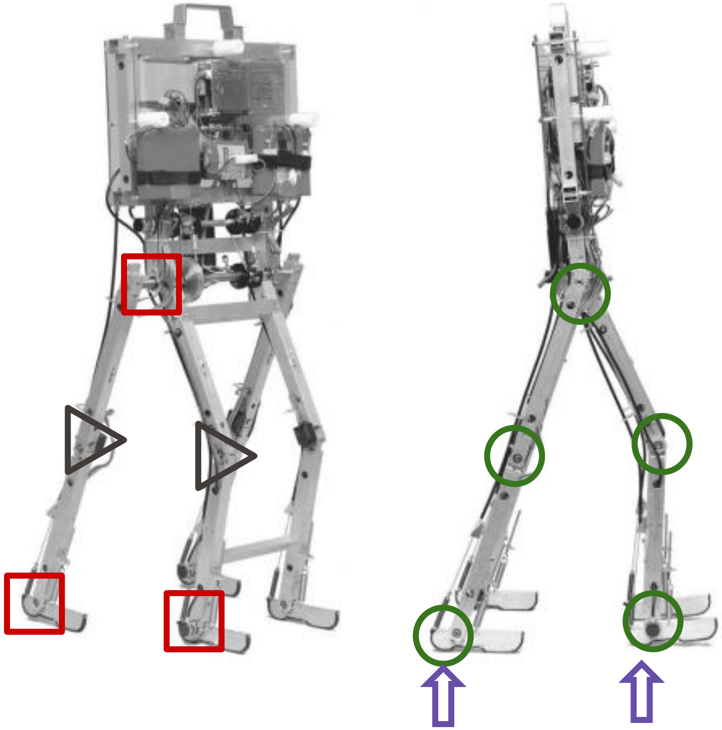
# Physical prototype “Meta”




Simple point mass model
Realistic model
Physical prototype
Inter-leg angle ϕ [rad]
Normalized step time
ΔT
-Δϕ
swing leg retraction
knee strike
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.2
0.4
0.6
0.8
1
(a)
(b)
(c)

# Physical prototype





- Legs are constructed in **pairs** to achieve 2-D behavior
- 3 actuated joints
- 2 passive knee joint, with a stop and latching mechanism
- 5 angular sensors
- 2 heel strike sensors

# Step-to-step approximation





$$\mathbf{v}^* + \Delta\mathbf{v}_{n+1} \approx \mathbf{v}^* + \mathbf{A}\,\Delta\mathbf{v}_n\,.$$

| | |
|---|---|
| $\mathbf{v}^*$ : | initial conditions |
| $\Delta\mathbf{v}_n$ : | deviation from initial conditions |
| $\Delta\mathbf{v}_{n+1}$ : | deviation at the next step |
| **A** : | Jacobian, *found through simulation* |

→ The eigenvalues of **A** are the **Floquet multipliers**.

*This is valid for small perturbations*

# Local stability

|**λ**| < 1 → shrinking deviation (stable)
|**λ**| > 1 → growing deviation (unstable)

**λ** < 0 → oscillatory behavior

2 multipliers with the point mass model :

$\Delta\theta^{+}$ : the stance-leg angle after heel strike

$\Delta\dot{\theta}^{+}$ : the stance-leg angular velocity

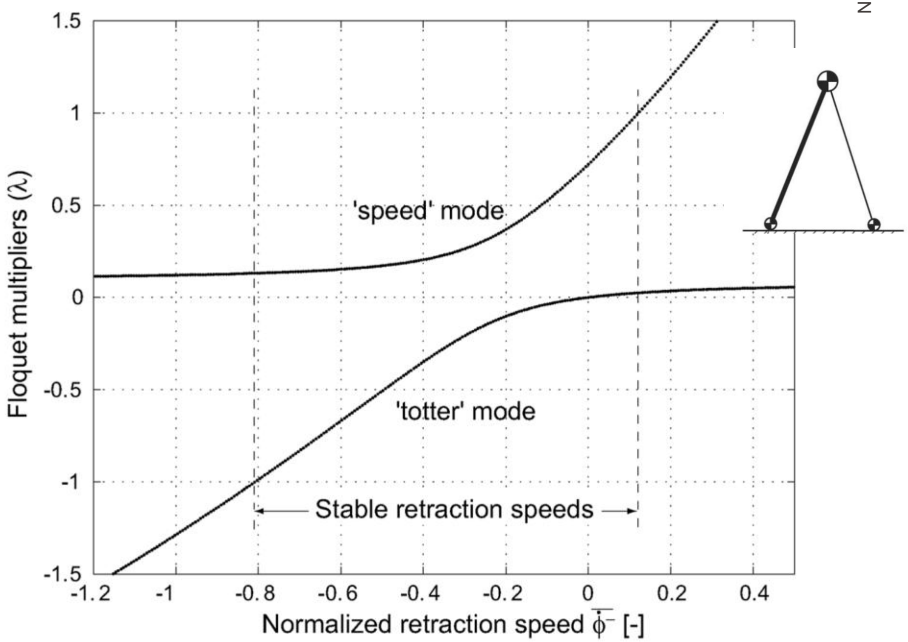



Fig. 7. Floquet multipliers of the simple point mass model for a range of swing-leg retraction speeds. The stable retraction speeds range from −0.80 to 0.12.

