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Hybrid Zero Dynamics of Planar Biped Walkers

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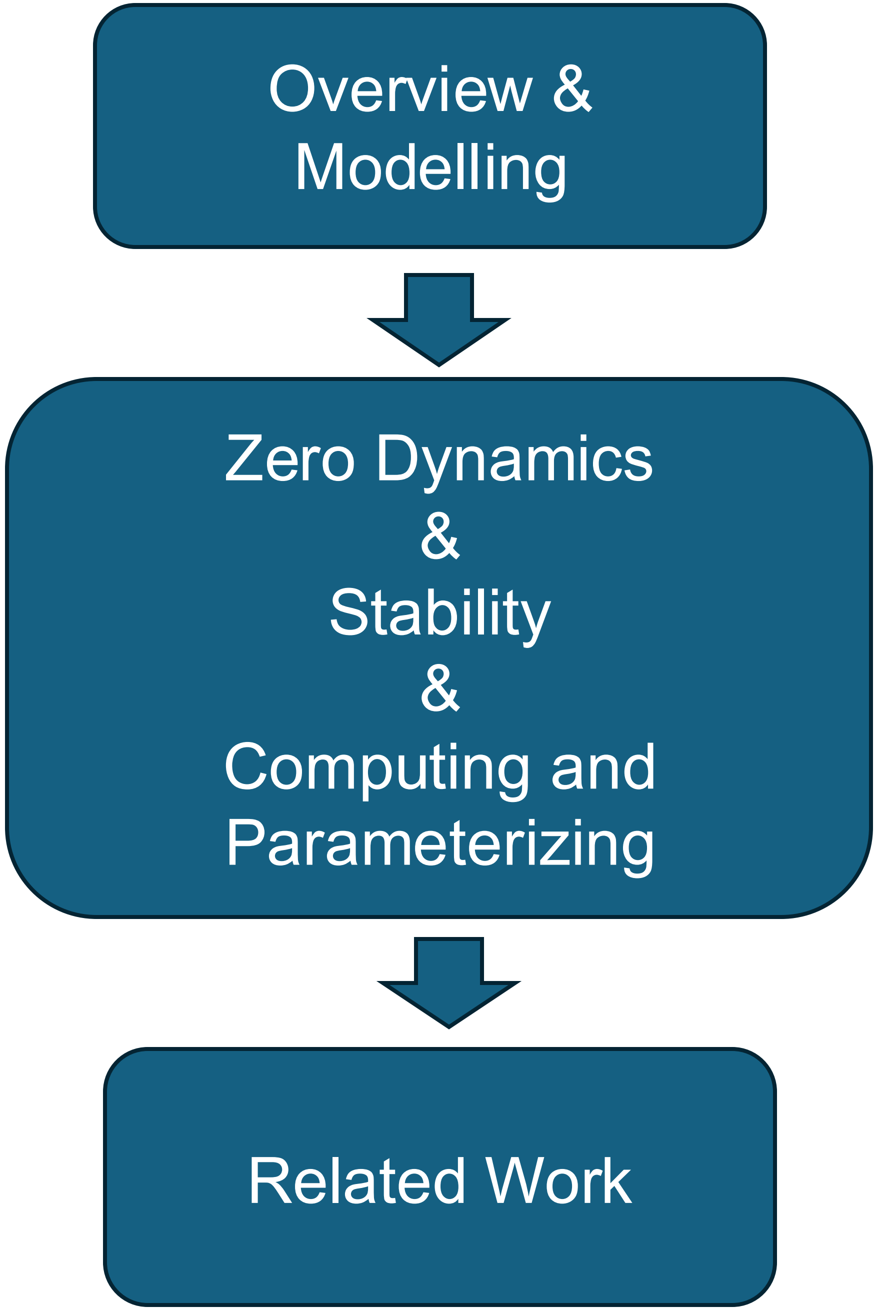
EPFL

Outline


Overview &
Modelling
Zero Dynamics
&
Stability
&
Computing and
Parameterizing
Related Work

