Legged Robots

Lecture 2

Gaits, Models, Stability criteria, and Locomotion metrics

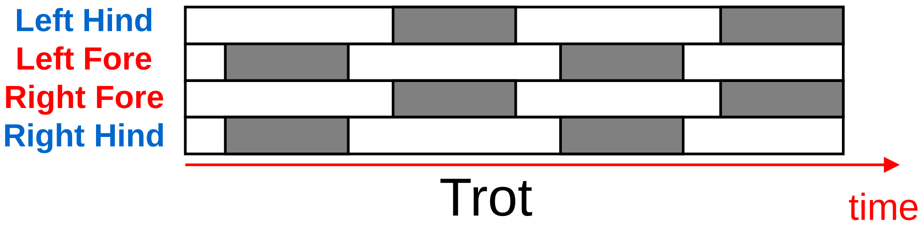
Auke Jan Ijspeert

Different types of gaits

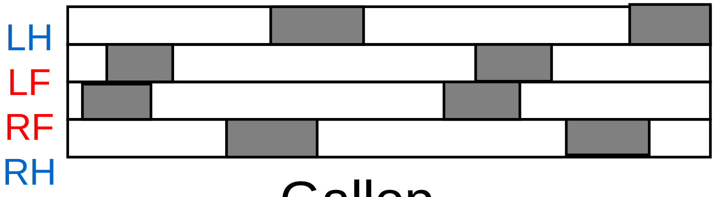
What is a gait?

- A **gait** is a **cyclic pattern of locomotion** for legged animals

- Different species of mammals use **different gaits for different speeds**: cats, dogs and horses can walk, trot and gallop while elephants and giraffes usually walk, pace and gallop
- Humans typically use two gaits, walk or run, but can also skip, or use other variants of walk and run (e.g. power walk, and jog)
- A terrestrial legged gait is mainly defined by the **sequence of footfall patterns**

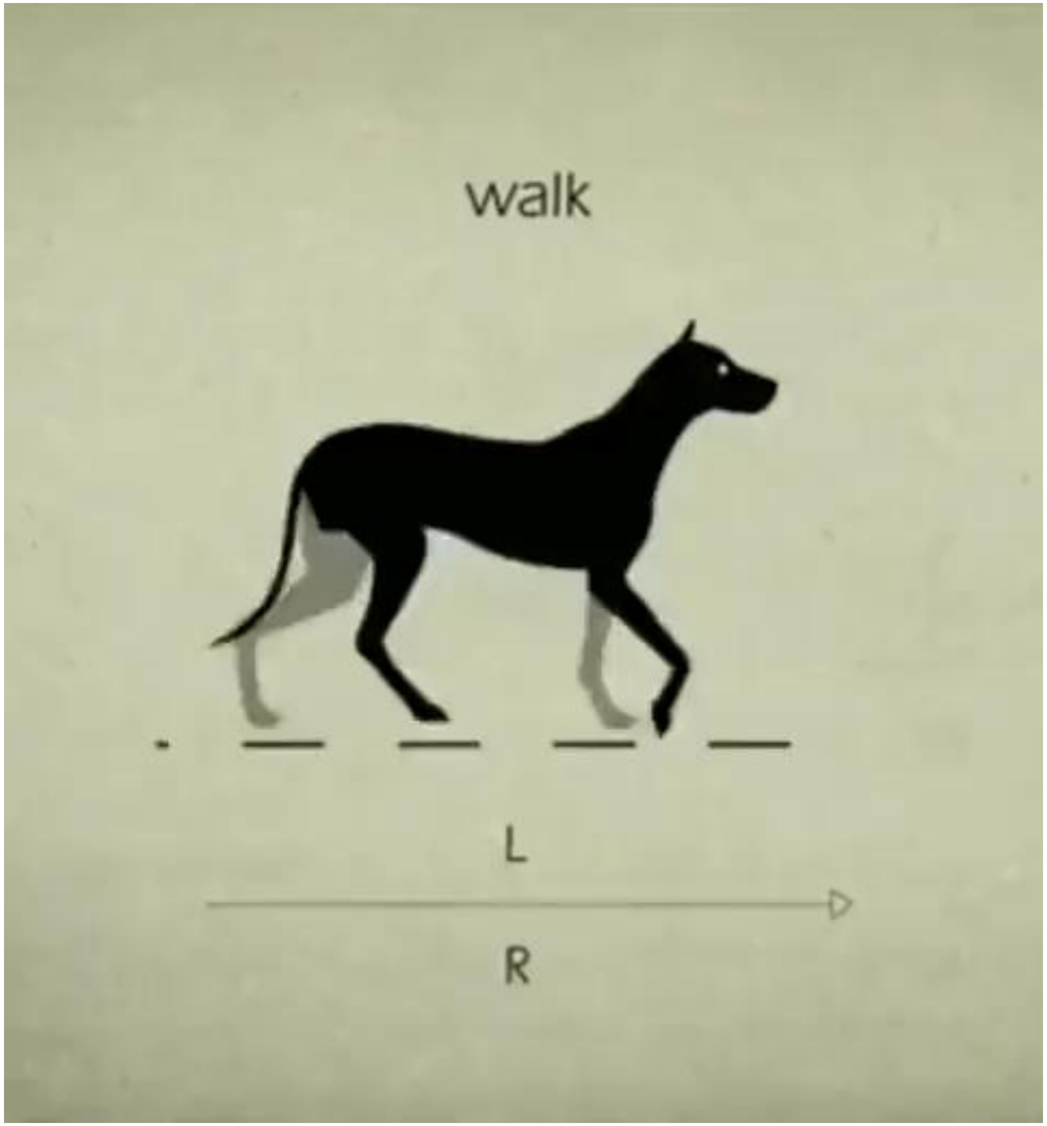


Trot



Gallop

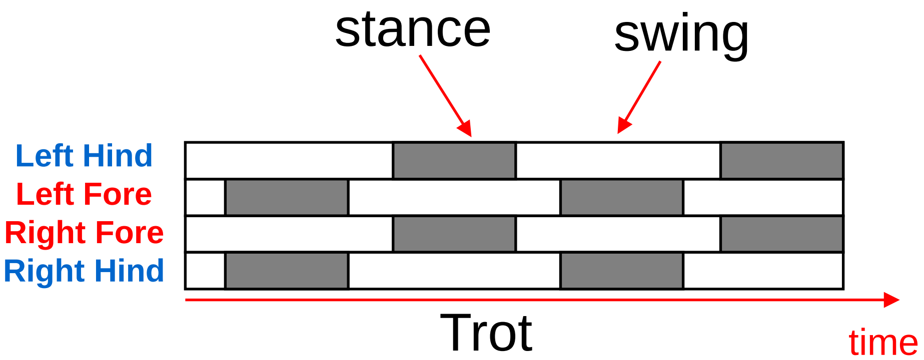
Some dog gaits

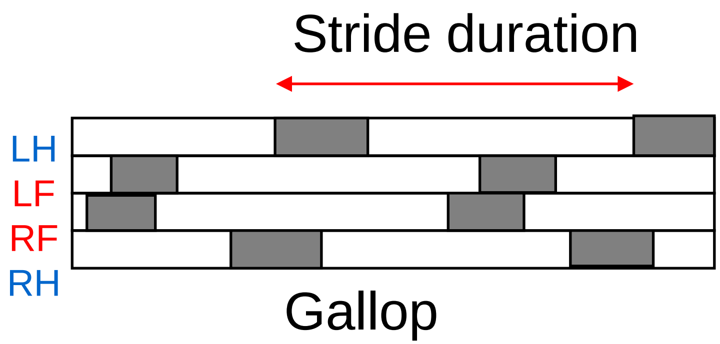


https://twitter.com/wonderofscience/status/1663175862622253058

Some useful terminology

- ***Stride duration*** = the duration of a complete cycle (the period)
- ***Swing phase*** of a limb (period during which the limb is off the ground)
- ***Stance phase*** (period during which the limb touches the ground)
- ***Duty factor*** = Stance duration / Stride duration



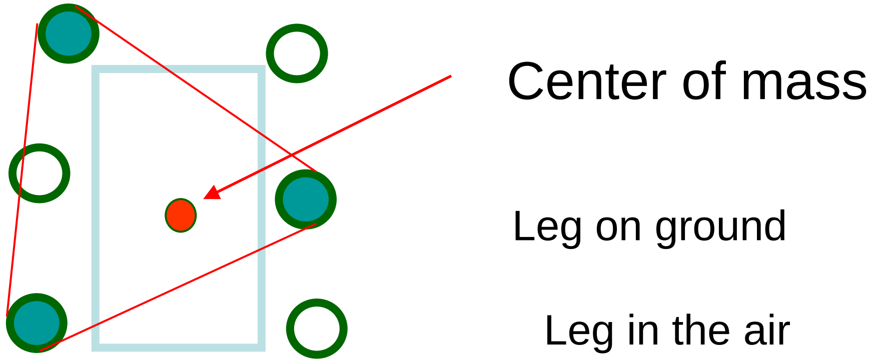


4

Statically versus dynamically stable gaits

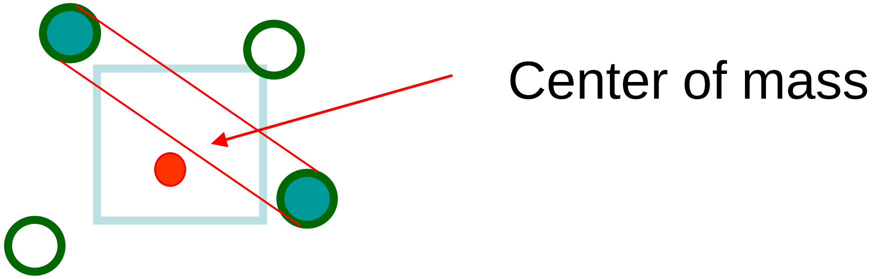
- ***Statically stable gait***: the center of mass is maintained at all times above the *support polygon* formed by the contacts between the limbs and the ground

Tripod gait in a hexapod robot:



- ***Dynamically stable gait***: the center of mass is maintained over the support polygon only *on average* over time

Trotting gait in a quadruped robot:

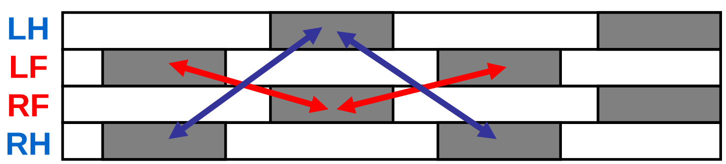


Hildebrand Classification of gaits

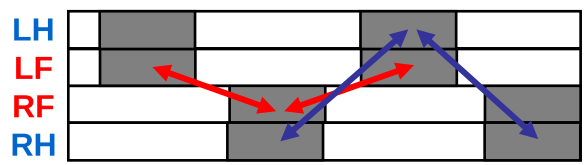
First complete classification of possible gaits by Hildebrand (1965)

We generally make the distinction between symmetric and asymmetric gaits

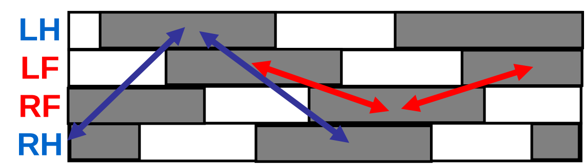
Symmetric gaits ➔ the footfalls of a pair of feet (fore or hind) are evenly spaced *in time* (e.g. walk, pace, trot)



Trot

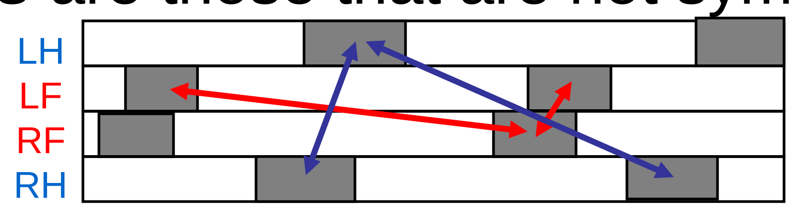


Pace

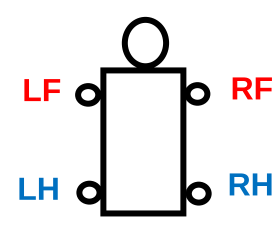


Lat. Sequence walk

Asymmetric gaits are those that are not symmetric (e.g. gallop)



Gallop

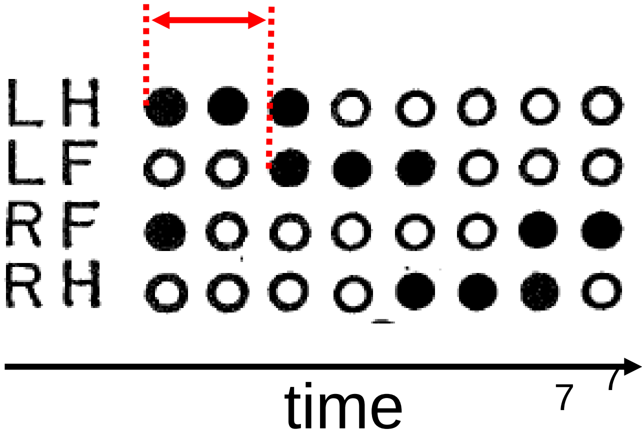


Classification of symmetrical gaits

Following Hildebrand classification of symmetrical gaits, it is sufficient to measure **two quantities** to classify all the possible symmetric gaits in quadrupeds, namely:

1. **Duty factor**
   - ***Walking* gait** if duty factor > 0.5
   - ***Running* gait** if duty factor < 0.5
   - Note: walking and running do therefore not specify a particular gait but are just a characteristic. E.g. one can have a *walking trot* (e.g. in salamander) or a *running trot* (in a horse).

2. **Percentage of stride interval** that the footfall of a forefoot lags behind the footfall of the hind foot on the same side of the body



Examples

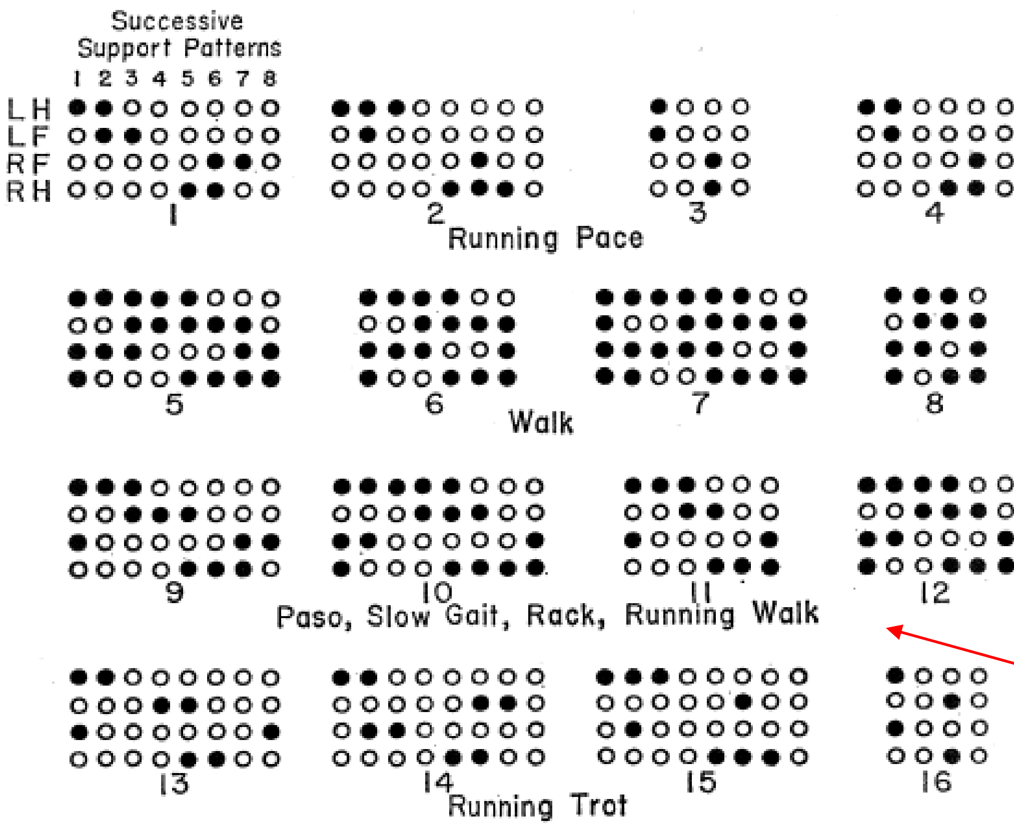
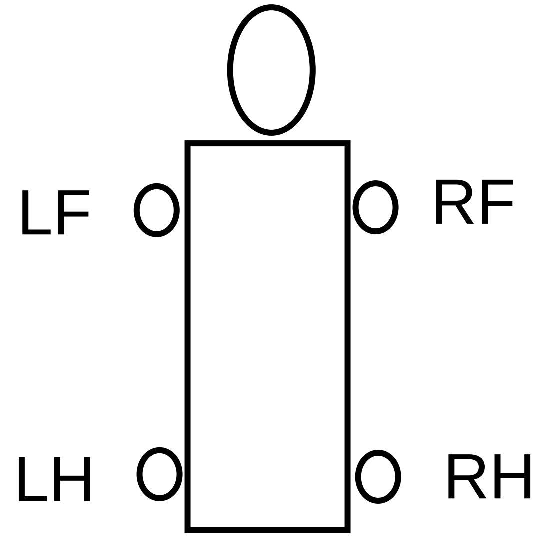


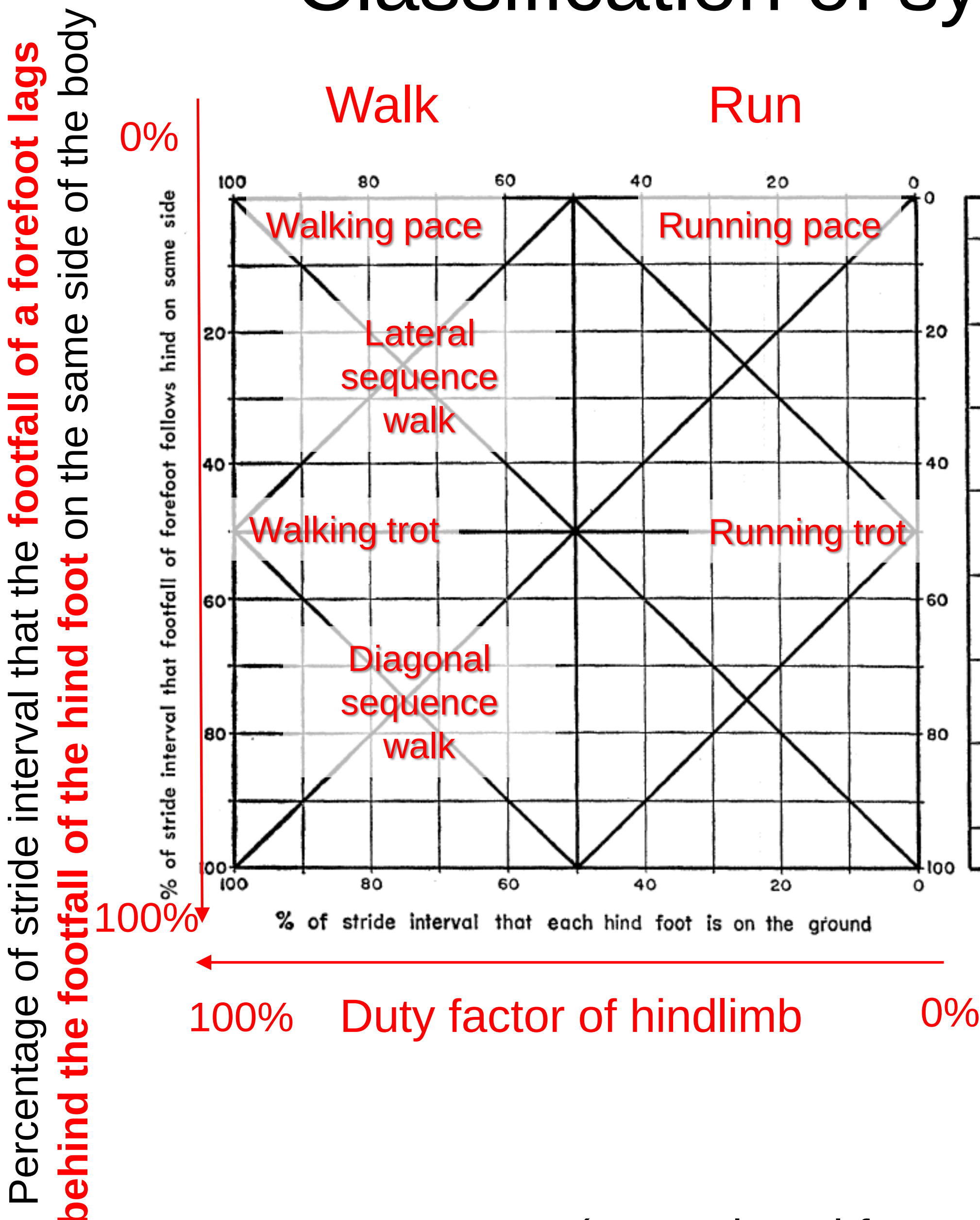
Fig. 3. Sixteen of the many support sequences that might be used by horses doing the gaits indicated. The initials L, R, F, and H stand for left, right, fore, and hind feet. Black circles indicate feet supporting weight; open circles, unweighted feet. Within each diagram, a vertical row of four circles shows a particular pattern of support. Thus, in the fifth support pattern of support sequence No. 1, only the RH foot is on the ground. Each sequence starts with the footfall of the LH foot. Sequences 1, 5, 9, 10, 13, 14, and 15 are relatively common for horses.



