EPFL

Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point (Shuuji KAJITA, et al.)

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Executive Summary

- Biped Walking Patterns
- Merge two control viewpoints of 2003
  - ZMP-based
    - Assumes ideal model
  - Inverted Pendulum approach
    - Feedback control
- LQ Preview Controller
  - Anticipate ZMP over a horizon
  - Tolerate arbitrary foot placements
  - Compensate the inaccuracies in the model

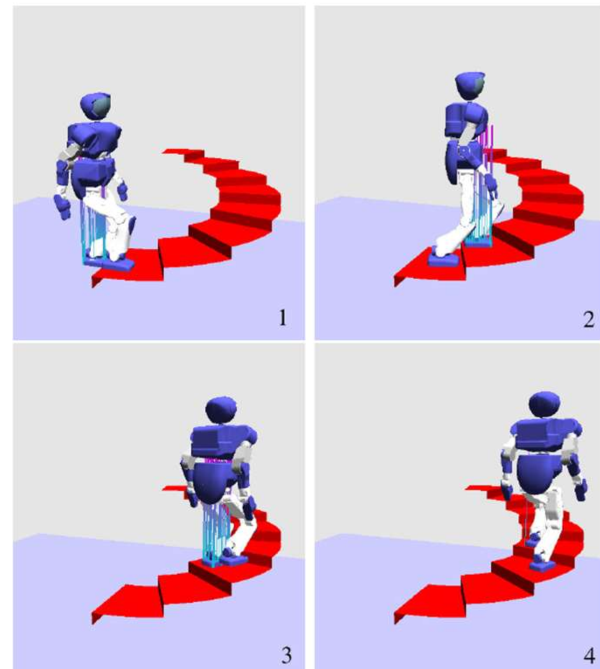


Fig. 1: Walking on spiral stairs (simulation)

Dynamic Model Used

- Inverted pendulum under constraint

$$\ddot{x} = \frac{g}{z_c}x + \frac{1}{mz_c}\tau_y \qquad \ddot{y} = \frac{g}{z_c}y + \frac{1}{mz_c}\tau_x$$

- ZMP

$$p_x = x - \frac{z_c}{g}\ddot{x} \qquad p_y = y - \frac{z_c}{g}\ddot{y}$$

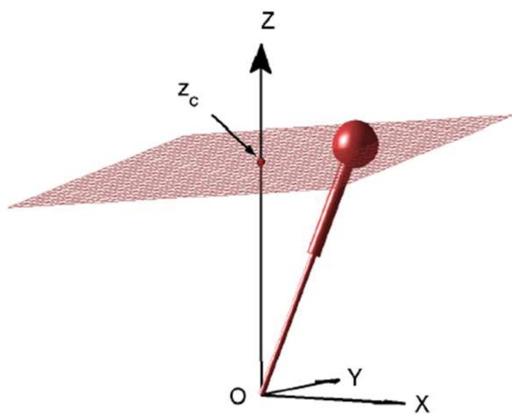


Fig. 2: Inverted pendulum under constraint

- Assumptions
  - Constant hip height (z_c)
  - Small angles

$$p = \frac{\tau}{mg}$$

Junseo

ZMP Control as a servo problem

- Walking pattern generator as a ZMP tracking control system
- Servo controller
  - Input: ZMP error
  - Output: Jerk u_x
- Dynamic ZMP equation
  - Input: u_x
  - Output: ZMP
  - Method: Kagami, Nishiwaki et al: Discretize ZMP equation

$$\frac{d}{dt}\ddot{x} = u_x \qquad p_x = x - \frac{z_c}{g}\ddot{x}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_x$$

$$p_x = \begin{bmatrix} 1 & 0 & -z_c/g \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}.$$