Overview

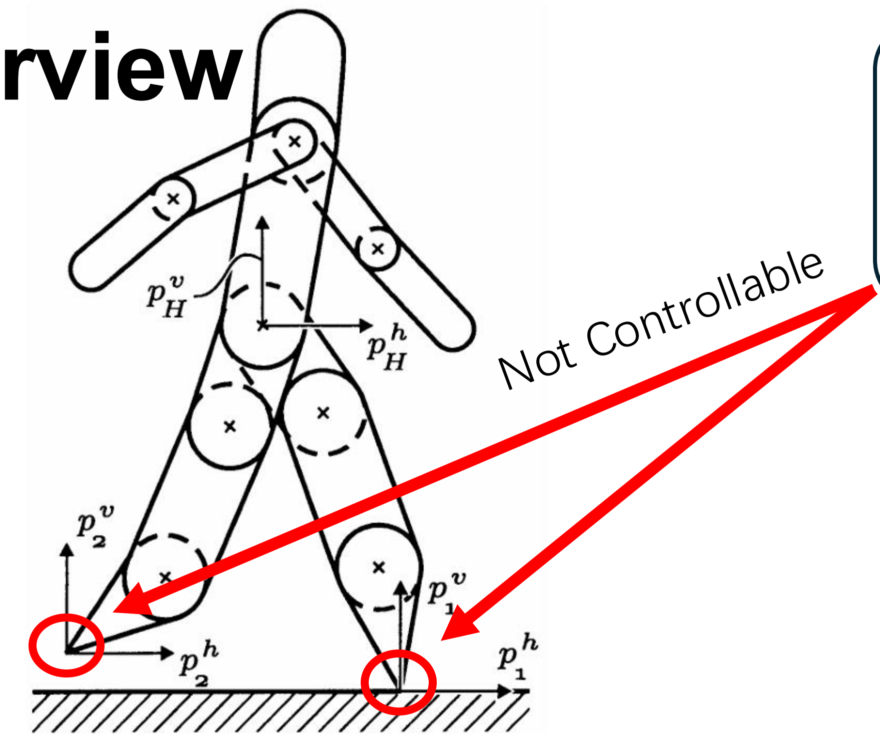


Fig. 1. Higher DOF robot model. Cartesian coordinates are indicated at the hips and the leg ends.

One degree of **Underactuation**, **planar, bipedal walker**

Observation : State Feedback $q, \dot{q}$

Torque Control

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu.$$

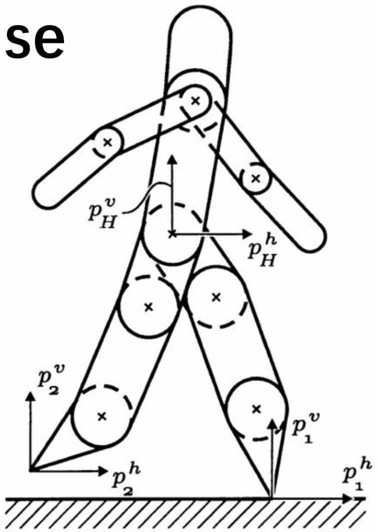
$$\dot{x} = \begin{bmatrix} \dot{q} \\ D^{-1}(q)\left[-C(q,\dot{q})\dot{q} - G(q) + Bu\right] \end{bmatrix} =: f(x) + g(x)u$$

Zero Dynamics & Stability & Computing and Parameterizing

Controller Parameterization

Modelling

Swing Phase



$$\dot{x} = f(x) + g(x)u$$

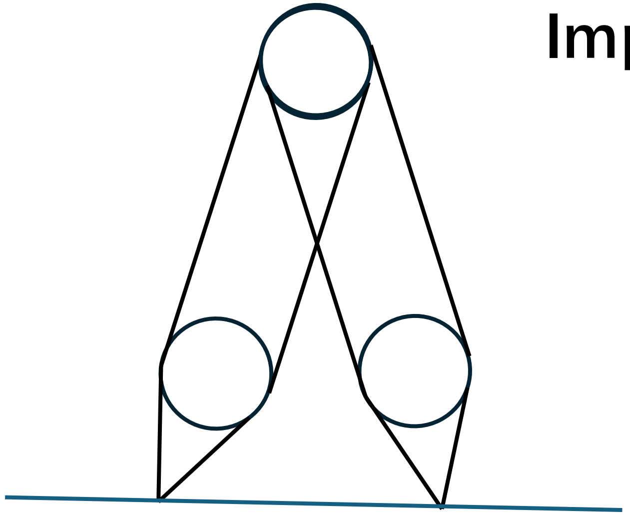
$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu.$$

$$\dot{x} = \begin{bmatrix} \dot{q} \\ D^{-1}(q)\left[-C(q,\dot{q})\dot{q} - G(q) + Bu\right] \end{bmatrix}$$
$$=: f(x) + g(x)u$$

$x := (q, \dot{q})$ State Variable

$q := (q_1, \ldots, q_N)$ Joint Variable

Impact Phase



$$D_e(q_e)\ddot{q}_e + C_e(q_e,\dot{q}_e)\dot{q}_e + G_e(q_e) = B_e u + \delta F_{\text{ext}}$$

$$q_e = \Upsilon(q) \text{ and } \dot{q}_e = \frac{\partial \Upsilon(q)}{\partial q}\dot{q}$$

where $\Upsilon(q) := (q', p_H^h(q), p_H^v(q))'$, and $p_H^h(q)$ and $p_H^v(q)$ are the horizontal and vertical positions of the hip, respectively.

Modelling

Assumptions

IH5) The impulsive forces may result in an instantaneous change in the velocities, but there is no instantaneous change in the configuration.

$$D_e(q_e^-)\dot{q}_e^+ - D_e(q_e^-)\dot{q}_e^- = \left(\frac{\partial E(q_e)}{\partial q_e}\right)' \begin{bmatrix} F_2^T \\ F_2^N \end{bmatrix}$$

velocities

$$\frac{\partial E(q_e)}{\partial q_e}\dot{q}_{e\_}^+ = 0$$

configuration

$$\Pi^{-1}(q_e^-)\begin{bmatrix} \dot{q}_e^+ \\ F_2^T \\ F_2^N \end{bmatrix} = \begin{bmatrix} D_e(q_e^-)\dot{q}_e^- \\ 0 \end{bmatrix}$$

$$\Delta(x^-) := \begin{bmatrix} \Delta_q q^- \\ \Delta_{\dot{q}}(q^-)\dot{q}^- \end{bmatrix}$$

$$\dot{x} = f(x) + g(x)u, \qquad x^- \notin S$$
$$x^+ = \Delta(x^-), \qquad x^- \in S$$

where

$$S := \{(q,\dot{q}) \in TQ | p_2^v(q) = 0, p_2^h(q) > 0\}$$

Zero Dynamics

For a system modeled by *ordinary differential equations* (in particular, no impact dynamics), the **maximal internal dynamics of the system** that are compatible with the **output being identically zero** is called the zero dynamics.