Note: these are names used by horse trainers

Hildebrand (1965)

Classification of symmetrical gaits



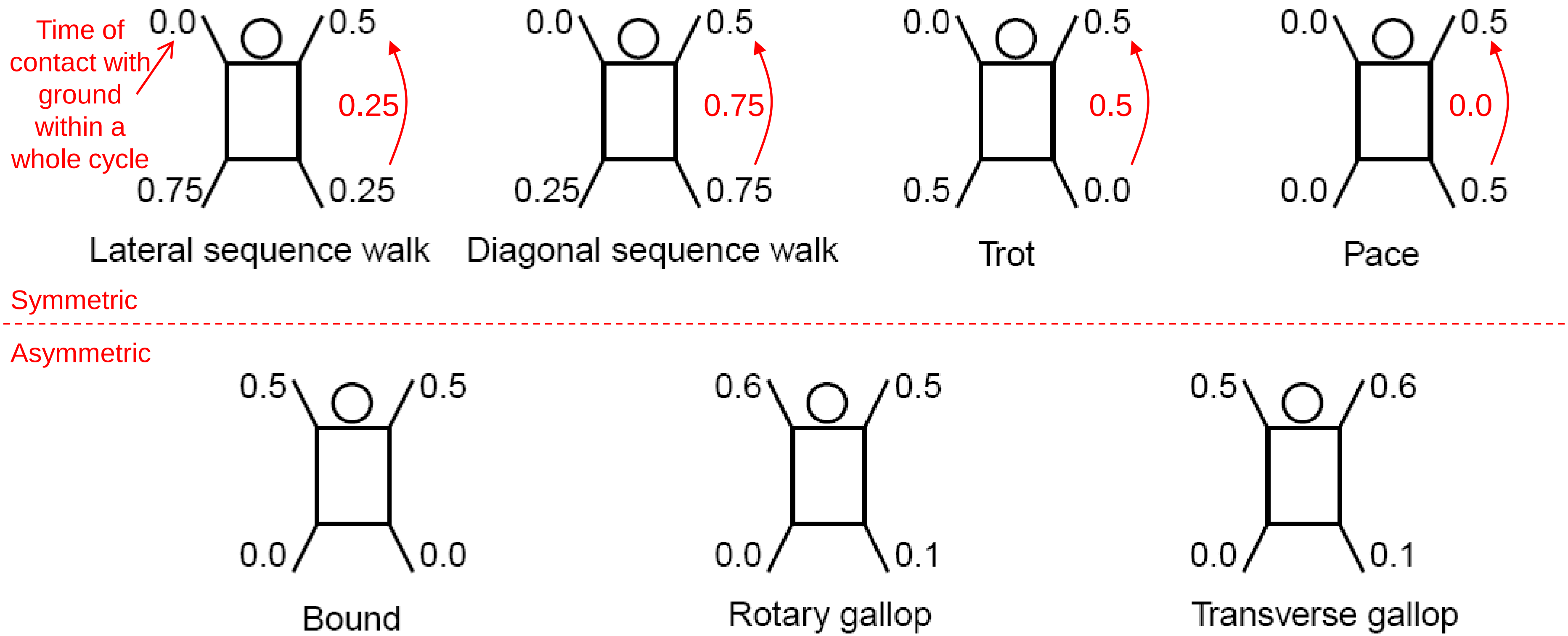
		WALK Very Slow	WALK Slow	WALK Moderate	WALK Fast	RUN Slow	RUN Moderate	RUN Fast
PACE			slow walking pace	mod. walking pace	fast walking pace	slow running pace	mod. running pace	fast running pace
LATERAL SEQUENCE	Lateral Couplets	very slow lat. seq. lat. cpts. walk	slow lat. seq. lat. cpts. walk	moderate lat. seq. lat. cpts. walk	fast lat. seq. lat. cpts. walk	slow lat. seq. lat. cpts. run	moderate lat. seq. lat. cpts. run	fast lat. seq. lat. cpts. run
LATERAL SEQUENCE	Single-foot	very slow walk. lat. seq. single foot	slow walk. lat. seq. single-foot	moderate walk. lat. seq. single-foot	fast walk. lat. seq. single foot	slow run. lat. seq. single foot	moderate run. lat. seq. single foot	fast run. lat. seq. single-foot
LATERAL SEQUENCE	Diagonal Couplets	very slow lat. seq. diag. cpts. walk	slow lat. seq. diag. cpts. walk	moderate lat. seq. diag. cpts. walk	fast lat. seq. diag. cpts. walk	slow lat. seq. diag. cpts. run	moderate lat. seq. diag. cpts. run	fast lat. seq. diag. cpts. run
TROT		very slow walking trot	slow walking trot	moderate walking trot	fast walking trot	slow running trot	moderate running trot	fast running trot
DIAGONAL SEQUENCE	Diagonal Couplets		slow diag. seq. diag. cpts. walk	moderate diag. seq. diag. cpts. walk	fast diag. seq. diag. cpts. walk	slow diag. seq. diag. cpts. run	moderate diag. seq. diag. cpts. run	fast diag. seq. diag. cpts. run
DIAGONAL SEQUENCE	Single-foot			moderate walk. diag. seq. single foot	fast walk. diag. seq. single-foot	slow run. diag. seq. single foot	moderate run. diag. seq. single foot	fast run. diag. seq. single-foot
DIAGONAL SEQUENCE	Lateral Couplets							
PACE								

(Reproduced from Hildebrand 1965)

Most common quadruped gaits

Lateral sequence walk

Trot



Rotary gallop

Other definitions of gaits

- Asymmetric gaits were also classified by Hildebrand (1977) using more variables to describe them

- Hildebrand classification is the most widespread way of classifying gaits

- But there exist other possibilities to classify gaits
  - Walking/running definition based on energetic considerations (e.g. Cavagna et al. 1976). In walking the kinetic and potential energy of the center of mass of the animal change in opposite phase while in running they change in phase
  - To represent gait variations under non-steady locomotion and to break the separation between symmetric and asymmetric gaits (Abourachid 2003)

Insect walking gaits

- Insect (legged) gaits have also been characterized, e.g. D. Wilson, *Insect Walking*, Annual Review of Entomology 11, pp103-122, 1966.

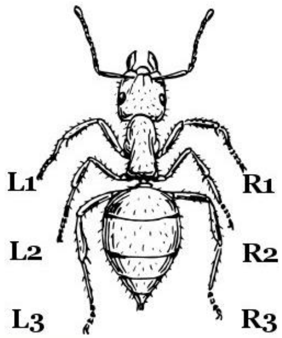
- See also a nice description here:
- http://blog.dave-wood.org/maths/insect-locomotion/

- Three main gaits used by insects:
- At very slow speeds: ***(slow) metachronal wave gait,*** with maximum one leg in swing at a given time

- At medium speeds: ***tetrapod gait, (or fast metachronal wave gait)*** with maximum two legs in swing at a given time

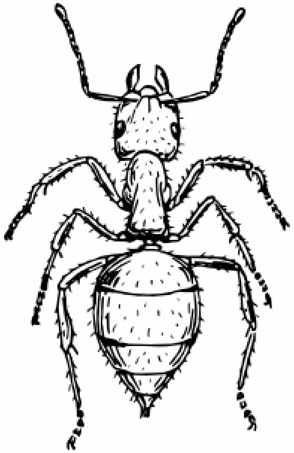
- At faster speeds: ***tripod gait,*** with three legs in swing.

Insect walking gaits



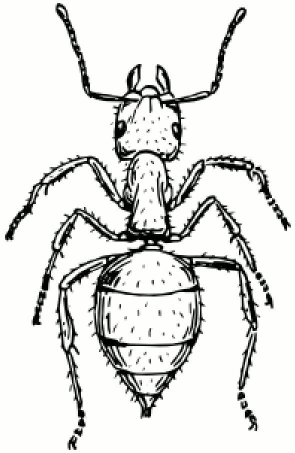
http://blog.dave-wood.org/maths/insect-locomotion/

Animated gif, with red dot showing limb in swing



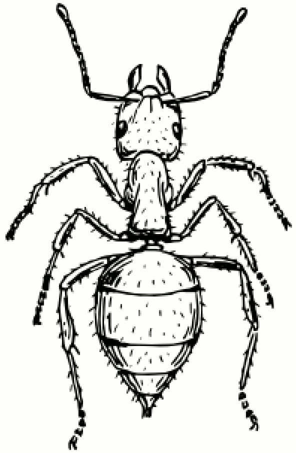
Slow metachronal wave

(L3)(L2)(L1)(R3)(R2)(R1)



Tetrapod gait
(or fast metachronal wave)

(L3 R1)(L2)(L1 R3)(R2)



Tripod gait

(L3 L1 R2)(R3 L2 R1)

Legs in brackets are in swing, the tetrapod gait (L3 R1)(L2)(L1 R3)(R2) means that L3 and R1 are in swing together, followed by L2 on its own, then L1 and R3 together and so on.

13

Use of models in legged robots

- Models (kinematic and dynamic) are very useful in legged robotics
- They are used when deciding and **designing the mechanical structure** of the robot
- They are used to design **physics-based robot simulations**
- They are **essential for model-based locomotion controllers**, where they are used for **feet placement**, **balance control**, and/or **full-body control**.
- They are **essential for optimization-based approaches**, e.g. for optimal control
- They are useful for **post-processing**, e.g. to characterize locomotion properties after experiments are run.

Different types of models

- **Kinematic models**. Pure geometrical models that are useful for determining the work space of the robot, for preventing collisions, for designing dynamic models.
- **Dynamic models,** i.e. models with masses and inertias to compute the forces and torques necessary for locomotion.
- **Forward and inverse models**. E.g. inverse kinematics to determine joint angles to reach a desired posture. Inverse dynamics to compute torques to perform a desired movement.
- **Different levels of complexity**, from simple to full models, from linear to nonlinear models. Today we will see simple models, also called **template models**.
- Here again these models are not only used in robotics, but **also to study animal and human locomotion**, in biomechanics and motor control.

Simple dynamic models of walking and running

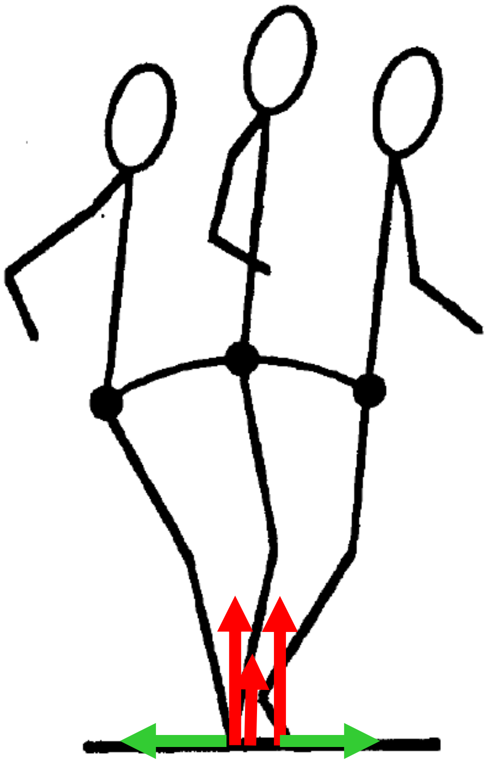
Interestingly many legged animals of different sizes, masses and number of limbs share similar features.

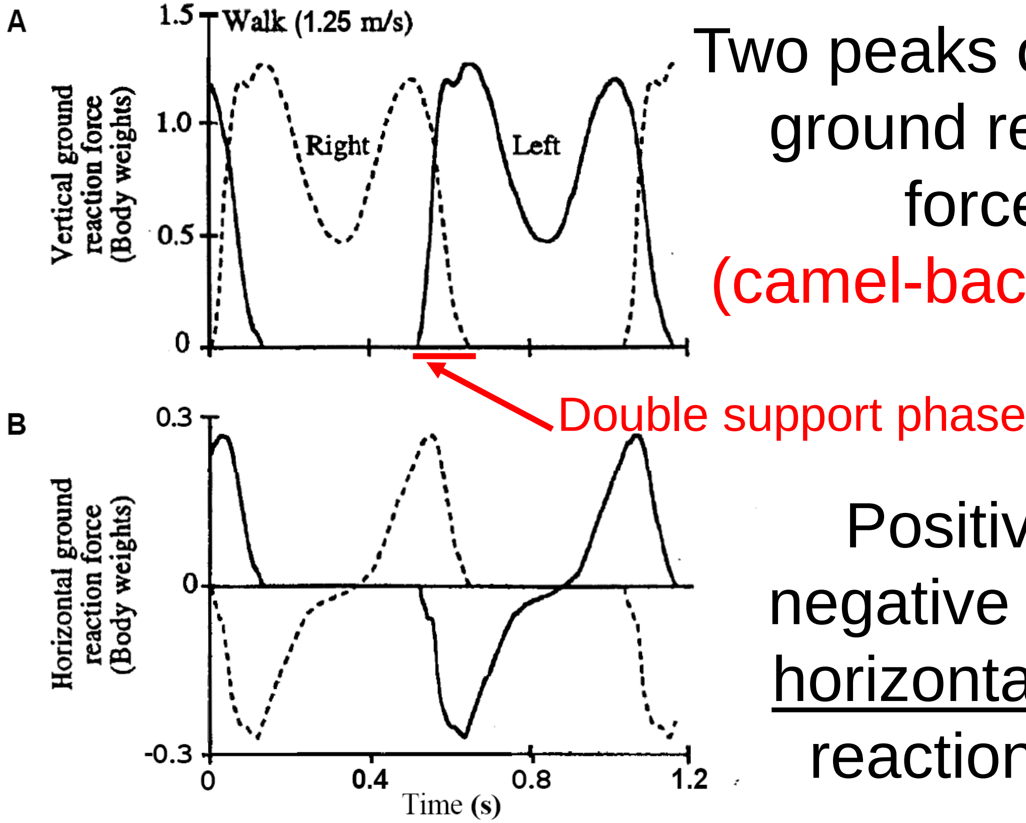
Simple **pendulum models** can explain several features of **walking** in a variety of animals and robots

Similarly, simple **spring-mass models** can explain several features of **running**

Note: **modeling running is simpler than walking!** This is because walking has double support phases (i.e. times during which both limbs have ground contacts, which creates a closed-kinematic chain)

Ground reaction forces during walking



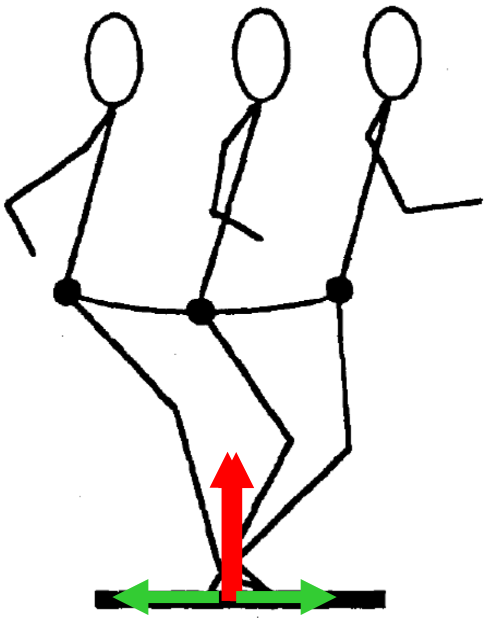


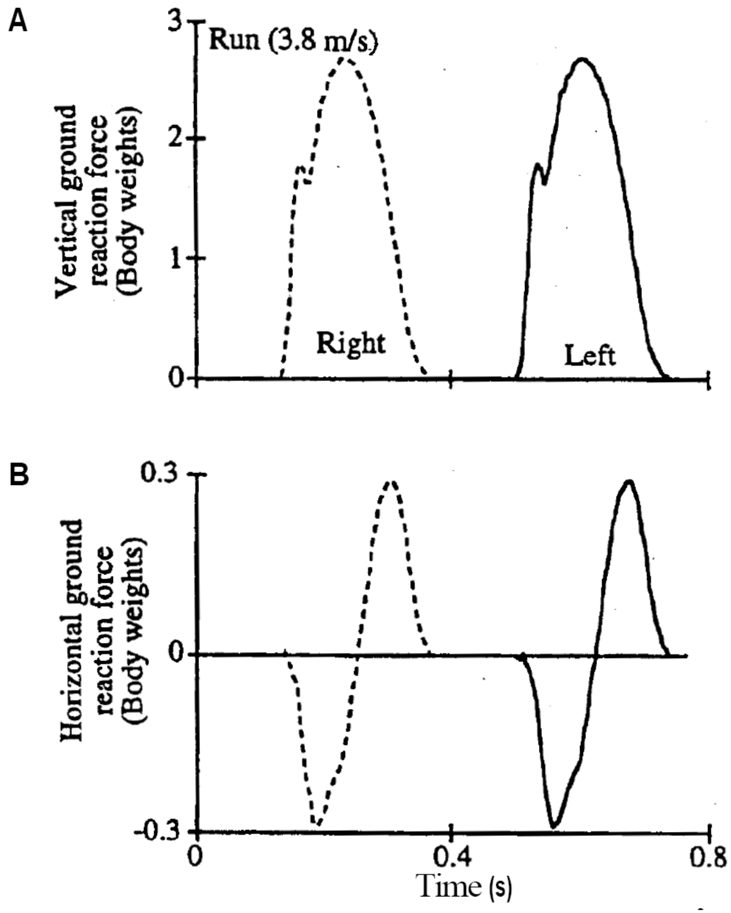
Two peaks of <u>vertical</u> ground reaction forces (camel-back shape)

Positive and negative peaks of <u>horizontal</u> ground reaction forces

From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

Ground reaction forces during running



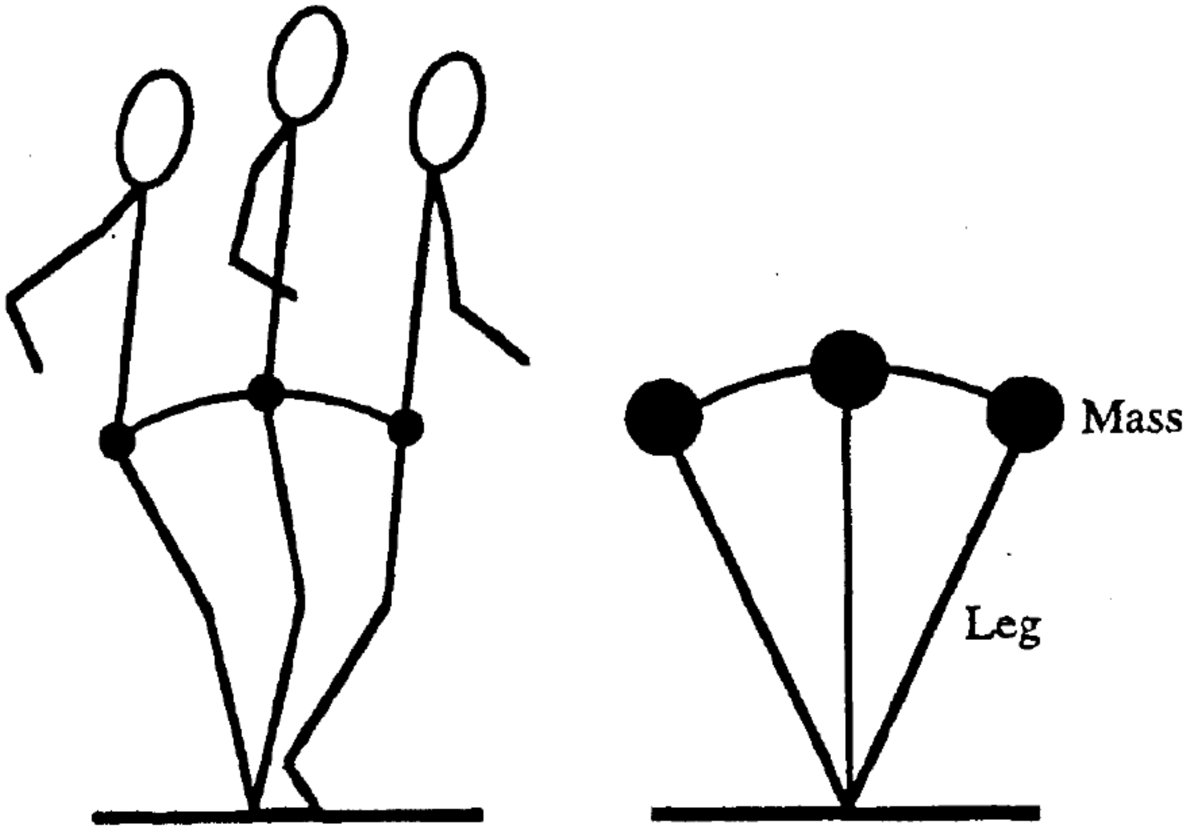


Unlike walking: Single peak of <u>vertical</u> ground reaction forces

Like walking: Positive and negative peaks of <u>horizontal</u> ground reaction forces