Eq. Dynamic ZMP equation

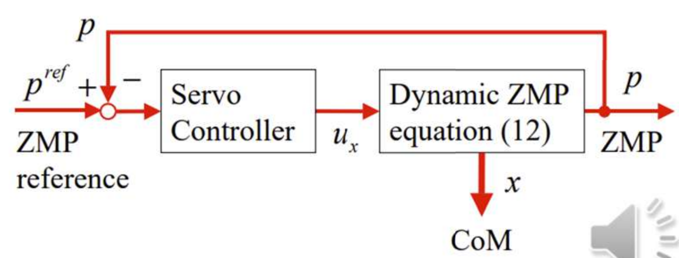
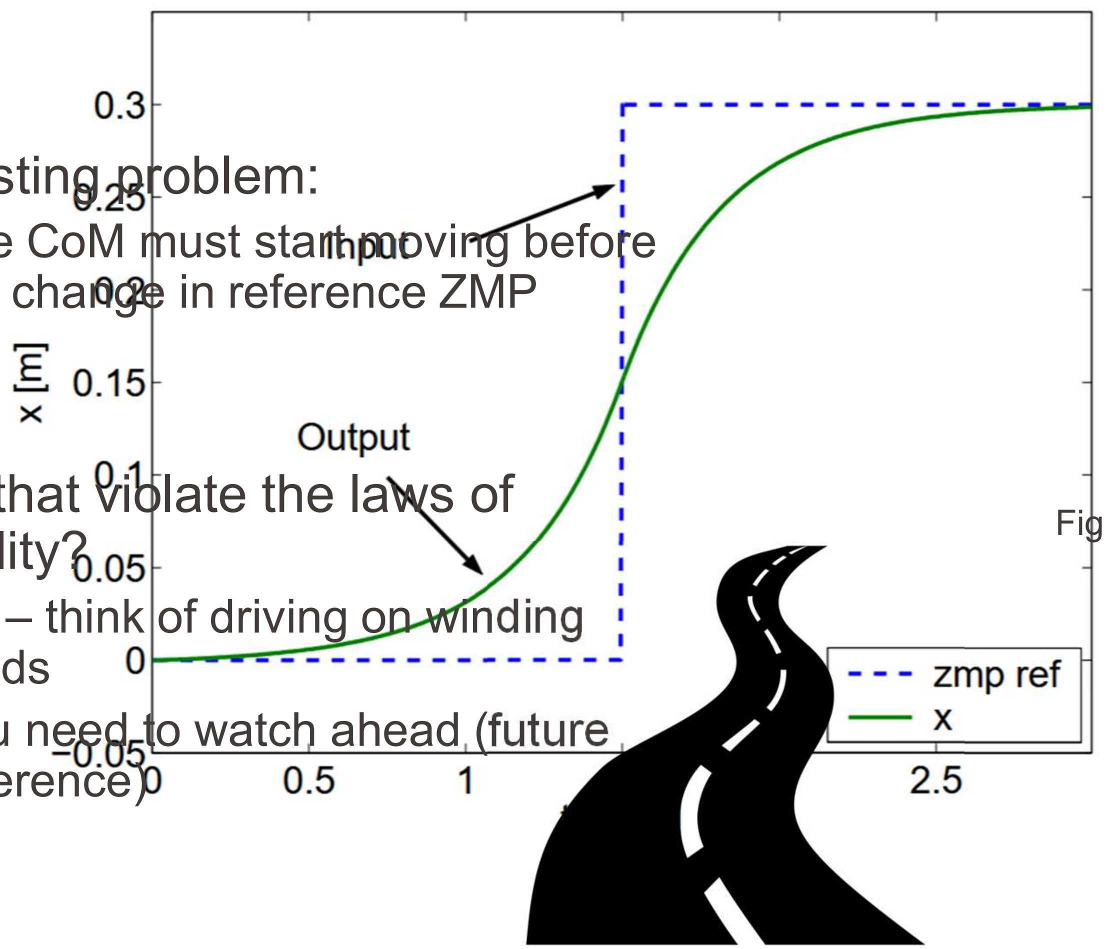


Fig. 3: Pattern generation as ZMP tracking control

Preview Control – Using future references

- Interesting problem:
  - The CoM must start moving before the change in reference ZMP
- Does that violate the laws of causality?
  - No – think of driving on winding roads
  - You need to watch ahead (future reference)