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x)$$

$\Longrightarrow$ When $y = h(x) \equiv 0$ and $\dot{y} = L_f h(x) \equiv 0$, we discuss how would x and u behave.

Note:

$$\dot{y} = \frac{\partial h}{\partial q}\dot{q} = \begin{pmatrix} \frac{\partial h}{\partial q} & 0 \end{pmatrix} f(x) = \frac{\partial h}{\partial x} f(x) = L_f h(x)$$

Lie Derivative

Swing Phase Zero Dynamics (Existence of Zero Dynamics in General Case)

$$\ddot{y} = \frac{\partial \dot{y}}{\partial x}(f(x) + g(x)u) = L_f^2 h(x) + L_g L_f h(x) u \equiv 0$$

$$u^*(x) = -\left(L_g L_f h(x)\right)^{-1} L_f^2 h(x)$$

Decoupling Matrix

$Z := \{x \in T\tilde{\mathcal{Q}} \mid h(x) = 0, L_f h(x) = 0\}$ is a smooth two-dimensional submanifold of TQ;

The existence of zero dynamics:

- Dimension of u = Dimension of y
- Decoupling Matrix is invertible
- There exist a smooth real value function $\theta(q)$, such that $h(q), \theta(q) \leftrightarrow q$ is a diffeomorphism.

We have a new coordinate system to descript the q: $h(q), \theta(q)$

My Understanding and Analogy:
In control theory, signal analysis, or analog circuit, we have something called:
Zero-input response and zero-state response; Particular solution and general solution in ODE

$$y(t) = y_{zi}(t) + y_{zs}(t)$$

Zero Dynamics

Swing Phase Zero Dynamics (Existence of Zero Dynamics in General Case)

$$\eta_1 = h(q) \quad \eta_2 = L_f h(q,\dot q)$$
$$\xi_1 = \theta(q) \quad \xi_2 = L_f \theta(q,\dot q)$$

$\Longrightarrow$

$$\dot\eta_1 = \eta_2 \quad \dot\eta_2 = L_f^2 h + L_g L_f h u$$ Output Dynamics($\eta_1 = y$)

$$\dot\xi_1 = \xi_2 \quad \dot\xi_2 = L_f^2\theta + L_g L_f \theta u$$ Zero Dynamics

$$\dot x = \begin{bmatrix} \dot q \\ D^{-1}(q)\left[-C(q,\dot q)\dot q - G(q) + Bu\right] \end{bmatrix} =: f(x) + g(x)u$$

Since the columns of $g(x)$ are involutive, the Frobenius theorem applies, and therefore one can always find a scalar function $\xi_2 = \gamma(x)$ such that $L_g\gamma = 0$

$$\dot\xi_1 = L_f\theta$$
$$\dot\xi_2 = L_f\gamma$$

Actually we are able to make:
$\gamma = D_N(q)\dot q$
where $D_N(q)$ is the last row of D

Zero Dynamics is invariant w.r.t all input directions [g(x)]

Using definition of the kinematic and potential energy:

$$\dot\xi_1 = \kappa_1(\xi_1)\xi_2$$
$$\dot\xi_2 = \kappa_2(\xi_1).$$

$$\xi_1 = \theta|_Z$$
$$\xi_2 = \left.\frac{\partial K}{\partial \dot q_N}\right|_Z$$
$$\kappa_1(\xi_1) = \left.\frac{\partial\theta}{\partial q}\begin{bmatrix} \frac{\partial h}{\partial q} \\ \gamma_0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right|_Z$$
$$\kappa_2(\xi_1) = -\left.\frac{\partial V}{\partial q_N}\right|_Z$$

Why do we do this whole thing?

On the next slide we will prove that:
Zero Dynamics Variable θ is continuous in both the swing phase and the impact phase.

Zero Dynamics

Hybrid Zero Dynamics (Existence of Zero Dynamics in Impact Phase)

$$\dot{z} = f_{\text{zero}}(z), \quad z^- \notin S \cap Z$$
$$z^+ = \Delta(z^-), \quad z^- \in S \cap Z$$

$\Longrightarrow$

$$\begin{pmatrix} h(q) \\ L_f h(q,\dot{q}) \\ p_2^v(q) \end{pmatrix} = 0$$

If a mapping $\begin{pmatrix} h \\ L_f h \\ p_2^v \end{pmatrix}$ has constant rank 2N-1, then the null-space S∩Z will be a smooth one-dim embedded manifold.

1. An open interval (a,b)
2. The circle S
3. The real line R

Hence, there exist a bijective mapping from R to S∩Z in impact phase.