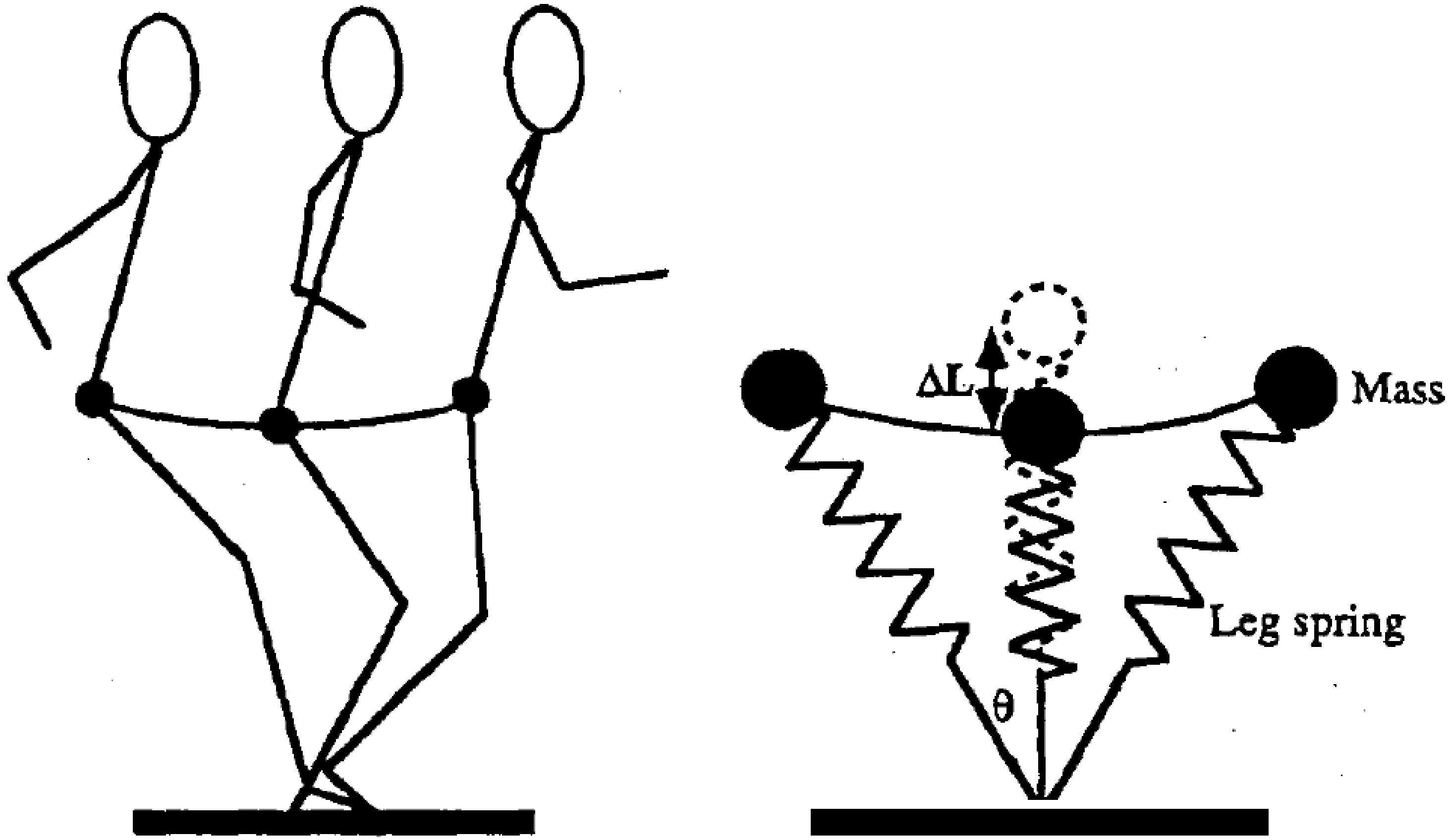
From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

Inverted pendulum model of walking



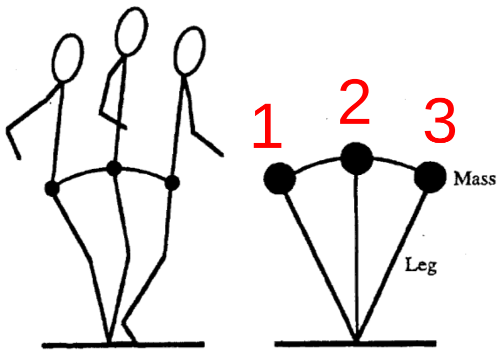
From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

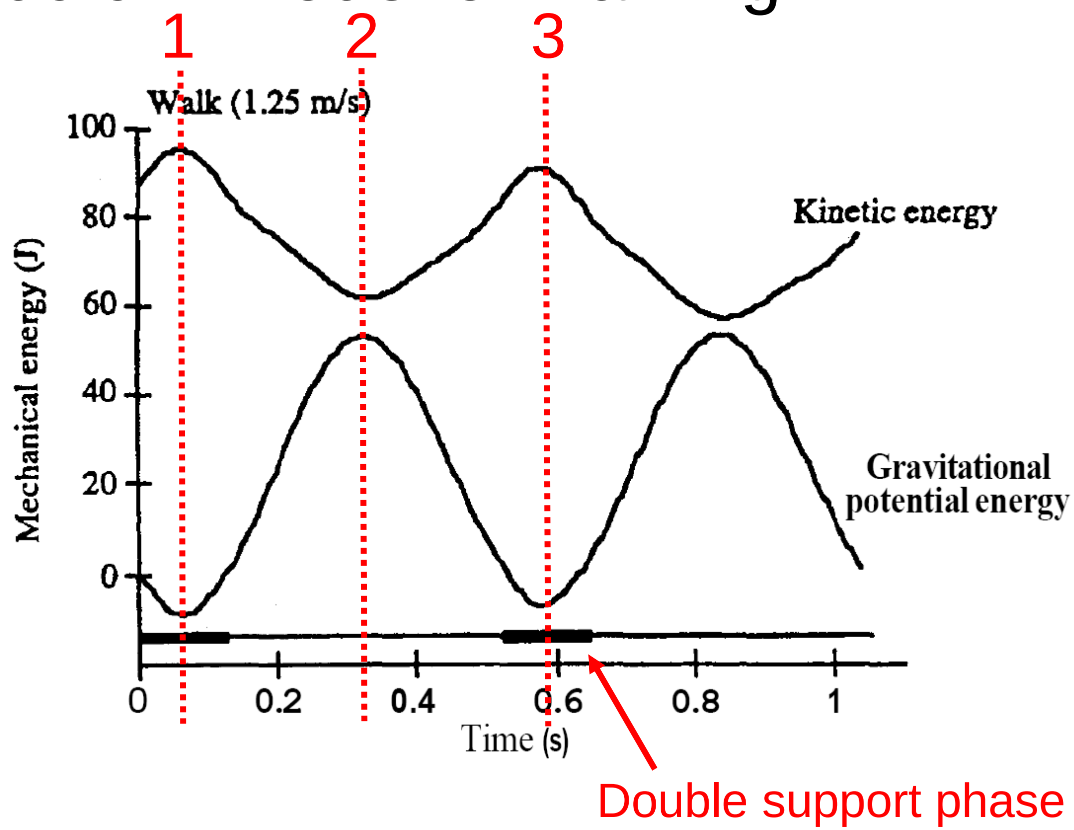
<u>Spring-mass</u> model of running



From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

Inverted pendulum model of walking

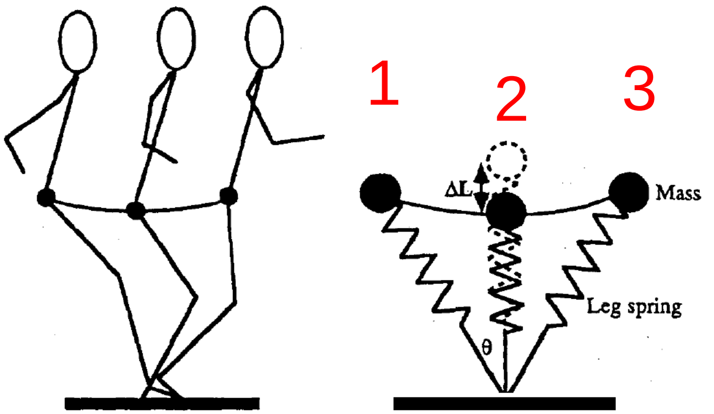


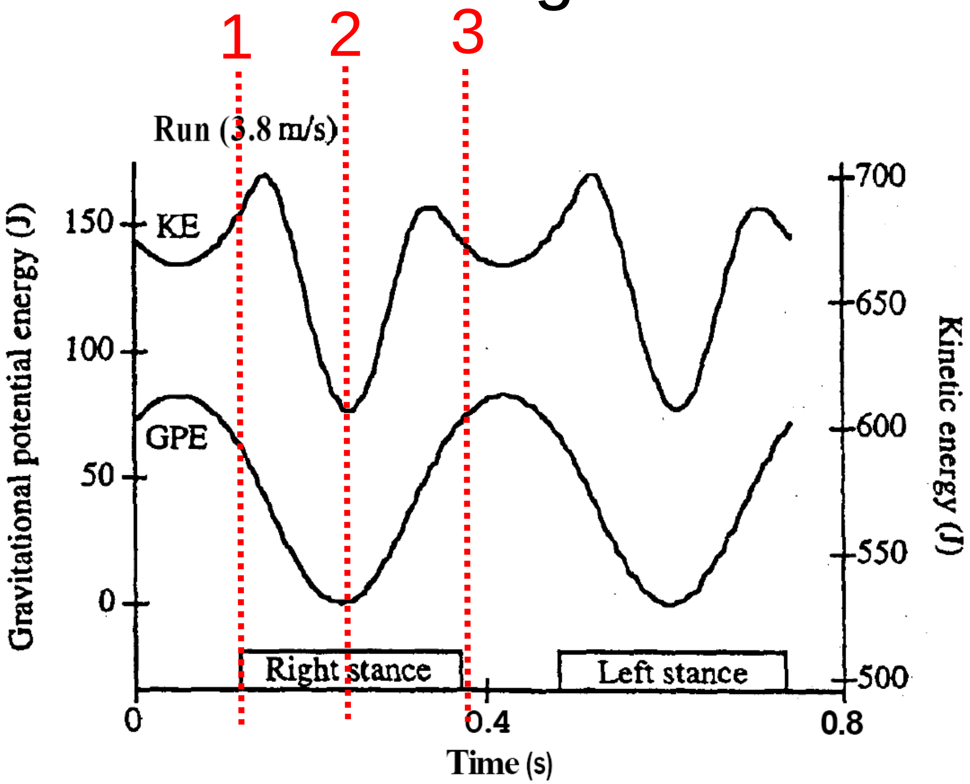


The model is consistent with the **energy exchange between kinetic and gravitational potential energy**. The two energy fluctuations are **out of phase**.

From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

Spring-mass model of running

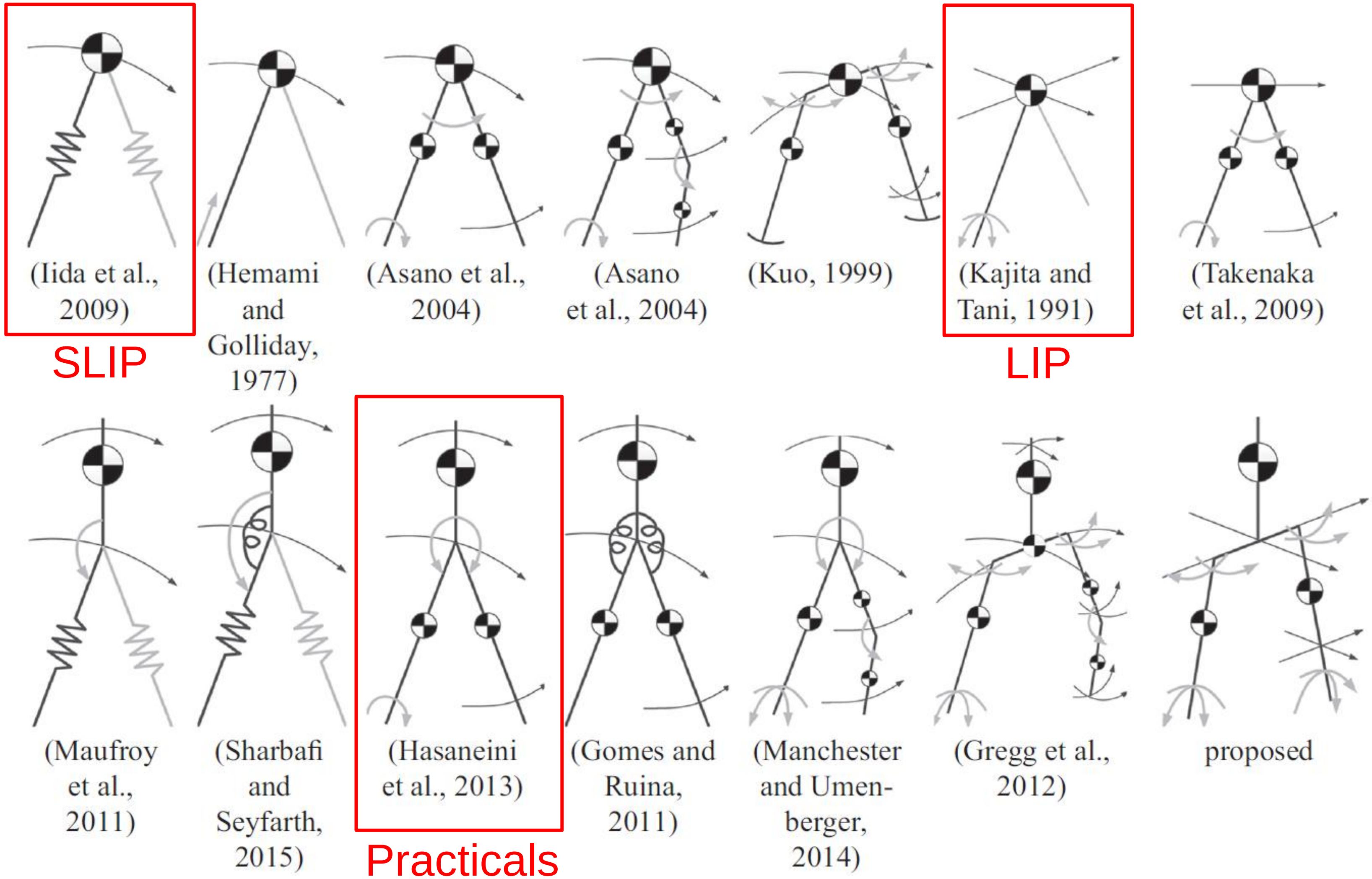




During running there is little energy exchange between kinetic and gravitational potential energy (they are in phase). **Energy is stored in elastic tissues instead**.

From: Farley C.T. and Ferris D.P. (1998) Biomechanics of walking and running: center of mass movements to muscle action, Exercise and Sport Sciences Reviews, 26:253-285.

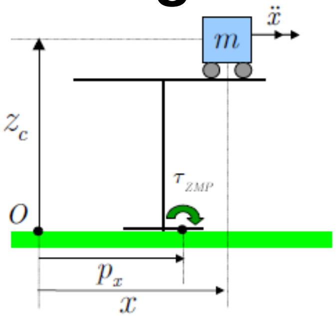
Many different types of dynamic models



Faraji, Salman, and Auke J Ijspeert. "3LP: A Linear 3D-Walking Model Including Torso and Swing Dynamics." *The International Journal of Robotics Research* 36, no. 4 (April 1, 2017): 436–55. https://doi.org/10.1177/0278364917708248.

Linear Inverted Pendulum (LIP) model

- Simple point-mass model proposed by Kajita and Tani 1991
- **Linear model** with **closed form solution**
- Used on many biped robots for **online footstep planning**
- The model is linked to the **cart-table model**



- It is a 3D model. Here we look at the 2D version, because *x* and *y* dynamics can be treated independently.

- Kajita, S., and K. Tani. "Study of Dynamic Biped Locomotion on Rugged Terrain-Derivation and Application of the Linear Inverted Pendulum Mode." In *1991 IEEE International Conference on Robotics and Automation Proceedings*, 1405–11 vol.2, 1991. https://doi.org/10.1109/ROBOT.1991.131811.

Forward acceleration in the linear inverted pendulum (2D)

In the linear inverted pendulum, the point mass is assumed to stay at a **constant height** z_0 and the (mass-less) **leg is telescopic**

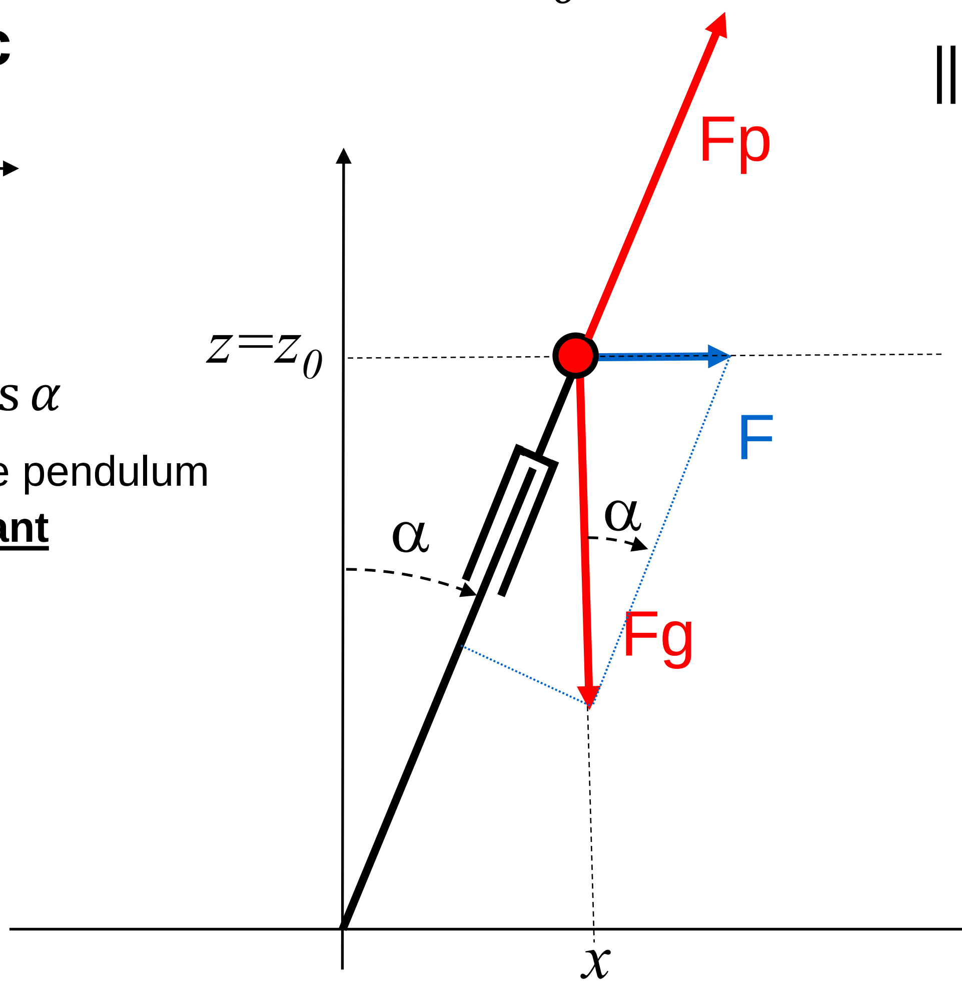
$$\|F\| = |mg \tan \alpha|$$

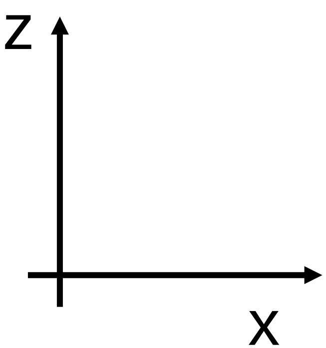
Direction of movement

$$x = l \sin \alpha$$

$$z = z_0 = l \cos \alpha$$

The length of the pendulum l is **not a constant**





Forward acceleration in the linear inverted pendulum (2D)