Nathan

# Effect of retraction speed



We see that most of the stable walking occurs for negative retraction speeds.

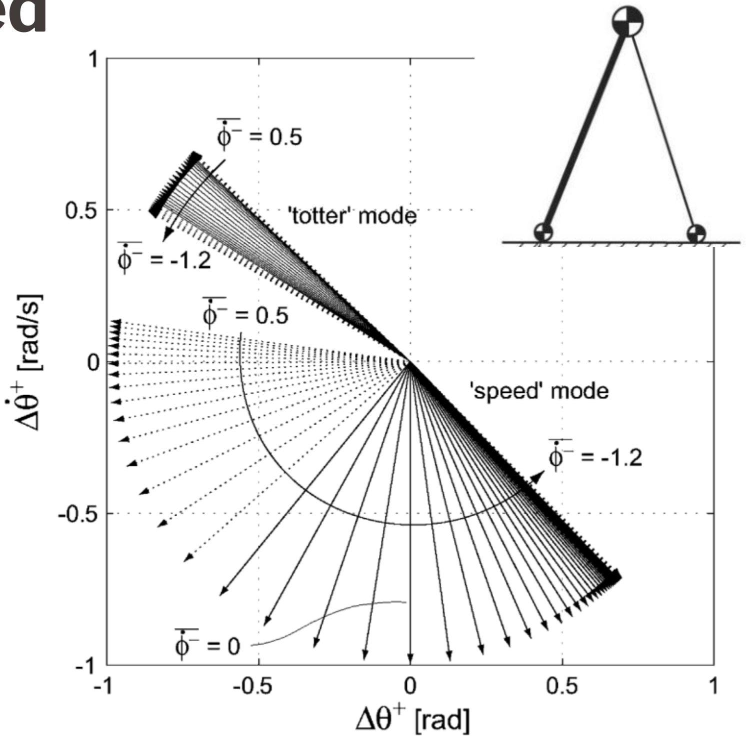


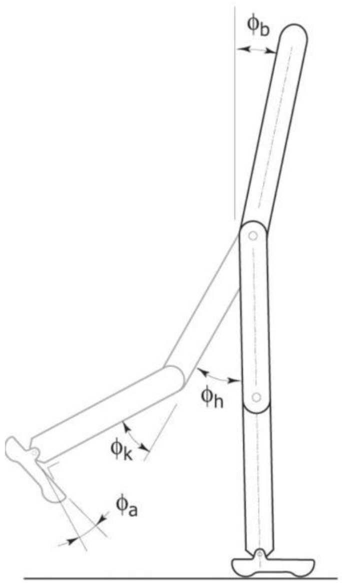

Fig. 8. Eigenvectors of the two eigenmodes of the simple point mass model for a range of swing-leg retraction speeds. The "speed" mode has a corresponding positive Floquet multiplier and the "totter" mode has a corresponding negative Floquet multiplier. The eigenvectors outside the stable range of retraction speeds are depicted as dotted arrows.