$\Longrightarrow$ Hybrid zero dynamics exists and is continuous in the whole process. (We can actually prove that θ is monotonically increasing along the step of hybrid zero dynamics)

$$\theta^- := \theta(q_0^-)$$
$$\theta^+ := \theta \circ \Delta_q(q_0^-).$$

Stability Analysis

In one sentence: We use Poincare analysis to find a fix point in the exponentially stable periodic orbit.

$$\zeta_2 = 1/2(\xi_2)^2$$

$$\frac{d\zeta_2}{d\xi_1} = \frac{\kappa_2(\xi_1)}{\kappa_1(\xi_1)}$$

$$V_{\text{zero}}(\xi_1) := -\int_{\theta^+}^{\xi_1} \frac{\kappa_2(\xi)}{\kappa_1(\xi)} d\xi$$

$$\zeta_2^- := \frac{1}{2}(\xi_2^-)^2$$
$$\zeta_2^+ := \delta_{\text{zero}}^2 \zeta_2^-.$$

$$\zeta_2^- = \zeta_2^+ - V_{\text{zero}}(\theta^-)$$

$\Longrightarrow$ $\zeta_2^- = \delta_{zero}^2 \zeta_2^- - V_{zero}(\theta^-)$ $\Longrightarrow$ $\zeta_2^- = -\dfrac{V_{zero}(\theta^-)}{1-\delta_{zero}^2}$ is a fix point.

In order to be exponentially stable, $0 < \delta_{zero}^2 < 1$.

We also need to prove that the controller/output is stability (method: there exist a mapping from zero-dynamic stability to get output stability)

Computing and Parameterizing Hybrid Zero Dynamics

Almost linear output function structure

$y = h(x) \coloneqq h_0(q) - h_d\big(\theta(q)\big)$ where $h_0(q) = H_0 q,\ \ \theta(q) = cq$

In zero dynamics: $h_0(q) = h_d\big(\theta(q)\big)$ $\Longrightarrow$ $q = \begin{pmatrix} H_0 \\ c \end{pmatrix}^{-1} \begin{pmatrix} h_d(\xi_1) \\ \xi_1 \end{pmatrix}$

Bezier Polynomial Parameterization $b_i(s) := \sum_{k=0}^{M} \alpha_k^i \frac{M!}{k!(M-k)!} s^k (1-s)^{M-k}.$

$$h_d(\xi_1) = \begin{pmatrix} b_1(\theta) \\ b_2(\theta) \\ b_3(\theta) \\ \cdots \\ b_{N-1}(\theta) \end{pmatrix}$$

θ as Bezier phase variable,
we also must add constraints to the Bezier polynomial and constrict Bezier coefficients.

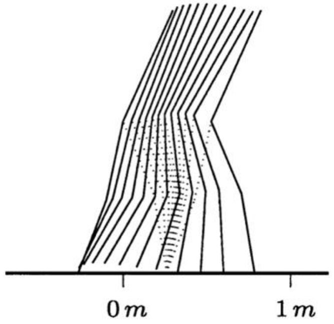
Turning this into an optimization problem:

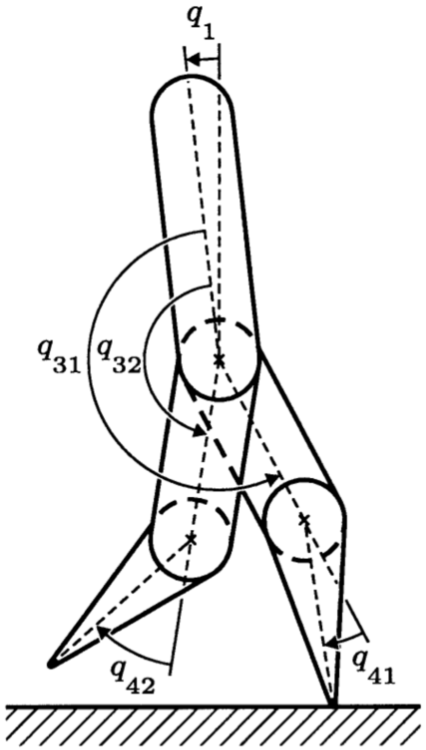
- The cost function: minimizing energy consumption of one step duration (one orbit)
- The variables: (N-1)*(M+1- 2) Bezier coefficients.
- The system constraints:
  - Hybrid Zero Dynamics
  - Hand-made constraints:
    - Nonlinear Inequality Constraints
    - Nonlinear Boundary Equality Constraints
    - Explicit Boundary Constraints

Implementation

5-link Biped Walker

$$y = h_0(q) - h_d \circ \theta(q) = \begin{bmatrix} q_{31} \\ q_{32} \\ q_{41} \\ q_{42} \end{bmatrix} - h_d \circ \theta(q).$$





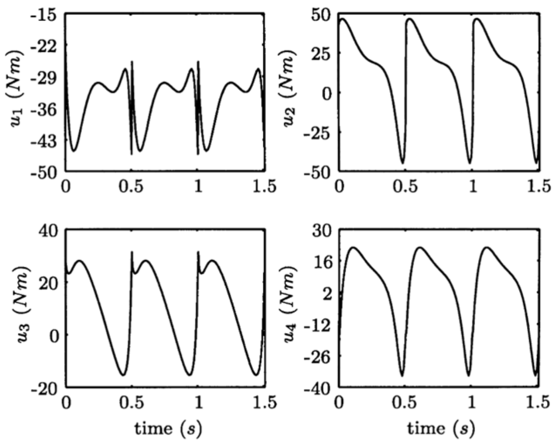
1. The first and second waypoint of Bezier curve is conditioned of the last one -> The Bezier curve length M > 3:

M chosen to be 6. (Problem: Too short for walk, too long for optimization vars)

2. There exists at least one point in where vanishes -> The smoothness of S∩Z needs to be guaranteed:

$$\frac{\partial}{\partial q}\begin{bmatrix} h(q) \\ p_2^v(q) \end{bmatrix}\Bigg|_{x \in S \cap Z} = \begin{bmatrix} H_0 - \frac{M}{\theta^- - \theta^+}\left(\alpha_M - \alpha_{M-1}\right) c \\ \frac{\partial p_2^v(q)}{\partial q}\Big|_{q=q_0^-} \end{bmatrix}$$

3. Other parameters are based on Assumptions.



Torque

Summary

Object:

- We want a **four-motor** biped [illegible] o walk stably.
- We hope to find a control m[illegible] stability is guaranteed mathematically.

Control the four motors to enforce $h(q)=0$

Lagrangian dynamics with **four joint** variables: too high-dimensional

Ground impact force

eliminate unnecessay DOF

Find a function, drive it to **zero**
This acts **4 constraints** that **reduces** the dimension to 1

During impact, the configuration stays continuous and **angular momentum is conserved**.

form a closed dynamics

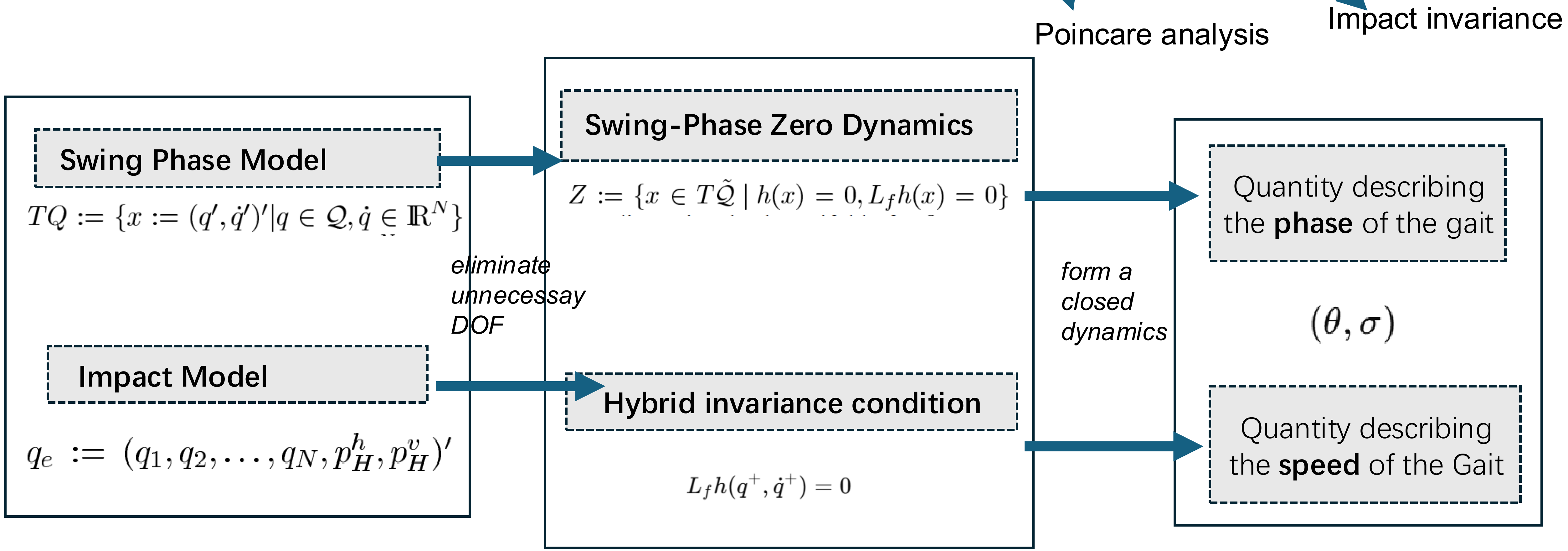
Quantity describing the **phase** of the gait