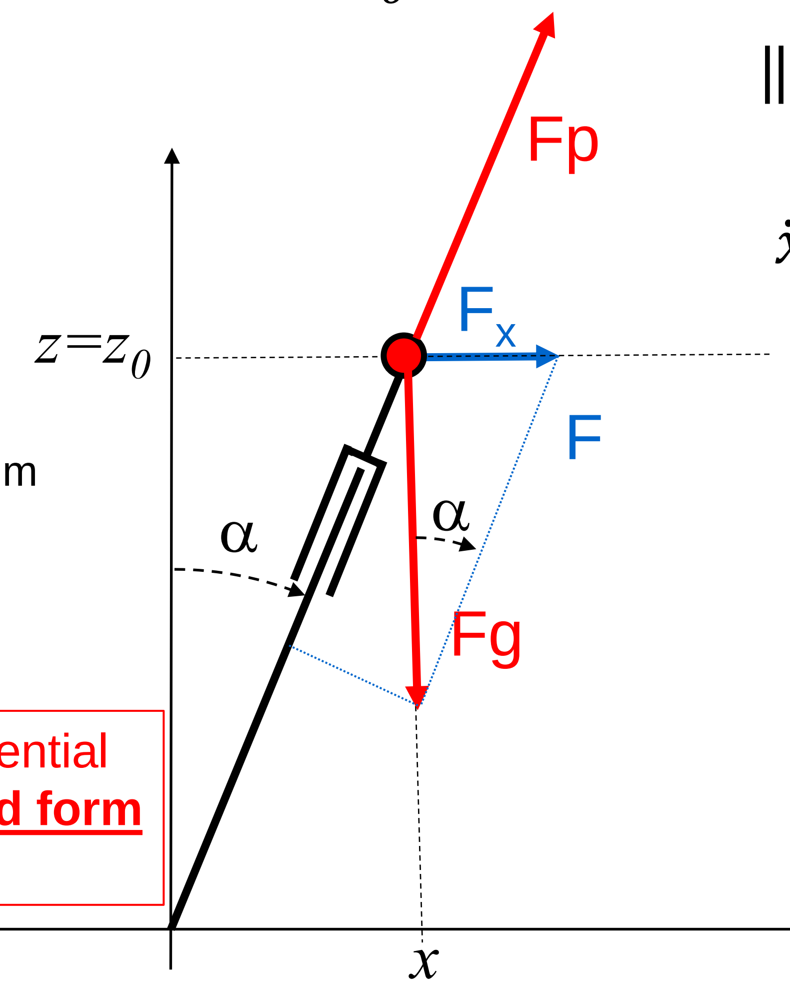
In the linear inverted pendulum, the point mass is assumed to stay at a **constant height** z_0 and the (mass-less) **leg is telescopic**

Direction of movement

$$x = l \sin\alpha$$

$$z = z_0 = l\cos\alpha$$

The length of the pendulum l is **<u>not a constant</u>**



$$\|F\| = |mg\tan\alpha|$$

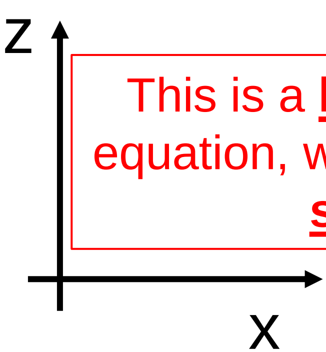
$$\ddot{x} = \frac{F_x}{m} \qquad \ddot{z} = 0$$

$$F_x = mg\tan\alpha$$

$$F_z = 0$$

$$\tan\alpha = \frac{x}{z_0}$$

$$\ddot{x} = \frac{g}{z_0}x$$



This is a **<u>linear</u>** differential equation, with a **<u>closed form solution</u>**

Linear Inverted Pendulum (LIP) model (2D)

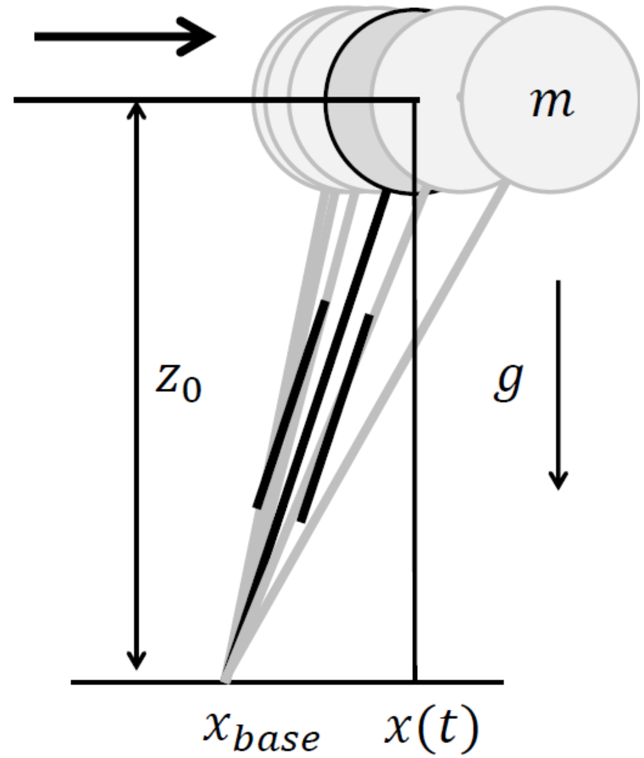
- Solving the equation of motion:

$$\ddot{x}(t) = \frac{g}{z_0}(x(t) - x_{base}),\ \omega = \sqrt{g/z_0}$$

$$x(t) = Ae^{-\omega t} + Be^{\omega t} + x_{base}$$

$$A = (-\dot{x}(0)/\omega + x(0) - x_{base})/2$$

$$B = (\dot{x}(0)/\omega + x(0) - x_{base})/2$$



- We thus have a **closed form (i.e. analytical) solution** that allows us to predict the forward movement of the center of mass

Linear Inverted Pendulum (LIP) model (2D)

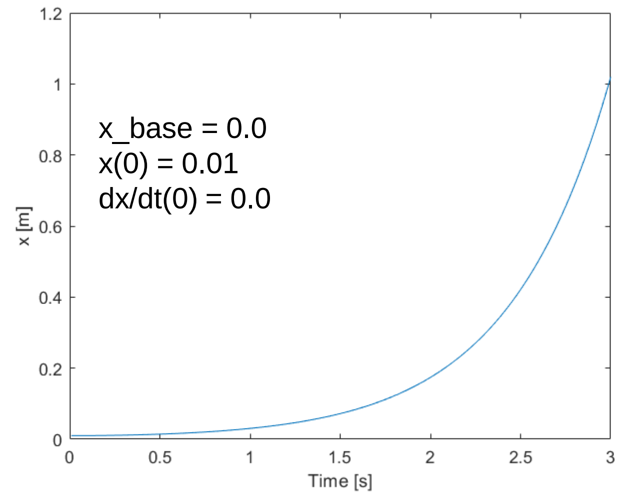
- The closed form solution is very **useful for foot-step planning**, and for moving forward at a desired average speed
- Typical **exponential behavior**:

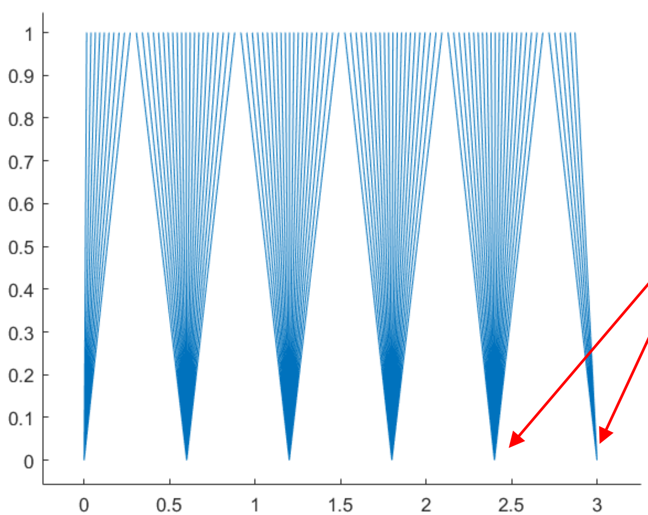
$$\ddot{x}(t) = \frac{g}{z_0}(x(t) - x_{base}),\ \omega = \sqrt{g/z_0}$$

$$x(t) = Ae^{-\omega t} + Be^{\omega t} + x_{base}$$

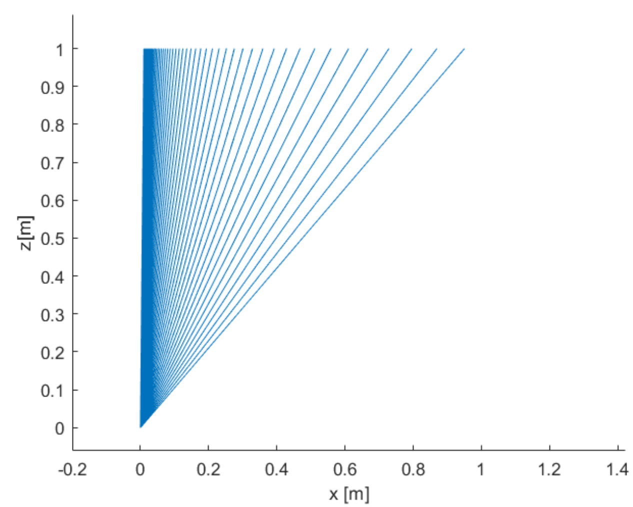
$$A = (-\dot{x}(0)/\omega + x(0) - x_{base})/2$$

$$B = (\dot{x}(0)/\omega + x(0) - x_{base})/2$$





Regular update of x_base



Linear Inverted Pendulum (LIP) model (2D)

- This is very **useful for footstep planning**, and for **moving forward at a desired average speed** (see the Divergent component of motion criterion in a few slides)

- Many humanoid robots used this to plan footsteps, and to keep stability using the Zero Moment Point criterion (see next)

- The robots that do this **typically move with crouched knees** to keep the center of mass at a constant height

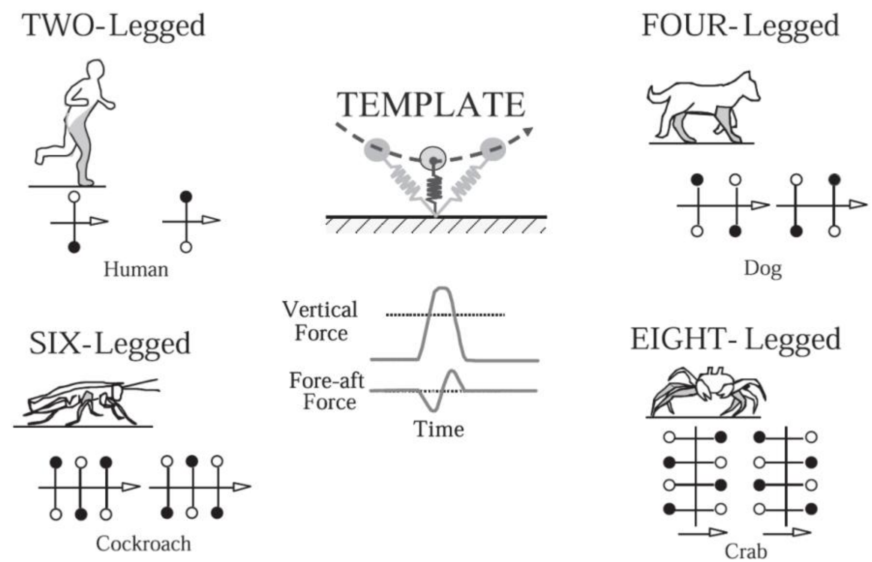
- This does not look human-like, and is not energy-efficient

SLIP model
Spring Loaded Inverse Pendulum

SLIP model is a well-known model, predicting correctly:

- Ground reaction forces
- Center of mass trajectories

And this for (running) animals with different leg numbers

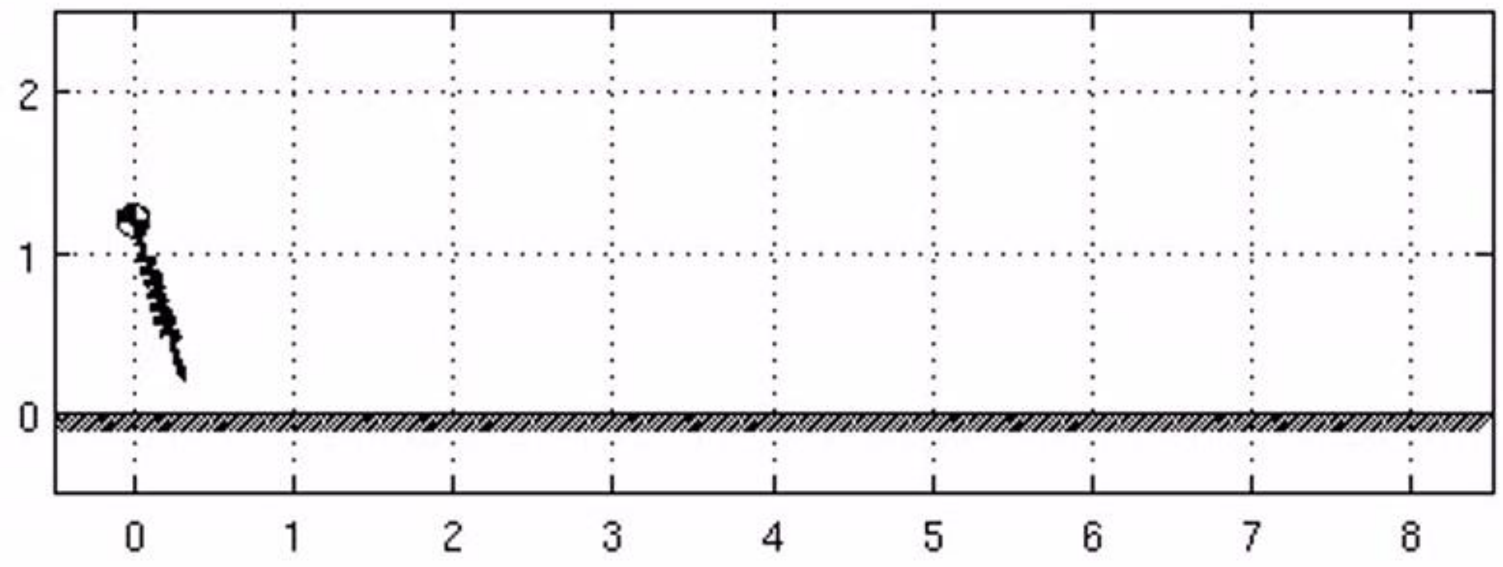


SLIP template in animals with different leg numbers

Holmes, P. & Full, Robert J. & Koditschek, D. & Guckenheimer, J. (2006). The Dynamics of Legged Locomotion: Models, Analyses, and Challenges. SIREV,48, 207—304.

It is also used to help the design of controllers for **hopping and running robots**, see the Virtual Leg control method of Marc Raibert

SLIP model
(Spring Loaded Inverse Pendulum)
Slides adapted from Soha Pouya



SLIP model
(Spring Loaded Inverse Pendulum)

The SLIP model has been used to model both hopping robots and human running.

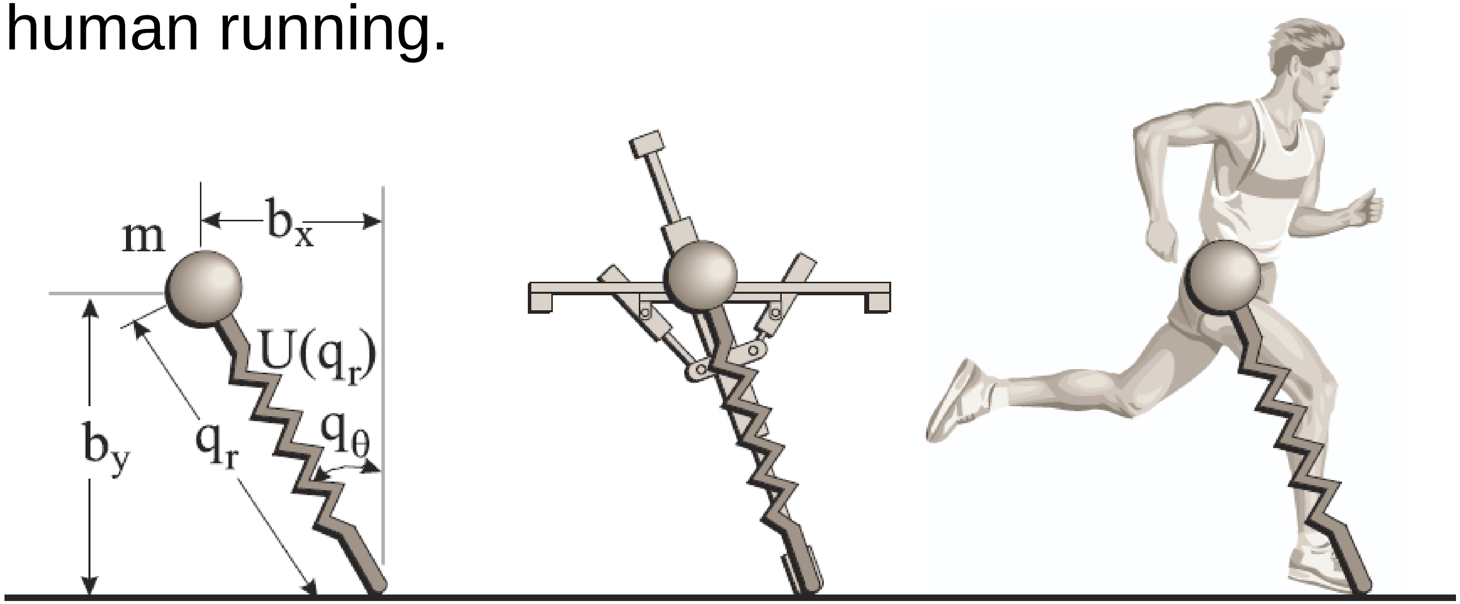


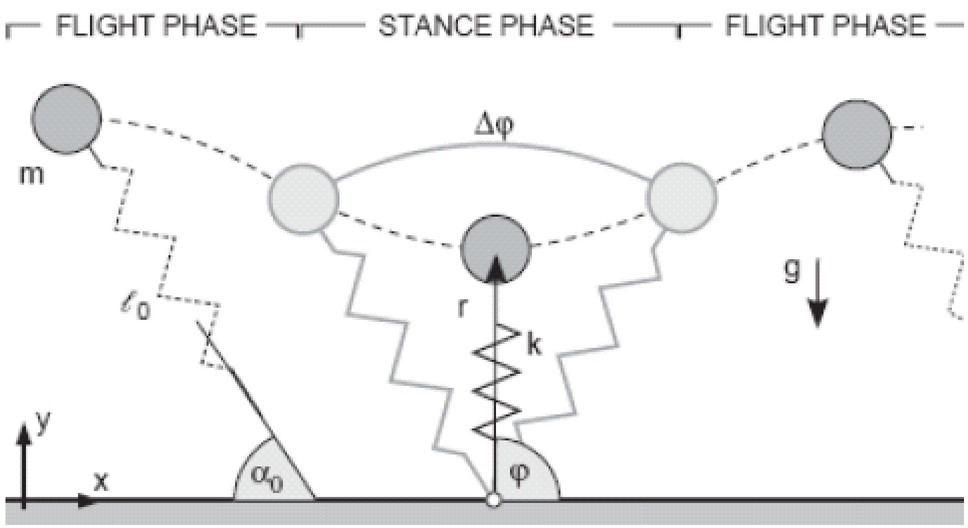
Fig. 1. (a) Left: The physical template: spring loaded inverted pendulum (SLIP) monoped with point mass, m, at hip and massless leg consisting of a spring with potential $U(q_r)$. By convention, both q_θ and b_x are defined to be negative to the left of vertical and positive to the right. (b) Middle: Illustration of the template's correspondence to Raibert's hopper. (c) Right: Illustration of the template's correspondence to a human runner.