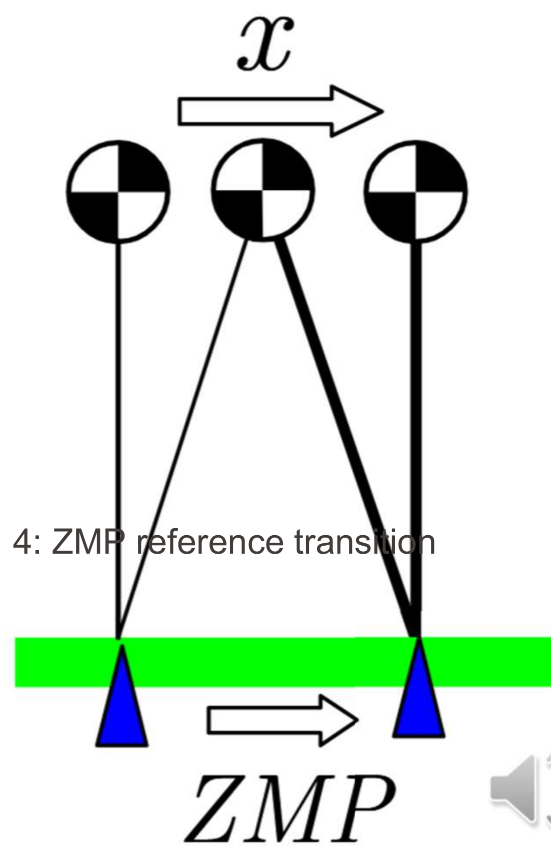


Fig. 4: ZMP reference transition

Preview Control – Optimal Control

- The servo controller must first be discretized as follows

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + Bu(k)$$
$$p(k) = C\boldsymbol{x}(k)$$

$$\boldsymbol{x}(k) \equiv \begin{bmatrix} x(kT) \\ \dot{x}(kT) \\ \ddot{x}(kT) \end{bmatrix}, \quad u(k) \equiv u_x(kT), \quad p(k) \equiv p_x(kT).$$

$$A \equiv \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad B \equiv \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix}, \quad C \equiv \begin{bmatrix} 1 & 0 & -z_c/g \end{bmatrix}$$

Preview Control – Optimal Control

- Then, with the provided reference ZMP: $p^{ref}(k)$ the authors defined a performance index (a cost function):

State changes

$$\Delta x(k) \equiv x(k) - x(k+1)$$

$$Q_x \in \mathbb{S}_+^{3\times 3}$$

$$J = \sum_{i=k}^{\infty} \left\{ Q_e e(i)^2 + \Delta x^T(i) Q_x \Delta x(i) + R \Delta u^2(i) \right\}$$

Tracking error

$$e(i) \equiv p(i) - p^{ref}(i)$$

$$Q_e > 0$$

Energy

$$\Delta u(k) \equiv u(k) - u(k+1)$$

$$R > 0$$

Preview Control – Optimal Control

- The optimal controller, derived with Linear Quadratic Integral control with Preview (LQI)

Future steps

$$u(k) = -G_e \sum_{i=0}^{k} e(i) \; - \; G_x \, \boldsymbol{x}(k) \; - \; \sum_{j=1}^{N_L} G_p(j) \, p^{ref}(k+j).$$