Quantity describing the **speed** of the Gait

Summary

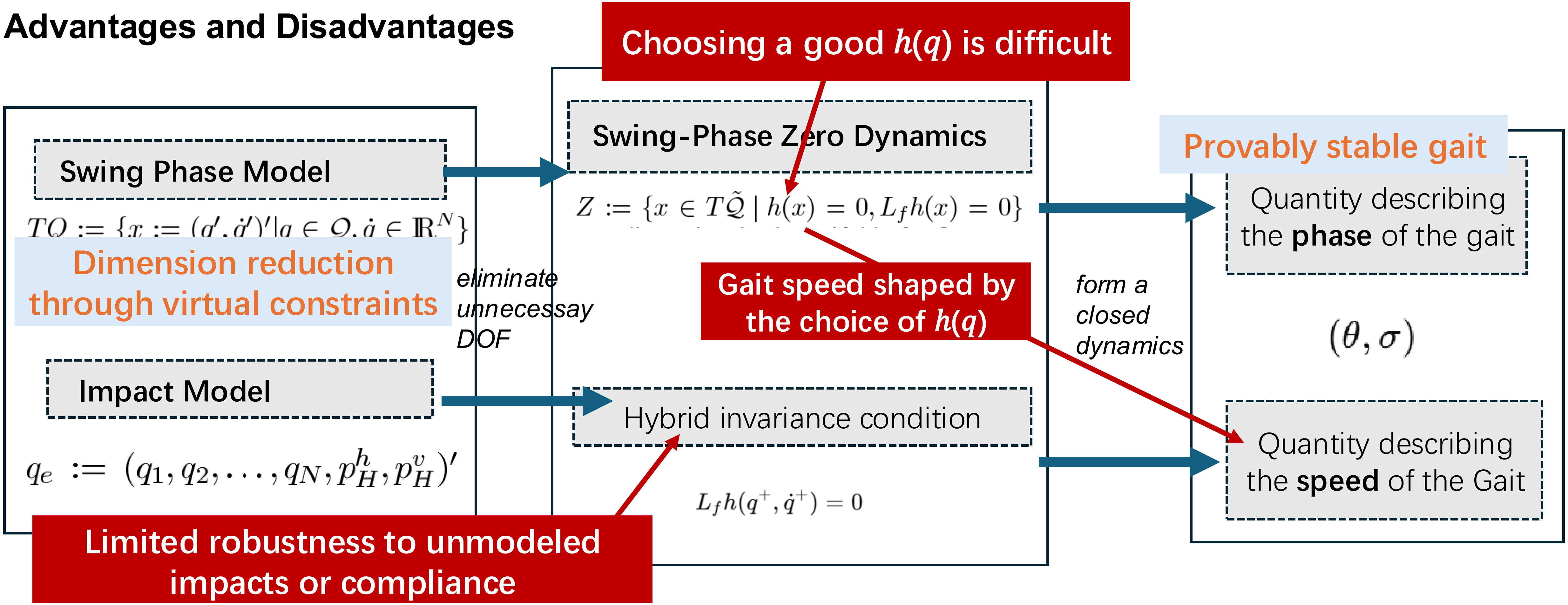
The Basic idea of Hybrid Zero Dynamics

- Restricting the dynamics of the robot on **a lower-dimensional** attractive and invariant subset of its state space.



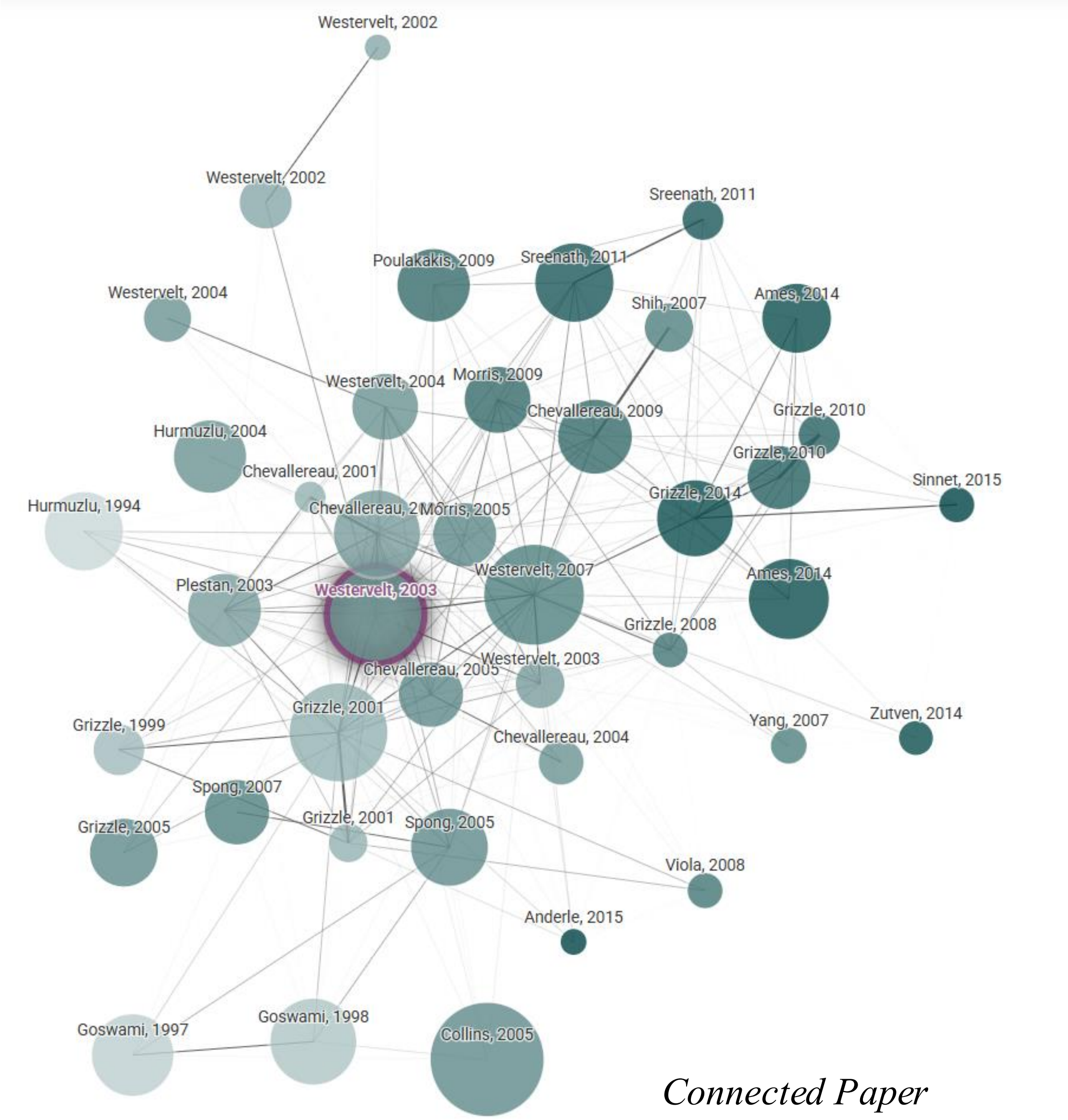
Summary

HZD provides a rigorous framework enabling provably stable dynamic walking, profoundly shaping modern legged-robotics control.