William J. Schwind and Daniel E. Koditschek. Approximating the stance map of a 2 DOF monoped runner. Journal of Nonlinear Science, 10(5):533{588, 2000.

Model Details

The model is kept very simple: The body is represented by a point mass m, the leg by massless linear spring of stiffness k leg and length L0 when fully extended (see Fig. 1) [1].

Symbols:
m — point mass,
L_0 — rest length,
α_0 —leg angle of attack during flight,
g —gravitational acceleration,
k—spring stiffness,
r—radial position of the point mass,
φ—angular position of the point mass,
$\Delta\varphi$—angle swept during stance



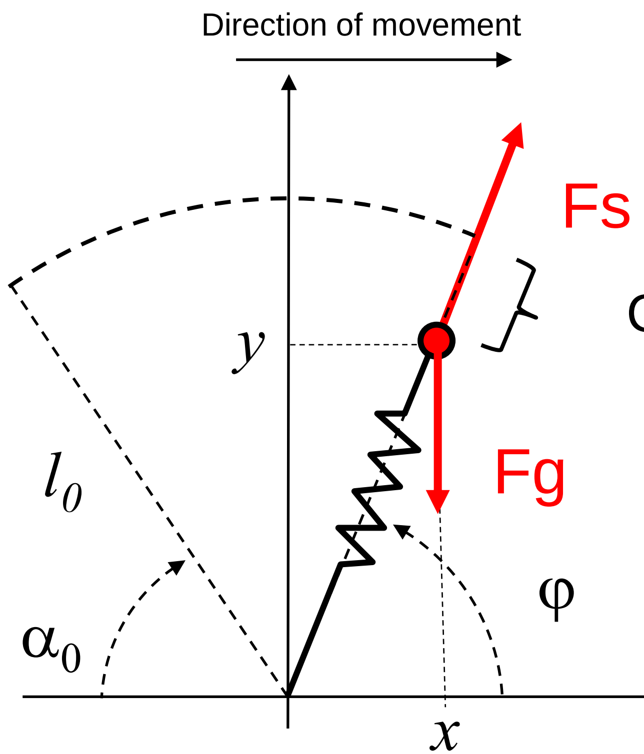
Hartmut Geyer, et al., " Spring-mass running: simple approximate solution and application to gait stability", Journal of Theoretical Biology, Aug. 2004.

Assumptions

- As the **leg is mass-less**, it has no moment of inertia (like the LIP model).
- The leg touches the ground with a **constant angle of attack α_0**.
- **During the stance phase the leg angle changes naturally with the motion**.
- In the flight phase the angle of attack is reset to α_0. Note: this would require a good controller on a hoping robot.
- Effects of friction or other non-conservative forces are neglected, therefore the system is **energy conservative**.

Mathematical Modeling

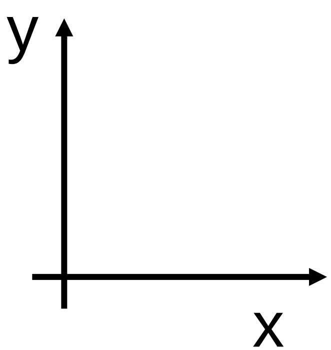
By using Newton-Euler method the dynamic equations of the leg in the SLIP model can be obtained as follows:



Compression of the spring:

$$F_{s,x} = k\Delta x$$
$$= k(l_0\cos(\varphi) - x)$$

$$F_{s,y} = k\Delta y$$
$$= k(l_0\sin(\varphi) - y)$$



Note that α_0 is measured from the left, and φ from the right.

Mathematical Modeling

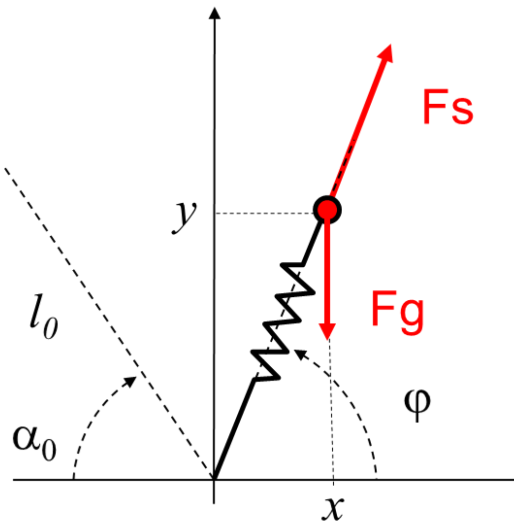
By using Newton-Euler method the dynamic equations of the leg in the SLIP model can be obtained as follows:

Fs,x

$$k(l_0 \cos(\varphi) - x) = m\ddot{x}$$

$$k(l_0 \sin(\varphi) - y)) - mg = m\ddot{y}$$

Fs,y



Equations of motion in **stance phase**:

Set of nonlinear differential equations. **Needs numerical integration**

$$\ddot{x} = \frac{k}{m}(l_0 \cos(\varphi) - x)$$

$$\ddot{y} = \frac{k}{m}(l_0 \sin(\varphi) - y) - g$$

Equations of motion in **flight phase**:

$$\ddot{x} = 0$$

$$\ddot{y} = -g$$

Open parameters:

$v_0 = \dot{x}_0$ Initial horiz. velocity

α_0 Angle of attack

k Leg stiffness

Mathematical Modeling

Conditions for switching between stance and flight:

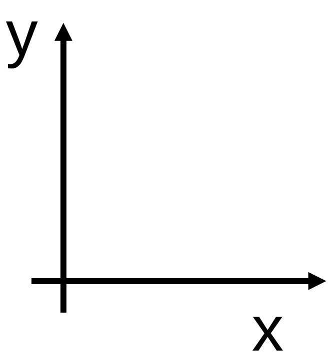
Direction of movement →

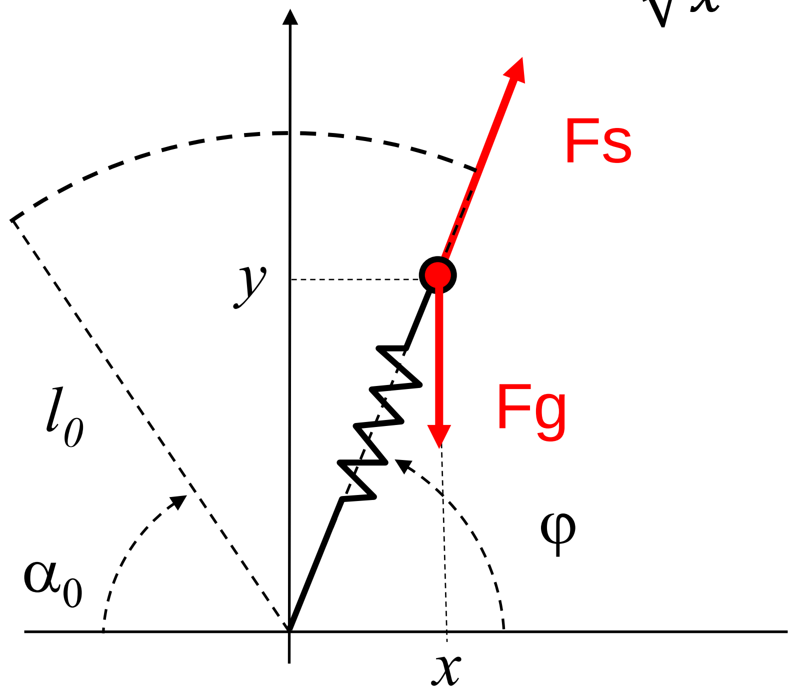
Condition for touch-down:

$$y \leq l_0 \sin(\alpha_0)$$

Condition for take-off:

$$\sqrt{x^2 + y^2} \geq l_0$$

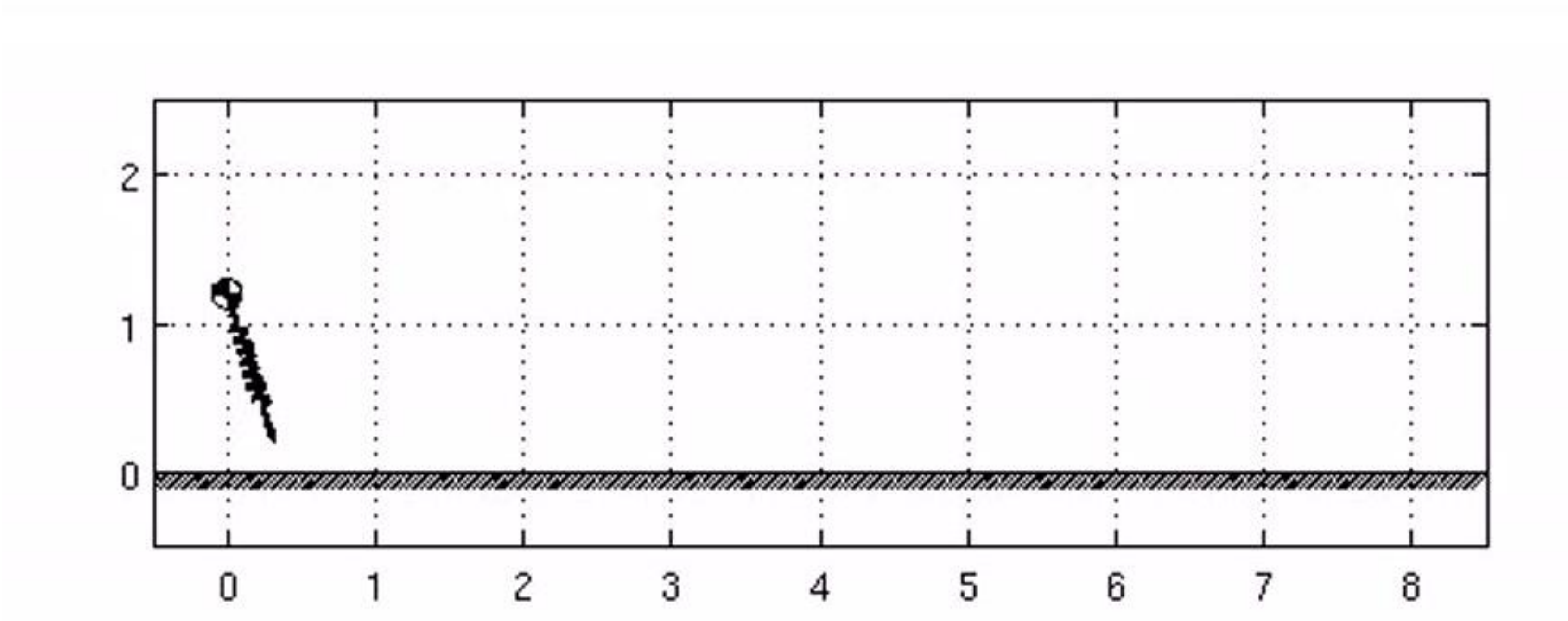




Note that α_0 is measured from the left, and φ from the right.

Simulation Results (by Soha Pouya)

The SLIP model can be solved using numerical integration



Example of a stable solution
With well-adjusted k (25 N/m) alpha (0.3 rad)
and v0 (1m/s)

Use in Biomechanics

Despite its simplicity, the dynamics of the SLIP model **closely resemble the running dynamics** of many animals and provide us with a convenient base to explore the dynamics of running systems [2].

Furthermore, gait patterns from the **SLIP model show self-stability** if the leg stiffness k and the angle of attack $\alpha 0$ (landing angle of spring) is adjusted properly [3].

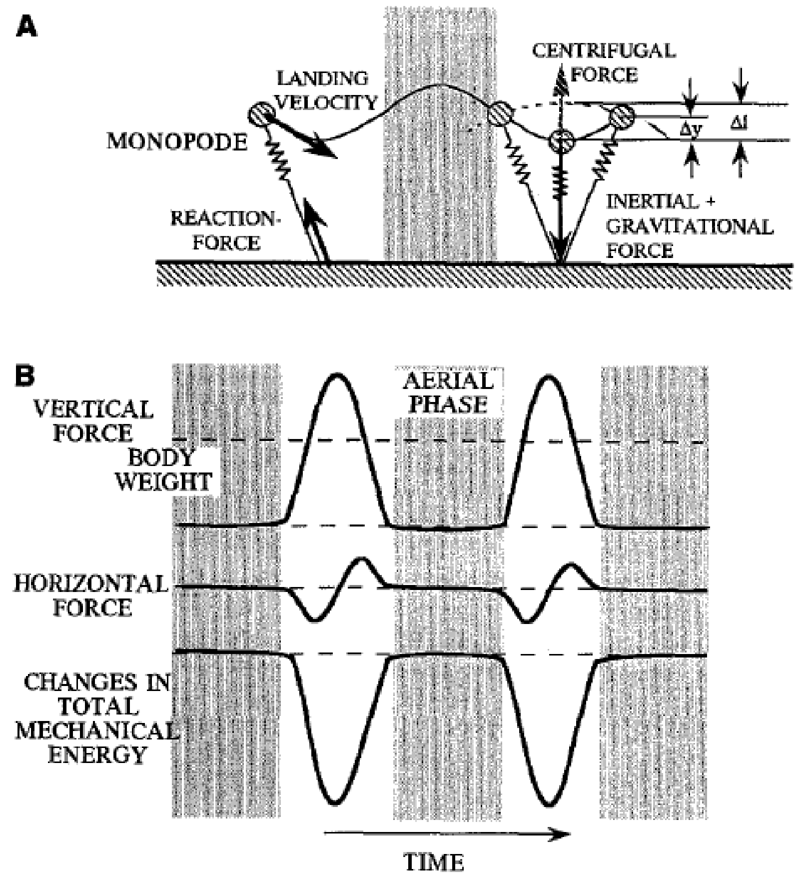
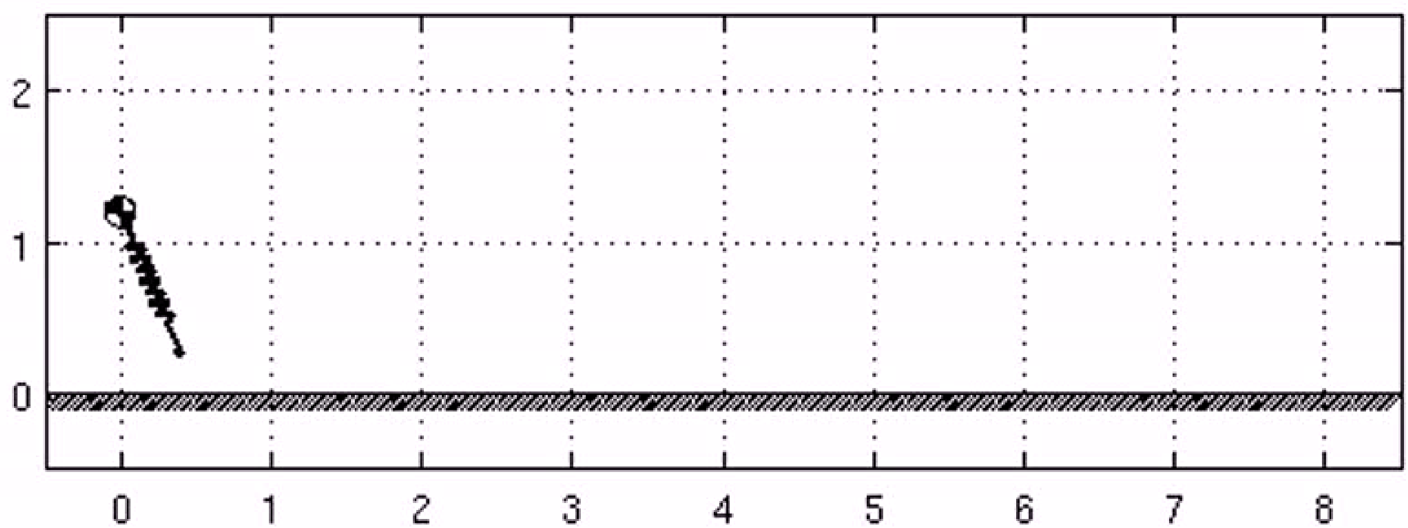


Fig. 1. A The bouncing monopode as model. Compression of the virtual leg (Δl) is different from the vertical oscillation of the center of mass (Δy). **B** Ground-reaction force and energy pattern observed for running, trotting, and hopping animals. During the aerial phase the total energy of the center of mass does not change, whereas it decreases during ground contact

Simulation Results (by Soha Pouya)

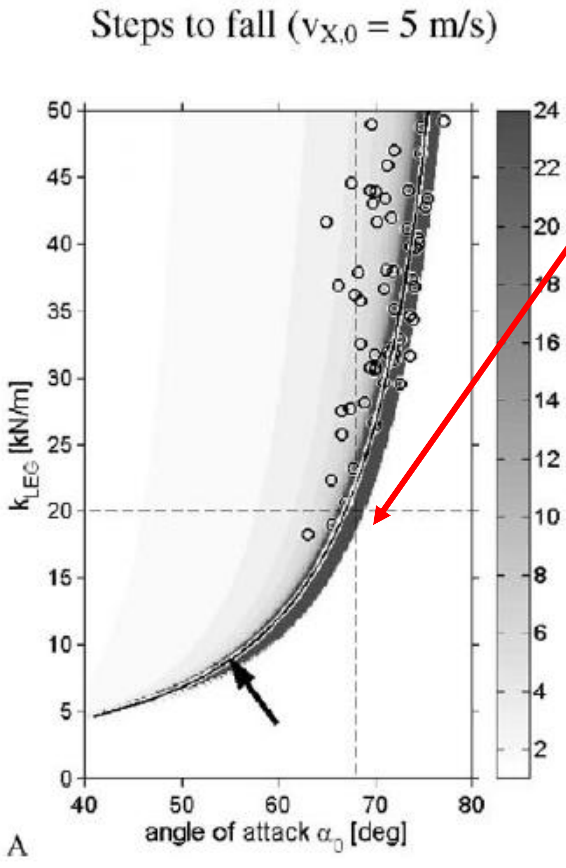
Not all values lead to stable hopping



alpha too small

Self-Stabilization:
Numerical Stability Measurement

The stability is measured by counting the number of steps that the SLIP model can perform before falling (i.e. a simple numerical method) [3]. The system shows ***self-stabilization*.**



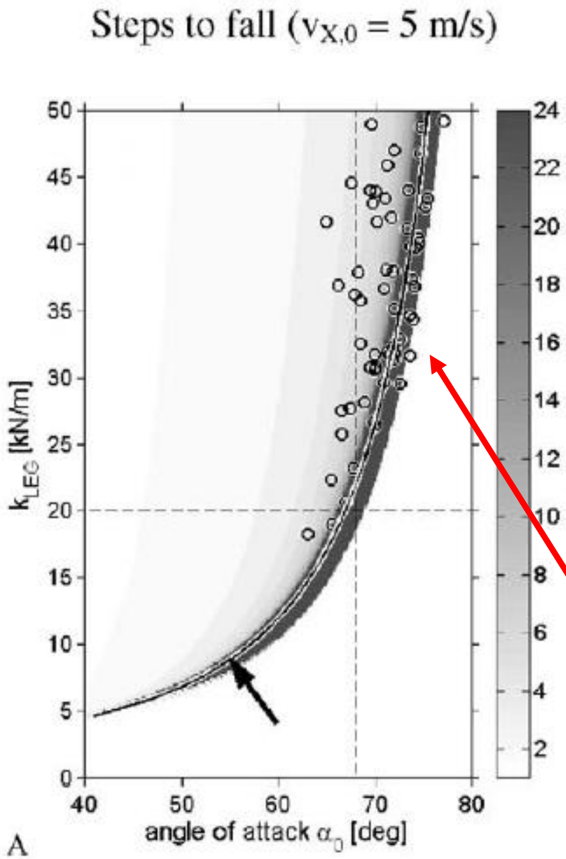
For a given leg stiffness k_{LEG}, the **angle of attack α_0 has to be carefully** chosen to obtain self-stabilization.