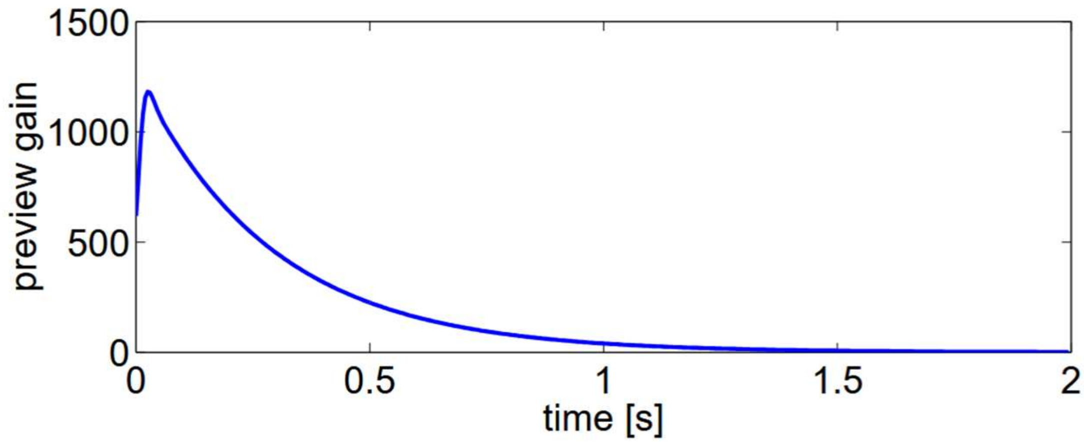


Fig. 5: Decrease of preview gain

Preview Control – Results

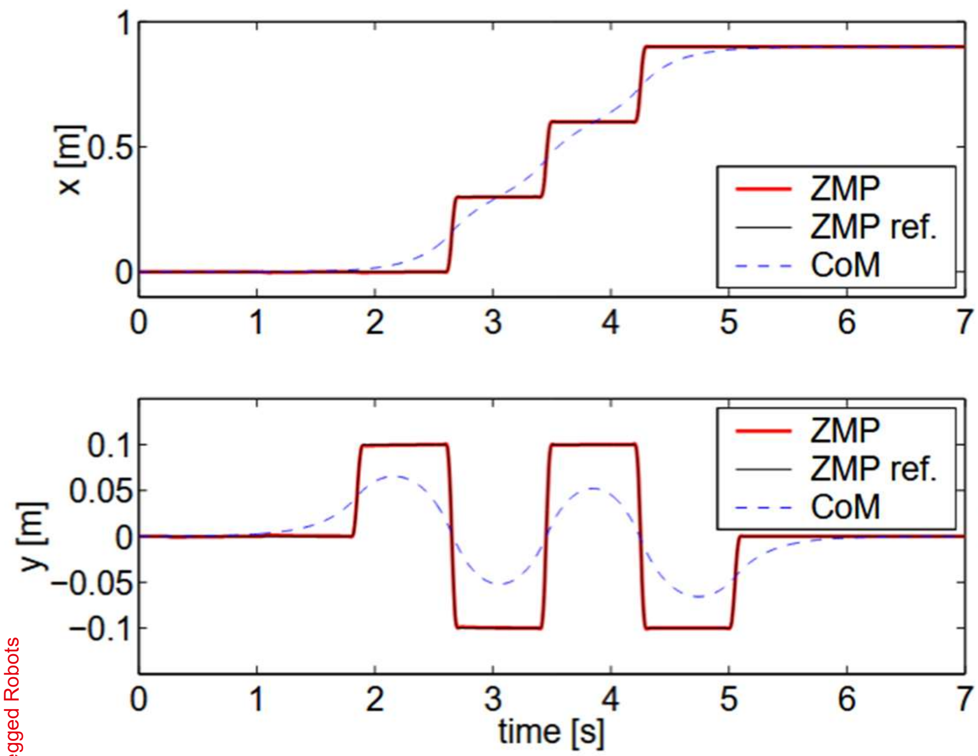


Fig. 6: Performance with $T * N_L = 1.6\ s$

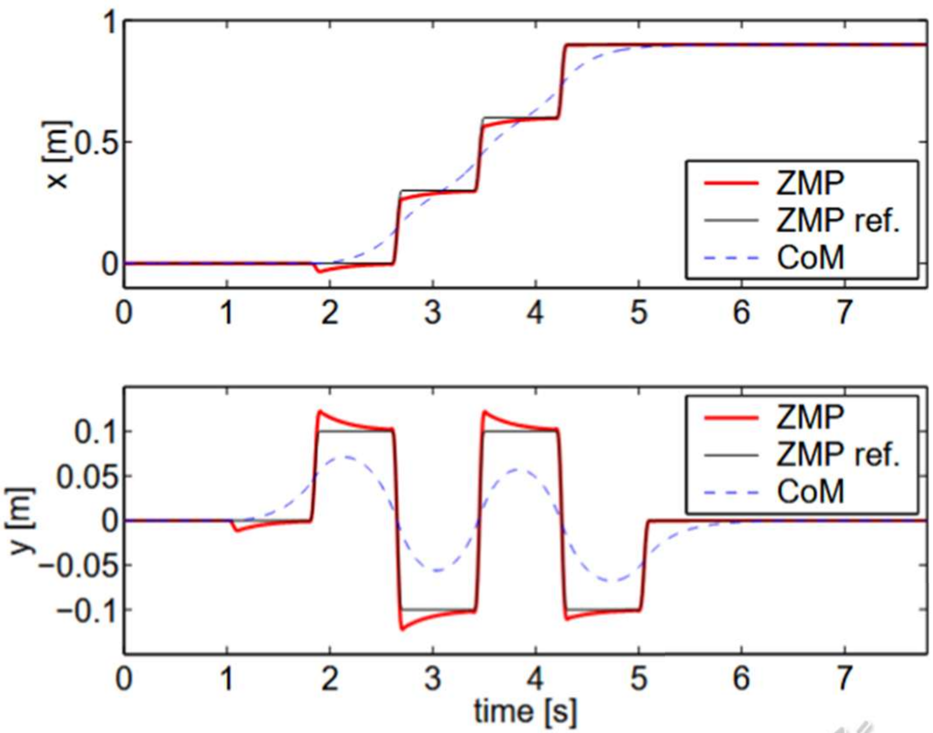


Fig. 7: Performance with $T * N_L = 0.8\ s$

From cart-table to multibody model

- What happens when we go from the simple cart-table model to the more complex multibody model?

Max x ZMP error: 2.3 cm!

Max y ZMP error: 1.6 cm!

How can we fix this?