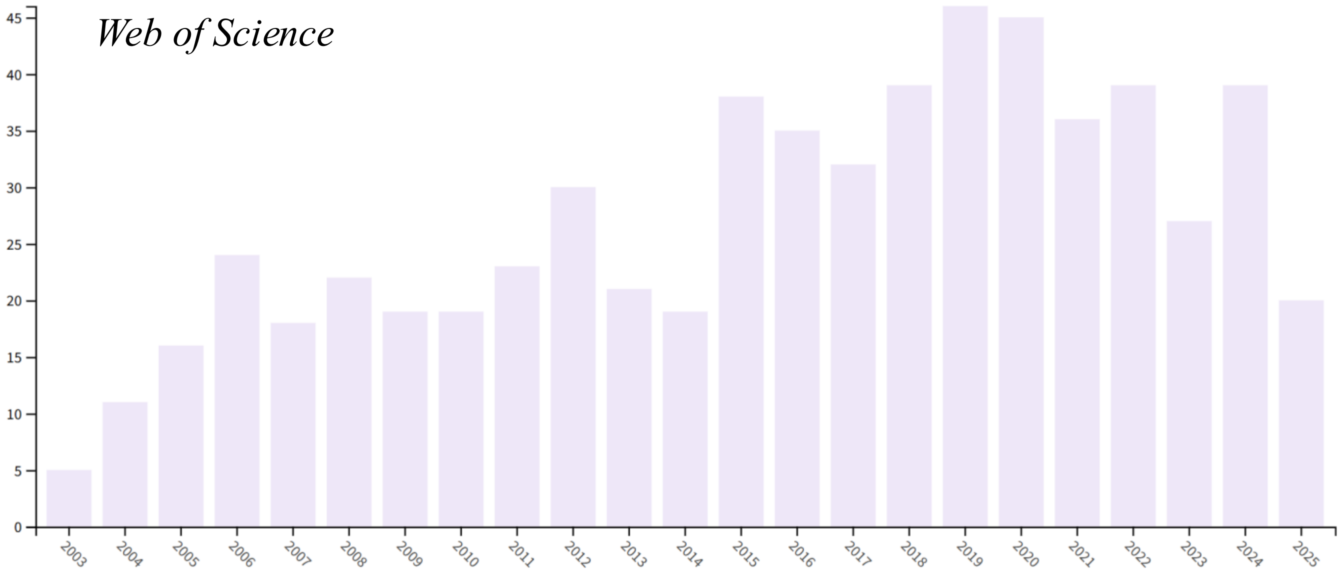


Summary

who is citing the article



Cited by
1185 (Google Scholar), 633 (Web of Science)



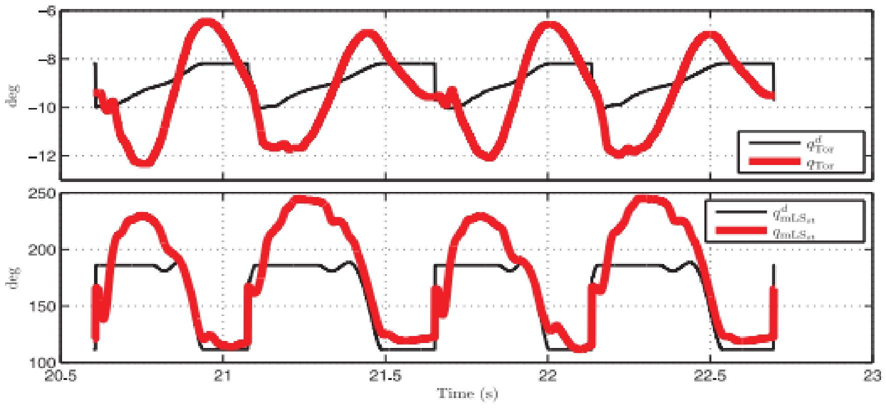
Summary

who is citing the article

MABEL

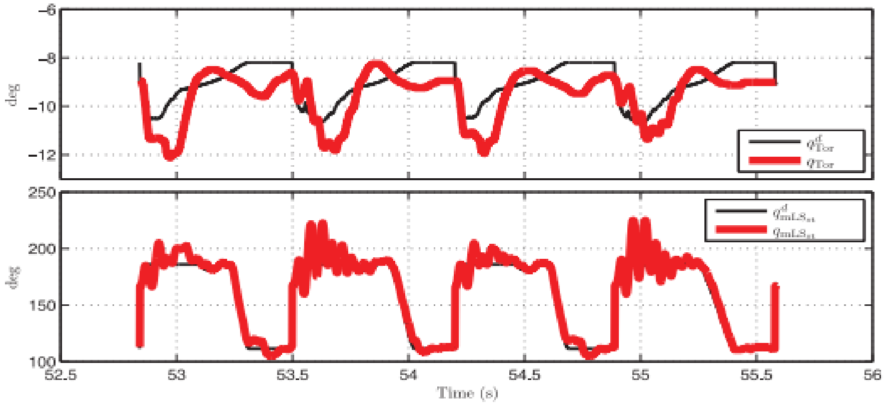
2010, developed by Jessy Grizzle at Umich