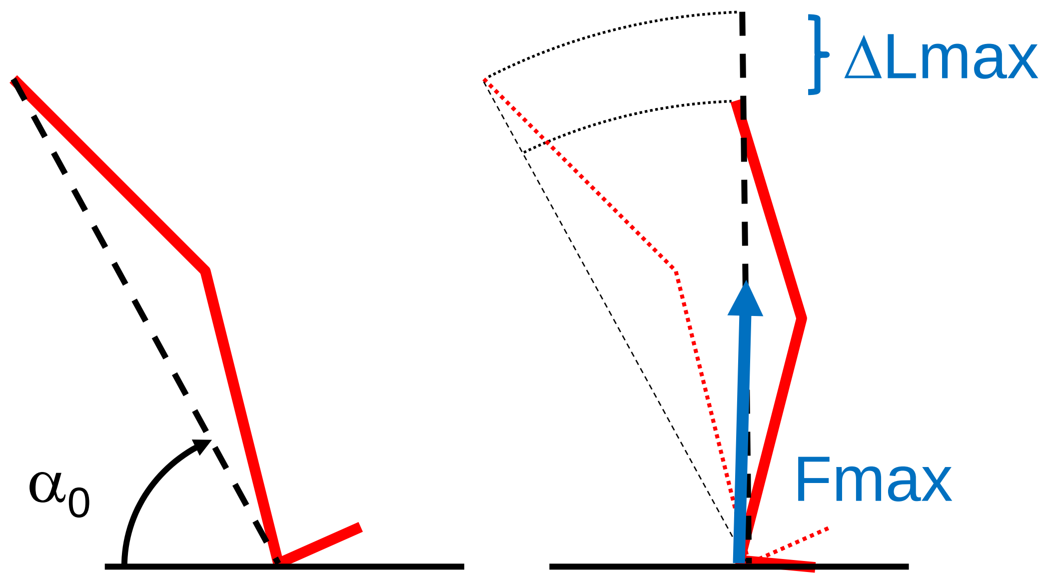
On a robot, this requires a controller to adjust the angle of attack during the flight phase (see Marc Raibert's Virtual Leg Control).

Good match to human running

Interestingly, the SLIP model presents a **good match to human running data**.
A. Seyfarth, H. Geyer, M. Günther, and R. Blickhan, "A movement criterion for running," *Journal of biomechanics,* vol. 35, pp. 649-55, May 2002

K_{human} = Fmax / ΔLmax





Good agreement with an **experimental study on human running**.

Simple models of walking and running

There are multiple very interesting papers using these types of models to analyze human locomotion and to compare gaits in different animals and robots.

Many surprising similarities have been found between very different animals.

References:

[1] Hartmut Geyer, et al., " Spring-mass running: simple approximate solution and application to gait stability", Journal of Theoretical Biology, Aug. 2004.

[2] Blickhan, R. and R.J. Full, *Similarity in multilegged locomotion: Bouncing like a monopode.* Journal of Comparative Physiology A: Neuroethology, Sensory, Neural, and Behavioral Physiology, 1993. **173**(5): p. 509-517.

[3] A. Seyfarth, H. Geyer, M. Günther, and R. Blickhan, "A movement criterion for running," *Journal of biomechanics,* vol. 35, pp. 649-55, May 2002

[4] C. David Remy, Keith Buffinton, and Roland Siegwart, A MATLAB Framework for Efficient Gait Creation, 2011.

[5] Holmes, P. & Full, Robert J. & Koditschek, D. & Guckenheimer, J. (2006). The Dynamics of Legged Locomotion: Models, Analyses, and Challenges. SIREV,48, 207—304.

Stability criteria

- There are many articles providing methods to quantify the stability of legged robot locomotion
- Statically stable gaits:
  - **Magnitude of static stability margin**
- Dynamically stable gaits (a.k.a. dynamic gaits):
  - **CoP, Center of Pressure**
  - **ZMP, Zero Moment Point**
  - **DCM Divergent component of motion** and **Capturability**
  - **Poincaré map analysis (return maps)**
  - FRI, Foot rotation Indicator
  - CMP, Centroidal Moment Pivot
  - Virtual pivot point

(FRI, CMP, Virtual pivot point:) No need to know

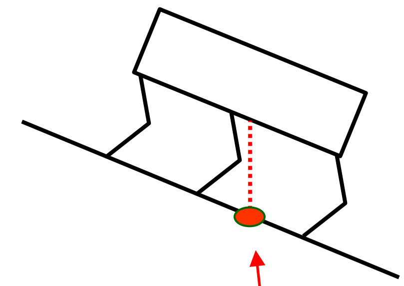
Stability criteria

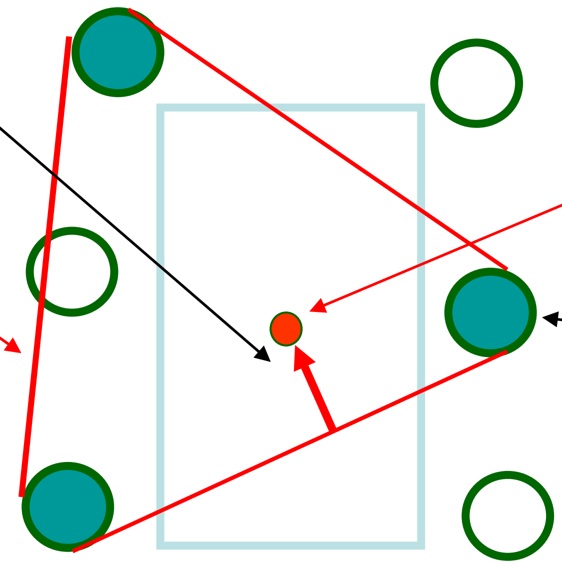
- Stability criteria are important.

- They can be used
  - To **help design your robot** (DOFs, mass distribution, sensor type and placement,…)
  - To **plan trajectories offline** (i.e. to generate reference trajectories in advance)
  - To perform **optimization** (online or offline)
  - To be part of **online control loops**
  - To carry out **post-processing and analyze locomotion patterns** (generated by robots or animals)

Statically stable gaits

- **Magnitude of static stability margin**:
  Shortest distance of the vertical projection of the center of mass to the support polygon





Definition 4. The magnitude of the *static stability margin at time t* for an arbitrary support pattern is equal to the shortest distance from the vertical projection of the center of gravity to any point on the boundary of the support pattern. If the pattern is statically stable, the stability margin is positive. Otherwise, it is negative.

- McGhee, R. B., & Frank, A. A. (1968). On the stability properties of quadruped creeping gaits. *Mathematical Biosciences*, *3*, 331–351. https://doi.org/10.1016/0025-5564(68)90090-4

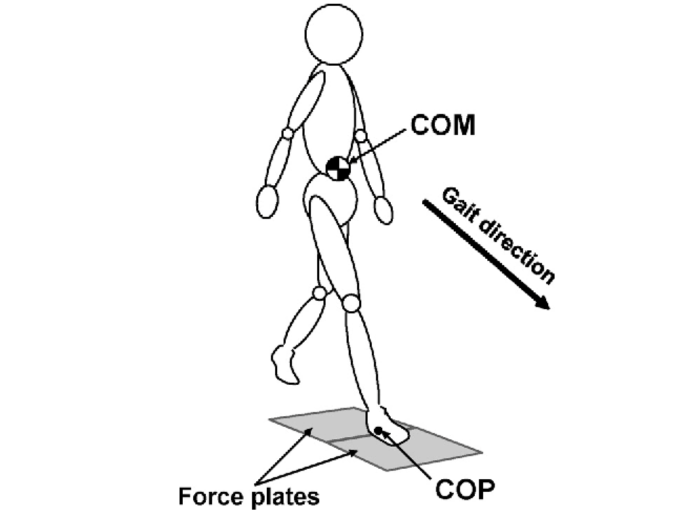
Center of mass (CoM) and center of pressure (CoP)

- At stand still (or in slow gaits), it is useful to monitor the projection of the ***center of mass (CoM)*** on the ground.

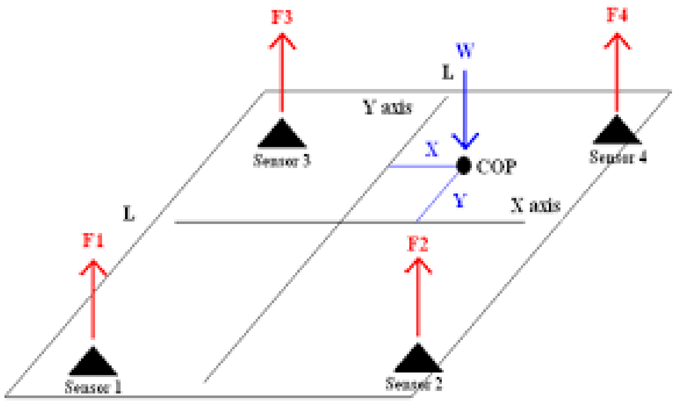
- In dynamic conditions, it is better to look at the ***Center of Pressure* (CoP),** the point of application of the ground reaction force vector. CoP and projected CoM are the same only at stand still.

- The **CoP can be measured with load sensors** under the feet of the robot (or with force plates):

- But the CoP cannot be measured when it goes out of the support polygon. One then needs a **more general criterion like the zero moment point (ZMP)**, see next.



Yamaguchi, et al. 2013. https://doi.org/10.1016/j.gaitpost.2012.11.007.



$$\overrightarrow{CoP} = \frac{\sum_{i=1}^{4} F_i \overrightarrow{X_i}}{\sum_{i=1}^{4} F_i}$$

Zero Moment Point

- ***Zero Moment Point (ZMP)*:** one of the most **widely use criterion** in biped robots

- *Zero Moment Point (ZMP)*: point on the ground at which the **net moment of all the forces** applied to the robot (ground reaction forces, inertial forces and gravity forces) has **no horizontal component**.

- ZMP ~ **projection** on the ground of the **point around which the robot is rotating.**

- This is an important point to monitor: **should stay in the support polygons over time** (can briefly go out)

ZMP computation

- ZMP is computed using the **dynamic model** of the robot
- $\mathbf{P}_{ZMP}$ is such that:

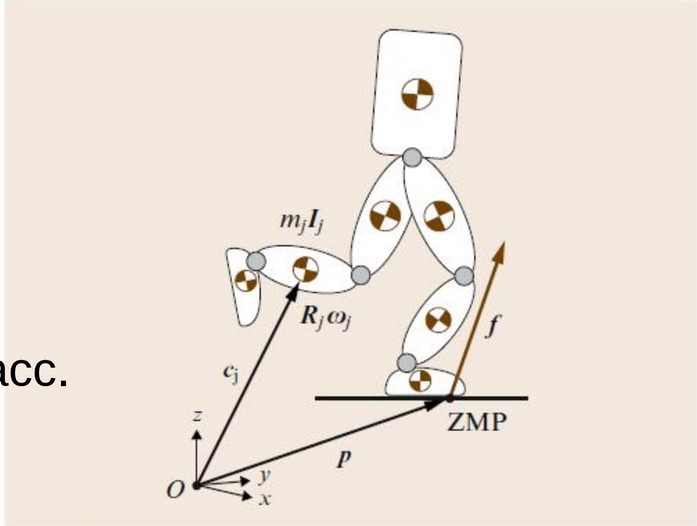


Fig. 16.13 Robot model and ZMP in three dimensions

$$\mathbf{r}_i = \mathbf{p}_i - \mathbf{p}_{ZMP}$$

Moment due to external acc. (including GRFs)

M. due to angular acceleration

$$\sum_i^N (\mathbf{r}_i \times m_i \mathbf{a}_i + \mathbf{I}_i \boldsymbol{\alpha}_i + \boldsymbol{\omega}_i \times \mathbf{I}_i \boldsymbol{\omega}_i - \mathbf{r}_i \times m_i \mathbf{g}) = (0,0,*)^T$$

M. due to Coriolis forces

M. due to gravity

N : number of links
p_i : position of link i
m_i : mass of link i
$\mathbf{a}_i$: external acceleration
$\mathbf{I}_i$: moment of inertia (matrix)
$\boldsymbol{\alpha}_i$: angular acceleration
$\boldsymbol{\omega}_i$: angular velocity
$\mathbf{g}$: gravity

Two independent equations,
Two unknowns: $P_{ZMP,X}$ $P_{ZMP,Y}$

Kajita and Espiau. 2008. "Legged Robots." In *Springer Handbook of Robotics*, edited by Bruno Siciliano and Oussama Khatib, 361–89. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-30301-5_17. https://link.springer.com/content/pdf/10.1007%2F978-3-540-30301-5_17.pdf

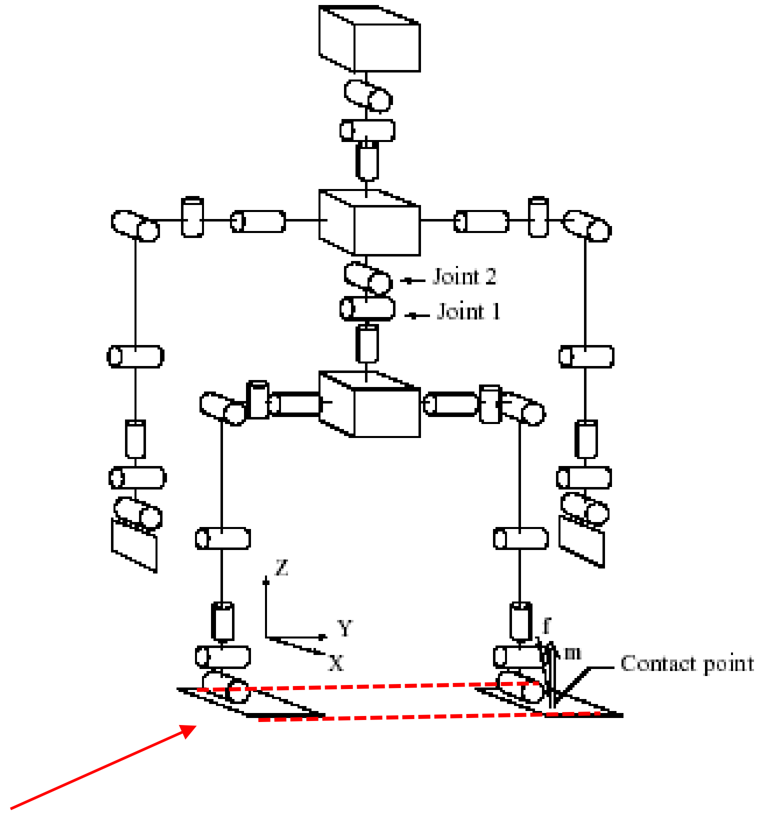
Zero Moment Point

- ZMP is <u>different</u> from the projection of the ***center of mass (CoM)*** on the ground (except at stand-still where it is the same).

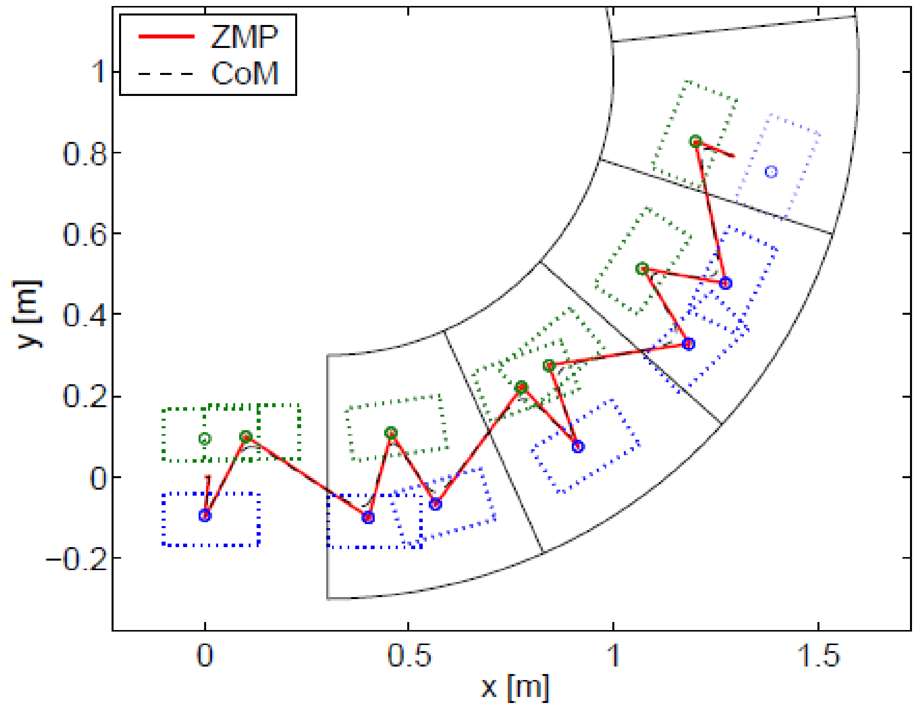
- When the ZMP is under one foot (or in a support polygon when several feet are on the ground), it is the <u>same as</u> the ***Center of Pressure*** **(CoP).** See Sardain, P., & Bessonnet, G. (2004). Forces acting on a biped robot. Center of pressure-zero moment point. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *34*(5), 630–637. https://doi.org/10.1109/TSMCA.2004.832811

- The ZMP exists even outside the support polygon (not the CoP)

ZMP



Foot-print polygon



Kajita, et al. 2003. “Biped Walking Pattern Generation by Using Preview Control of Zero-Moment Point.” In *ICRA 2003* https://doi.org/10.1109/ROBOT.2003.1241826.

Locomotion is stable if the ZMP remains within the foot-print polygons over time

Zero Moment Point references

Vukobratović and Borovac. 2004. “Zero-Moment Point — Thirty Five Years of Its Life.” *International Journal of Humanoid Robotics* 01 (01): 157–73. https://doi.org/10.1142/S0219843604000083.

Kajita and Espiau. 2008. “Legged Robots.” In *Springer Handbook of Robotics*, edited by Bruno Siciliano and Oussama Khatib, 361–89. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-30301-5_17.

Popovic, Goswami, and Herr. 2005. “Ground Reference Points in Legged Locomotion: Definitions, Biological Trajectories and Control Implications.” *The International Journal of Robotics Research* 24 (12): 1013–32. https://doi.org/10.1177/0278364905058363.

Sardain, P., and G. Bessonnet. 2004. “Forces Acting on a Biped Robot. Center of Pressure-Zero Moment Point.” *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans* 34 (5): 630–37. https://doi.org/10.1109/TSMCA.2004.832811.

Other similar criteria: FRI and CMP

There exists other criteria similar to ZMP: the **Foot rotation indicator (FRI)** and the **Centroidal moment pivot (CMP)**

These three types of criteria are related. For instance: The FRI point coincides with the ZMP when the foot is stationary, and **diverges from the ZMP for non-zero rotational foot accelerations**.

See a comparison:

- Popovic, M. B., Goswami, A., & Herr, H. (2005). Ground Reference Points in Legged Locomotion: Definitions, Biological Trajectories and Control Implications. *The International Journal of Robotics Research*, *24*(12), 1013–1032. https://doi.org/10.1177/0278364905058363
- But note: some controversies about ZMP definitions. Sardain, P., & Bessonnet, G. (2004). Forces acting on a biped robot. Center of pressure-zero moment point. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *34*(5), 630–637. https://doi.org/10.1109/TSMCA.2004.832811

Capture point and capture region

- **Capture point** and **capture region**. A *Capture Point* is a point on the ground where the robot can step to in order to **bring itself to a complete stop**. A *Capture Region* is the collection of all Capture Points.

- See Pratt, J., Carff, J., Drakunov, S., & Goswami, A. (2006). Capture Point: A Step toward Humanoid Push Recovery. *2006 6th IEEE-RAS International Conference on Humanoid Robots*, 200–207. https://doi.org/10.1109/ICHR.2006.321385
- Koolen, T., de Boer, T., Rebula, J., Goswami, A., & Pratt, J. (2012). Capturability-based analysis and control of legged locomotion, Part 1: Theory and application to three simple gait models. *The International Journal of Robotics Research*, *31*(9), 1094–1113. https://doi.org/10.1177/0278364912452673

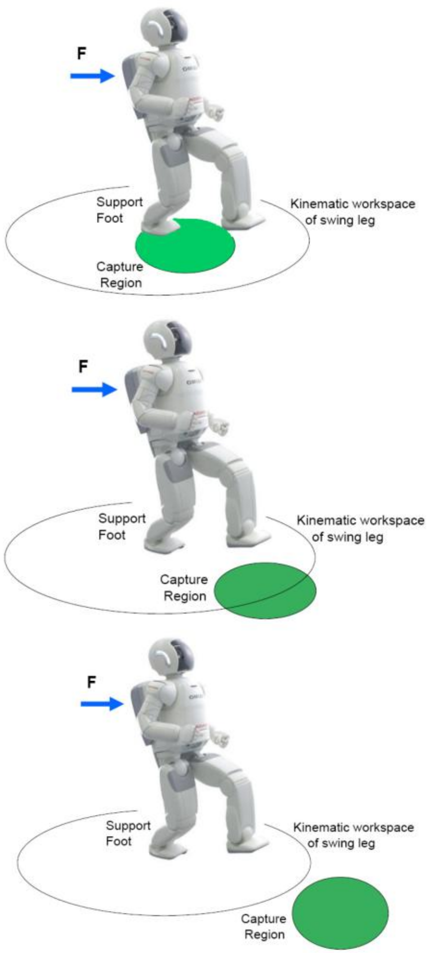
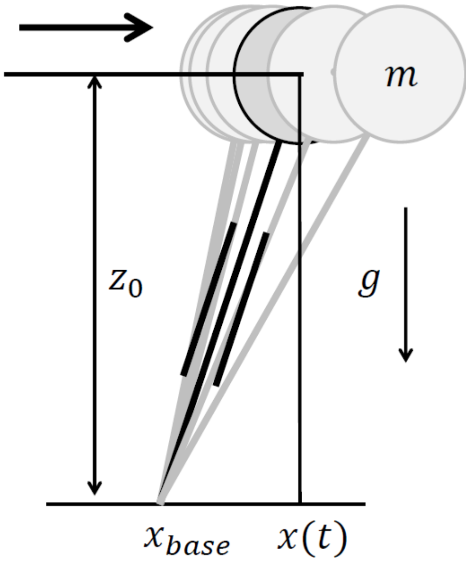


Fig. 2. When the Capture Region intersects the Base of Support, a humanoid can modulate its Center of Pressure to balance and does not need to take a step (top). When the Capture Region and Base of Support are disjoint, the humanoid must take a step to come to a stop (middle). If the Capture Region is outside the kinematic workspace of the swing foot, the humanoid cannot stop in one step (bottom figure).