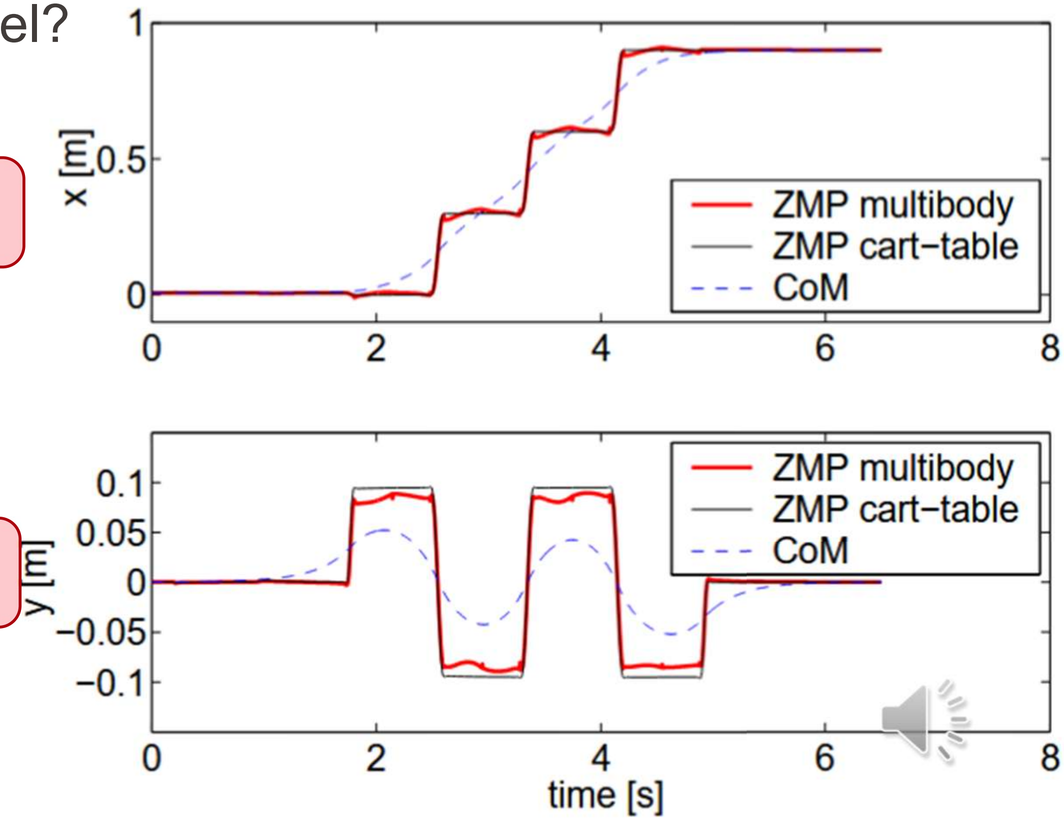
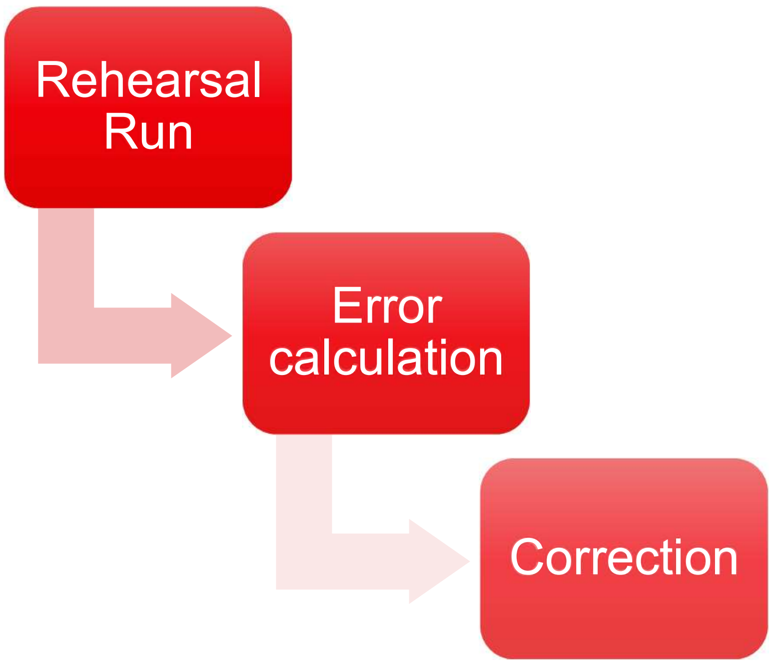


Fig. 8: ZMP calculated by the table cart model and the multibody model

Preview Control for error mitigation

- Solution: Use preview control once again to reduce the error in the multibody model