PD+Feedforward



(a)

HZD



(b)

leg motor positions

Sreenath K, Park H-W, Poulakakis I, Grizzle JW. A Compliant Hybrid Zero Dynamics Controller for Stable, Efficient and Fast Bipedal Walking on MABEL. *The International Journal of Robotics Research*. 2010;30(9):1170-1193. doi:10.1177/0278364910379882

Summary

Biped robot

use human locomotion data as inspiration, achieving walking behaviors on the prosthesis

extend to walking behaviors that more clearly resemble humans.

extend to three dimensional (3D) walking

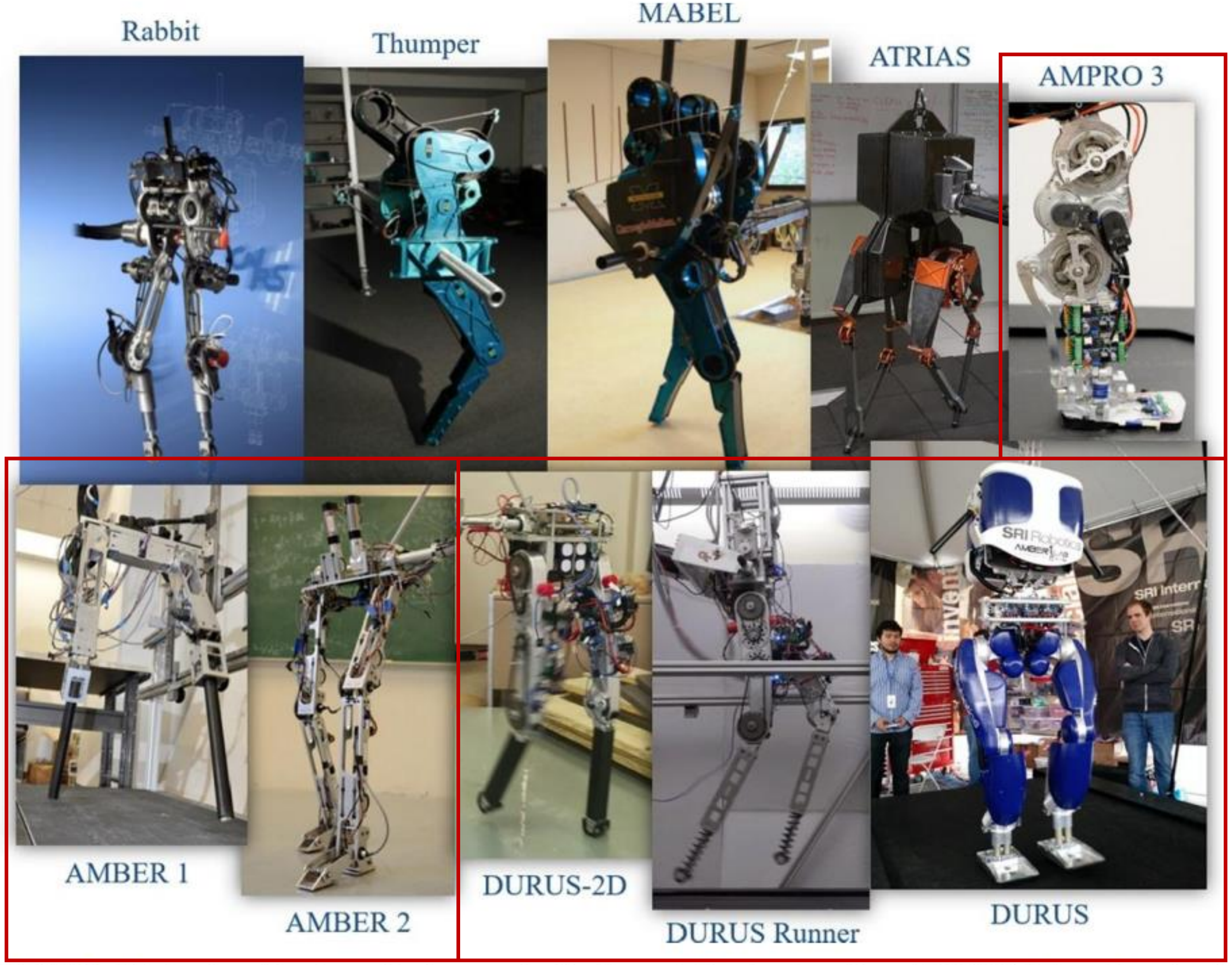


FIGURE 4.7.1 A collection of robots and robotic assistive devices for which HZD-based methods have successfully resulted in stable bipedal locomotion.

[2]Aaron D. Ames∗∗ and Ioannis Poulakakis††, Hybrid Zero Dynamics Control of Legged Robots, Chapter 4.7