Divergent component of motion (DCM)

- Remember the (2D) LIP model:

$$\ddot{x}(t) = \frac{g}{z_0}(x(t) - x_{base}),\ \omega = \sqrt{g/z_0}$$



- It is useful to introduce a new state variable:

$$\boxed{\xi = x + \frac{\dot{x}}{\omega}}$$

ξ: lower case xi

- Then we have two first order linear differential equations:

$$\dot{\xi} = \omega(\xi - x_{base}) \qquad \dot{x} = -\omega(x - \xi)$$

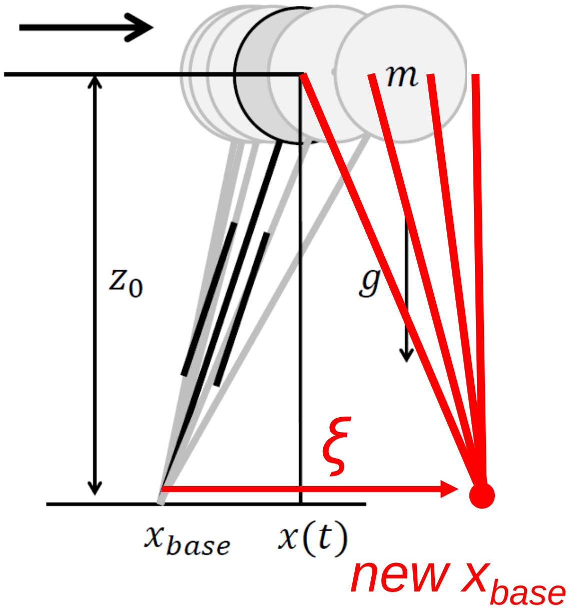
Divergent component of motion (DCM). ξ is the DCM. It is pushed away from x_{base}

Convergent component of motion. x is attracted to ξ

Divergent component of motion (DCM)

- It is therefore important to monitor the DCM, and to plan the foot steps (i.e. x_{base}) to bring the robot (the CoM) where you want and to prevent falling.

$$\xi = x + \frac{\dot{x}}{\omega}$$



- The DCM corresponds to the ***instantaneous capture point.***
- Indeed by setting the next foot step (i.e. x_{base}) to ξ at the beginning of a step, the time evolution of x goes to zero (i.e. the pendulum stops).

$$\dot{\xi} = \omega(\xi - x_{base})$$

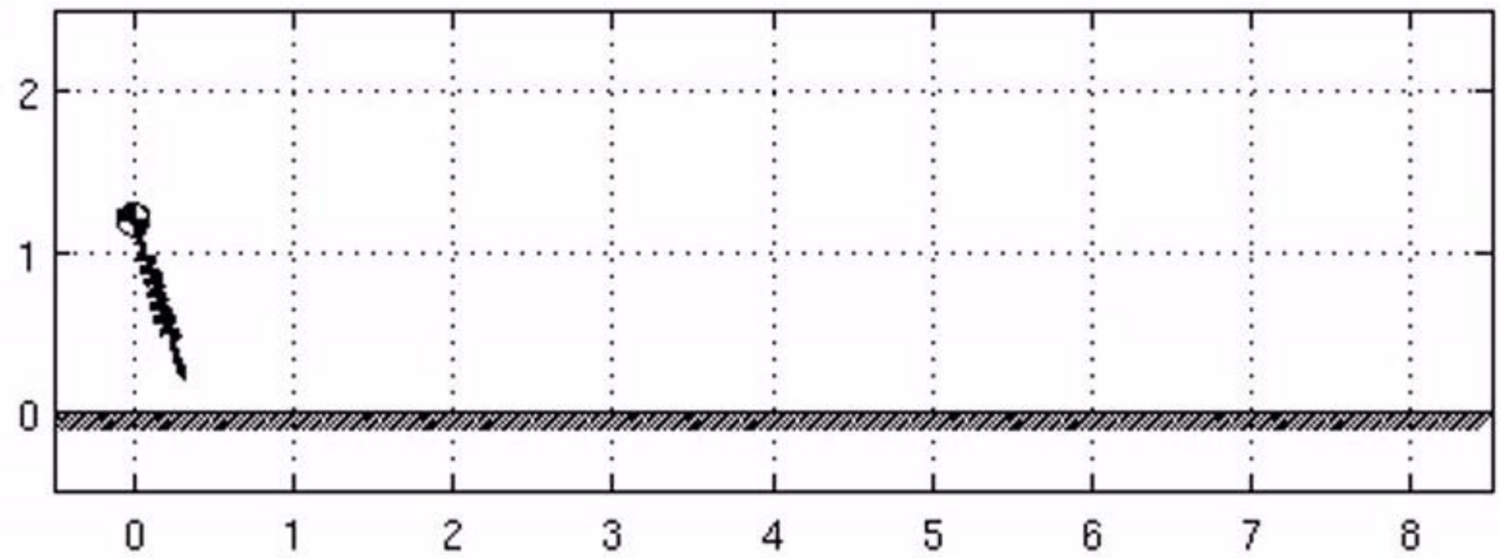
$$\dot{x} = -\omega(x - \xi)$$

- Takenaka, Toru, Takashi Matsumoto, and Takahide Yoshiike. 2009. “Real Time Motion Generation and Control for Biped Robot: 1st Report: Walking Gait Pattern Generation.” In *Proceedings of* IROS 2009.
- Englsberger et al (2014). Trajectory generation for continuous leg forces during double support and heel-to-toe shift based on divergent component of motion. In *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems* (pp. 4022-4029). IEEE.

Note: DCM is also known as “**extrapolated center of mass**,” or XCoM

Poincaré and Return map analysis

- For very dynamic gaits, e.g. hopping and running, one can **estimate how stable the limit cycle behavior** is.
- Example hopping in the spring-loaded inverted pendulum (SLIP) model



- The stability of hopping can be assessed using a **return map**, a specific type of **Poincaré map**

Poincaré maps

Strogatz, S. (1994). Nonlinear Dynamics and Chaos. With applications to Physics, Biology, Chemistry, and Engineering. Addison Wesley Publishing Company.

Poincaré maps are useful for studying swirling flows, such as the flow near a periodic orbit (or as we'll see later, the flow in some chaotic systems). Consider an n-dimensional system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$. Let S be an $n-1$ dimensional ***surface of section*** (Figure 8.7.1). S is required to be transverse to the flow, i.e., all trajectories starting on S flow through it, not parallel to it.

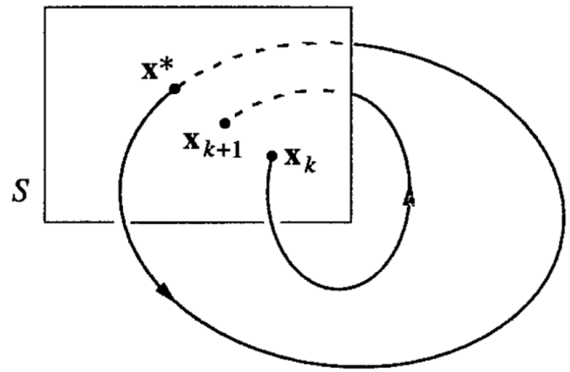


Figure 8.7.1

The ***Poincaré map*** P is a mapping from S to itself, obtained by following trajectories from one intersection with S to the next. If $\mathbf{x}_k \in S$ denotes the kth intersection, then the Poincaré map is defined by

$$\mathbf{x}_{k+1} = P(\mathbf{x}_k).$$

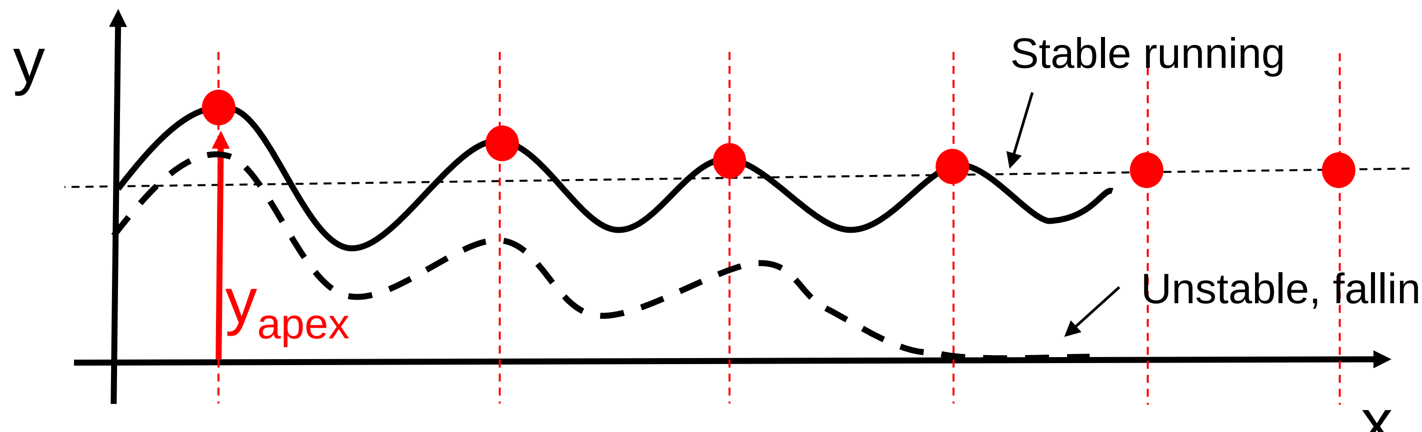
Suppose that $\mathbf{x}^*$ is a ***fixed point*** of P, i.e., $P(\mathbf{x}^*) = \mathbf{x}^*$. Then a trajectory starting at $\mathbf{x}^*$ returns to $\mathbf{x}^*$ after some time T, and is therefore *a closed orbit* for the original system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$. Moreover, by looking at the behavior of P near this fixed point, we can determine the stability of the closed orbit.

Thus the Poincaré map converts problems about closed orbits (which are difficult) into problems about fixed points of a mapping (which are easier in principle, though not always in practice). The snag is that it's typically impossible to find a formula for P.

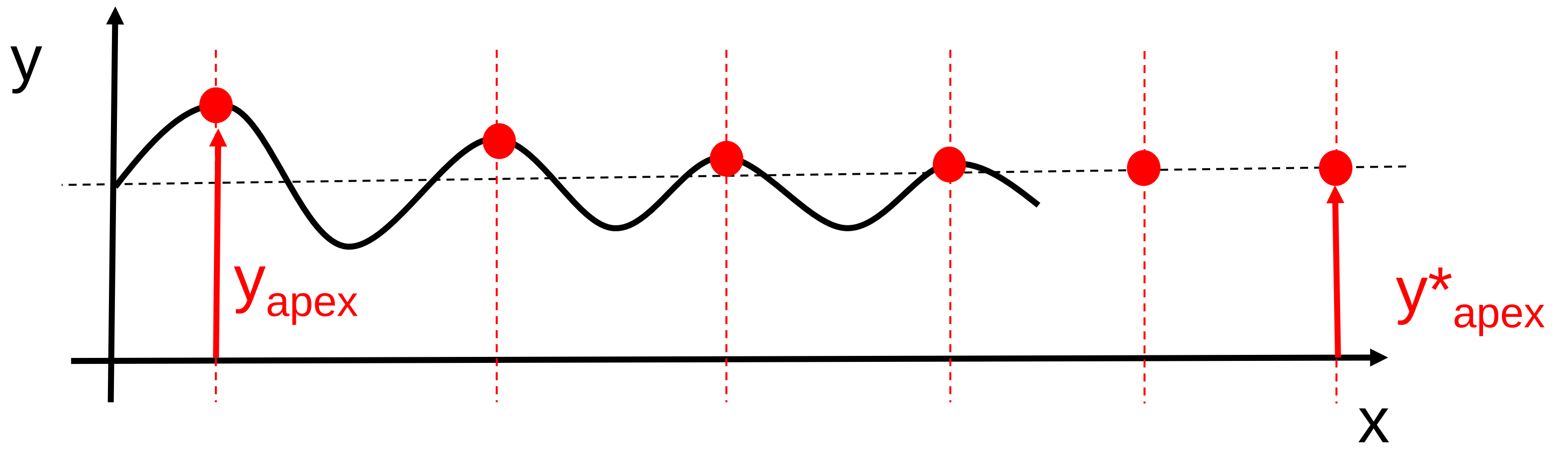
This leads to a discrete dynamical system with a state space that is one dimension smaller than the original continuous dynamical system.

Return map analysis

- A kind of Poincaré map: the ***return map***
- To analyze the stability: look at a particular event and **investigate how repetitive it is**, and **how it reacts after perturbations**.
- Choice: **apex height** (i.e. maxima of height *y*)
- The *return map* investigates how this apex height changes from step to step.



Return map analysis



The system converges to a stable hopping when two conditions are met:

1) The solution is **periodic** $y_{i+1} = y_i = y^*_{APEX}$,

2) **deviations** from this solution **must diminish** step-by-step $\left|\frac{dy_{i+1}}{dy_i}\right|_{y^*_{APEX}} < 1$.

Steady state apex height

The larger the basin of attraction, and the faster the convergence to steady state, the more stable the hopping is

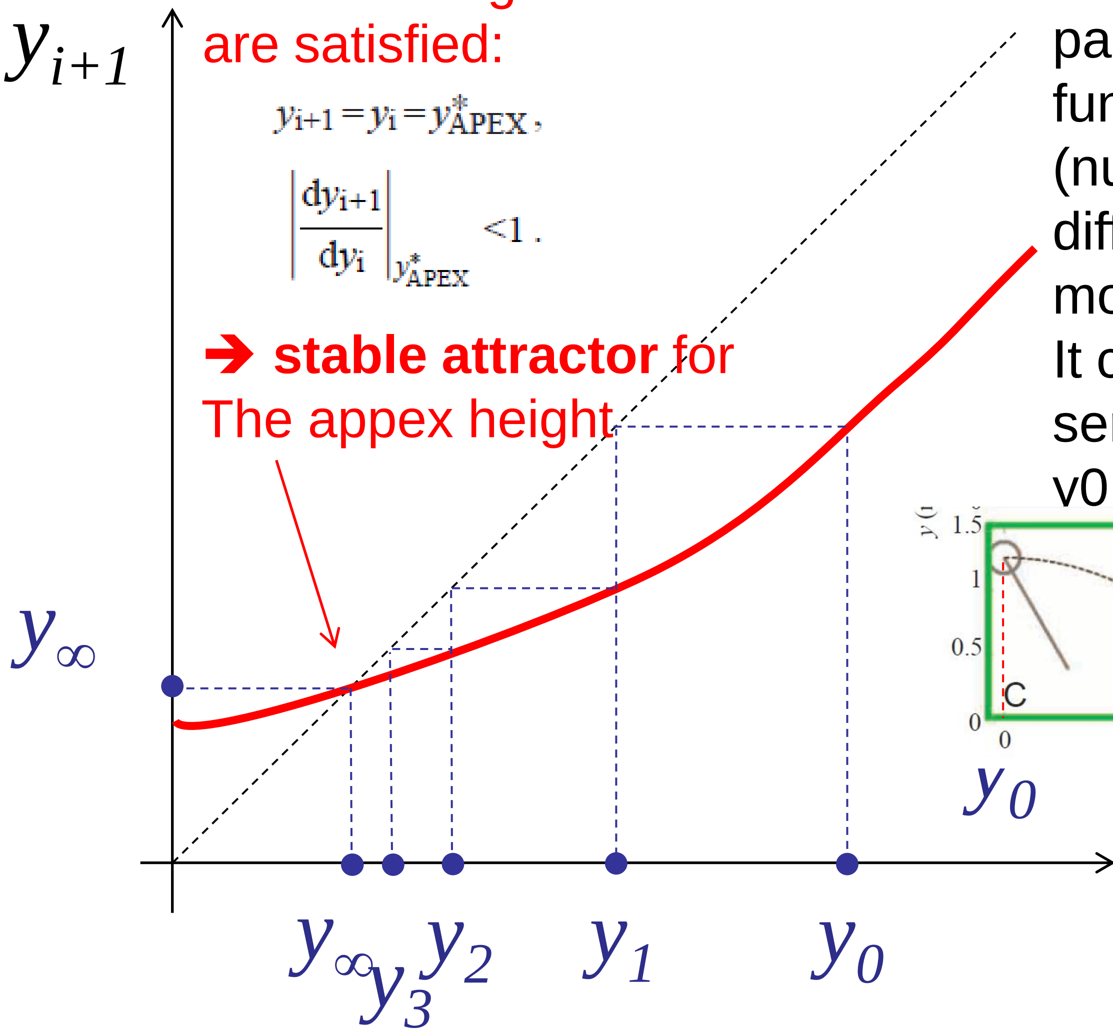
Example: **Return map** for the Spring-Loaded inverted pendulum (SLIP) model

The following conditions are satisfied:

$$y_{i+1} = y_i = y^*_{APEX},$$

$$\left| \frac{dy_{i+1}}{dy_i} \right|_{y^*_{APEX}} < 1 .$$

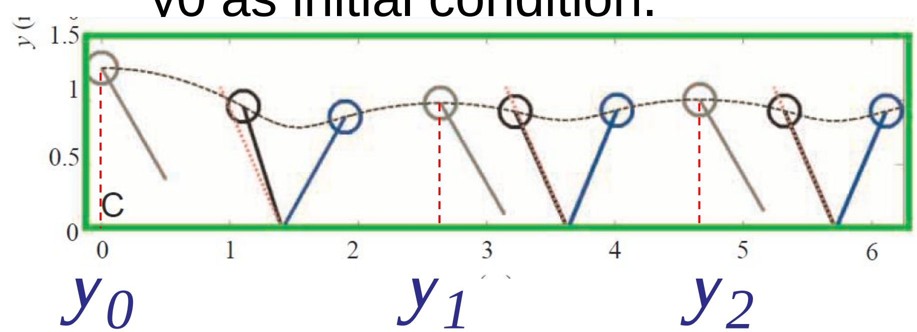
➔ **stable attractor** for The appex height



Initial condition

The **return map** is a $y_{i+1}(y)$ function for a given set of open parameters (v_0, α_R, w_R, k). This function is **obtained numerically** (numerical integration of the differential equations of the SLIP model).

It can be used to compute the series of apex heights from any v0 as initial condition.