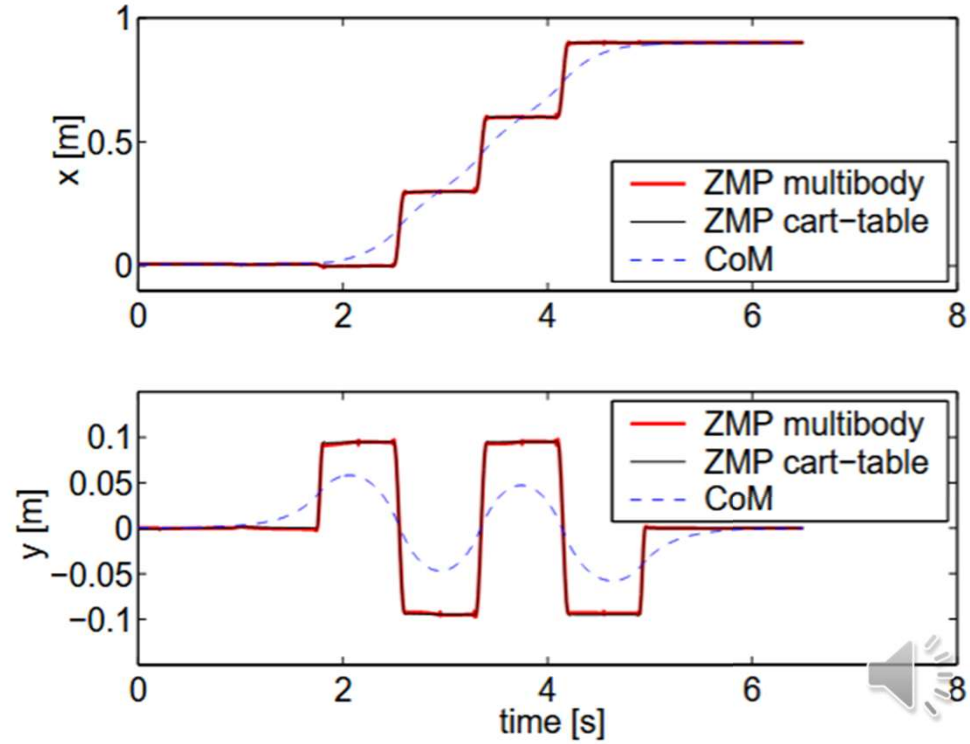


Fig. 9: Corrected ZMP of multibody model

Citations

- Most influential paper by S. KAJITA: **2970 citations**
- **Technology enabler**: it was implemented of many robots such as the HRP