# Comparison with the realistic model





We have 4 multipliers,
but only two of them are dominant



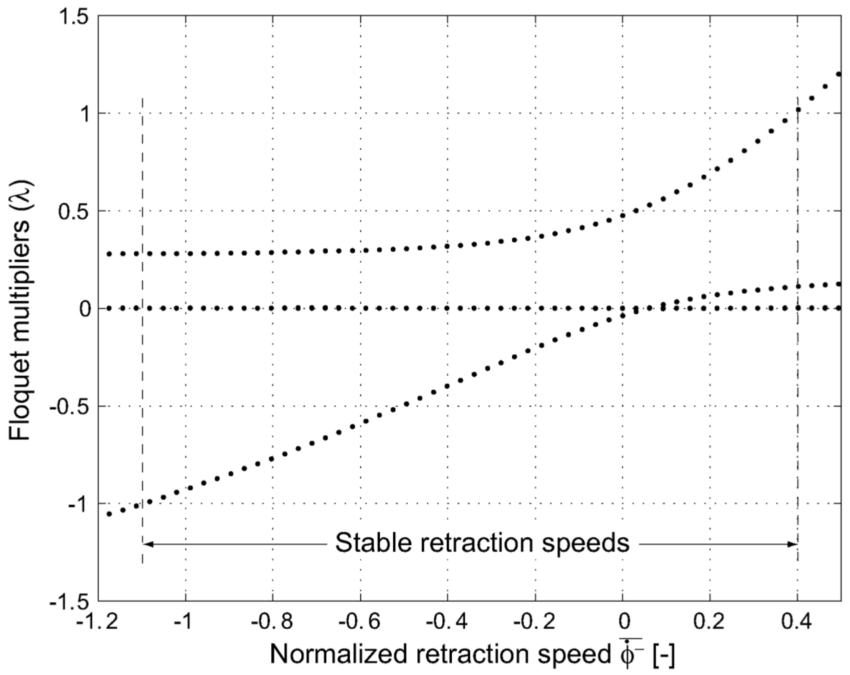



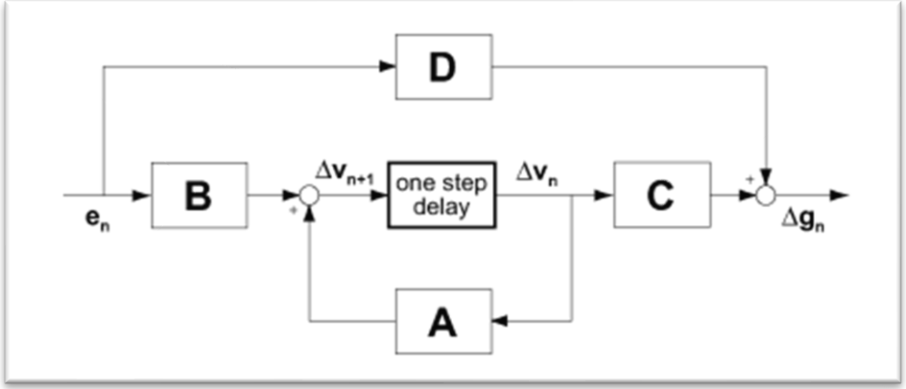

Fig. 11. Real part of the four Floquet multipliers of the realistic model for a range of swing-leg retraction speeds, the imaginary part of all Floquet mutlipliers is close to zero. The stable retraction speeds range from −1.1 to 0.4.

# Disturbance Response

Mahdy

Summarized by:

- Floquet Multipliers
- Step-to-step dynamic response
- Adding influence of distribution



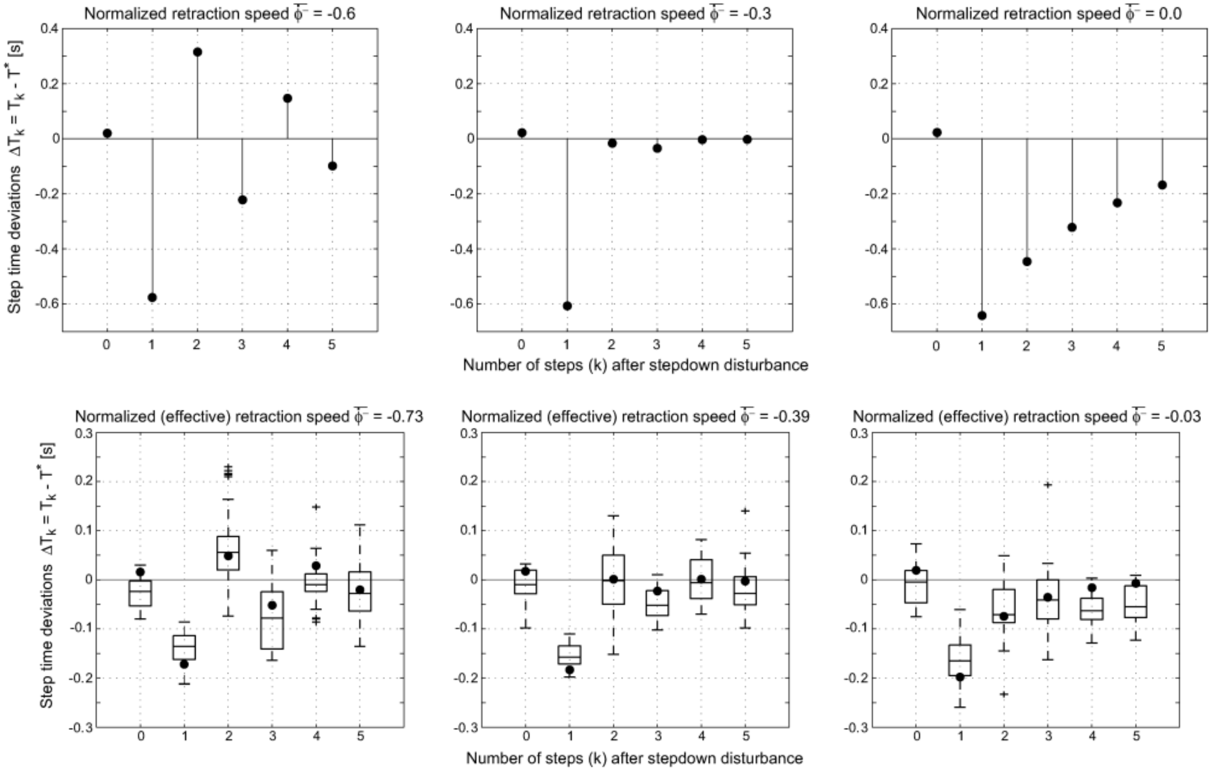

System description:

$$\Delta \mathbf{v}_{n+1} = \mathbf{A}\,\Delta \mathbf{v}_n + \mathbf{B}\,\Delta \mathbf{e}_n$$
$$\mathbf{g}_n = \mathbf{C}\,\Delta \mathbf{v}_n + \mathbf{D}\,\Delta \mathbf{e}_n$$

$$\Delta T_k = \mathbf{D}, \quad \text{for} \quad k = 0$$
$$\Delta T_k = \mathbf{C}\mathbf{A}^{k-1}\mathbf{B}, \quad \text{for} \quad k > 0$$

# Simulation Results



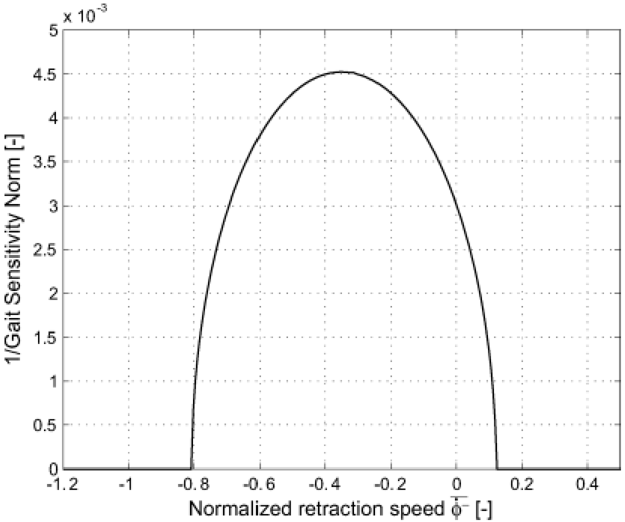
# Gait Sensitivity Norm

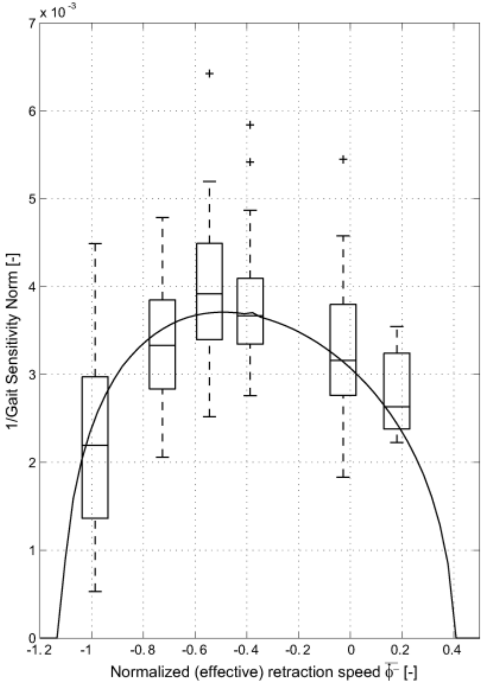
- A good prediction of the disturbance rejection ability of a limit cycle
- Calculated using sensitivity matrices
- Approximated using system's impulse response

$$\left\| \frac{\partial \mathbf{g}}{\partial \mathbf{e}} \right\|_2 = \sqrt{\text{trace}(\mathbf{D}^T\mathbf{D}) + \sum_{k=0}^{\infty} \text{trace}(\mathbf{B}^T(\mathbf{A}^T)^k\mathbf{C}^T\mathbf{C}\mathbf{A}^k\mathbf{B})}.$$

$$\left\| \frac{\partial \mathbf{g}}{\partial \mathbf{e}} \right\|_2 \approx \frac{1}{\Delta h} \sqrt{\sum_{k=0}^{6} \left( \frac{T_k - T^*}{0.2T^*} \right)^2}.$$

# Simulation Results



# Pros and Cons

## Pros

- Swing-leg retraction dramatically improves stability
- Stability can be predicted cheaply using the Gait Sensitivity Norm (GSN)
- The simple point-mass model is surprisingly predictive

## Cons

- The totter mode introduces oscillation. The simple model breaks down at extremes. Swing-leg retraction cannot influence the first disturbed step