Other locomotion metrics

- In addition to stability criteria, the following locomotion metrics are often used in legged robots:

- **Cost of Transport (CoT)**
- **Stride length and stride frequency**
- **Froude number**
- **Benchmarks and agility scores**

- These criteria are very **useful to compare the performance of different robots, and also to compare robots to animals.**

Cost of Transport

- The Cost of Transport is a **dimension-less measure of the energy efficiency** for an animal or a robot to move a given distance

$$COT = \frac{E}{mgd}$$

E = Energy [J]
m = mass [Kg]
g = gravity [m/s2]
d = distance traveled [m]

- Equivalently:

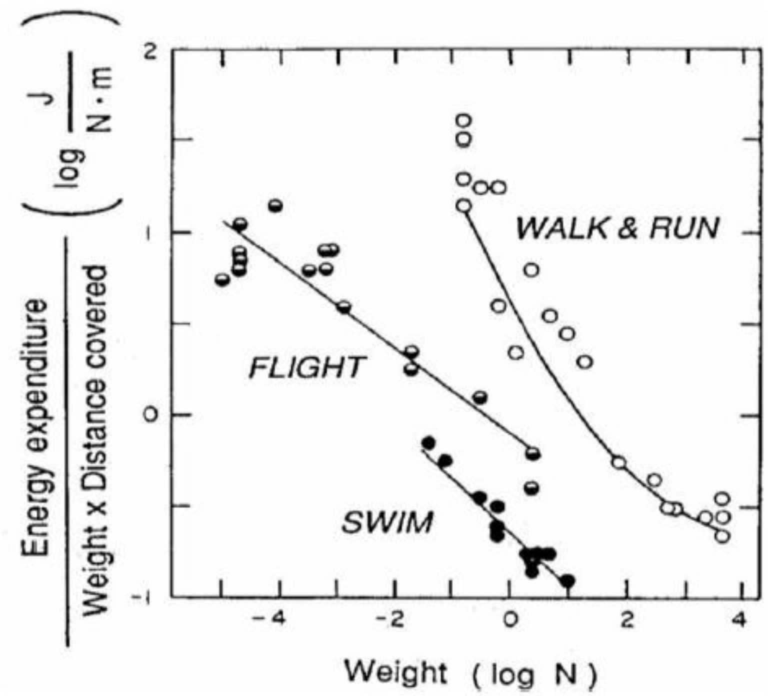
$$COT = \frac{P}{mgv}$$

P = Power [W=J/s]
m = mass [Kg]
g = gravity [m/s2]
v = velocity[m/s]

- It is very useful to compare the energy efficiency of different modes of locomotion and of different robots

Comparison of CoTs

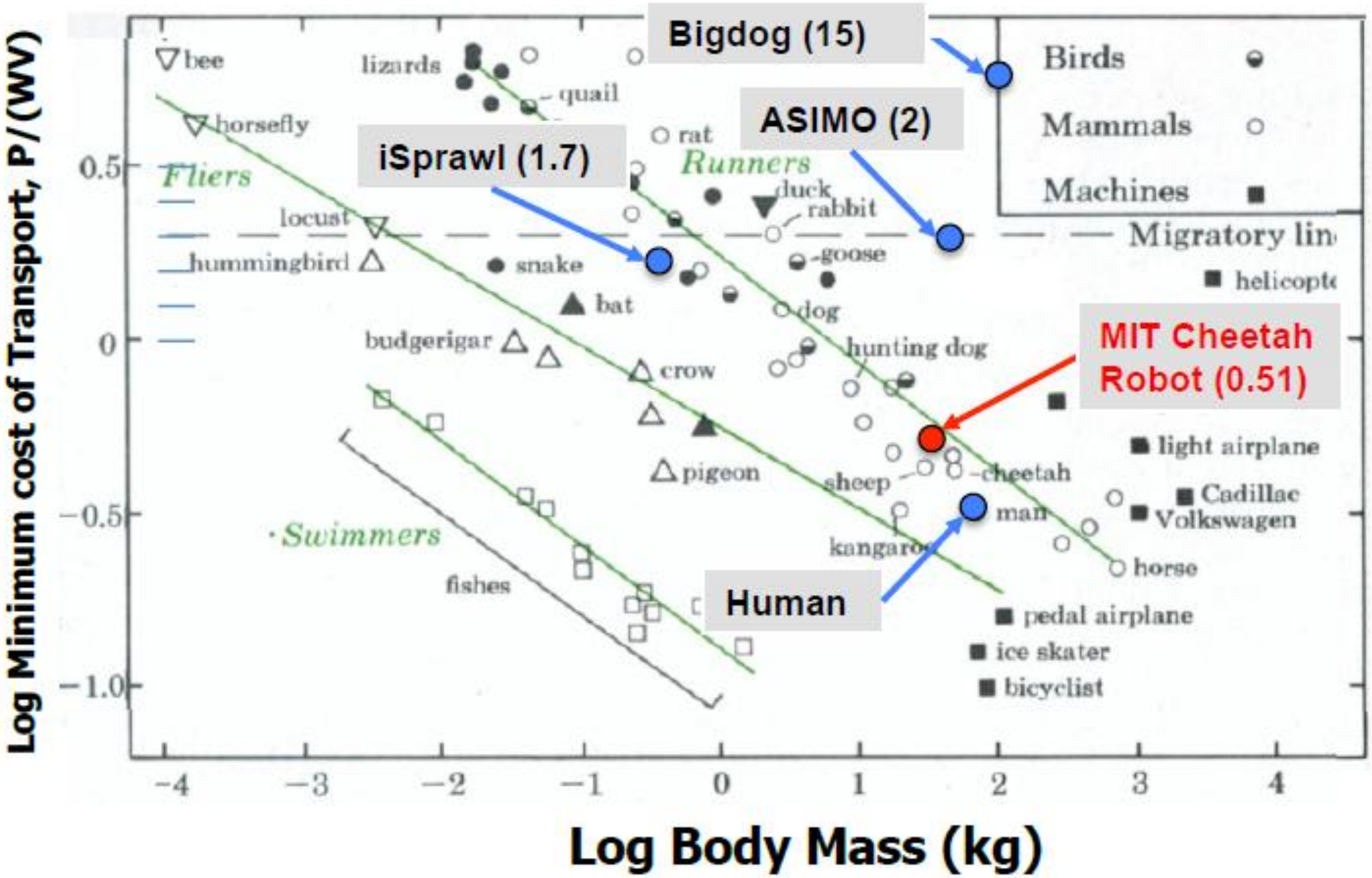
- For instance, the **CoT of human walking is around 0.4**
- The lowest CoT of legged robots is 0.19 (Ranger robot from Cornell)
- Among animals, for a given body weight, the cost of transport is greater in terrestrial locomotion than in flying and swimming due to a lower overall efficiency



Cavagna, Giovanni A. “Symmetry and Asymmetry in Bouncing Gaits.” *Symmetry* 2, no. 3 (September 2010): 1270–1321. https://doi.org/10.3390/sym2031270.

Tucker, V. A. 1975. “The Energetic Cost of Moving About: Walking and Running Are Extremely Inefficient Forms of Locomotion. Much Greater Efficiency Is Achieved by Birds, Fish—and Bicyclists.” *American Scientist* 63 (4): 413–19.

Comparison of CoTs



COT Ranger: 0.19, for 9.9Kg

Seok, Sangok, Albert Wang, Meng Yee Chuah, David Otten, Jeffrey Lang, and Sangbae Kim. 2013. "Design Principles for Highly Efficient Quadrupeds and Implementation on the MIT Cheetah Robot." In *2013 IEEE International Conference on Robotics and Automation*, 3307–12. https://doi.org/10.1109/ICRA.2013.6631038.

Comparison of CoTs

- The **lowest CoT of legged robots is 0.19** by the Ranger robot from Cornell.
- Bhounsule, Pranav A., Jason Cortell, and Andy Ruina. 2012. “Design and Control of Ranger: An Energy-Efficient, Dynamic Walking Robot.” In *Adaptive Mobile Robotics*, 441–448. World Scientific.
- Walked a **40.5 mile (65 Km) ultra-marathon on a single battery charge** and without human touch!

Froude Number

- The Froude number is a **dimension-less measure of velocity**, that **allows to compare dynamically similar gaits between small and large animals/robots**
- It can be used to study trends in animal gait patterns. In analyses of the dynamics of legged locomotion, a walking limb is often modeled as an inverted pendulum, where the center of mass goes through a circular arc centered at the foot. The Froude number is the **ratio of the centripetal force around the center of motion (the foot) and the weight of the animal walking**:

$$\mathrm{Fr} = \frac{\text{centripetal force}}{\text{gravitational force}} = \frac{\frac{mv^2}{l}}{mg} = \frac{v^2}{gl}$$

Typically: Fr < 1.0 means walking gaits, and Fr > 1.0 means running gaits (with flight phases, because high centripedal forces)

- The Froude number may also be calculated from the stride frequency *f* as follows

$$\mathrm{Fr} = \frac{v^2}{gl} = \frac{(lf)^2}{gl} = \frac{lf^2}{g}$$

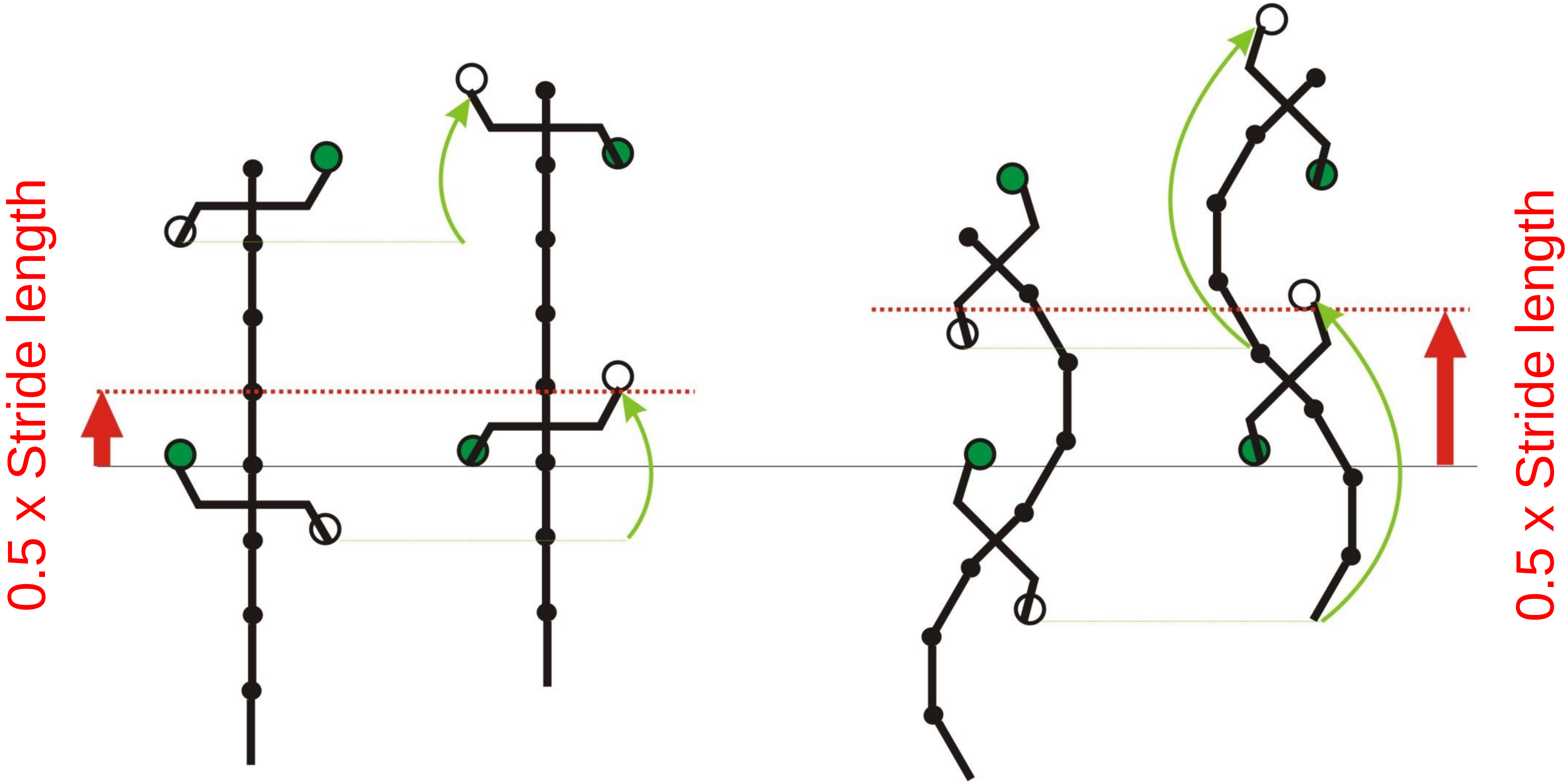
l is the characteristic length, typically the total leg length

- https://en.wikipedia.org/wiki/Froude_number#Walking_Froude_number
- See the following paper for interesting comparisons between animals:
- Alexander, R. McN. 1984. "The Gaits of Bipedal and Quadrupedal Animals." *The International Journal of Robotics Research* 3 (2): 49–59. https://doi.org/10.1177/027836498400300205.

Stride length

- The ***stride length*** is simply the distance made over a whole cycle.
- For given *stride frequency* (i.e. stepping frequency), it is worth making movements that optimize the stride length
- For instance, the **salamander uses a body-limb coordination that optimizes the stride length**, see next slide.

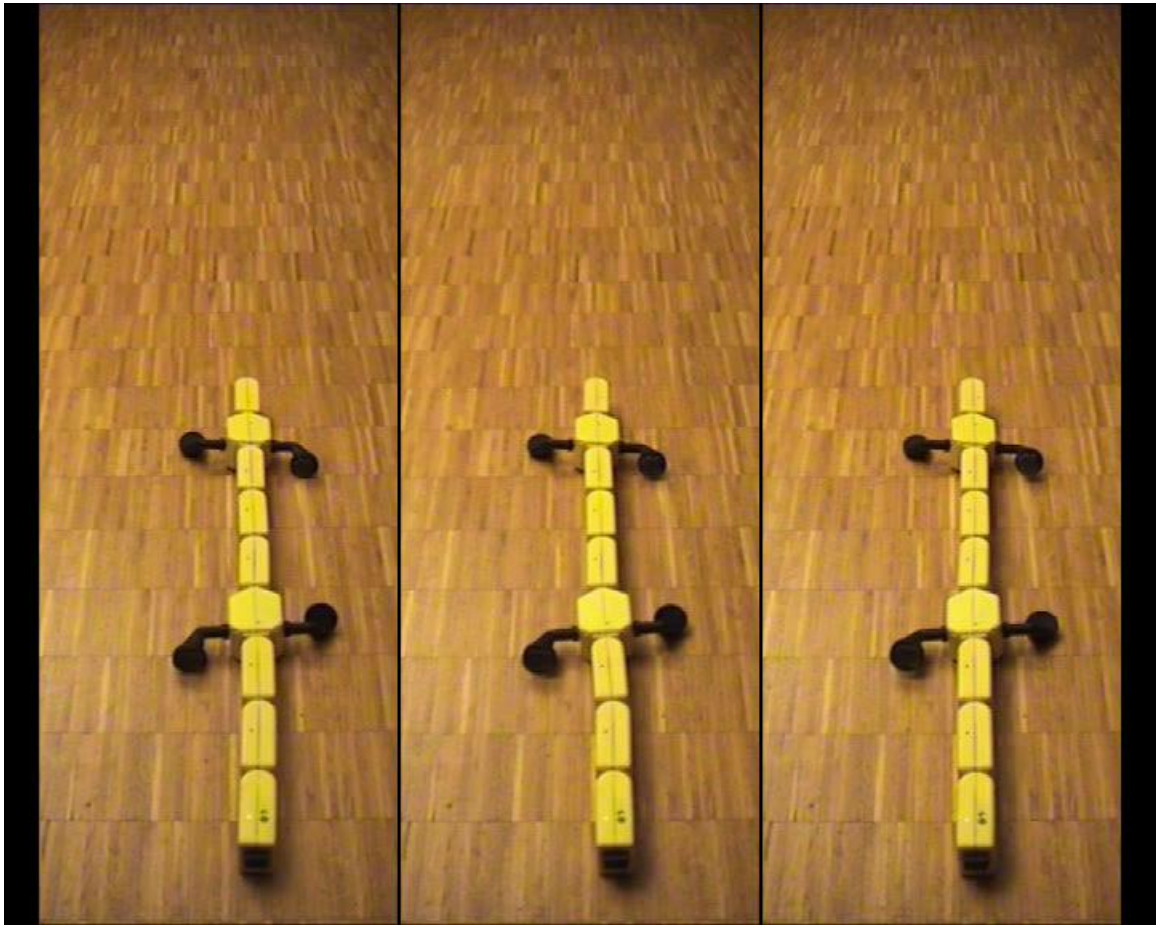
The salamander uses a body-limb coordination that optimizes the stride length

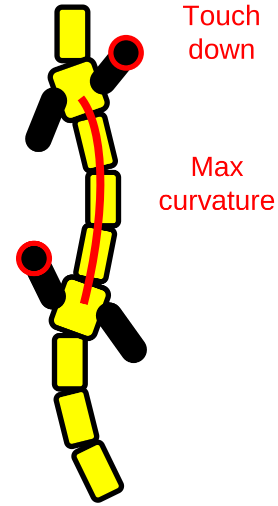


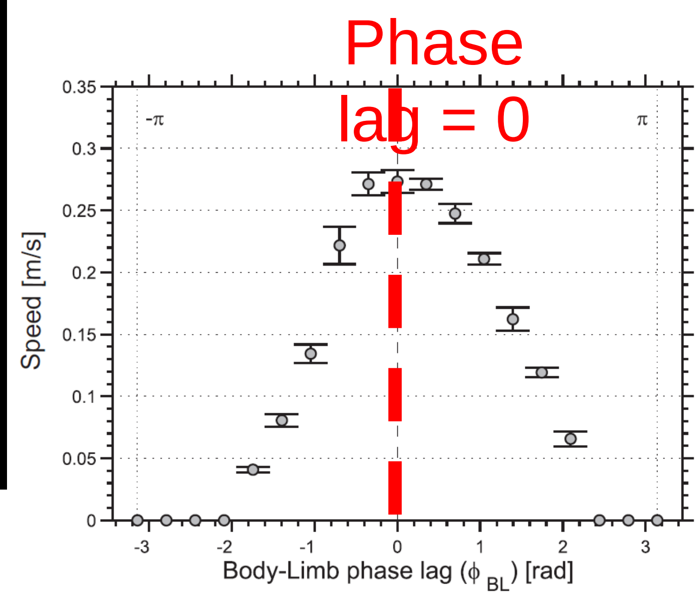
Straight spine

Spine undulating with
an S-shaped standing wave

The salamander uses a body-limb coordination that optimizes the stride length







The three examples have the same frequency.
The speed is optimized thanks to the stride length.

Benchmarks and agility scores

- There start to be several **benchmarks and competitions** that allow robotic teams to compare the performance of their robots.
- See for instance:
  - Nie, Chenghui, Xavier Pacheco Corcho, and Matthew Spenko. 2013. “Robots on the Move: Versatility and Complexity in Mobile Robot Locomotion.” *IEEE Robotics Automation Magazine* 20 (4): 72–82. https://doi.org/10.1109/MRA.2013.2248310.
  - Eckert, P., and A. J. Ijspeert. 2019. “Benchmarking Agility For Multilegged Terrestrial Robots.” *IEEE Transactions on Robotics*, 1–7. https://doi.org/10.1109/TRO.2018.2888977.

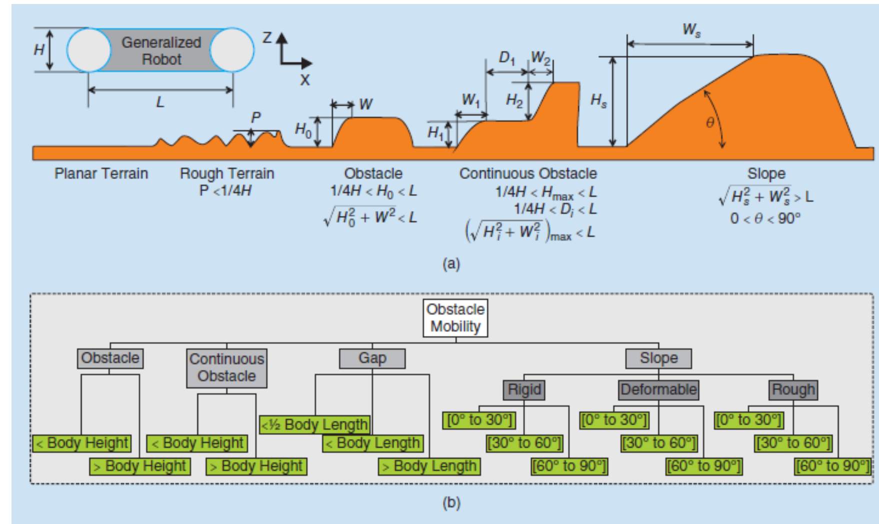


Figure 2. The classification of obstacle type based on (a) the size relative to the robot in question and the (b) binning of each obstacle type. A (single) obstacle’s geometry is limited by the robot’s body length, which is defined as the wheel or leg-base of the robot. Obstacles that occur before the termination of a previous obstacle are classified as continuous. Slopes have an inclination $<90°$. Inclinations $\geq 90°$ are considered scansorial.

- Such efforts are really important to:
  - be able to **quantitatively compare the performance of different robots and controllers** in the same conditions
  - to provide **measures for improving performance over time** (e.g. for iterative design of robots and controllers)
  - possibly, to be used as **reward functions in reinforcement learning** approaches

Possible exam questions

- Explain which **different types of models** exists, and why they are useful
- Derive the **equations of motion** of the **LIP and SLIP** models, and explain why they are useful
- Present some **stability criteria**, discuss why stability criteria are important and how they can be used
- Explain different possible **locomotion metrics** and why they are useful.
- Explain the **cost of transport**
- Explain the **Froude number**