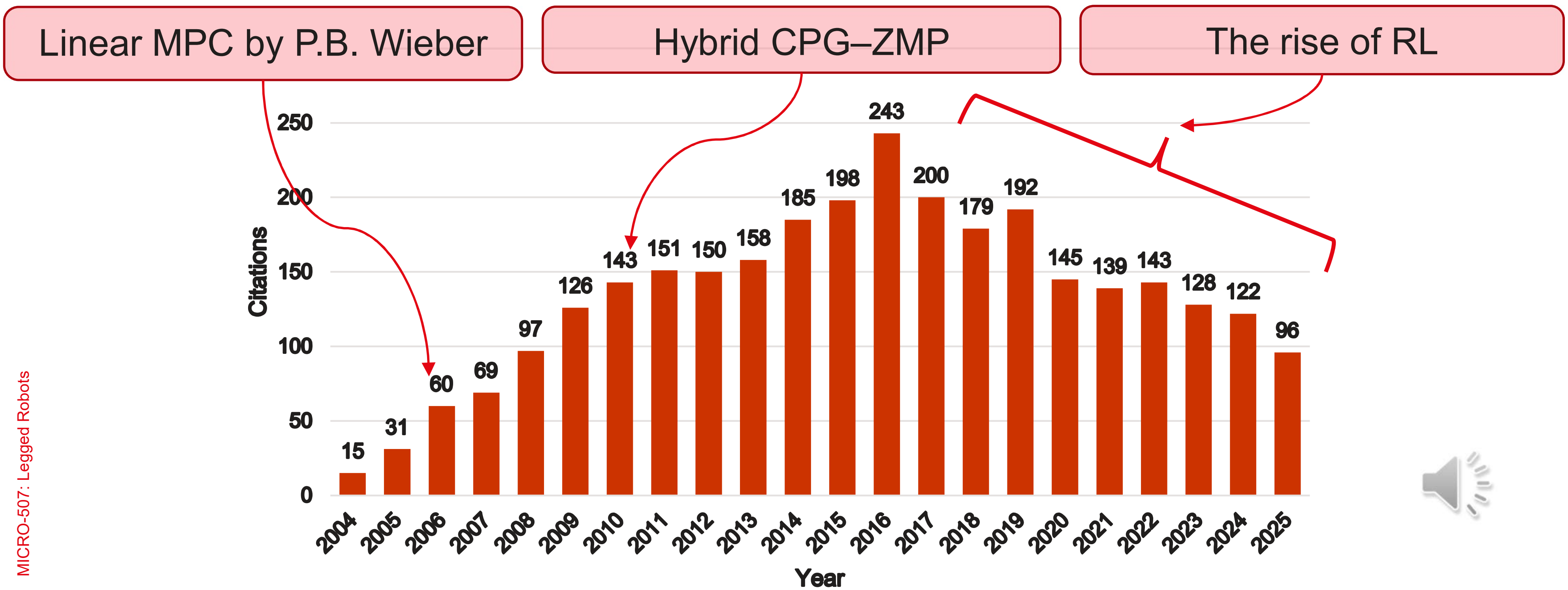


Fig .10: Citations count

Conclusion

- **Pros**
  - Good performance of walking
  - Simple and solvable model
  - Flexibility of preview period

- **Cons**
  - Not robust to different environments
  - Assumes constant hip height
  - Needs explicit foot position input
  - Outdated: Reinforcement Learning used more in legged robotics

Appendix

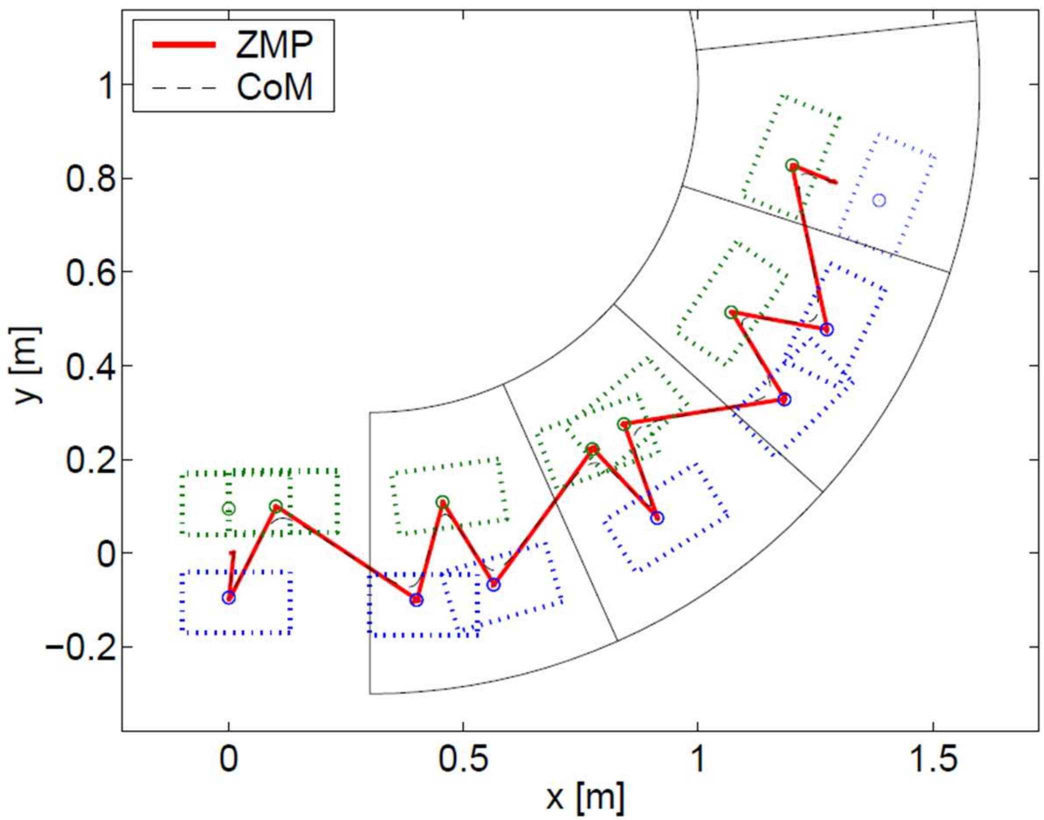


Fig 11: Planned trajectory for a walk on spiral stairs: ZMP and CoM projected on the horizontal plane

Citations

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Speaker

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